Chiral nuclear dynamics with three-body forces

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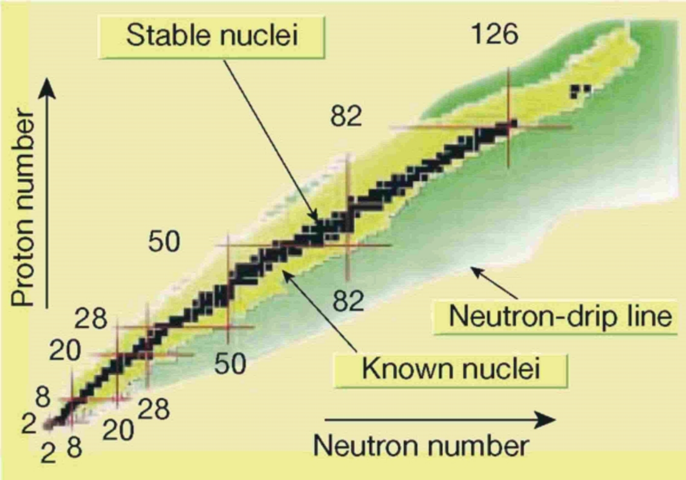
Nuclear landscape and chiral nuclear interactions

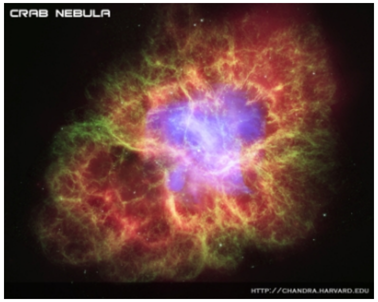
Nuclei near the drip-line

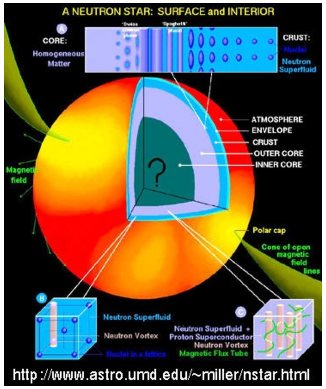
- New generation of rare-isotope experiments
- Understanding ***r*-process nucleosynthesis**



Dense stellar objects

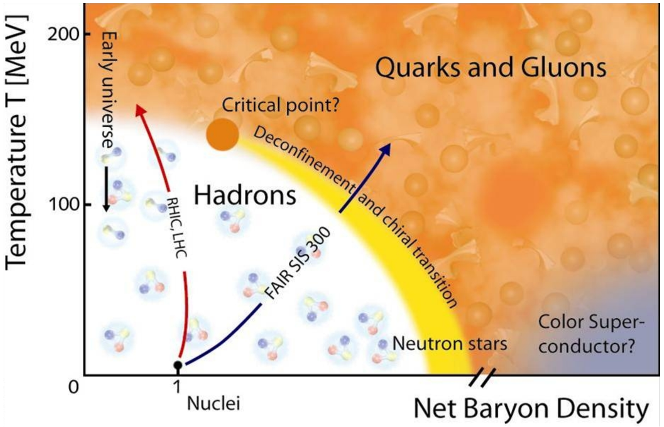
- Structure of **neutron stars**
- Dynamics of core-collapse supernovae





QCD thermodynamics

- Medium dependence of chiral condensate
- Implications for **QCD phase diagram**

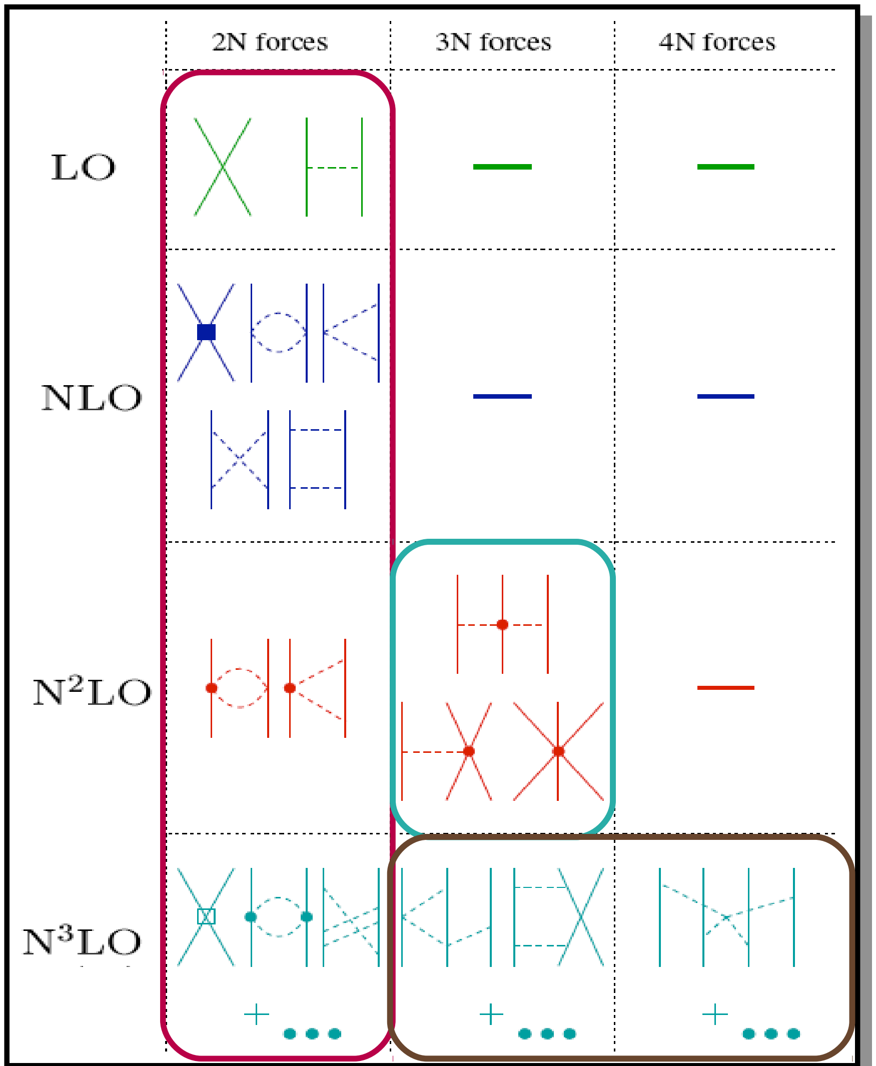


Chiral nuclear interactions

Exploit **symmetry structure** of fundamental theory of QCD: **chiral symmetry** and the resulting **separation of scales** arising from its spontaneous breaking

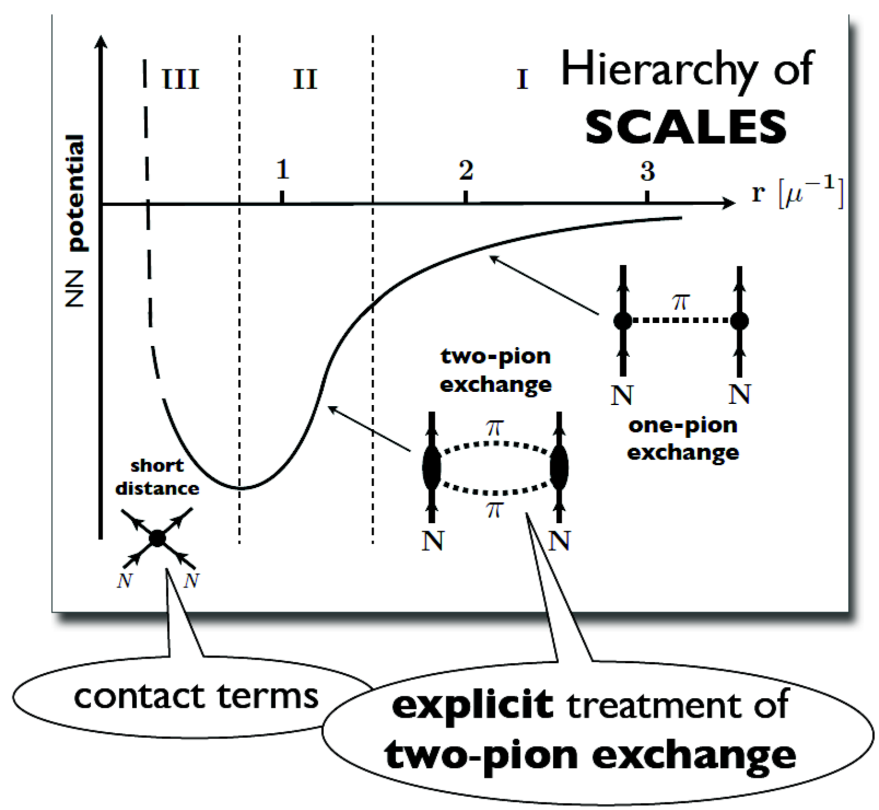
SYSTEMATIC EXPANSION in powers of Q/Λ_χ : $\mathcal{L}_{\rm eff} = \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(0)}_{NN} + \cdots$

$Q = p, m_\pi$



Weinberg, van Kolck, Epelbaum, Kaiser, Machleidt

Early history: M.Taketani et al. (1951)



In-medium nucleon-nucleon interactions

Explicit three-nucleon forces become **COMPUTATIONALLY PROHIBITIVE**

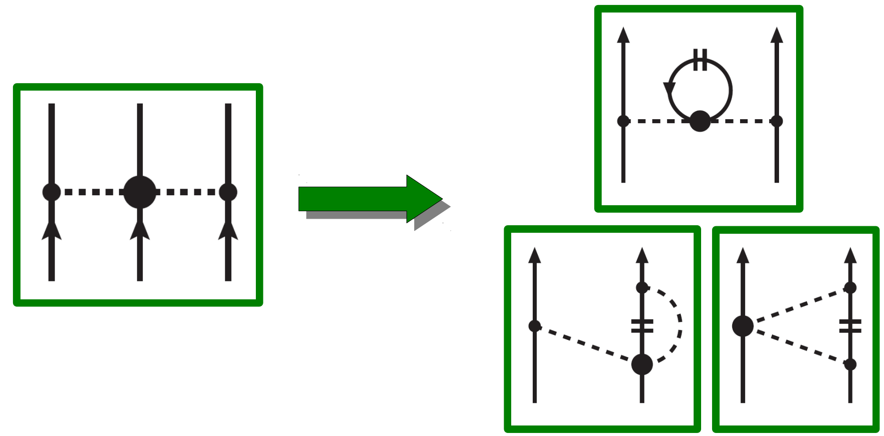
Is there a viable alternative?

Effective, in-medium 2-body interaction from leading-order **CHIRAL 3NF**

$$V_{3N}^{(2\pi)} = \sum_{i\neq j\neq k} \frac{g_A^2}{8f_\pi^4} \frac{\vec{\sigma}_i \cdot \vec{q}_i \, \vec{\sigma}_j \cdot \vec{q}_j}{(\vec{q_i}^2 + m_\pi^2)(\vec{q_j}^2 + m_\pi^2)} F_{ijk}^{\alpha\beta} \tau_i^\alpha \tau_j^\beta$$

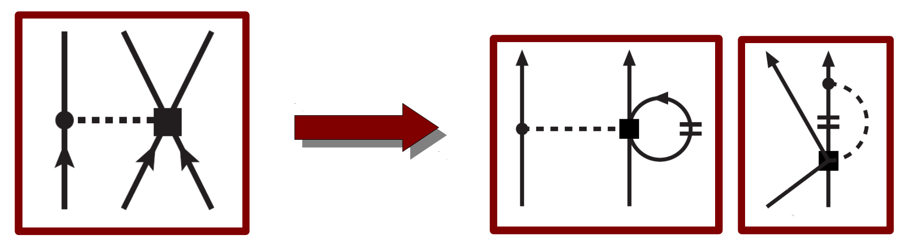
$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta}\left(-4c_1 m_\pi^2 + 2c_3 \vec{q}_i \cdot \vec{q}_j\right) + c_4 \epsilon^{\alpha\beta\gamma} \tau_k^\gamma \vec{\sigma}_k \cdot (\vec{q}_i \times \vec{q}_j)$$

$$\mathrm{N^3LO}: \ c_1 = -0.81, \ c_3 = -3.2, \ c_4 = 5.4 \ [\mathrm{GeV}^{-1}]$$



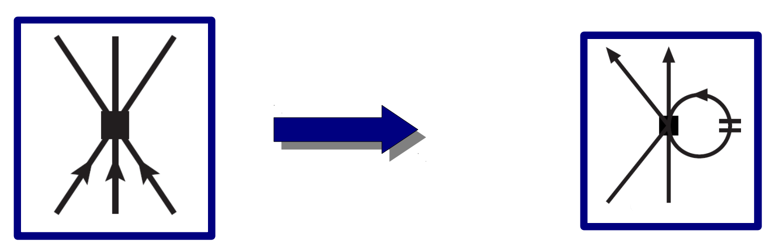
$$V_{3N}^{(1\pi)} = -\sum_{i\neq j\neq k} \frac{g_A c_D}{8f_\pi^4 \Lambda_\chi} \frac{\vec{\sigma}_j \cdot \vec{q}_j}{\vec{q_j}^2 + m_\pi^2} \vec{\sigma}_i \cdot \vec{q}_j \, \vec{\tau}_i \cdot \vec{\tau}_j$$

$$c_D(\mathrm{N^3LO}) = -0.2$$

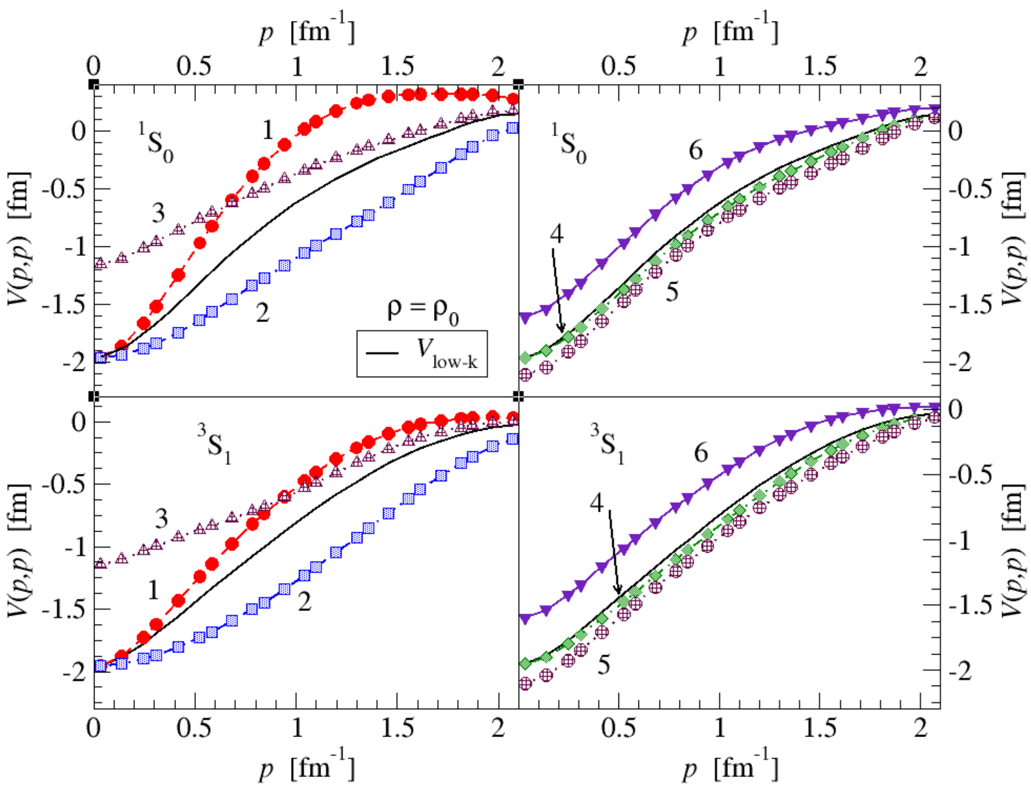


$$V_{3N}^{(\mathrm{ct})} = \sum_{i\neq j\neq k} \frac{c_E}{2f_\pi^4 \Lambda_\chi} \vec{\tau}_i \cdot \vec{\tau}_j$$

$$c_E(\mathrm{N^3LO}) = -0.205$$



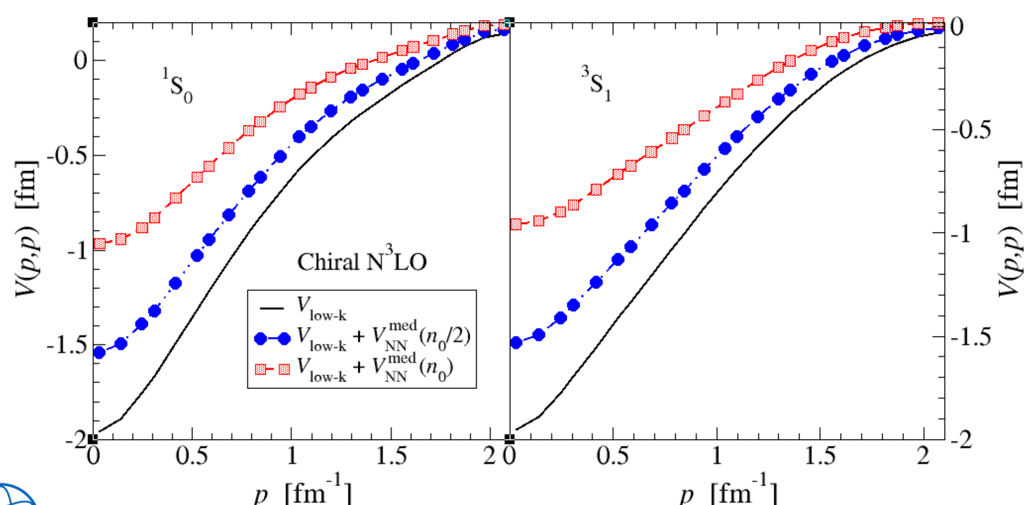
Partial wave matrix elements

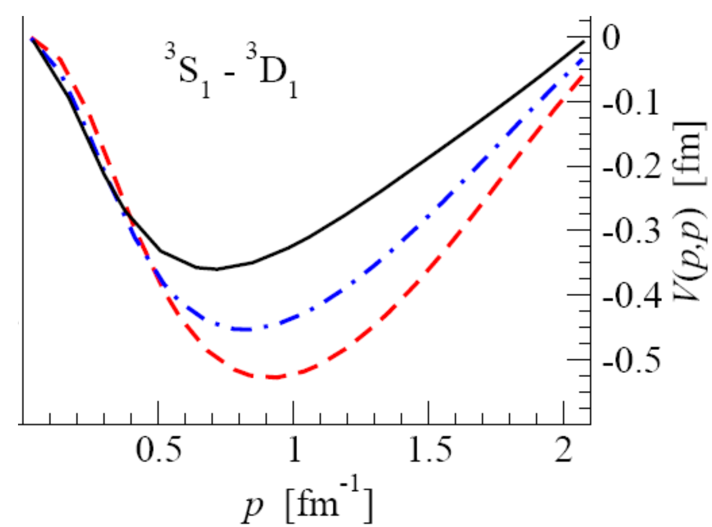


JWH, N. Kaiser, W. Weise, PRC 79 (2009)
JWH, N. Kaiser, W. Weise, PRC 81 (2010)

- Large cancellations between components of 2π-exchange 3NF

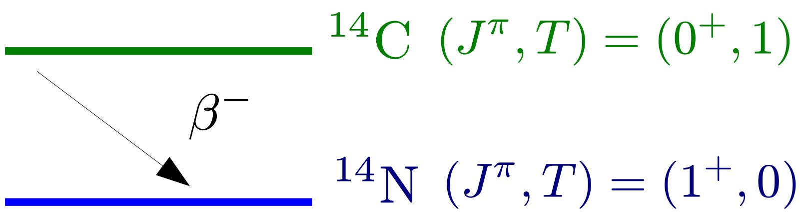
- Increase in attractive tensor force





Anomalously-long half-life of ^{14}C

- Selection rules for allowed Gamow-Teller transition



- $^{14}\mathrm{C}$ beta decay half-life = ***5730 years***
- **Necessary for radiocarbon dating!**
- Typical *p*-shell half-lives on the order of **MINUTES**
- **ACCIDENTAL CANCELLATION** in matrix element

$$M_{GT} = \langle \psi_f || \mathcal{O}(GT) || \psi_i \rangle \approx \pm 0.002$$

- **No satisfactory theoretical explanation**

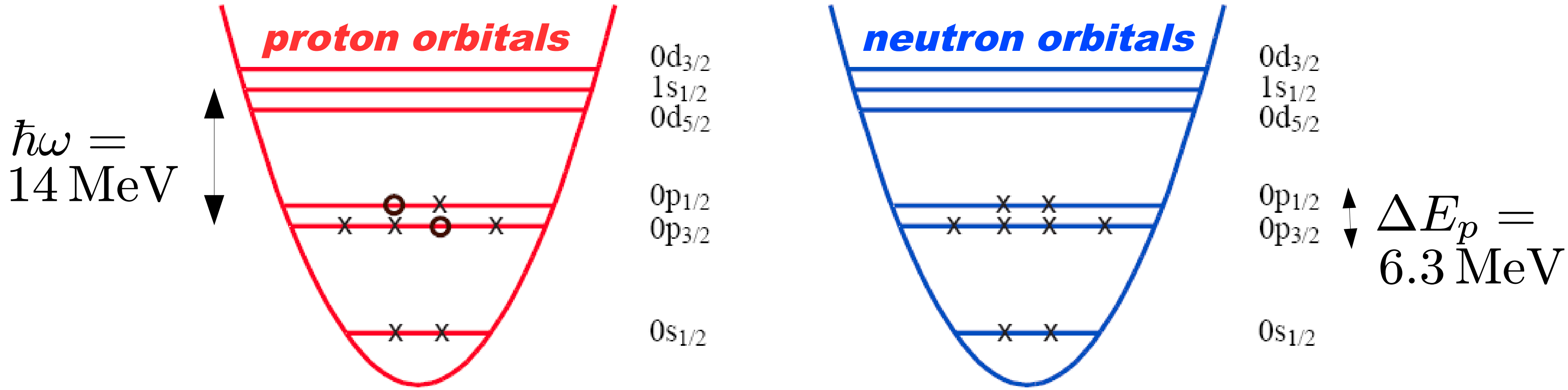
Decay	Q (keV)	$t_{1/2}$ (sec)
^{1}n$(\beta^-)^{1}$H	782.346	6.166E+02
^{3}H$(\beta^-)^{3}$He	18.596	3.887E+08
^{6}He$(\beta^-)^{6}$Li	3507.76	8.067E−01
^{7}Be$(EC)^{7}$Li	861.835	4.604E+06
^{8}He$(\beta^-)^{8}$Li	10653.7	1.110E−01
^{8}Li$(\beta^-)^{8}$Be	16003.71	8.403E−01
^{8}B$(\beta^+)^{8}$Be	17978.5	7.70E−01
^{9}Li$(\beta^-)^{9}$Be	13606.0	1.783E−01
^{9}C$(\beta^+)^{9}$B	16497.9	1.265E−01
^{10}C$(\beta^+)^{10}$B	3647.82	1.9290E+01
^{11}Li$(\beta^-)^{11}$Be	20675	8.5E−03
^{11}Be$(\beta^-)^{11}$B	11506.1	1.381E+01
^{11}C$(\beta^+)^{11}$B	1982.20	1.2234E+03
^{12}Be$(\beta^-)^{12}$B	11707	2.13E−02
^{12}B$(\beta^-)^{12}$C	13369.4	2.020E−02
^{12}N$(\beta^+)^{12}$C	17338.0	1.1000E−02
^{13}B$(\beta^-)^{13}$C	13437.2	1.736E−02
^{13}N$(\beta^+)^{13}$C	2220.45	5.979E+02
^{13}O$(\beta^+)^{13}$N	17766.2	8.55E−03
^{14}B$(\beta^-)^{14}$C	20644	1.28E−02
^{14}C$(\beta^-)^{14}$N	156.472	1.807E+11
^{14}O$(\beta^+)^{14}$N	5143.064	7.0606E+01
^{15}C$(\beta^-)^{15}$N	9771.68	2.449E+00
^{15}O$(\beta^+)^{15}$N	2753.95	1.2224E+02
^{16}C$(\beta^-)^{16}$N	8012.1	7.47E−01
^{16}N$(\beta^-)^{16}$O	10419.1	7.13E+00
^{16}N$^*(\beta^-)^{16}$O	10539.5	1.571
^{17}N$(\beta^-)^{17}$O	8680	4.174E+00
^{17}Ne$(\beta^+)^{17}$F	14536	1.093E−01
^{18}C$(\beta^-)^{18}$N	11810	9.5E−02
^{18}N$(\beta^-)^{18}$O	13899	6.24E−01

[Chou et al., PRC (1993)]

Shell model description

- Too many particles for *ab initio* methods (2009) → use **shell model**

Two $0p$ holes in an $^{16}\mathrm{O}$ core

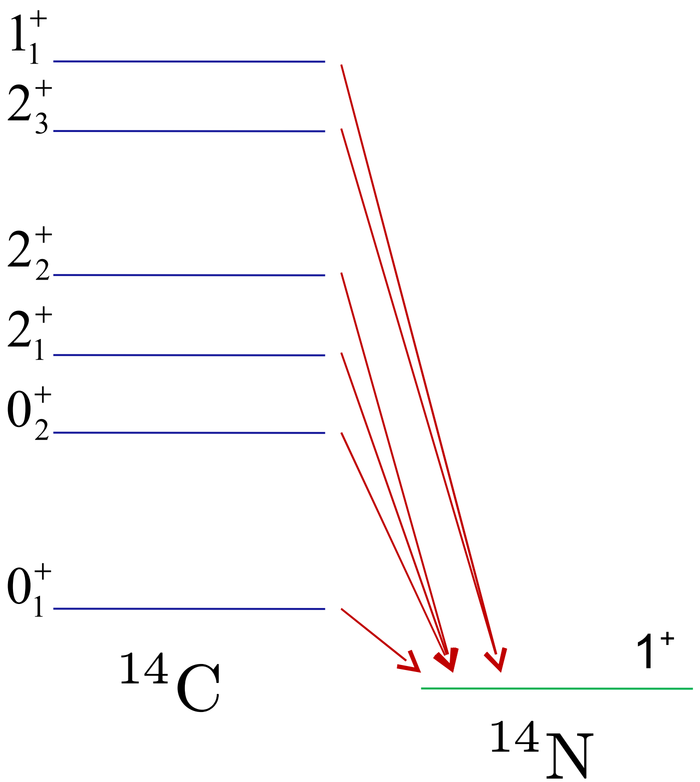


- **Higher-order configurations** included **perturbatively**

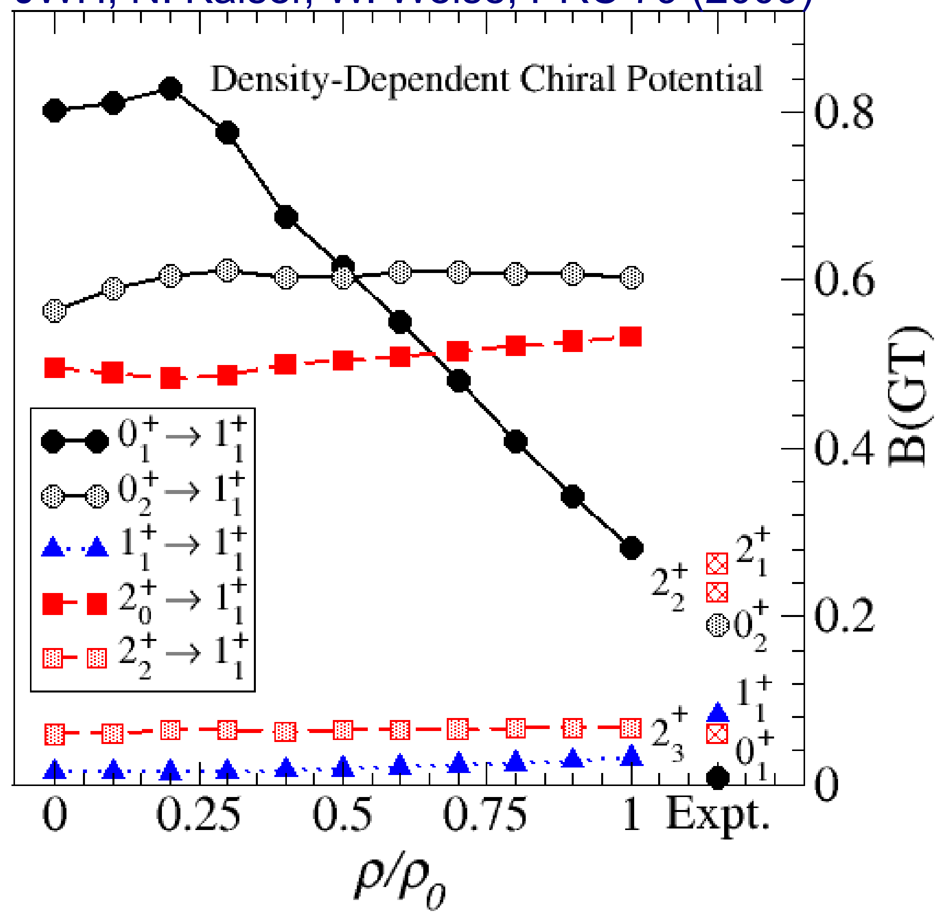
- **Ground state** of $^{14}\mathrm{N}$ **modified strongly** by 3NF

Experimental Gamow-Teller strengths

- Charge-exchange reaction: $^{14}\mathrm{N}(d,{}^{2}\mathrm{He})^{14}\mathrm{C}$ [Negret *et al.*, PRL (2006)]



JWH, N. Kaiser, W. Weise, PRC 79 (2009)



- **DRAMATIC SUPPRESSION** of <u>**only**</u> the **GROUND-STATE** Gamow-Teller transition

- **Challenge to ab-initio many-body methods** [P. Maris *et al.*, PRL (2011)]

Nuclear energy density functionals

$$E_{SHF}[\rho,\tau,\vec{J}] = \int d^3x \left[\frac{1}{2M}\tau + \frac{3}{8}t_0\rho^2 + \frac{1}{16}t_3\rho^{2+\alpha} + \frac{1}{16}(3t_1+5t_2)\rho\tau \right.$$
$$\left. + \frac{1}{64}(9t_1-5t_2)(\vec{\nabla}\rho)^2 - \frac{3}{4}W_0\rho\vec{\nabla}\cdot\vec{J} + \frac{1}{32}(t_1-t_2)\vec{J}^2\right]$$

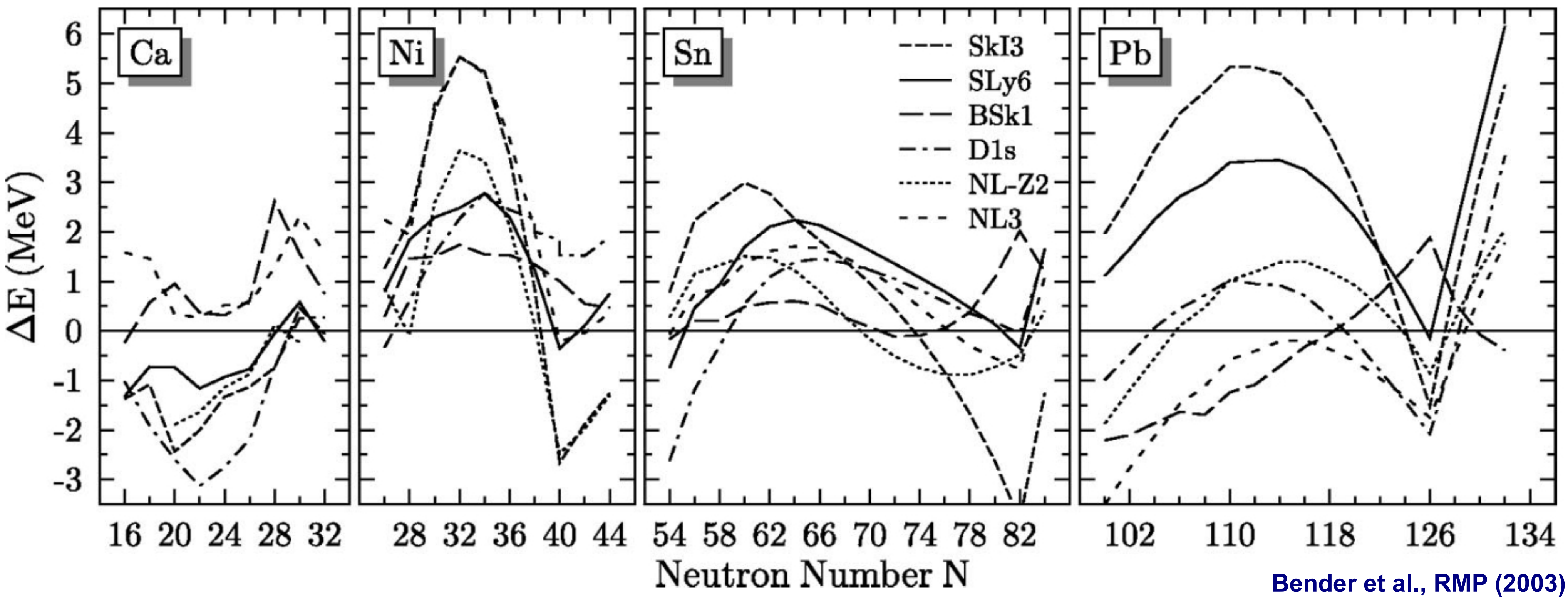
- Hartree-Fock expression for the energy is highly nonlocal
- Energy written as a functional of **local densities** and their derivatives only:
  - Nucleon density $\rho(\vec{x}) = \sum_j |\phi_j(\vec{x})|^2$
  - Kinetic density $\tau(\vec{x}) = \sum_j |\vec{\nabla}\phi_j(\vec{x})|^2$
  - Spin-orbit density $\vec{J}(\vec{x}) = \sum_j \phi_j^\dagger(\vec{x})(-i\vec{\nabla}\times\vec{\sigma})\phi_j(\vec{x})$

(1) Vary energy with respect to orbitals to obtain Schrödinger-like equation

(2) Obtain new orbitals from Schrödinger equation

(3) Iterate until self-consistency is obtained

Binding energies across periodic table



Bender et al., RMP (2003)

- Binding energies across the periodic table accurate to within ~ 5 MeV
- Charge radii, single-particle energies, collective excitations (time-dependent SCMF)

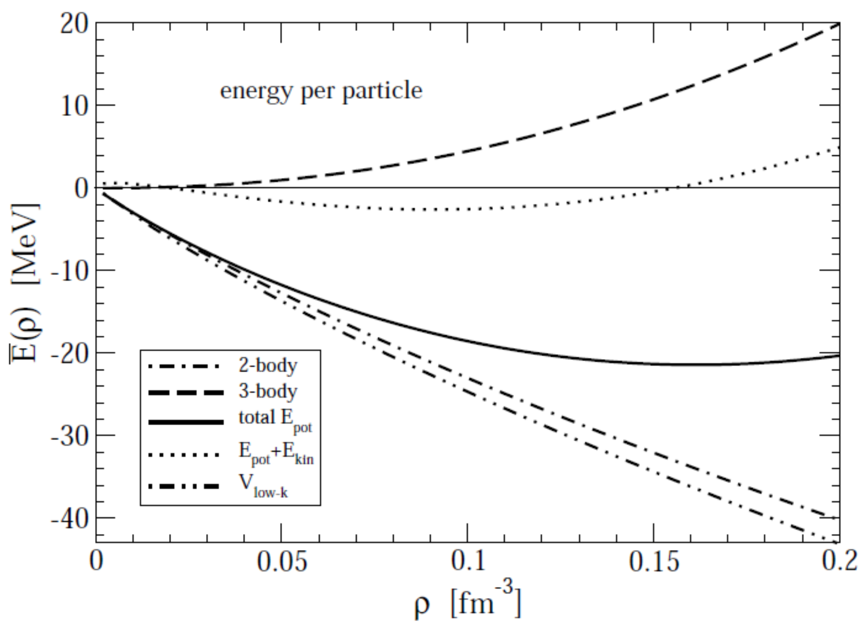
How can we derive successful Skryme energy functionals from microscopic two- and three-body forces?

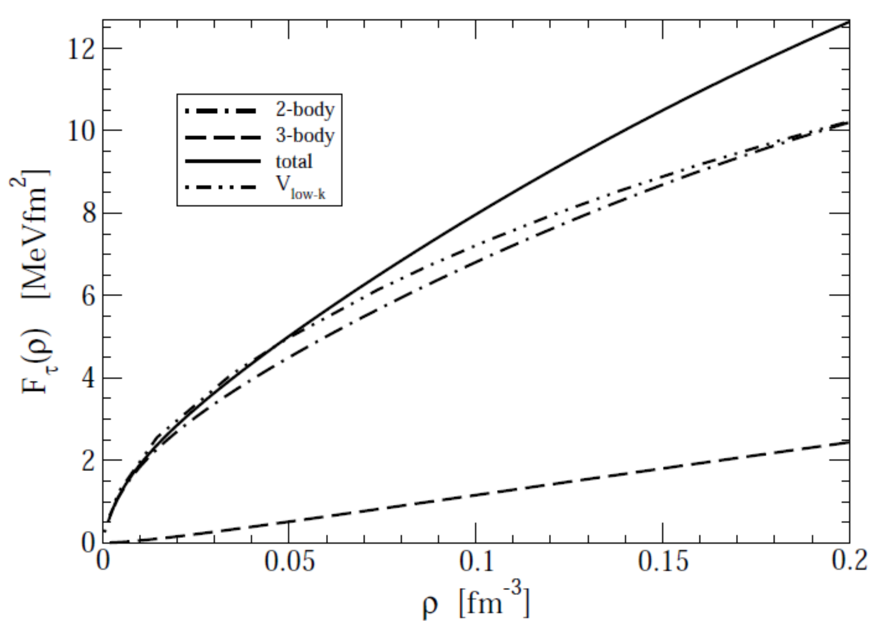
Microscopic energy density functional

$$E_{HF} = \int d\vec{r}\; \mathcal{E}[\rho, \tau, \vec{J}]$$

- Derive from **density matrix expansion**: compare to phenomenological models
- General form of energy density up to **second order in gradients:**

$$\mathcal{E}[\rho,\tau,\vec{J}] = \rho \underline{\bar{E}(\rho)} + \left[\tau - \frac{3}{5}\rho k_f^2\right]\left[\frac{1}{2M} - \frac{k_f^2}{4M^3} + \underline{F_\tau(\rho)}\right] + (\vec{\nabla}\rho)^2 \underline{F_\nabla(\rho)} + \vec{\nabla}\rho\cdot\vec{J}\underline{F_{so}(\rho)} + \vec{J}^2 \underline{F_J(\rho)}$$

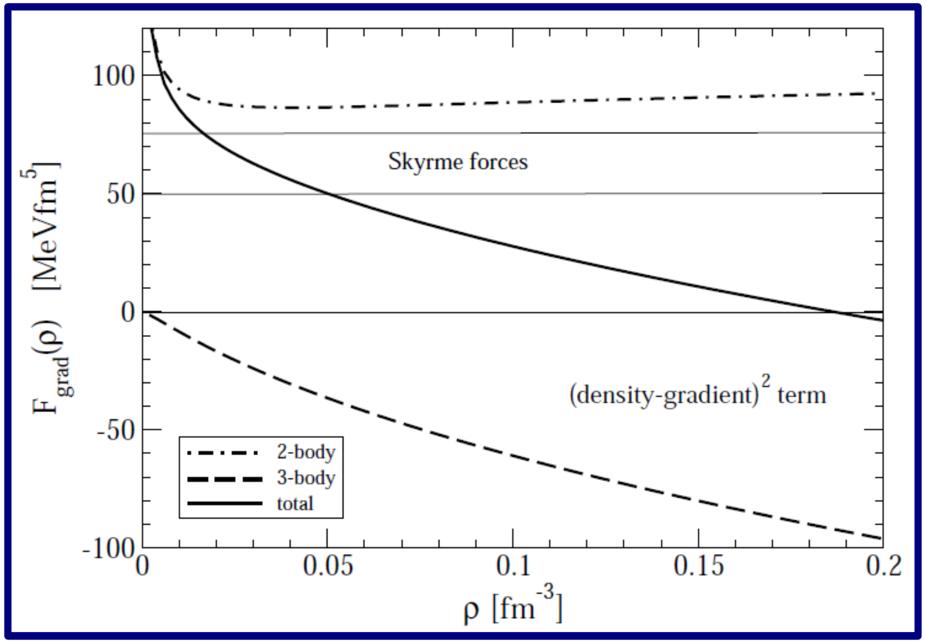




JWH, N. Kaiser, W. Weise, arXiv:1107.5966

Density dependent couplings

JWH, N. Kaiser, W. Weise, arXiv:1107.5966



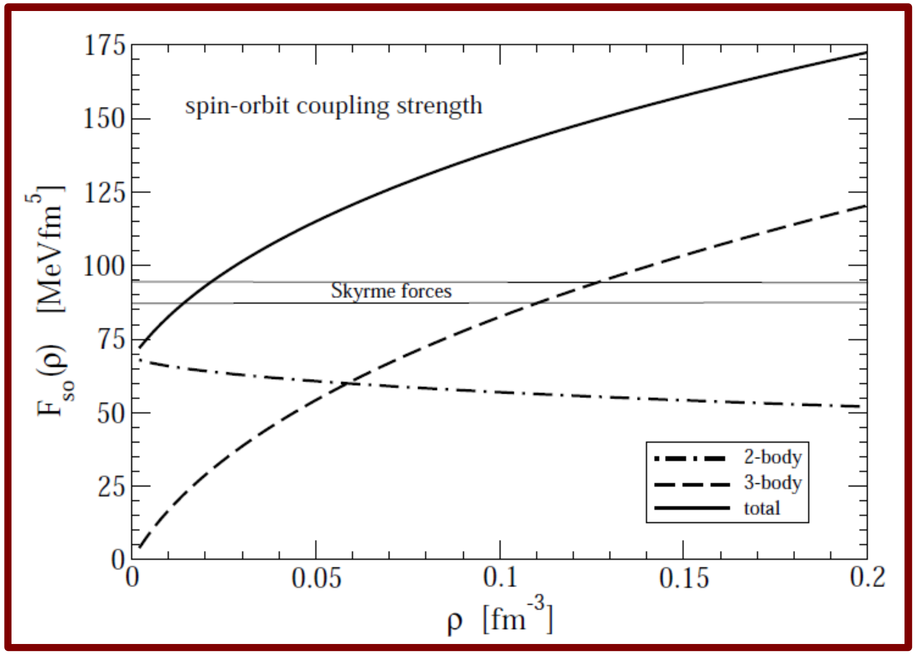
- Realistic Skryme density functionals:

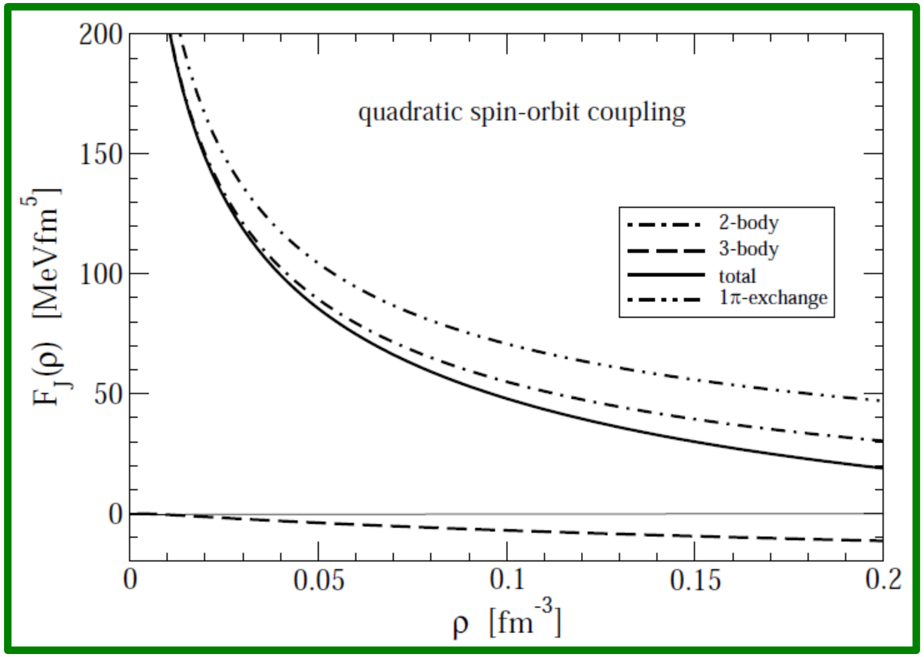
$$F_\nabla \simeq 50 - 75 \text{ MeV fm}^5$$

- Empirically, $F_{so} \simeq 90 \text{ MeV fm}^5$

- Three-nucleon force results in a **spin-orbit strength** that **increases** with density

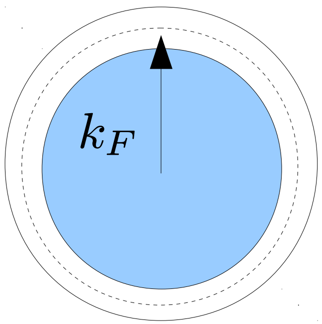
- Pion exchange gives dominant contribution to F_J , particularly at small densities





Quasiparticle interaction in nuclear matter

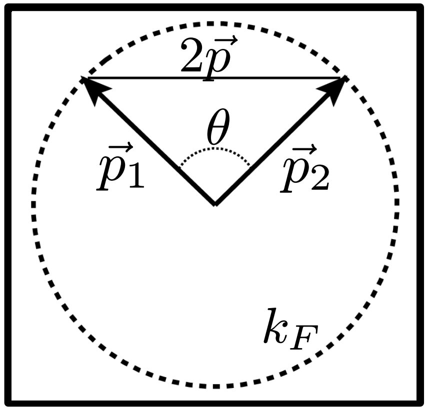
- ***Strongly-interacting,*** *normal* Fermi at low *T* described in terms of ***weakly-interacting* QUASIPARTICLES**
  - Quasiparticle lifetime $\tau \sim (\epsilon - \epsilon_F)^{-2}$
  - Characterize **low-energy excitations** about the interacting ground state



$$\mathcal{F}(\vec{p}_1,\vec{p}_2) = f(\vec{p}_1,\vec{p}_2) + f'(\vec{p}_1,\vec{p}_2)\vec{\tau}_1\cdot\vec{\tau}_2 + [g(\vec{p}_1,\vec{p}_2) + g'(\vec{p}_1,\vec{p}_2)\vec{\tau}_1\cdot\vec{\tau}_2]\,\vec{\sigma}_1\cdot\vec{\sigma}_2$$

- Legendre polynomial expansion:

$$f(\vec{p}_1,\vec{p}_2) = \sum_L f_L\, P_L(\cos\theta)$$
$$f'(\vec{p}_1,\vec{p}_2) = \sum_L f'_L\, P_L(\cos\theta)$$
$$\vdots$$



  - **Symmetric nuclear matter**: Bulk equilibrium properties, collective excitations
  - **Neutron matter**: Magnetic susceptibility, response to weak probes: neutrino scattering and absorption

Role of three-nucleon forces?

(1^{st} + 2^{nd} order 2NF) + (1^{st} order 3NF)

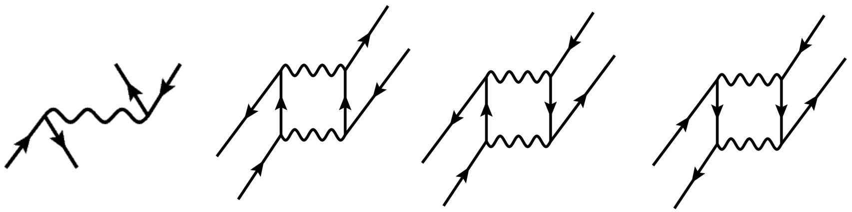
$V_{\rm N3LO}^{(1+2)} \quad (k_F = 1.33 \text{ fm}^{-1})$

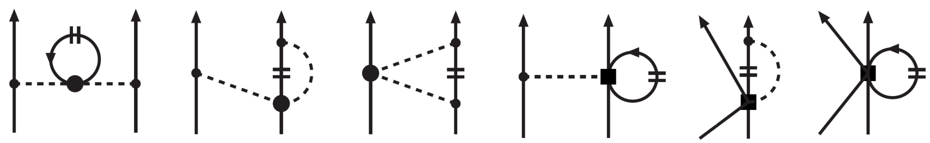
l	F_l	G_l	F'_l	G'_l
0	-1.64	0.35	1.39	1.59
1	-0.13	0.50	0.58	0.47

$V_{\rm N3LO}^{(1+2)} + V_{3N}^{(1)} \quad (k_F = 1.33 \text{ fm}^{-1})$

l	F_l	G_l	F'_l	G'_l
0	-0.15	0.35	1.36	1.20
1	-0.22	0.21	0.29	0.24

Observables ($V_{\rm N3LO}^{(1+2)}$)	**Empirical**	**Observables** ($V_{\rm N3LO}^{(1+2)} + V_{3N}^{(1)}$)
$\frac{M^*}{M_N} = 1 + \frac{F_1}{3} = 0.96$	[0.7 – 1.0]	$\frac{M^*}{M_N} = 1 + \frac{F_1}{3} = 0.93$
$\mathcal{K} = \frac{3\hbar^2 k_F^2}{M^*}(1+F_0) = -148$ MeV	[200 – 300 MeV]	$\mathcal{K} = \frac{3\hbar^2 k_F^2}{M^*}(1+F_0) = 203$ MeV
$\beta = \frac{\hbar^2 k_F^2}{6M^*}(1+F'_0) = 31$ MeV	[30 – 36 MeV]	$\beta = \frac{\hbar^2 k_F^2}{6M^*}(1+F'_0) = 31$ MeV
$\delta g_l = \frac{F'_1 - F_1}{3(1+F_1/3)} = 0.12$	[0.20 – 0.26]	$\delta g_l = \frac{F'_1 - F_1}{3(1+F_1/3)} = 0.11$
$g'_{NN} = 0.67$	[0.6 – 0.7]	$g'_{NN} = 0.52$





JWH, N. Kaiser, W. Weise, to appear in NPA

Conclusions/Outlook

- **Chiral effective field theory** used to connect fundamental theory of strong interactions to the nuclear force and nuclear structure physics

- Implementation of three-nucleon forces a *frontier* in medium-mass and heavy nuclei

  - Facilitated by **density-dependent NN interactions:** Nuclear shell model calculation for **carbon-14 lifetime**

  - Microscopic **energy density functionals**: binding energies and charge radii across the periodic table

- **Quasiparticle interaction** with chiral two- and three-nucleon interactions

  - Satisfactory description of bulk nuclear matter properties

  - Future: **neutron star matter**