Muon $g - 2$ and the pion form factor in lattice QCD

International School of Nuclear Physics
33rd course
From Quarks and Gluons to Hadrons and Nuclei
Erice-Sicily
September 16-24, 2011

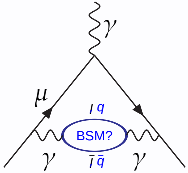
Andreas Jüttner

CERN Theory Division

Introduction: elm. pion FF and $a_\mu^{\mathrm{LO,had}}$

Some cartoons:

μ LO$^{\mathrm{elm}}$ vacuum polarisation



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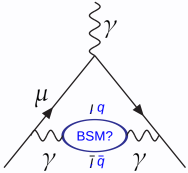
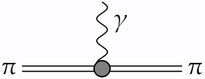


pion form factor



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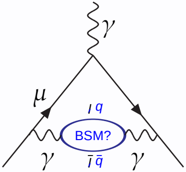

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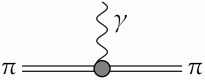

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non-perturbative QCD-contributions to both processes

- talks by Tobias Goecke and Christian Fischer for analytical approaches
- this talk: lattice QCD
  in principle a well defined problem in field Euclidean field theory
  in practice brute force neither feasible nor nice
  new ideas in the following
- Mainz group: Bastian Brandt, Michele Della Morte, Benjamin Jäger, A.J., Hartmut Wittig

Motivation

- lattice QCD has seen continous progress
(e.g. FLAG summary for low energy physics results)
Eur.Phys.J. C71 (2011) 1695, arXiv:1011.4408

- a number of yet unsolved obstacles, some of which affect a_μ^{LHV} and $f^{\pi\pi}$

- in particular for a_μ^{LHV} the impact on particle physics potentially huge:
$(a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{th}}) \times 10^{-10} = 29(9)$ while $a_\mu^{\mathrm{LHV}} = 690(5)$
according to Jegerlehner and Nyffeler, Physics Reports 477(2009)1-110, arXiv:0902.3360
(of similar importance: light-by-light scattering but not covered here)

Lattice QCD

What is . . . ?

	QCD
N_c	3
N_f, fundamental	1+1+1+1+1+1
$SU(2)$ iso-spin brk.	✔
m_π	135MeV
V	∞
a	0

Lattice QCD

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	QCD	Lattice QCD
N_c	3	3
N_f, fundamental	1+1+1+1+1+1	0, 2, 2+1, 2+1+1
$SU(2)$ iso-spin brk.	✓	✗
m_π	135MeV	$\lesssim m_\pi^{\text{sim}}$
V	∞	2-4fm
a	0	0.05-0.1fm

for a_μ^{LHV} and $f^{\pi\pi}(q^2)$ severe systematic effects due to

- finite volume
- unphysically heavy pion mass

Difficulties (and solutions): finite volume

- in the finite lattice volume hadron momenta are discrete: $\vec{p} = \frac{2\pi}{L}\vec{n}$
  $\rightarrow$ hadron matrix elements can only be computed for these Fourier momenta:

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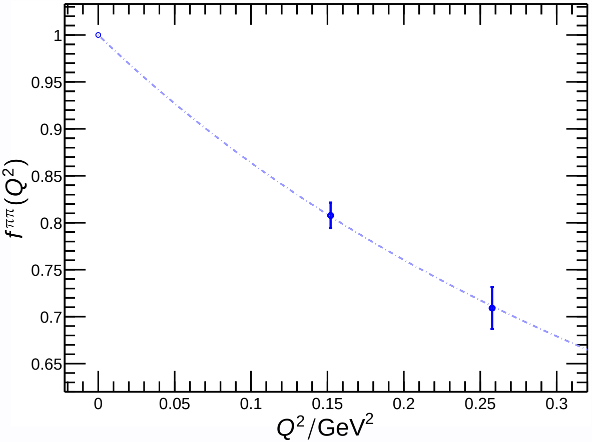
pion form factor in $N_f = 2 + 1$ lattice QCD by RBC/UKQCD



UKQCD JHEP 05(2007)016, JHEP 0807(2008)112

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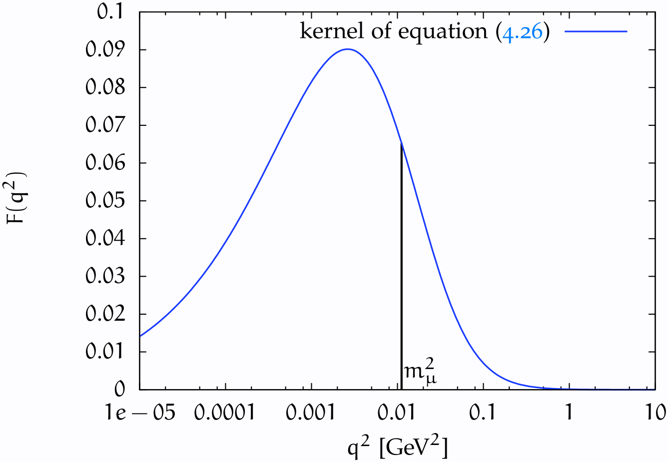

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Tuning the fermion boundary conditions allows to induce arbitrary hadron momenta: (partially) twisted boundary conditions
Bedaque PLB539(2004), Divitiis et al., PLB 595 (2004) 408, Bedaque, Chen, PLB 616:208-214,2005, Sachrajda, Viladoro, PLB 609:73-85,2005, . . .

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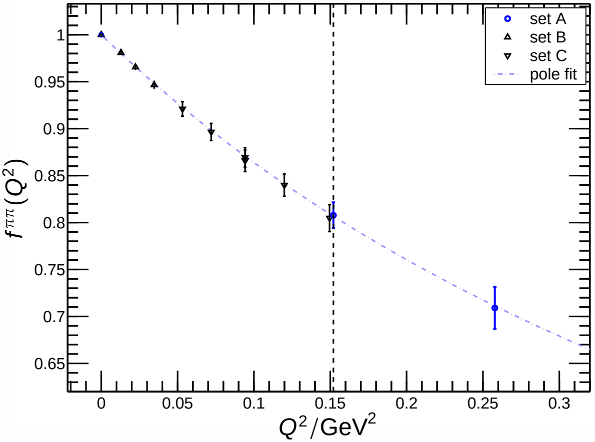

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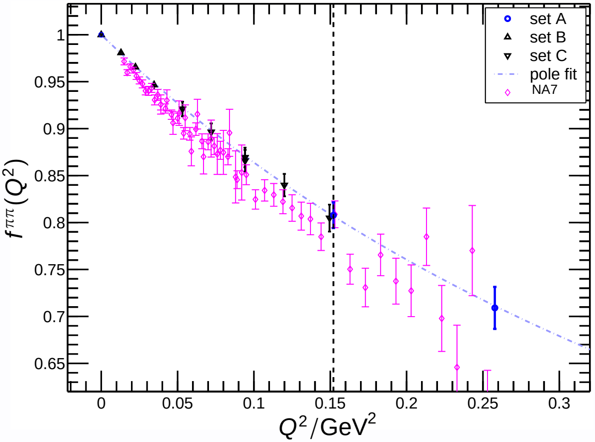

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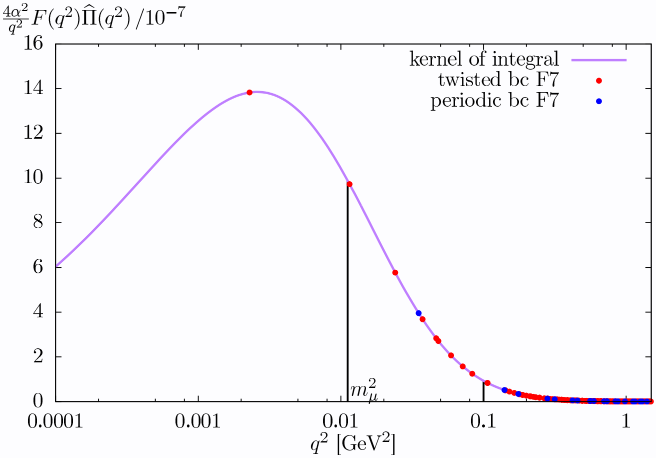
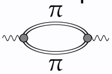


CLS preliminary, plot by Benjamin Jäger, Mainz

Difficulties (and solutions): Quark mass dependence

- to date no single simulation of a_μ^{LHV} and $f^{\pi\pi}(q^2)$ with phyiscal quark masses exists
- for making a *prediction* at the physical point in LQCD one resorts to extrapolations in the quark mass based on chiral perturbation theory

a_μ^{LHV}



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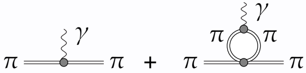

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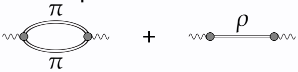

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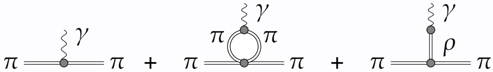

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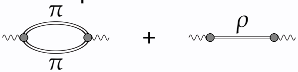

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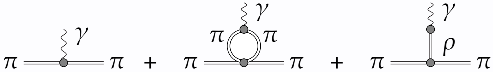

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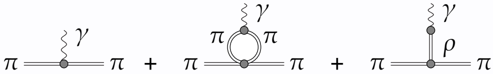

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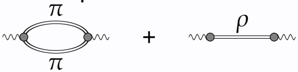

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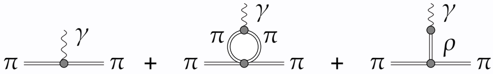

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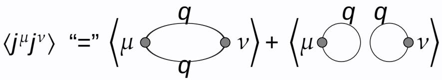
- currently only model independent way out: simulate directly with physical pion masses (i.e. wait)

Difficulties (and solutions): disconnected diagrams



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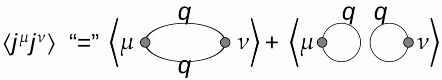

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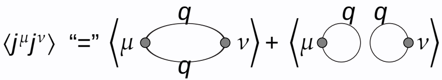

- since simulations iso-spin symmetric here only relevant for a_μ^{LHV}
- can be done, but numerically *expensive* and therefore neglected in all current simulations!!
- new technique allows their prediction in chiral perturbation theory; crude estimate at NLO: -10% shift in VP $\Pi(q^2)$
  Della Morte & A.J. JHEP 1011 (2010) 154, arXiv:1009.3783

Summary of difficulties

- ✓ momentum resolution (twisted boundary conditions)
- ✗ mass dependence (extrapolation must be avoided)
- ? disconnected diagrams (they are small and we have a rough estimate)

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Remainder: Status of ongoing calculations

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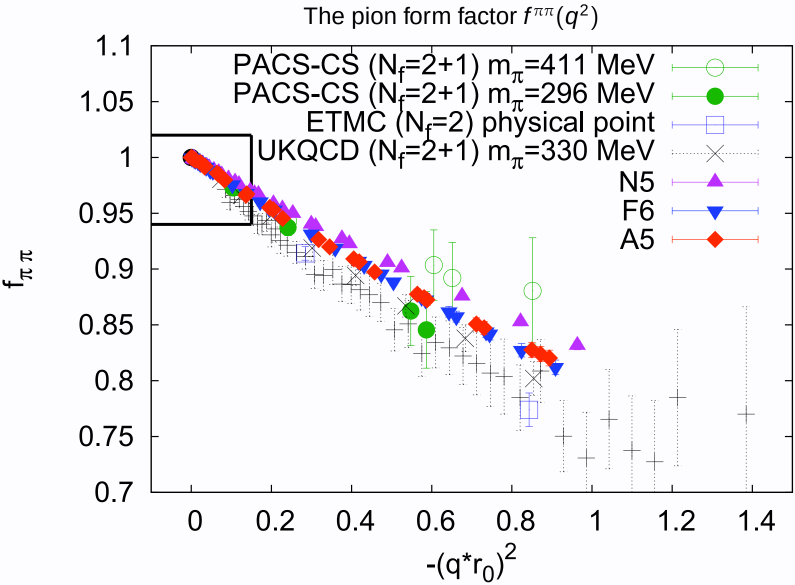

CLS preliminary, plot by Bastian Brandt, Mainz, arXiv:1109.0196

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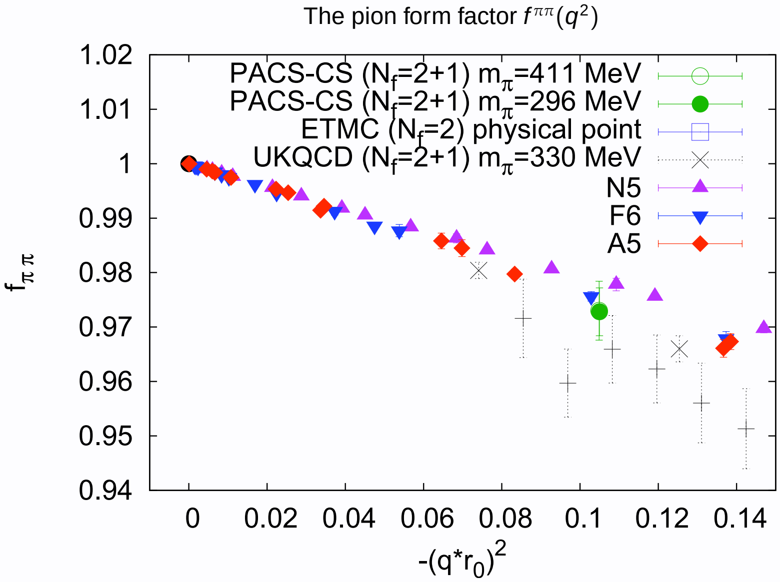

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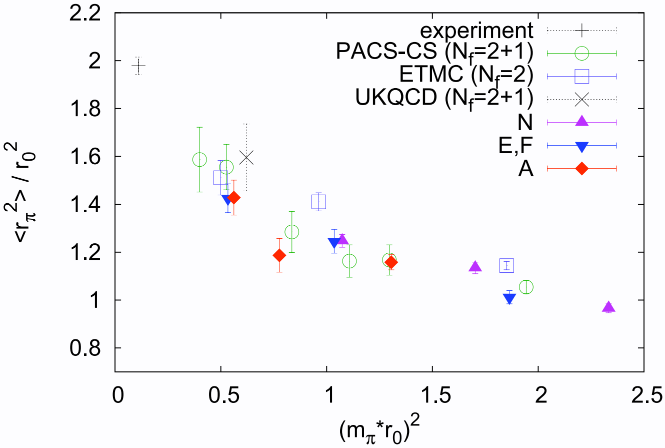
$$f^{\pi\pi}(q^2) = 1 + \frac{1}{6} < r_\pi^2 > q^2 + \dots$$



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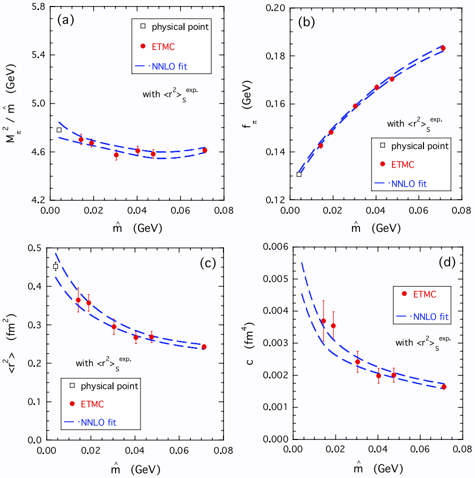

- all collaborations try to extrapolate the lattice data using χPT or some model
  example here ETM *Phys.Rev. D79 (2009) 074506, arXiv:0812.4042*

Summary $f^{\pi\pi}(q^2)$ - FLAG report

- G. Colangelo, S. Dürr, A. J., L. Lellouch, H. Leutwyler, V. Lubicz, S. Necco, C. Sachrajda, S. Simula, A. Vladikas, U. Wenger, H. Wittig
  Eur.Phys.J. C71 (2011) 1695, arXiv:1011.4408

$\rightarrow$ quantities we consider: m_u, m_d, m_s, $f_+(0)$, f_K/f_π, B_K, NLO LEC's, potentially more in the future

Collaboration	N_f	publication status	chiral extrapolation	continuum extrapolation	finite volume	$< r^2 >^{\pi}_V$ [fm^2]	c_V (GeV^{-4})	$\bar{\ell}_6$
RBC/UKQCD 08A	2+1	A	●	■	★	0.418(31)	—	12.2(9)
LHP 04	2+1	A	●	■	●	0.310(46)	—	—
JLQCD/TWQCD 09	2	A	●	■	■	0.409(23)(37)	3.22(17)(36)	11.9(0.7)(1.0)
ETM 08	2	A	●	●	●	0.456(30)(24)	3.37(31)(27)	14.9(1.2)(0.7)
QCDSF/UKQCD 06A	2	A	★	●	●	0.441(19)(56)(29)	—	—
BCT 98						0.437(16)	3.85(60)	16.0(0.5)(0.7)
NA7 86						0.439(8)		
GL 84								16.5(1.1)

Outlook $f^{\pi\pi}(q^2)$

- only twisted boundary conditions allow for model-independent extraction of the charge radius (derivative at $q^2 = 0$)
- a *final* word on the lattice on $f^{\pi\pi}(q^2)$ musn't depend on χPT in the way current results do

Status a_μ^{LHV}

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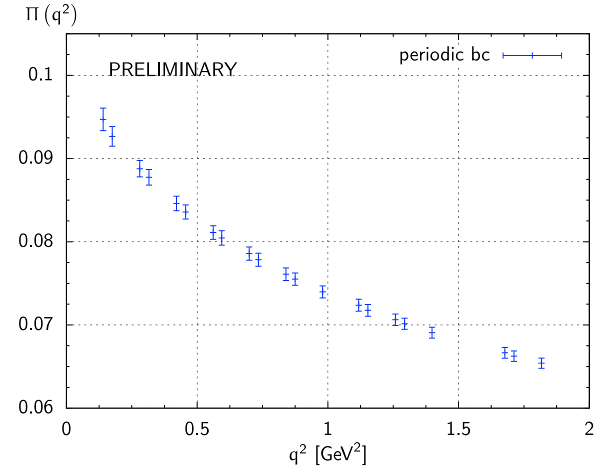

plots by Benjamin Jäger, Mainz, CLS-collaboration

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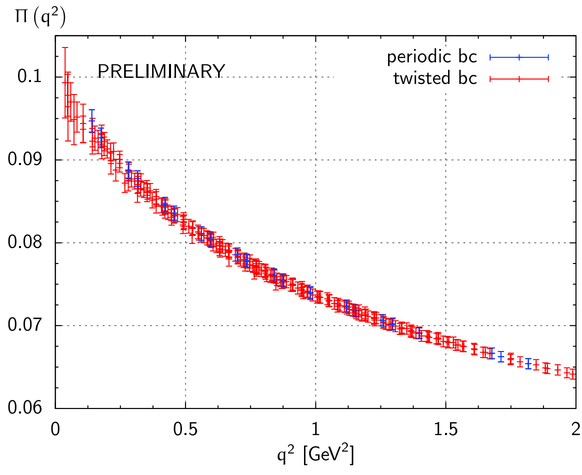

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- twisting helps - many more data points in q^2

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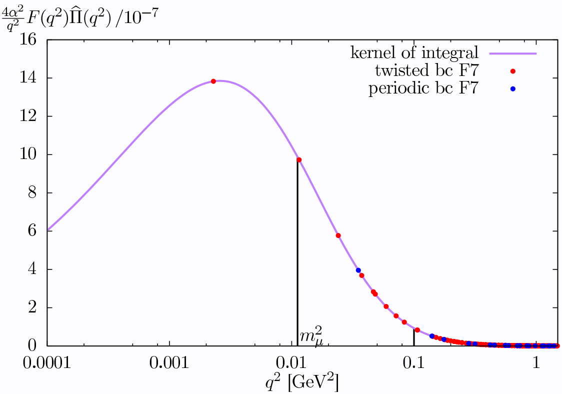

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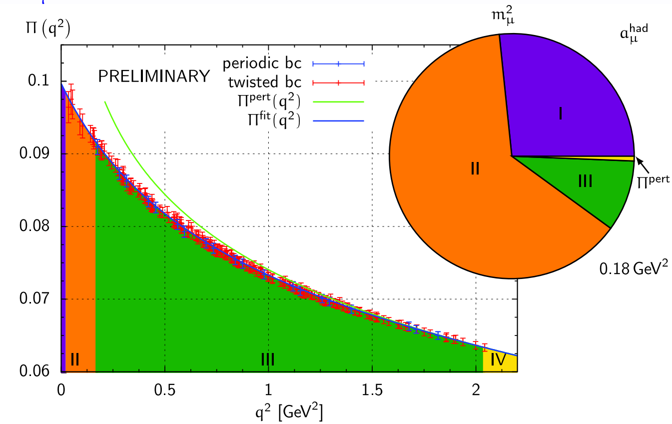

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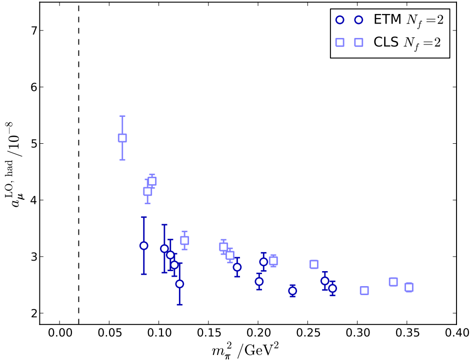
$$a_\mu^{\mathrm{LHV}} \propto \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 K(Q^2)\left(\Pi(Q^2) - \Pi(0)\right)$$

- twisting helps - many more data points in q^2
- fit raw lattice data to various ansätze (pole, polynomial, Padé)
- match with PT (vary matching point)
- integrate: fold with QED PT

Status a_μ^{LHV}

a_μ^{LHV} in $N_f = 2$ flavour QCD



(thanks to Dru Renner for providing ETM's data)

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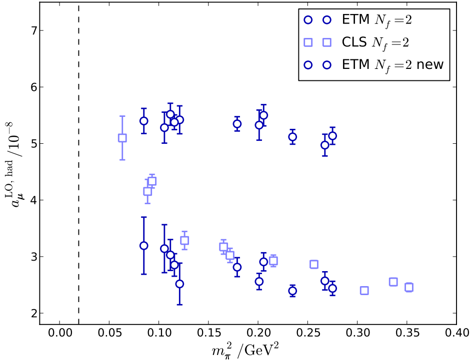

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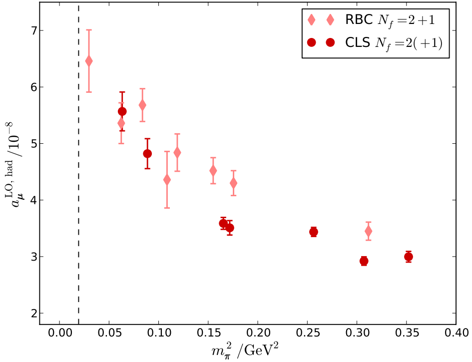
- $m_\pi \neq m_\pi^{\mathrm{phys}}$ in all simulations (ETM divide out ρ-pole and they then extrapolate linearly in m_π^2, *arXiv:1103.4818*)

Status a_μ^{LHV}

a_μ^{LHV} in $N_f = 2+1$ flavour QCD



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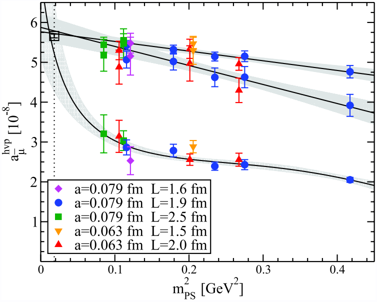
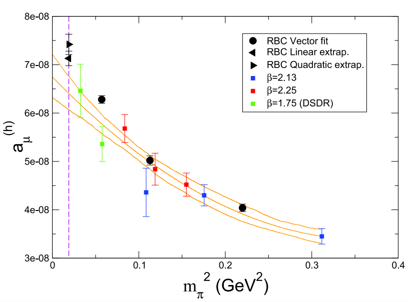
Status a_μ^{LHV}

parameterising the mass dependence



ETM arXiv:1103.4818



UKQCD arXiv:1107.1497

Numbers

Collaboration	N_f	$a_\mu^{\mathrm{LHV}}/10^{-10}$
UKQCD *arXiv:1107.1497*	2+1	641(33)(32)
ETM *arXiv:1103.4818*	2	572(16)
exp. *e.g. Jegerlehner & Nyffeler Physics Reports 477(2009)1-110*		690(5)

(for older efforts see: Aubin and Blum, Phys.Rev. D75 (2007) 114502, hep-lat/0608011 QCDSF, Nucl.Phys. B688 (2004) 135-164 hep-lat/0312032)

TODO:

- stabilise momentum depdendence by using twisting
- compute contribution of disconnected diagrams
- simulate closer/at the physical pion
- iso-spin breaking effects?

Summary

- recent summaries provide an impressive account of the advanced status of lattice QCD *(FLAG and Laiho, Lunghi, van de Water Phys.Rev. D81 (2010) 034503, arXiv:0910.2928)*
- efforts to predict $f^{\pi\pi}(q^2)$ and $a_\mu^{\rm LHV}$ are still struggling to meet phenomenologists/experimentalist expectations/needs
- there has been progress:
  - twisted boundary conditions
  - estimates for the disconnected diagrams
  - simulations closer to the physical point
- for $a_\mu^{\rm LHV}$ ETM and RBC have recently pubished status reports and the Mainz group will soon present their results
- for $f^{\pi\pi}(q^2)$ Mainz will also soon provide new results