Pseudoscalar meson transition form factors

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Outline

The **muon anomalous magnetic moment** $a_\mu = (g_\mu - 2)/2$ is one of the **most precisely measured** quantities in particle physics

$$a_\mu^{exp} = 11659208.0(6.3) \times 10^{-10} \quad (1)$$

- G.W.Bennet et al, Phys. Rev. D73 (2006) 072003

In **SM theoretical evaluations** it consists of the 3 contributions:

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{had} \quad (2)$$

- F.Jegerlehner and A.Nyffeler, Phys. Reports 477 (2009) 1-110 and the **latest reevaluation**

- M.Davier, A.Hoecker, B.Malaescu and Z.Zhang, Eur. Phys. J. C71 (2011) 71:1515

gives

$$a_\mu^{th} = 11659180.2(4.9) \times 10^{-10} \qquad (3)$$

Then

$$a_\mu^{exp} - a_\mu^{th} = 27.8(8.0) \qquad (4)$$

and **Standard Model predictions differ by** 3.5σ **from the experimental value**.

Anom.magnetic moment is **caused** mainly by the **leading order (LO) hadronic contributions** in the low-energy region $m_{\pi_0^2} < t < t_{max}$ by the **exclusive hadronic final states**

$$a_\mu^{had.LO} = \frac{1}{3}(\frac{\alpha}{\pi})^2\{\int_{m^2_{\pi^0}}^{t_{max}} \frac{dt}{t}K(t)\frac{3t}{4\pi\alpha^2}\sum_i \sigma_{tot}(e^+e^- \to i) + + \int_{t_{max}}^{\infty} \frac{dt}{t}K(t)R(t)\} \quad (5)$$

Results can be improved by two ways:

- by a **more precise measurement of** $\sigma_{tot}(e^+e^- \to i)$ and then by an **integration over the experimental points** as it is realized in almost all existing evaluations
- in the case of binary final states in e^+e^- -annihilation processes, **fitting all existing data on the corresponding FF by a sophisticated model** in space-like and time-like regions simultaneously and then **integrating over** $\sigma_{tot}(e^+e^- \to i)$ to be given through FF dependent on few **physically interpretable parameters with transferred errors**.

Further, we are concerned in the **contributions of the** $e+e^- \to P\gamma$**,** $(P=\pi^0,\eta,\eta')$ **processes** to $a_\mu^{had.LO}$

$$\sigma_{tot}(e^+e^- \to P\gamma) = \frac{\pi\alpha^2}{6}(1-\frac{m_P^2}{s})^3 \mid F_P\gamma(s) \mid^2 \qquad (6)$$

.

With the aim of diminishing the error of the contribution, we shall **exploit the second way of improvements**.

One of the first measurements of π^0, η and η' **transition FFs in the space-like region** was carried out by

- H.J.Behrend et al (CELLO Collab.), Z. Phys. C49 (1991) 401-409.

where really the π^0 **transition FF in the space-like region was observed for the first time**.
An **extension to higher** Q^2 was achieved by

- J.Gronberg et al (CLEO Collab.), Phys. Rev. D57 (1998) 33-54

to be **recently supplemented for** π^0 **up to** $Q^2 = 34.36 GeV^2$ by

- B.Aubert et al (BABAR Collab.), Phys. Rev. D80 (2009) 052002

These data can be completed for η' by 6 points of L3 Collab.

- M.Acciarri et al, Phys. Lett. B418 (1998) 399

and recent preliminary BABAR η and η' transition FFs

- V.P.Druzhinin, arXiv:1011.6159 [hep-ex] 6 Dec 2010,

however to be **presented only graphically**.

For a **measurements of π^0, η and η' in time-like region** commonly the annihilation processes $e^+e^- \to \gamma P$ are used.

Especially for π^0 and η a lot of data was obtained on colliding $e^+ - e^-$ beams in Novosibirsk by SND detector.

- M.N.Achasov et al, Eur. Phys. J. C12 (2000) 25
- M.N.Achasov et al, Phys. Lett. B559 (2003) 171-178

and by CMD-2 detector for η transition FF in

- R.R.Akhmetsin et al, Phys. Lett. B509 (2001) 217-226

The **latter corrected** and published together with π^0 in

- R.R.Akhmetsin et al, Phys. Lett. B605 (2005) 26-36.

Note:

$1/3$ **of the presented data** on $\sigma_{tot}(e^+e^- \to \eta\gamma)$ gives **zero information on** $F_{\gamma\eta}(t)$ - only **upper boundary estimations are presented** - or the **values are charged by the error equal, even larger, than the central value**.

These data can be completed by BABAR η and η' transition FFs at $t = 112 GeV^2$

- B.Aubert et al, Phys. Rev. D74 (2006) 012002,

by NA60 η FF

- R.Arnaldi et al, Phys. Lett. B677 (2009) 260

and by MAMI-C also η FF

- H.Berghauser et al, Phys. Lett. B701 (2011) 562

the **last two again presented only graphically**.

Further, our intention will be to achieve **optimal description** of all these $t < 0$ and $t > 0$ data on $F_{\gamma\pi^0}(t)$, $F_{\gamma\eta}(t)$, $F_{\gamma\eta'}(t)$ always **by one analytic function** explicitly known on the real axis of t-plane from $-\infty$ to $+\infty$.

These functions **respect all known FF properties** like

- the asymtotic behavior

$$\lim_{Q^2\to\infty} Q^2 F_{\gamma P}(Q^2) = 2f_P \tag{7}$$

- the normalization

$$lim_{Q^2\to 0} F_{\gamma P}(Q^2) = \frac{1}{4\pi^2 f_P} \tag{8}$$

- the reality condition $F^*_{\gamma P}(t) = F_{\gamma P}(t^*)$
- analytic properties with the **lowest branch point** $t_0 = m^2_{\pi^0}$ and **one effective inelastic branch point** t_{inl}
- unitarity condition, i.e. $ImF_{\gamma P}(t) \neq 0$ only for $m^2_{\pi^0} < t < \infty$

There is **single FF for each** $\gamma^* \to \gamma P$ **transition** defined by

$$< P(p)\gamma(k) \mid J^{EM}_\mu \mid 0 >= \epsilon_{\mu\nu\alpha\beta} p^\nu \epsilon^\alpha k^\beta F_{\gamma P}(q^2), \qquad (9)$$

A straightforward **calculation of** $F_{\gamma P}(Q^2)$ **behavior for** $-\infty < Q^2 < +\infty$ **in QCD is impossible**, therefore we construct a **sophisticated** $U\&A$ **model**.

The **QCD motivated models** for a description of $F_{\gamma P}(Q^2)$ **in space-like region** can be found in

- P.Kroll, Eur. Phys. J. C (2011) 71:1623

$F_{\gamma P}(t)$ - suitable to **split into two terms** depending on the isotopic character of the photon

$$F_{\gamma P}(t) = F_{\gamma P}^{I=0}(t) + F_{\gamma P}^{I=1}(t) \tag{10}$$

$F_{\gamma P}^{I=0}(t)$ can be **saturated by only isoscalar vector mesons**
$F_{\gamma P}^{I=1}(t)$ can be **saturated by only isovector vector mesons**
whereby **both sets possess photon quantum numbers**.

How much resonances will be considered ?
It is **prescribed by the interval of existing data in $t > 0$ region**.
The data on π^0 - **allow to consider all 3 ground state vector mesons** $\rho(770)$, $\omega(782)$, $\phi(1020)$ - adding also $\omega'(1420)$ and $\rho'(1450)$ **in order to obtain automatically normalized models**.

The **same number of resonances** is considered for η and η'.

Resonance parameters are fixed at the TABLE values.
Then **normalized 5 resonance VMD parametrization** is

$$
\begin{aligned}
F_{P\gamma}^{I=0}(t) &= \frac{1}{2}\mathbf{F_{P}}_{\gamma}(\mathbf{0})\frac{m_\omega'^2}{m_\omega'^2 - t} \\
&+ \{\frac{m_\omega^2}{m_\omega^2 - t} - \frac{m_\omega'^2}{m_\omega'^2 - t}\}(f_{\gamma P\omega}/f_\omega) \\
&+ \{\frac{m_\phi^2}{m_\phi^2 - t} - \frac{m_\omega'^2}{m_\omega'^2 - t}\}(f_{\gamma P\phi}/f_\phi)
\end{aligned}
$$

$$
\begin{aligned}
F_{P\gamma}^{I=1}(t) &= \frac{1}{2}\mathbf{F_{P}}_{\gamma}(\mathbf{0})\frac{m_{\rho'}^2}{m_\rho'^2 - t} \\
&+ \{\frac{m_\rho^2}{m_\rho^2 - t} - \frac{m_\rho'^2}{m_\rho'^2 - t}\}(f_{\gamma P\rho}/f_\rho)
\end{aligned}
$$

BUT $F_{P\gamma}(0) = \frac{2}{\alpha m_P}\sqrt{\frac{\Gamma(P\to\gamma\gamma)}{\pi m_P}}$, where $\Gamma(P \to \gamma\gamma)$ are **fixed at the world averaged values** from TABLE.

The analytic properties of $F_{\gamma P}(t)$:

- consist in the assumption - $F_{\gamma P}(t)$ is **analytic in the whole complex** t**-plane** besides two cuts on the positive real axis
- generated by **branch points** t_0 **and** t_{in}.

Practically it is achieved by **nonlinear transformations**

$$t = t_0 - \frac{4(t^s_{in} - t_0)}{[1/V - V]^2} \tag{11}$$

$$t = t_0 - \frac{4(t^v_{in} - t_0)}{[1/W - W]^2} \tag{12}$$

in normalized VMD parametrizations, respectively.

The **inelastic square-root branch points t^s_{in} and t^v_{in} include in average contributions of all higher important thresholds effectively** and are left to be **free parameters** of *U&A* model.

Variable V (W) is **conformal mapping**

$$V(t) = i\frac{\sqrt{q^s_{in} + q} - \sqrt{q^s_{in} - q}}{\sqrt{q^s_{in} + q} + \sqrt{q^s_{in} - q}} \tag{13}$$

$$q = [(t - t_0)/t_0]; \quad q^s_{in} = [(t^s_{in} - t_0)/t_0]$$

of the **four-sheeted Riemann surface in t-variable onto one V-plane** (W-plane).

In order to **demonstrate the reality condition** $F^*_{\gamma P}(t) = F_{\gamma P}(t^*)$ explicitly, one can utilize relations between complex conjugate values of the corresponding zero-width VMD model pole positions in V (W) plane

$$V_{\omega 0} = -V^*_{\omega 0}, \quad W_{\rho 0} = -W^*_{\rho 0} \tag{14}$$

and

$$V_{i0} = 1/V^*_{i0}, \quad i = \phi, \omega' \quad W_{\rho' 0} = 1/W^*_{\rho' 0} \tag{15}$$

following from the experience that in a fitting procedure of existing data on $F_{\gamma P}(t)$ such numerical value of t^s_{in} (t^v_{in}) is found that

$$(m_i^2 - \Gamma_i^2/4) < t^s_{in}, t^v_{in} \quad i = \omega, \rho \tag{16}$$

and

$$(m_j^2 - \Gamma_j^2/4) > t_{in}^s, t_{in}^v \quad j = \phi, \omega', \rho'. \tag{17}$$

Finally, incorporating $\Gamma \neq 0$ by a substitution

$$m_r^2 \to (m_r - i\Gamma_r/2)^2 \tag{18}$$

one comes to $U\&A$ **model** of $F_{\gamma P}(t)$ in the form

$$\begin{aligned} F_{\gamma P}^{I=0}[V(t)] = (\frac{1-V^2}{1-V_N^2})^2\{\frac{1}{2}F_{\gamma P}(0)H(\omega') \\ +[L(\omega) - H(\omega')]a_\omega \\ +[H(\phi) - H(\omega')]a_\phi\} \end{aligned}$$

$$F^{I=1}_{\gamma P}[W(t)] = (\frac{1-W^2}{1-W_N^2})^2\{\frac{1}{2}F_{\gamma P}(0)H(\rho') + [L(\rho) - H(\rho')]a_\rho\}$$

with

$$L(\omega) = \frac{(V_N - V_\omega)(V_N - V^*_\omega)(V_N - 1/V_\omega)(V_N - 1/V^*_\omega)}{(V - V_\omega)(V - V^*_\omega)(V - 1/V_\omega)(V - 1/V^*_\omega)}$$

$$H(i) = \frac{(V_N - V_i)(V_N - V^*_i)(V_N + V_i)(V_N + V^*_i)}{(V - V_i)(V - V^*_i)(V + V_i)(V + V^*_i)}, i = \phi, \omega'$$

$$L(\rho) = \frac{(W_N - W_\rho)(W_N - W^*_\rho)(W_N - 1/W_\rho)(W_N - 1/W^*_\rho)}{(W - W_\rho)(W - W^*_\rho)(W - 1/W_\rho)(W - 1/W^*_\rho)}$$

$$H(\rho') = \frac{(W_N - W_{\rho'})(W_N - W^*_{\rho'})(W_N + W_{\rho'})(W_N + W^*_{\rho'})}{(W - W_{\rho'})(W - W^*_{\rho'})(W + W_{\rho'})(W + W^*_{\rho'})}$$

and normalization points $V(t)_{t=0} = V_N$, $W(t)_{t=0} = W_N$.
It depends - on **5 free parameters**

$$t^s_{in}, t^v_{in}, a_j = (f_{\gamma Pj}/f_j) \quad j = \rho, \omega, \phi \tag{19}$$

determined in an optimal **description of existing data**.

In optimal description of existing data one finds the free parameters of the $U\&A$ models

for π^0: (see Fig.1) $q_{in}^s = 5.5210 \pm 0.0084$

$q_{in}^v = 5.6120 \pm 0.1414$
$a_\omega = 0.0063 \pm 0.0013$
$a_\phi = -0.0004 \pm 0.0001$
$a_\rho = 0.0212 \pm 0.0006$
$\chi^2/ndf = 121/75 = 1.61$

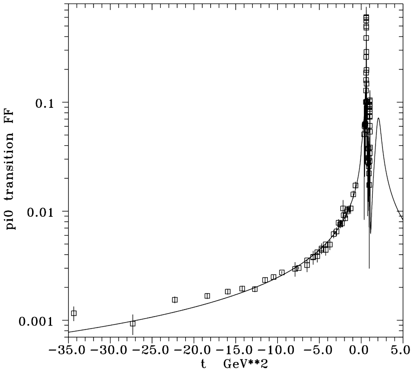

Figure: 1 A description of data on $\gamma - \pi^0$ transition form factor.

for η: (see Fig.2)

$q^s_{in} = 6.7104 \pm 0.0190$
$q^v_{in} = 5.5006 \pm 0.0632$
$a_\omega = 0.0002 \pm 0.0014$
$a_\phi = -0.0020 \pm 0.0003$
$a_\rho = 0.0250 \pm 0.0013$
$\chi^2/ndf = 52/52 = 1.00$

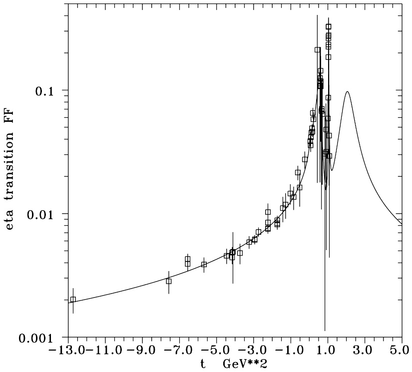

Figure: 2 A description of data on $\gamma - \eta$ transition form factor.

for η': (see Fig.3)

$q^s_{in} = 5.5366 \pm 0.0891$
$q^v_{in} = 7.7554 \pm 0.0158$
$a_\omega = -0.1134 \pm 0.0078$
$a_\phi = 0.0098 \pm 0.0091$
$a_\rho = 0.1241 \pm 0.0026$
$\chi^2/ndf = 59/50 = 1.18$

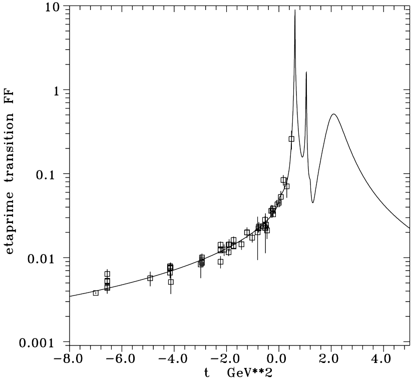

Figure: 3 A description of data on $\gamma - \eta'$ transition form factor.

MUON g-2

Obtaining in a such way **behavior of FFs in time-like resonant region**, one can calculate corresponding $\sigma_{tot}(e^+e^- \to P\gamma)$ and subsequently to **evaluate contributions**

$$\Delta a_\mu^{P\gamma} = \frac{1}{4\pi^3} \int_{m^2_{\pi^0}}^{t_{max}} ds\sigma_{tot}^{P\gamma}(t)K(t) \qquad (20)$$

to muon $g-2$.

The **upper boundary** of the low-energy integral is taken at the value $t_{max} = 2.0449 GeV^2$.

In such way the **region with relative good data** at least on π_0 and η **is covered.**

The following results are determined:

$$\begin{aligned} a_\mu^{\pi^0\gamma} &= 5.372(036) \times 10^{-10} \\ a_\mu^{\eta\gamma} &= 1.155(008) \times 10^{-10} \\ a_\mu^{\eta'\gamma} &= 2.069(965) \times 10^{-10} \end{aligned}$$

to be compared with the recent values of

- M.Davier, A.Hoecker, M.Malaescu and Z.Zhang, Eur. Phys. J. C71 (2011) 71:1515

$$\begin{aligned} a_\mu^{\pi^0\gamma} &= 4.420(194) \times 10^{-10} \\ a_\mu^{\eta\gamma} &= 0.640(024) \times 10^{-10} \\ a_\mu^{\eta'\gamma} &= - - - - - \end{aligned}$$

One can leave the norm $F_{P\gamma}(0)$ at the $U\&A$ models of transition FFs to **be free parameters** and determine them from the fit of the data.

Other free parameters are slightly changed, better description of data is achieved and the **following values of the norms** are found

$$
\begin{aligned}
F_{\gamma\pi^0}(0) &= 0.0352 \pm 0.0070[m_\pi^{-1}] \\
F_{\gamma\eta}(0) &= 0.0348 \pm 0.0026[m_\pi^{-1}] \\
F_{\gamma\eta'}(0) &= 0.0469 \pm 0.0016[m_\pi^{-1}]
\end{aligned}
$$

Finally, recalculated **values of two-photon decay widths** from the obtained normalization points $F_{\gamma P}(0)$ by means of

$\Gamma(P \to \gamma\gamma) = \frac{\pi\alpha^2 m_P^3}{4} F_{P\gamma}^2(0)$
and are

$$
\begin{aligned}
\Gamma(\pi^0 \to \gamma\gamma) &= (5.28 \pm 0.26) eV \\
\Gamma(\eta \to \gamma\gamma) &= (428.33 \pm 63.70) eV \\
\Gamma(\eta' \to \gamma\gamma) &= (4142.88 \pm 274.01) eV
\end{aligned}
$$

to be compared with TABLE values

$$
\begin{aligned}
\Gamma_{exp}(\pi^0 \to \gamma\gamma) &= (7.84 \pm 0.56) eV \\
\Gamma_{exp}(\eta \to \gamma\gamma) &= (511.03 \pm 27.79) eV \\
\Gamma_{exp}(eta' \to \gamma\gamma) &= (4305.00 \pm 424.95) eV
\end{aligned}
$$

The largest disagreement is found for the π^0 value, indicating that **something is wrong**:

- may be the value in TABLE is incorrect
- may be the BABAR data in space-like region up to $Q^2 = 35 GeV^2$ are not reliable
- if not, then the **expressions for the norm and asymptotic behavior** of the pseudoscalar meson transition FFs derived by **S.Brodsky(1981) from QCD are incorrect**....(**HARDLY!**)

- **Existing data** on pseudoscalar meson transition FFs are **described by the sophisticated *U&A* model**
- Knowing transition FFs, $\sigma_{tot}(e^+e^- \to P\gamma)$ are found and the **contributions of $e^+e^- \to P\gamma$ processes to muon g-2 anomaly** are evaluated
- **By an alternative method two-gamma decay widths** of π^0, η and η' pseudoscalar mesons have been **determined**