# Effective field theory approach to nuclear matter

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# Motivation

Nuclei and nuclear matter:
Effective theory: Skyrme(1960) Energy functional $E = E(\psi\dagger, V_{eff}, \psi)$

$$V_{eff} = t_0 + t_1 k^2 + t_2 k \cdot k' + t_3/6\rho$$

perturbative (no loops); saturation by three-body force
Bender, Heenen, Reinhard, RevModPhys 2003
Nucleon-Nucleon interaction and few-nucleon systems:
Effective field theory
Epelbaum, Hammer, Meißner RevModPhys 2009
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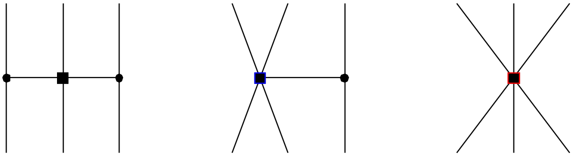
Nuclear matter in effective field theory:
Kaiser, Fritsch, Weise, NPA 2002
Bogner, Schwenk, Furnstahl, Nogga, NPA 2005

# Nucleon-Nucleon interaction

$$
\begin{aligned}
V_{eff} &= V_{OPEP} + V^{(0)} + V^{(2)} , \\
V_{OPEP} &= -\left(\frac{g_A}{2F_\pi}\right)^2 \frac{\tau_1 \cdot \tau_2 \boldsymbol{\sigma_1} \cdot \mathbf{q} \boldsymbol{\sigma_2} \cdot \mathbf{q}}{q^2 + M_\pi^2} \\
V^{(0)} &= C_S + C_T \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2} \\
V^{(2)} &= C_1 \mathbf{q^2} + \mathbf{C_2 k^2} + (\mathbf{C_3 q^2} + \mathbf{C_4 k^2}) \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2} \\
&\quad + i C_5 \frac{1}{2} (\boldsymbol{\sigma_1} + \boldsymbol{\sigma_2}) \cdot \mathbf{k} \times \mathbf{q} \\
&\quad + C_6 \boldsymbol{\sigma_1} \cdot \mathbf{q} \boldsymbol{\sigma_2} \cdot \mathbf{q} + \mathbf{C_7} \boldsymbol{\sigma_1} \cdot \mathbf{k} \boldsymbol{\sigma_2} \cdot \mathbf{k}
\end{aligned}
\tag{1}
$$

EFFECTIVE FIELD THEORY: Counting Scheme
LO = Leading Order
NLO = Next-to-Leading Order

# Three-Nucleon interaction:



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# Three-Nucleon interaction: NNLO long range

$$V_c = \sum_{i \neq j \neq k} \frac{1}{2} \left(\frac{g_A}{2f_\pi}\right)^2 \frac{\sigma_{\mathbf{i}} \cdot \mathbf{q_i}}{q_i^2 + m_\pi^2} \frac{\sigma_{\mathbf{j}} \cdot \mathbf{q_j}}{q_j^2 + m_\pi^2} F^{\alpha,\beta}_{i,j,k} \tau_i^\alpha \tau_j^\beta, \quad (2)$$

abbreviation:

$$F^{\alpha,\beta}_{i,j,k} = \delta^{\alpha,\beta}\left(-\frac{4c_1 m_\pi^2}{f_\pi^2} + \frac{2c_3}{f_\pi^2} \mathbf{q_i} \cdot \mathbf{q_j} + \sum_\gamma \frac{\mathbf{c_4}}{\mathbf{f_\pi^2}} \varepsilon^{\alpha,\beta,\gamma} \tau^\gamma_{\mathbf{k}} \sigma_{\mathbf{k}} \cdot \mathbf{q_i} \times \mathbf{q_j}\right). \quad (3)$$

$g_A = 1.26$ axial decay constant
$c_1 = -0.81\, GeV^{-1}$ pion-nucleon scattering
$c_3 = -3.40\, GeV^{-1}$
$c_4 = +3.40\, GeV^{-1}$
Buttiker,Meißner, NPA668(1998)252 N$\pi$
Entem,Machleidt, PRC66(2002)014002 N$^3$LO pion diagrams NN

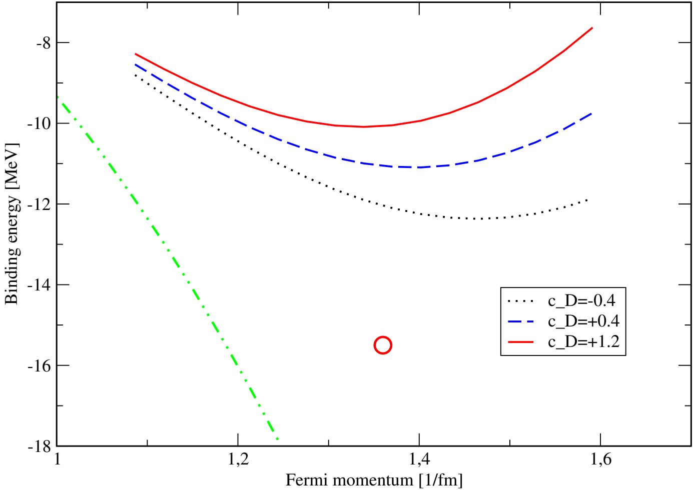
# Three-Nucleon interaction: NNLO short range

$$V_D = -\sum_{i\neq j\neq k} \frac{g_A}{8f_\pi^2}\frac{c_D}{f_\pi^2\Lambda_\chi}\frac{\boldsymbol{\sigma_j}\cdot\mathbf{q_j}}{\mathbf{q_j^2}+\mathbf{m_\pi^2}}\tau_i\cdot\tau_j\boldsymbol{\sigma_i}\cdot\mathbf{q_j} \tag{4}$$

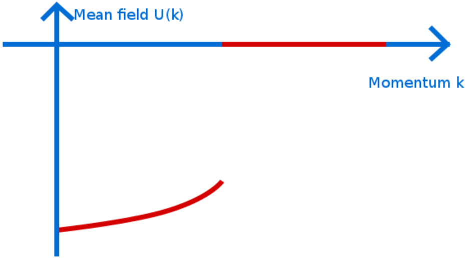
$$V_E = \frac{1}{2}\sum_{j\neq k}\frac{c_E}{f_\pi^4\Lambda_\chi}\tau_j\cdot\tau_k \tag{5}$$

$\Lambda_\chi = \Lambda_{Lippmann-Schwinger}$

# Problem: Nuclear matter, standard Brueckner Theory



# Analysis: Standard mean field in Brueckner Theory




- Note: Mean field is not an observable.
- Observables should not depend on U(k)!

# Hint I: Two and three-body correlations

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**Solution to the Bethe-Faddeev equation within the continuous version of the hole-line expansion**

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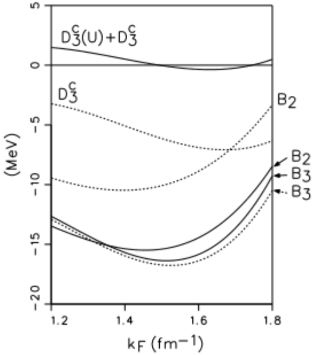


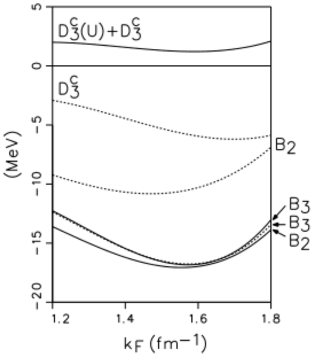


Reid Soft Core.................................................Argonne v14

# Hint II: Analytical Structure

G.A.Baker, Rev.Mod.Phys.(1971), Singularity Structure of the Perturbation Series of Ground State Energy of a Many-Fermion System.

Problem: Attractive contributions in two-nucleon interaction may generate bound states, Deuteron!

Consequence: In the low density limit, nuclear matter has at least two phases,
(i)deuteron gas
(ii)Fermi liquid.

Phase transition implies analytical singularity in the theory.

1. Replace K-matrix by R-matrix!
2. Perturbative expansion in terms of R-matrix!

# R-matrix model of nuclear matter

(i) R-matrix from Lippmann-Schwinger equation:[as in vacuum]

$$R(K, q_f, q_i) = V(q_f, q_i) - \int dq'\, q'^2\, VQgR(K, q', q_i) \tag{6}$$

$$g(q', q) = \frac{q'^2 - q^2}{(q'^2 - q^2)^2 + \Gamma^2} \tag{7}$$

(ii) Binding energy from two- and three-body interaction:[Perturbative]

$$(E/A)_2 = \frac{4k_F^3}{\pi m_N} \sum_{\alpha, L} \int dx w(x) R(K(x), q(x), q(x))^{\alpha}_{L,L} \tag{8}$$

$$(E/A)_3 = \frac{g_A^2}{(2\pi f_\pi)^4} \frac{36}{\pi k_F^3} \int d^3P \int d^3p \, (c_1\, T_1 + c_3\, T_3 + c_4\, T_4 + c_D\, T_D + c_E\, T_E) \tag{9}$$

(iii) here:

$c_D = -0.2$

$c_E = +0.2$

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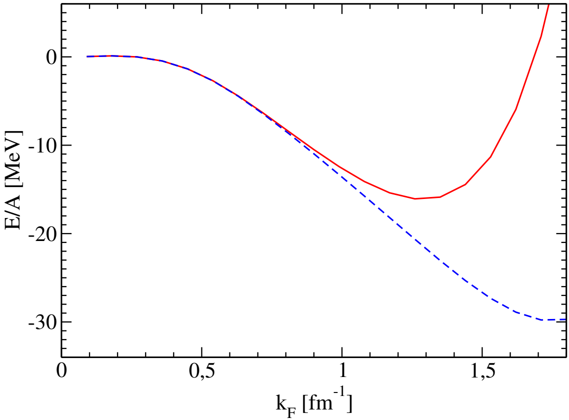
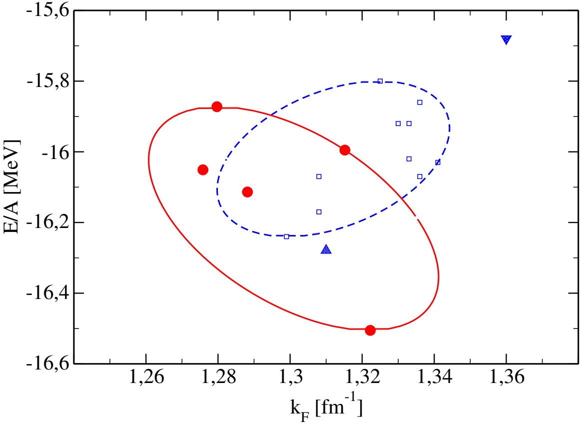
# Nuclear matter, NNLO

EFT in NNLO, R-matrix, $\Lambda_{Lippman-Schwinger} = 550$MeV, $\Lambda_{spectralfct} = 600$MeV
Three nucleon interaction



# Saturation points of nuclear matter

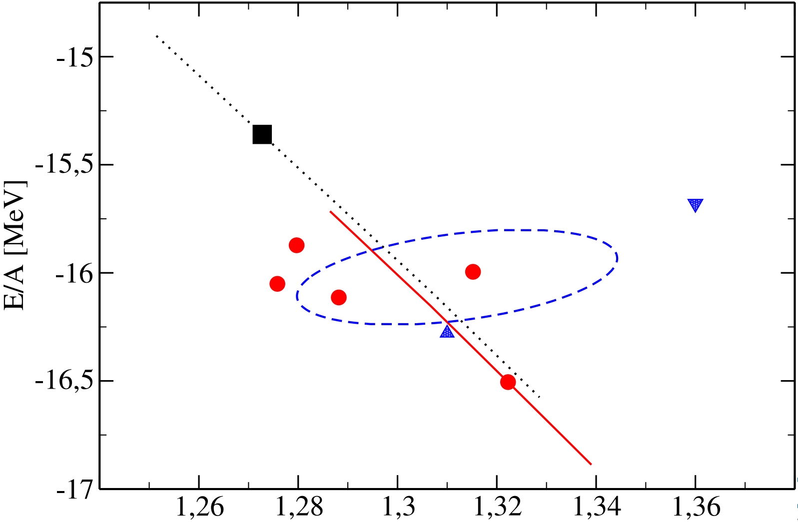
microscopic mean field calculations; minimal $\chi^2$ for nuclear masses.
triangle: Möller, Swiatecki, Treiner; Bethe



# Coester line from three-body contact interaction

microscopic mean field calculations; minimal $\chi^2$ for masses.

square: $c_D = 0, c_E = 0$; dots: $-1.1 < c_D < 0.5$



# Conclusion

- The NNLO effective field theory nucleonic interactions produce saturation of nuclear matter
  with E/A $= -16.2 \pm 0.3$ MeV and $k_F = 1.30 \pm 0.03$ fm$^{-1}$
  using the R-matrix approach and using $c_D = -0.2$ and $c_E = +0.2$
- The compression modulus is K $= 197 \pm 37$ MeV.
- Below $k_F = 0.51 \text{fm}^{-1}$, nuclear matter is instable and may form a deuteron gas.

An approach unifying few-body systems and heavy nuclei requires the explicit treatment of three-body correlations.