Fermi-Liquid Theory for Strong Interactions

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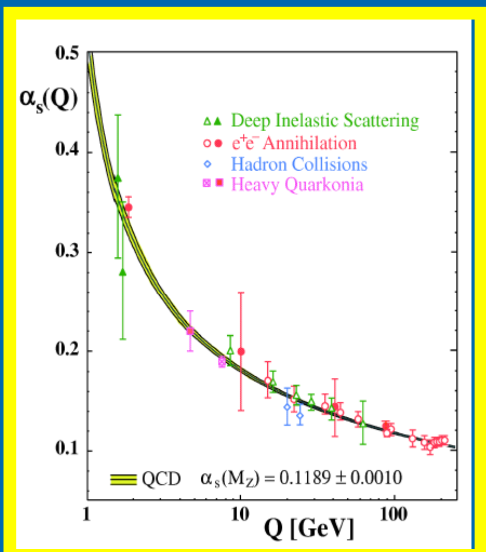
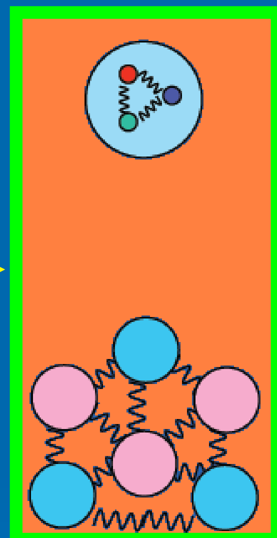
The theorist's dream…

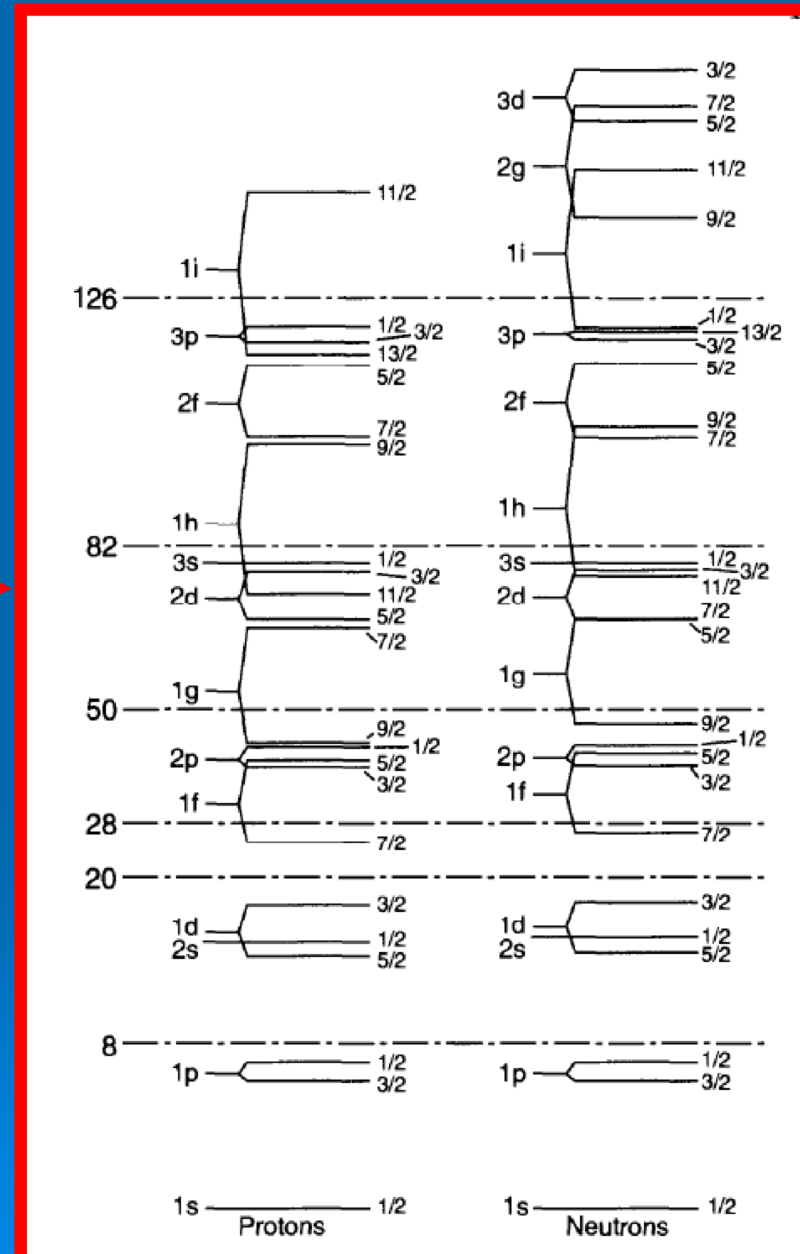
...from asymptotic freedom...

to confinement...

to the nuclear shell model



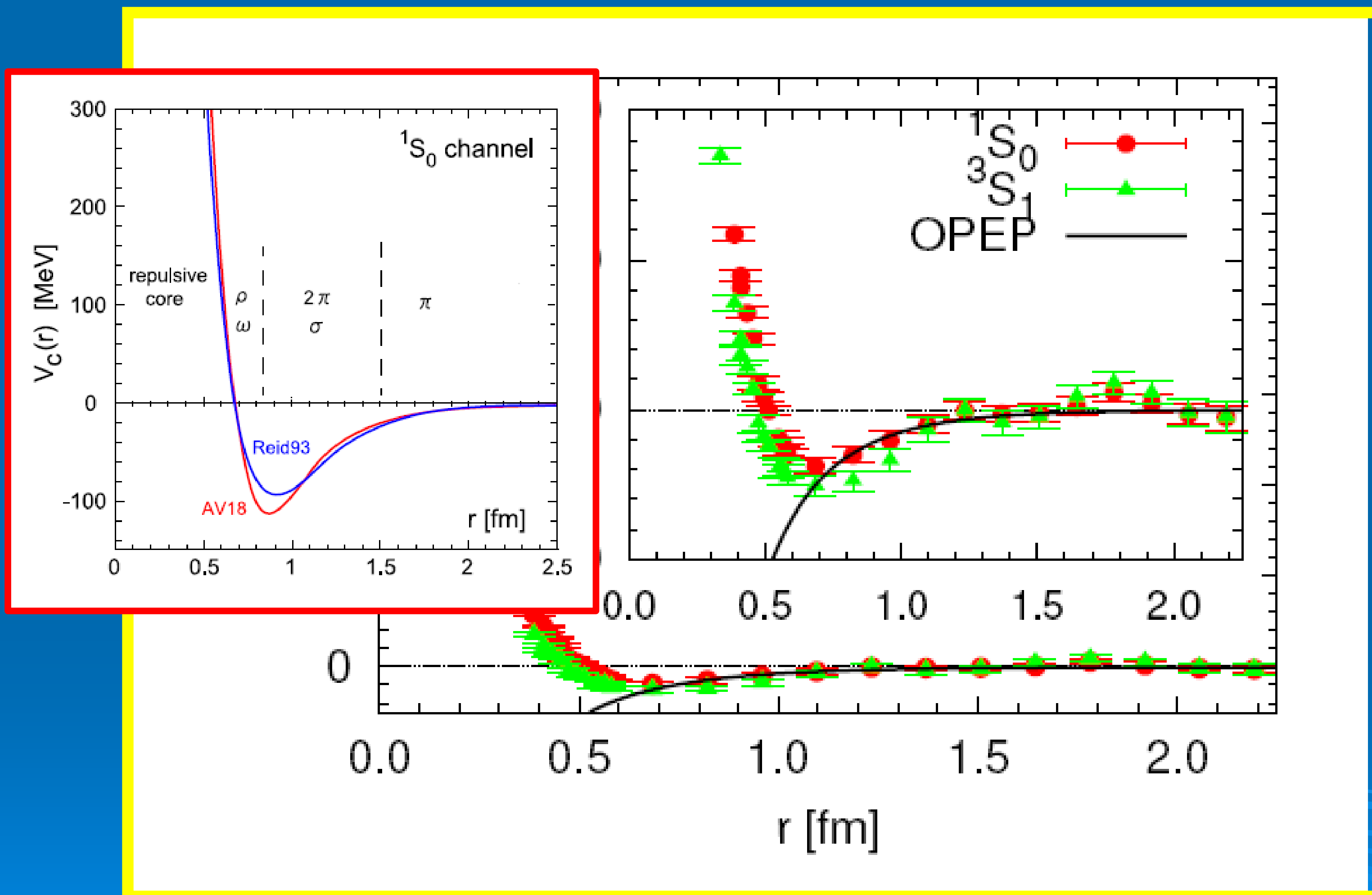




- Nucleus ~ cold, degenerate Fermi-Gas of Quasiparticles
- $U = U_0 + U_{so} \ell \cdot \sigma$

NN-Interaction from the Lattice

N. Ishii, S. Aoki, T. Hatsuda, Phys. Rev. Lett. 99, 022001 (2007).



$m_\pi/m_\rho = 0.595$

Agenda:

- Density Functional & Fermi Liquid Theory
- Landau-Migdal Parameters and Nuclear Matter
- Landau-Migdal Parameters and Nuclear Dynamics
- Hypermatter and Neutron Stars
- Summary

II. Density Functional Theory for Fermi Liquids

Existence Theorems on the Dynamics of Interacting Quantum Many-Body Systems:

Kohn-Sham	**(~1960) :**	**QM many-body systems ≅ DFT of E[ρ]**
Kohn-Hohenberg	**(~1963) :**	**DFT → E[ρ,τ]**
Nuclei	**:**	**$E[\rho_p,\rho_n,\tau_p,\tau_n,\kappa_p,\kappa_n\ldots]$**

Nuclei: Energy Density Functional depending in proton (q=p) and neutron (q=n) densities, currents…

$$E\left[\rho_q,\tau_q,\kappa_q\ldots\right]=T^{00}\left[\rho_q,\tau_q,\kappa_q\ldots\right]=T(\tau_q)+\frac{1}{2}E_{\rm int}(\rho_q,\tau_q,\kappa_q\ldots)$$

…in Infinite Nuclear Matter:

$$E(\rho)=\sum_{\substack{q=p,n\\ s=\pm 1/2}}\int\frac{d^3k}{(2\pi)^3}\frac{\hbar^2k^2}{2m_q}n_{qs}(k)+\frac{1}{2}\sum_{q,q',s,s'}\int\frac{d^3k_1}{(2\pi)^3}\int\frac{d^3k_2}{(2\pi)^3}n_{qs}(k_1)n_{q's'}(k_2)\Gamma_{qs,q's'}(k_1-k_2\,|\,\rho)$$

Cold (T=0) Nuclear Matter:

$$n_{qs}(k)=\left\langle\Psi^\dagger\Psi\right\rangle=\Theta(\lambda_{qs}-e_{qs}(k))\underset{unpolarized}{\rightarrow}\Theta(\lambda_q-e_q(k))\quad\wedge\quad\lambda_q=e_{qF}=\frac{\hbar^2k_F^2}{2m_q}+U_q(k_F)$$

Spin-Saturated Nuclear Matter → $\Sigma_s\ldots=N_s=2$:

$$E(\rho)=\sum_{q=p,n}N_s\int\frac{d^3k}{(2\pi)^3}\frac{\hbar^2k^2}{2m_q}n_q(k)+\frac{1}{2}\sum_{q,q'}N_sN_{s'}\int\frac{d^3k_1}{(2\pi)^3}\int\frac{d^3k_2}{(2\pi)^3}n_q(k_1)n_{q'}(k_2)\Gamma_{q,q'}(k_1-k_2\,|\,\rho)$$

Isopin-Saturated (symmetric) Nuclear Matter → $\Sigma_q\ldots=N_q=2$:

$$E(\rho)=N_sN_q\int\frac{d^3k}{(2\pi)^3}\frac{\hbar^2k^2}{2m}n(k)+\frac{1}{2}(N_sN_q)^2\int\frac{d^3k_1}{(2\pi)^3}\int\frac{d^3k_2}{(2\pi)^3}n(k_1)n(k_2)\Gamma_{00}(k_1-k_2\,|\,\rho)$$

Elements of Density Functional Theory

$$E(\rho,\kappa) \approx E(\rho_0,\kappa_0) + \sum_{q=p,n}\left(\left(T_q + U_q(\rho_0)\right)\delta n_q + \Delta_q \delta\kappa_q\right) + \sum_{q,q'=p,n} f_{qq'}(\rho_0)\delta n_q \delta n_{q'} + \sum_{q,q'=p,n} d_{qq'}(\rho_0)\delta\kappa_q \delta\kappa_{q'} \ldots$$

The quasi-particle Self-Energy:

$$U_q = \frac{\delta}{\delta\rho_q}\frac{1}{2}E_{int} = \sum_{q'}\Gamma_{qq'}(\rho)n_{q'} + \frac{1}{2}\sum_{q'q''} n_{q'}n_{q''}\frac{\delta}{\delta n_q}\Gamma_{q'q''}(\rho) + \frac{1}{2}\sum_{q'q''}\kappa_{q'}\kappa_{q''}\frac{\delta}{\delta n_q}\Gamma^{(pair)}_{q'q''}(\rho)$$

The residual interaction (restoring force):

$$f_{qq'} = \Gamma_{qq'}(\rho) + 2\sum_{q''} n_{q''}\frac{\delta}{\delta n_q}\Gamma_{q'q''}(\rho) + \frac{1}{2}\sum_{k'k''} n_{k'}n_{k''}\frac{\delta^2}{\delta n_q \delta n_{q'}}\Gamma_{k'k''}(\rho) + \ldots$$

…the simplest case - E(ρ) ~ E^{HF}(ρ):

$$\frac{\delta E^{\rm HF}}{\delta n_{p\sigma}\delta n_{p'\sigma'}} = f^{\rm HF}_{\sigma,\sigma'}(p,p') = \langle p\sigma, p'\sigma'|V|p\sigma, p'\sigma'\rangle - \langle p\sigma, p'\sigma'|V|p'\sigma', p\sigma\rangle.$$

…but:

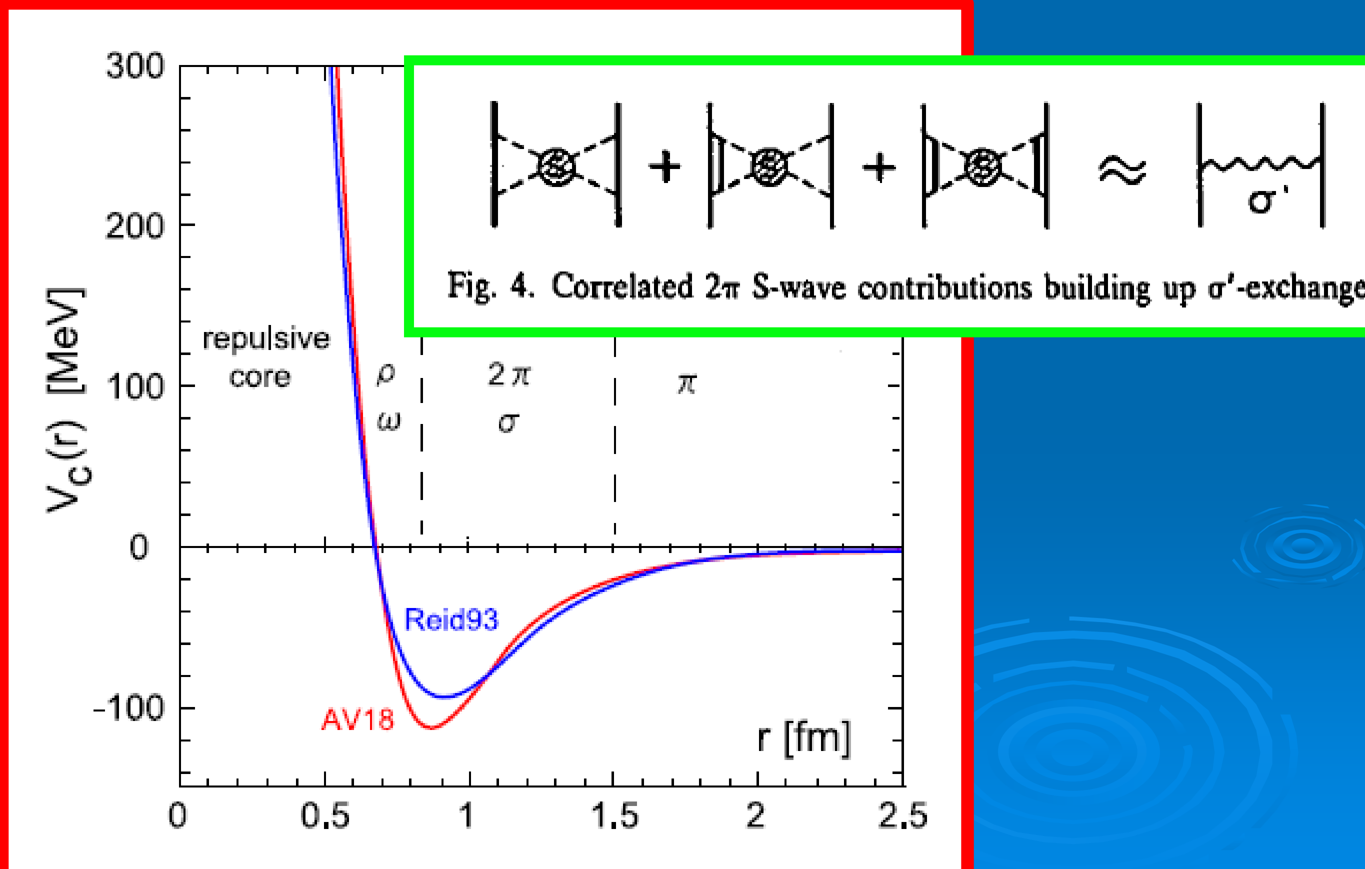


Fig. 4. Correlated 2π S-wave contributions building up σ'-exchange.

…two ways to solve the nuclear many-body problem:

The „ab initio" shell model choice:

• simple („bare") interaction → complicated wave function

The DFT choice:

• complicated („effective") interaction → simple wave function:

$$\left(T+V-E\right)\chi_{12}=0\mapsto\left(T+\Gamma-E\right)\phi_{12}=0$$

$$\Gamma=V+\int Vg_{12}Q_F\Gamma$$

$$M=\left\langle\phi_{12}\left|V\right|\chi_{12}\right\rangle\equiv\left\langle\phi_{12}\left|\Gamma\right|\phi_{12}\right\rangle$$

Brueckner G-Matrix Theory (K-Matrix Theory)

ALRIGHT RUTH, I ABOUT GOT THIS ONE RENORMALIZED.

The Quasi-Particle Scattering Amplitude and Landau-Migdal Parameters

$$A(k_1, k_2, \cos\vartheta) = N(k_F)\, f(\vec{k}_1, \vec{k}_2)$$

$$A = F + F'\tau_1 \bullet \tau_2 + G\sigma_1 \bullet \sigma_2 + G'\sigma_1 \bullet \sigma_2 \tau_1 \bullet \tau_2 + \ldots$$

$$A(k_1, k_2, \cos\vartheta) = \sum_{\ell} (2\ell + 1)\left(F_\ell + F_\ell'\tau_1 \bullet \tau_2 + G_\ell \sigma_1 \bullet \sigma_2 + G_\ell'\sigma_1 \bullet \sigma_2 \tau_1 \bullet \tau_2\right) P_\ell(\cos\vartheta)$$

Density of States at the Fermi-surface:

$$N(k_F) = \frac{m k_F}{\hbar^2 \pi^2} \underset{k_F^{eq}}{\longrightarrow} \sim \frac{1}{300}\left[\frac{1}{MeV fm^3}\right]$$

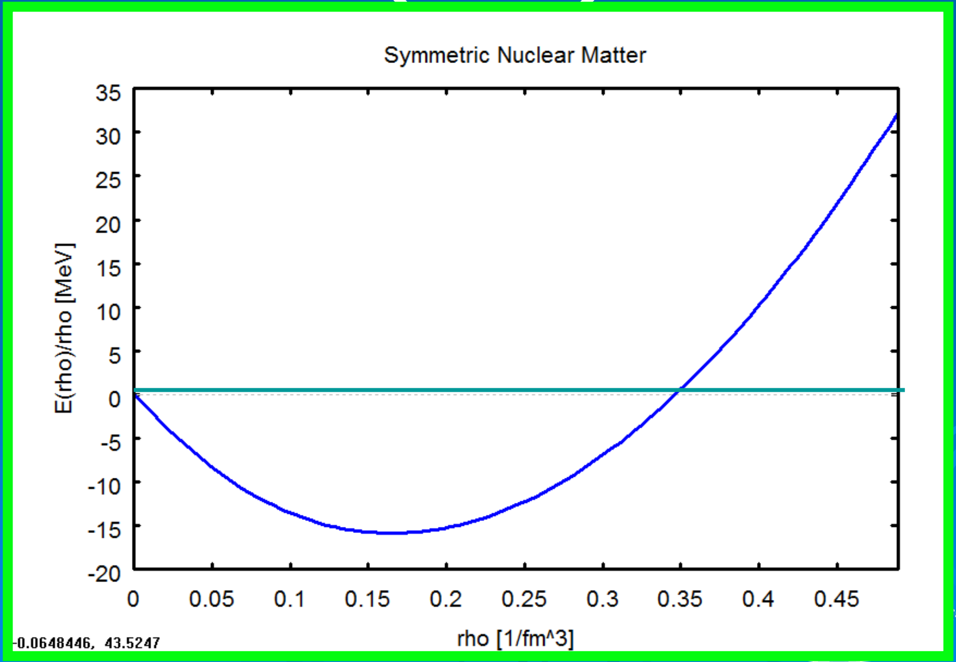
(LM Parameters are constrained by the Landau Sum Rules)

Landau-Migdal Parameters

Relation to Static (ground state) Properties

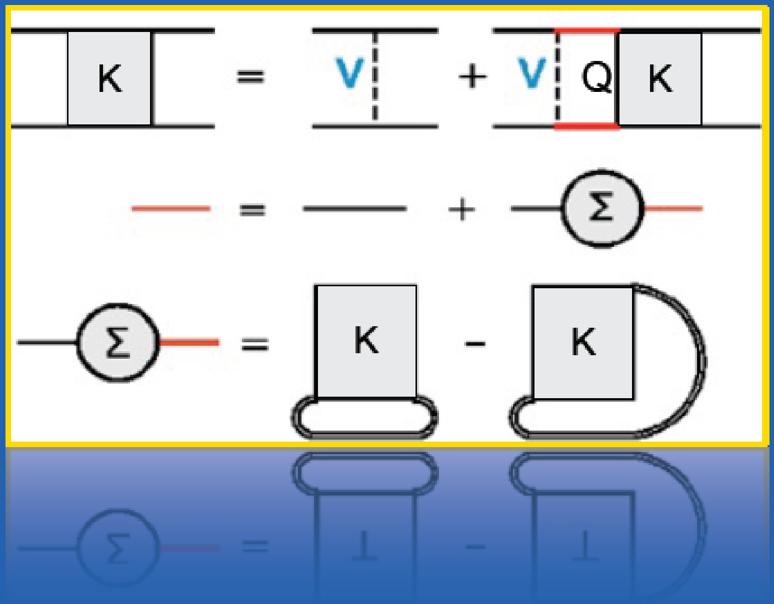
$$K = \frac{3\hbar^2 k_F^2}{m^\star}(1+F_0), \qquad \frac{m^\star}{m} = 1 + F_1/3, \quad \text{and} \quad E_{sym} = \frac{\hbar^2 k_F^2}{6m^\star}(1+F_0').$$

Empirical values @saturation density ρ_{sat}=0.16/fm^3:
K~250MeV ; m*/m~0.7 ; E_{sym}~30 MeV
(±10%)



The Giessen Approach to In-Medium Interactions…

$$K = V + \int V g_{NN} Q_F K$$



- Ladder Kernel
- Map the ab-initio calculations on an effective Lagrangian
- Medium dependent renormalization

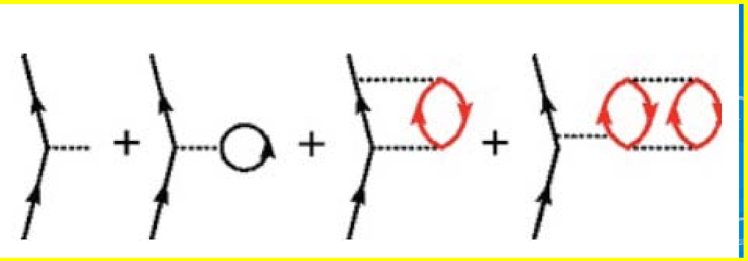
$$V_{OBE} = \sum_\alpha g_\alpha^2 D_\alpha(t) \langle \overline{u}_1 \hat{O}_\alpha u_3 \rangle \langle \overline{u}_2 \hat{O}_\alpha u_4 \rangle$$

The DDRH-DFT Lagrangian

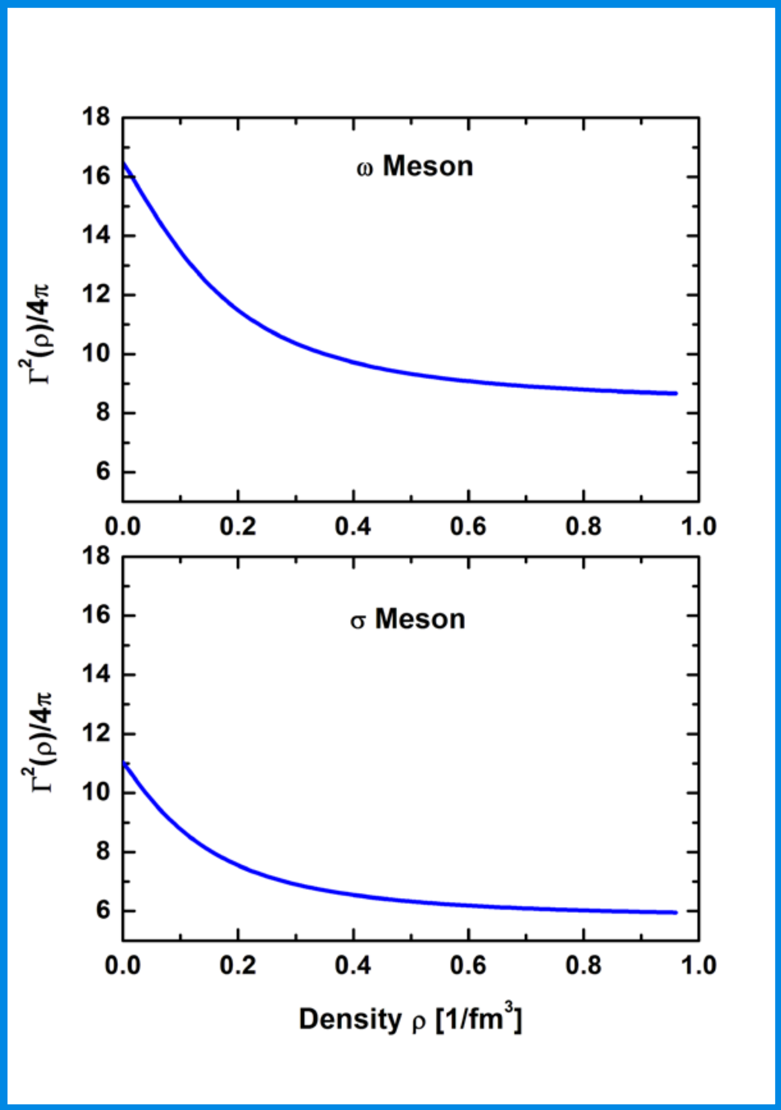
$$\mathcal{L}_{int} = \overline{\Psi}\hat{\Gamma}_\sigma(\hat{\rho})\Psi\Phi_\sigma - \overline{\Psi}\hat{\Gamma}_\omega(\hat{\rho})\gamma_\mu\Psi A^{(\omega)\mu} + \overline{\Psi}\hat{\Gamma}_\delta(\hat{\rho})\tilde{\tau}\Psi\mathbf{\Phi}_\delta - \overline{\Psi}\hat{\Gamma}_\rho(\hat{\rho})\gamma_\mu\tilde{\tau}\Psi\mathbf{A}^{(\rho)\mu} - \overline{\Psi}\hat{\Gamma}_\eta(\hat{\rho})\gamma_5\Psi\Phi_\eta - \bar{\Psi}\hat{\Gamma}_\pi(\hat{\rho})\gamma_5\gamma_\mu\tilde{\tau}\Psi\partial^\mu\mathbf{\Phi}_\pi - e\overline{\Psi}\hat{Q}\gamma_\mu\Psi A^{(\gamma)\mu} .$$

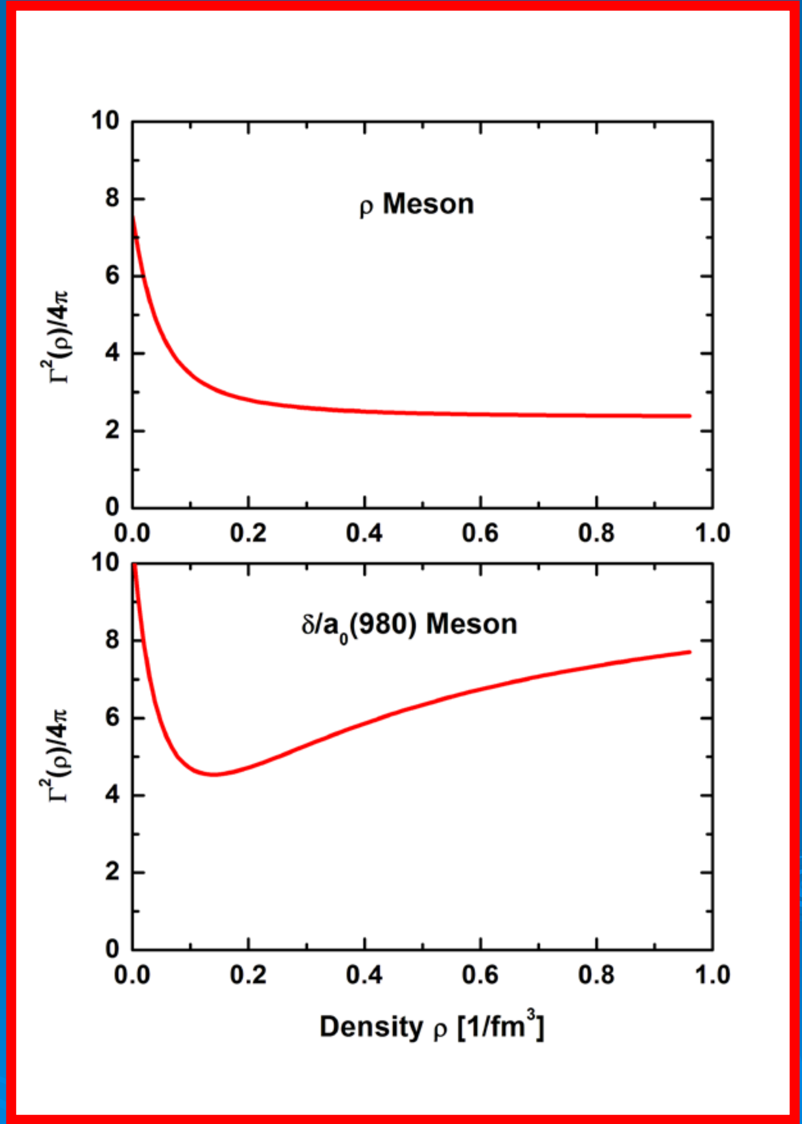
$$\frac{\delta\mathcal{L}_{int}}{\delta\overline{\Psi}} = \frac{\partial\mathcal{L}_{int}}{\partial\overline{\Psi}} + \frac{\partial\mathcal{L}_{int}}{\partial\hat{\rho}}\frac{\delta\hat{\rho}}{\delta\overline{\Psi}}$$

$$\hat{\Sigma}^\mu = \hat{\Sigma}^{\mu(0)} + \hat{\Sigma}^{\mu(r)}$$



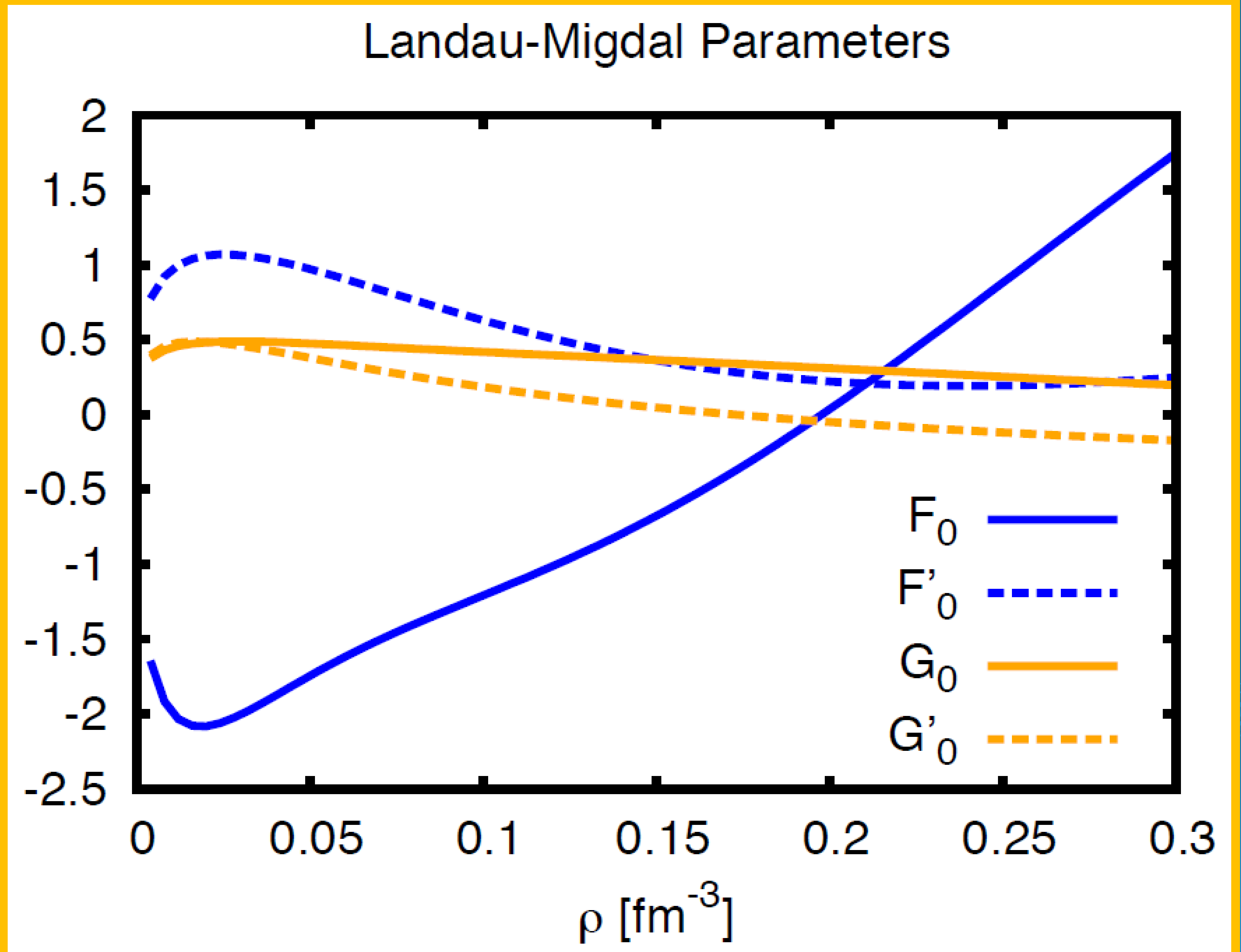
Nuclear Matter DBHF Vertices





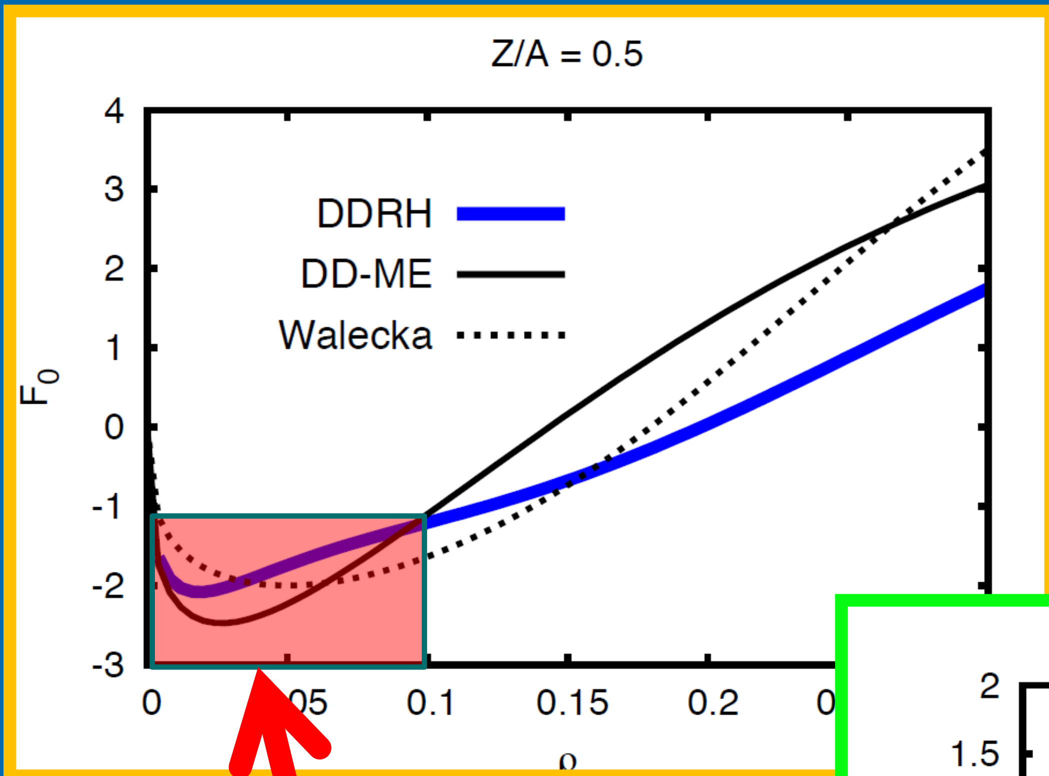
Isoscalar Vertices

Isovector Vertices

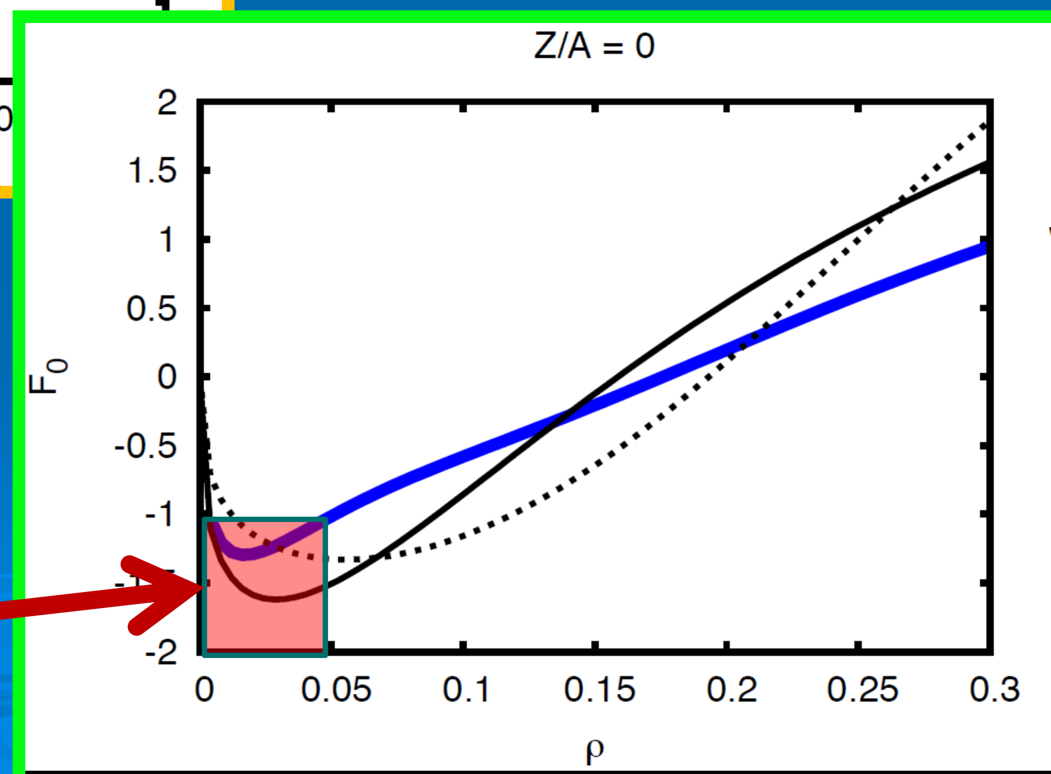


Landau-Migdal Parameters
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
0.05
0.1
0.15
0.2
0.25
0.3
ρ [fm^-3]
F_0
F'_0
G_0
G'_0

Symmetric Nuclear Matter



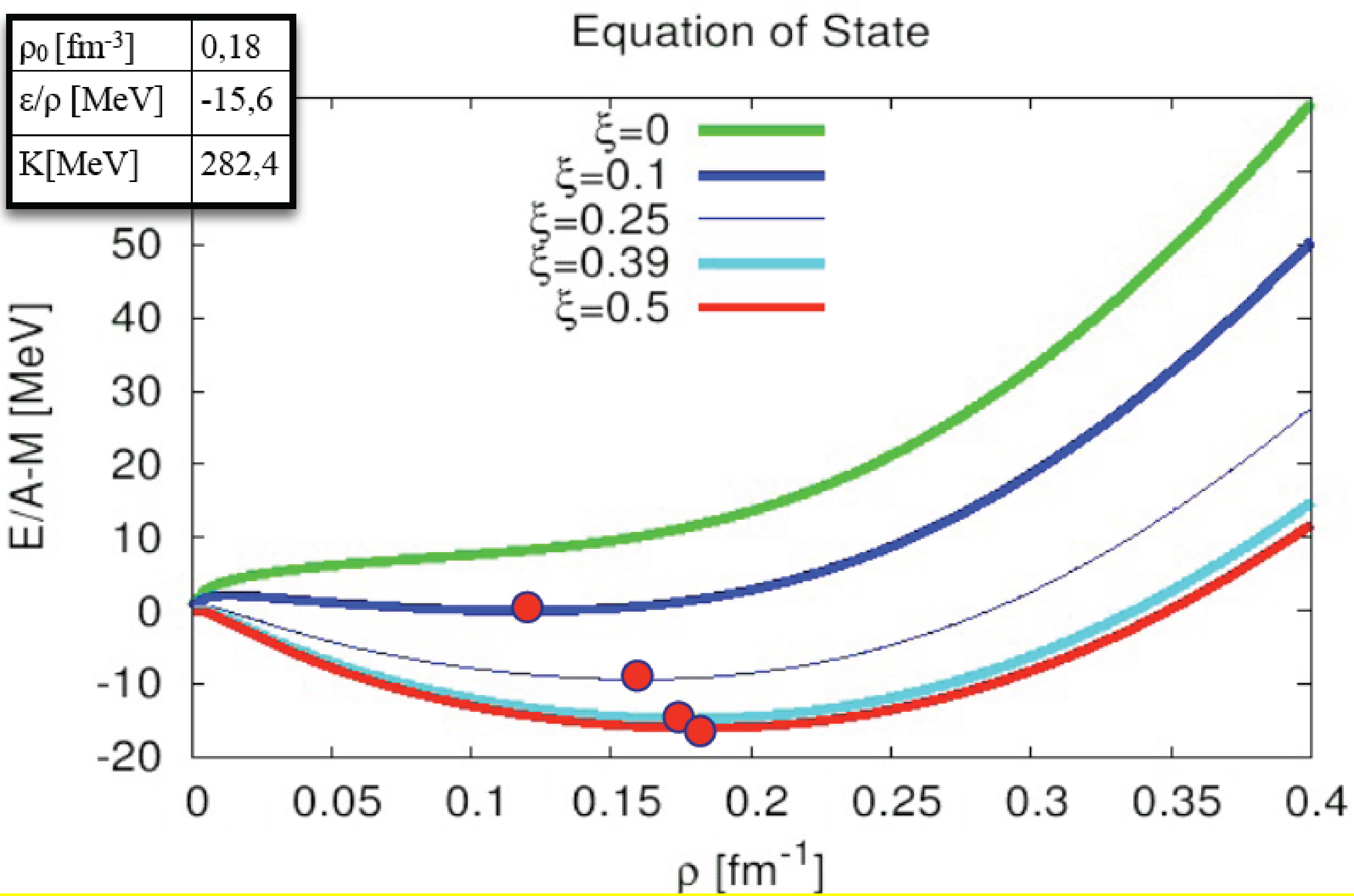
Pure Neutron Matter



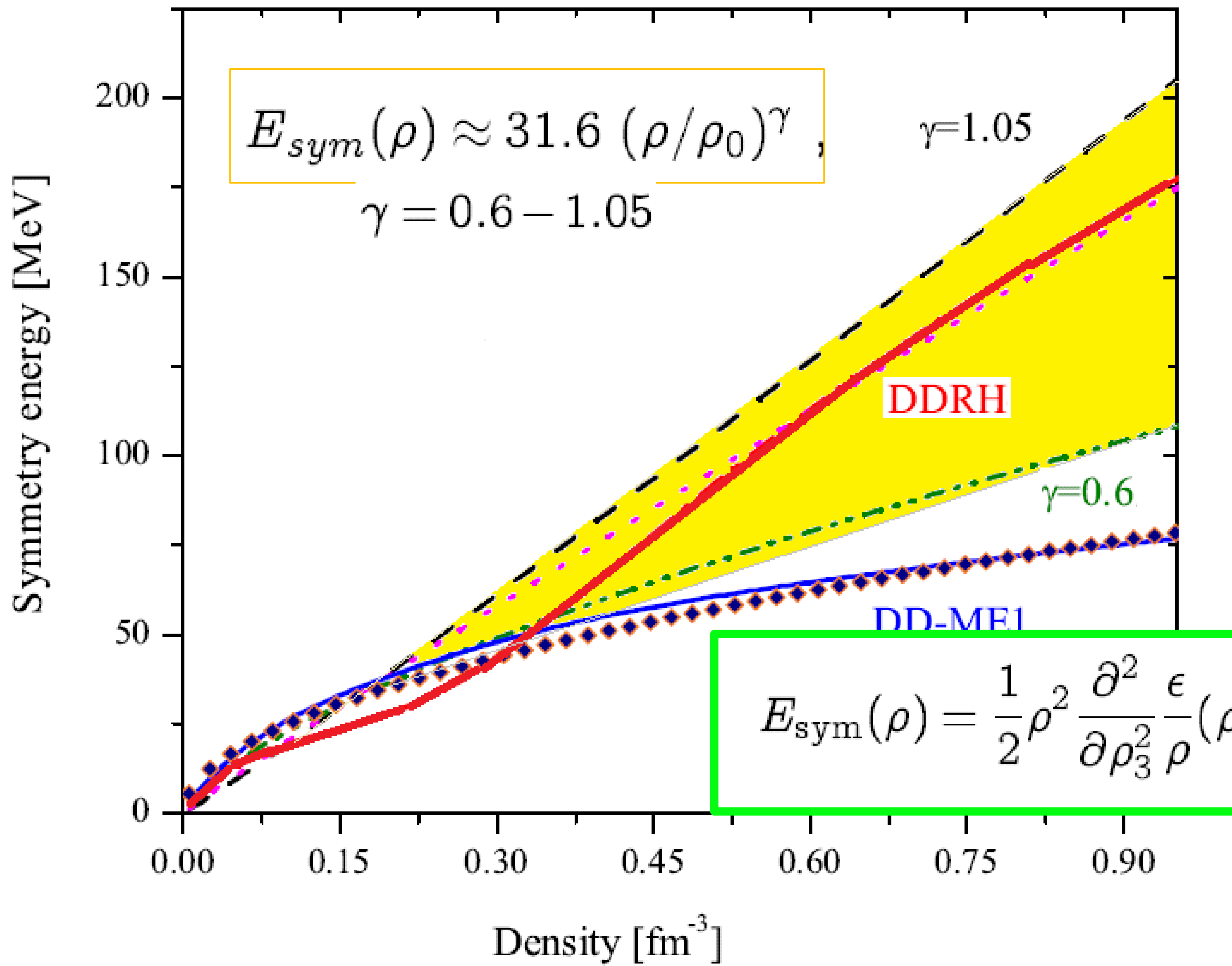
Region of Instability!

The Nuclear Equation of State (F_0,F_1,F'_0)

ρ_0 [fm^{-3}]	0,18
ε/ρ [MeV]	-15,6
K[MeV]	282,4



Density Dependence of the Symmetry-Energy (F'_0)

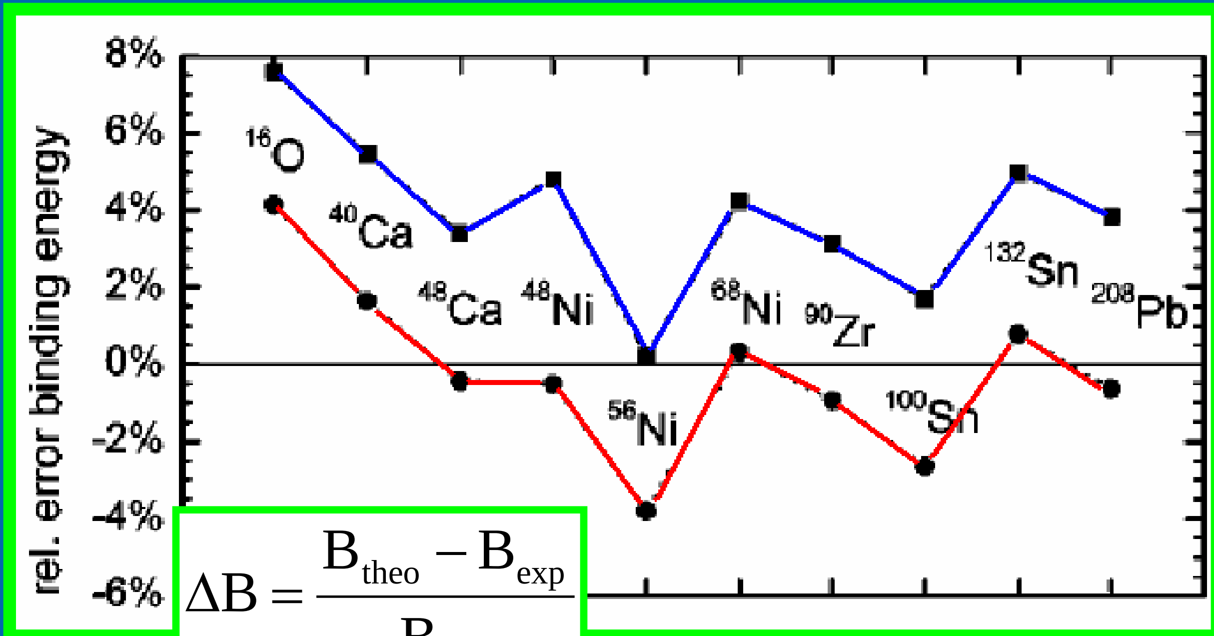


$$E_{\rm sym}(\rho) = \frac{1}{2}\rho^2 \frac{\partial^2}{\partial \rho_3^2}\frac{\epsilon}{\rho}(\rho, \rho_3)\ .$$

DDRH Results:

B(A) and Charge Radii

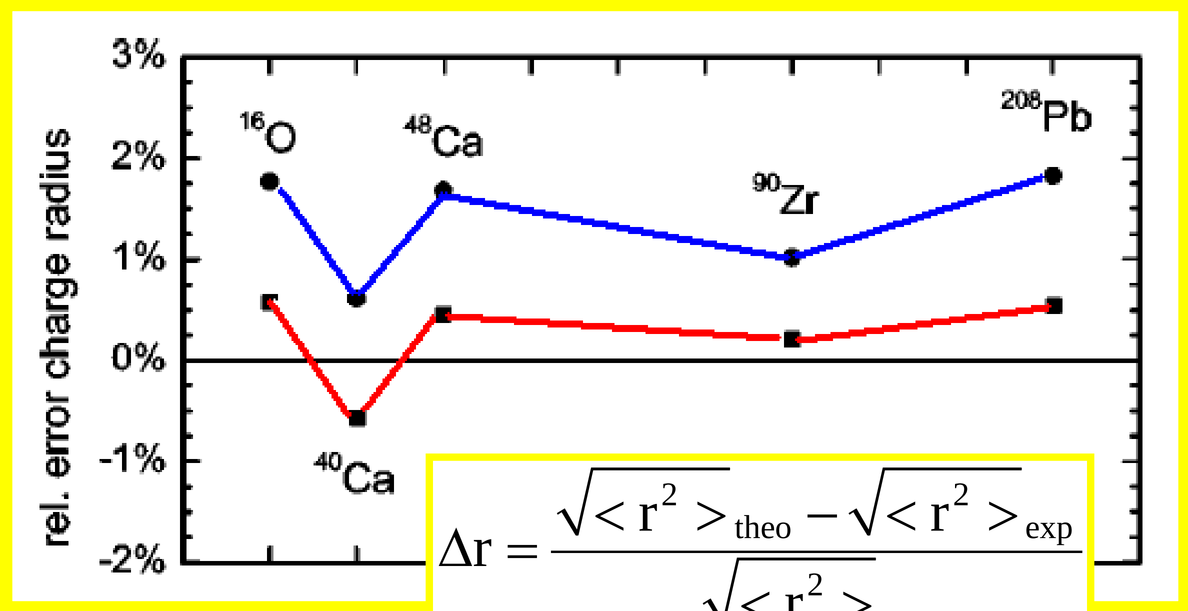
(F_0, F'_0)



$$\Delta B = \frac{B_{theo} - B_{exp}}{B_{exp}}$$

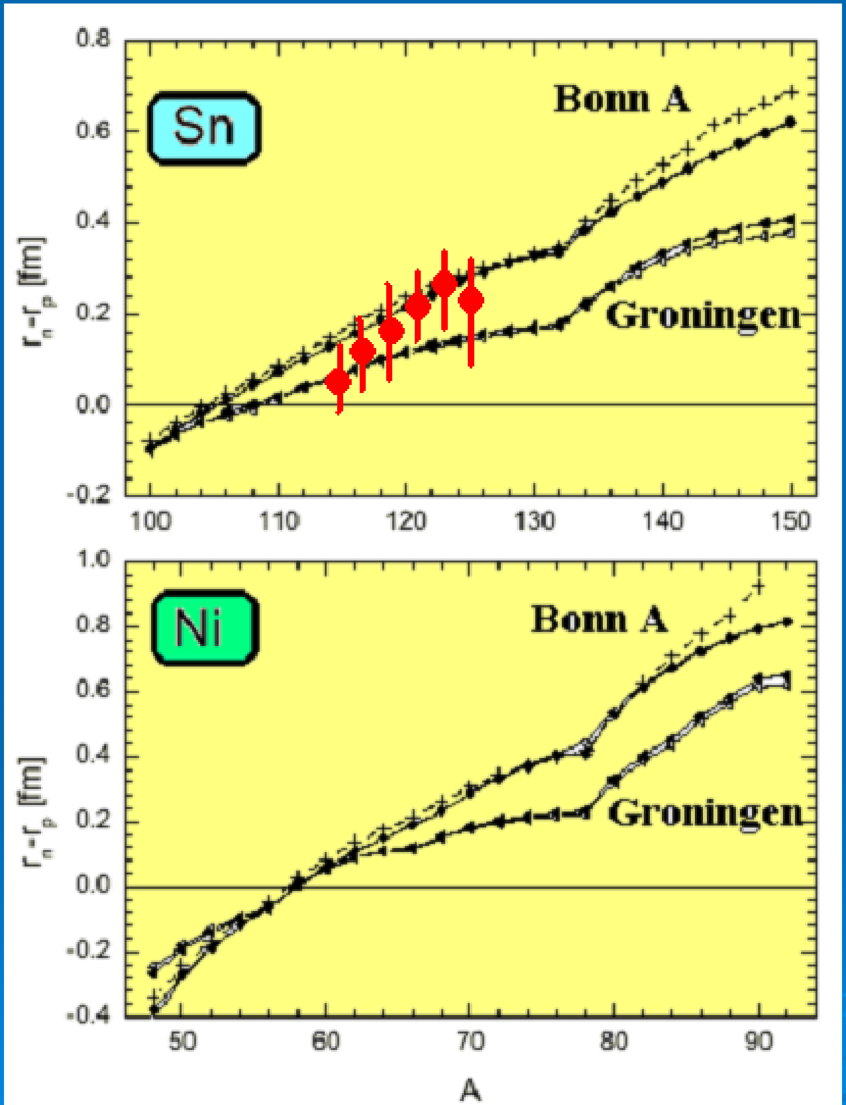
"Hartree" Vertices

(DME-Method: F. Hofmann, HL, PRC 1998)



$$\Delta r = \frac{\sqrt{<r^2>}_{theo} - \sqrt{<r^2>}_{exp}}{\sqrt{<r^2>}_{exp}}$$

Neutron Skins in Ni and Sn Isotopes



Neutron Skin and Symmetry Energy:

Bonn A : a_4 = 32 MeV

Groningen : a_4 = 26 MeV

Sn Data:
Krasnahorkay et al. PRL 82 (1999) 3216
(from Charge Exchange Spin-Dipole sum rules)

F. Hofmann et al., PR C64 (2001)
N. Tsoneva. H.L., PLB586 (2004), PRC77 (2008)...

III. Nuclear Dynamics

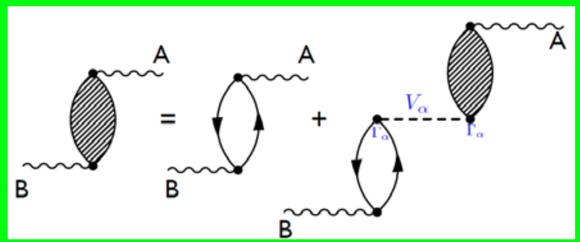
Relation to dynamical Properties
Dyson Equation for the 4-point Function

$$\Pi_{\alpha\beta} = \Pi^{(0)}_{\alpha\beta} + \sum_{\gamma} \Pi^{(0)}_{\alpha\gamma} \Gamma_{\gamma} \Pi_{\gamma\beta}$$

$$\Pi^{(0)}_{\alpha\beta} = \left\langle 0 \,|\, T^{+}_{\alpha} G^{(0)}_{\alpha\beta} T_{\beta} \,|\, 0 \right\rangle$$

$$R_{\lambda}(\omega, q) = \frac{1}{\pi} \Im \Pi_{\lambda\lambda}(\omega, q)$$

$$G^{(0)}_{\alpha\beta}(1,2\,|\,\omega) = \sum_{n} \frac{\rho_{\alpha n}(1)\rho^{+}_{\beta n}(2)}{E_n - \omega - i\eta} + \int dE \frac{\rho_{\alpha E}(1)\rho^{+}_{\beta E}(2)}{E - \omega - i\eta} + \ldots$$

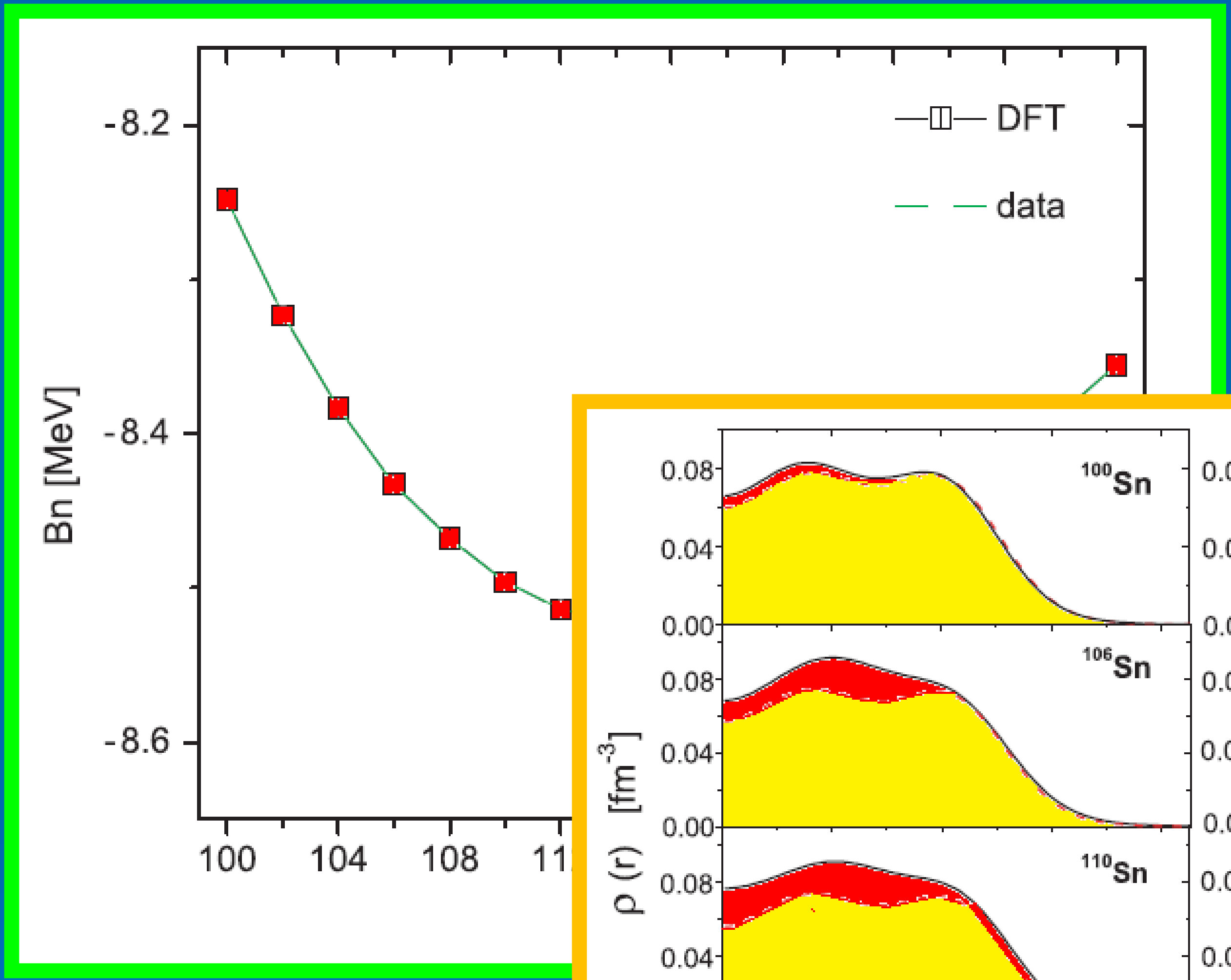


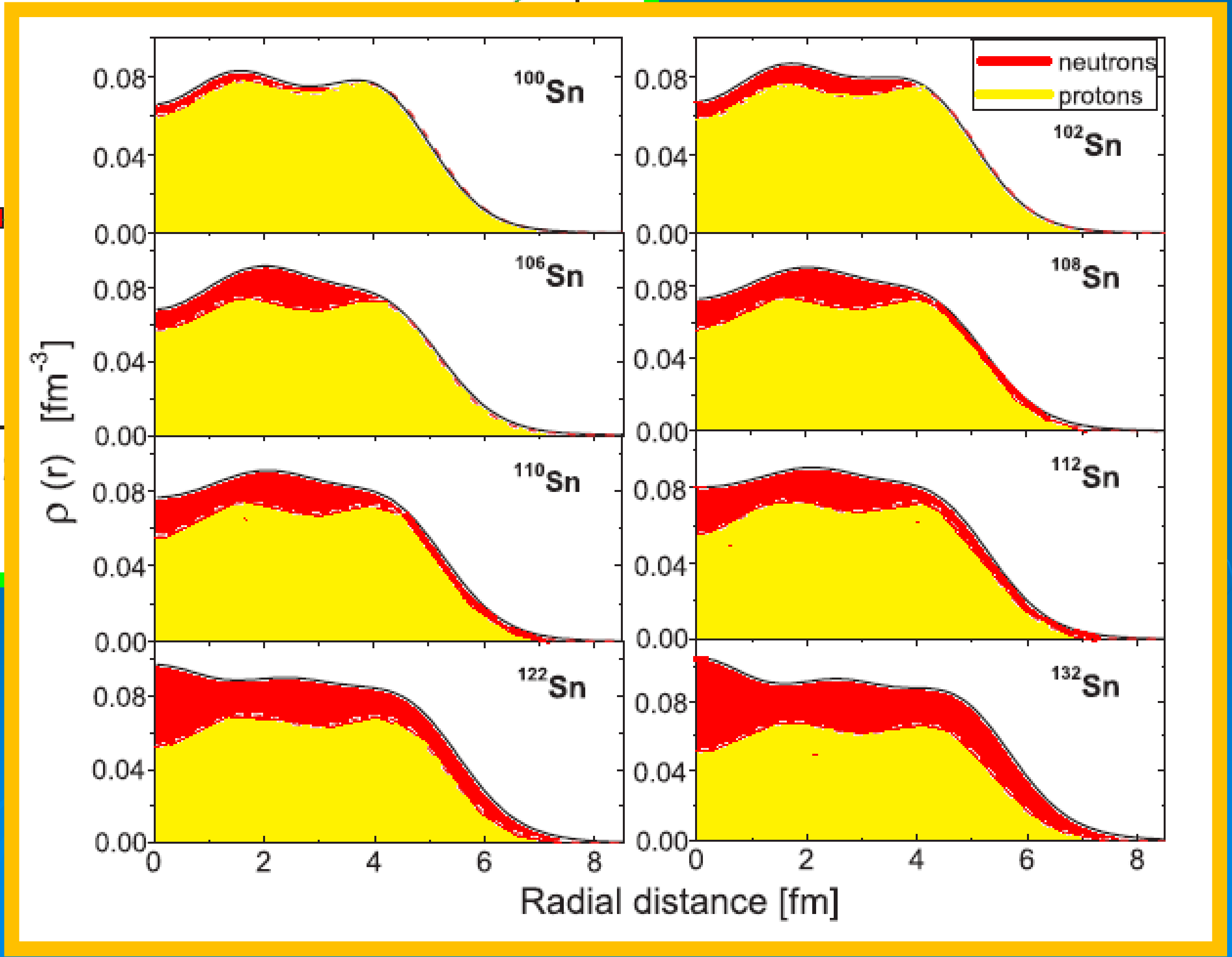
…how to probe LM-Dynamics:

- $F_\ell \rightarrow$ surface vibrations, isoscalar giant resonances
- $F'_\ell \rightarrow$ pygmy resonances, isovector giant resonances
- $G_\ell \rightarrow$ magnetic moments, isoscalar spin excitations
- $G'_\ell \rightarrow$ pionic correlations, Gamov-Teller resonances

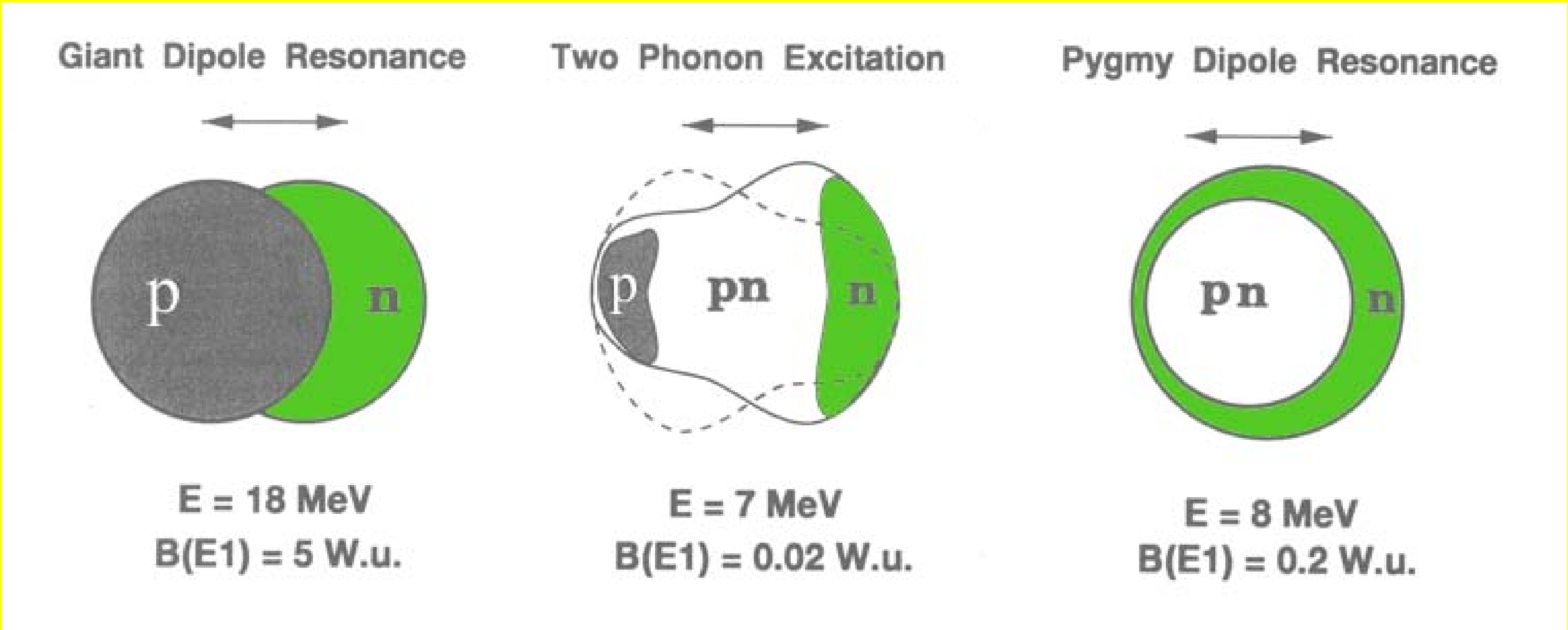
Sn Isotopes: DFT-HFB Results

(N. Tsoneva, HL, PRC77 (2008), PRL 2010, PRL 2011)





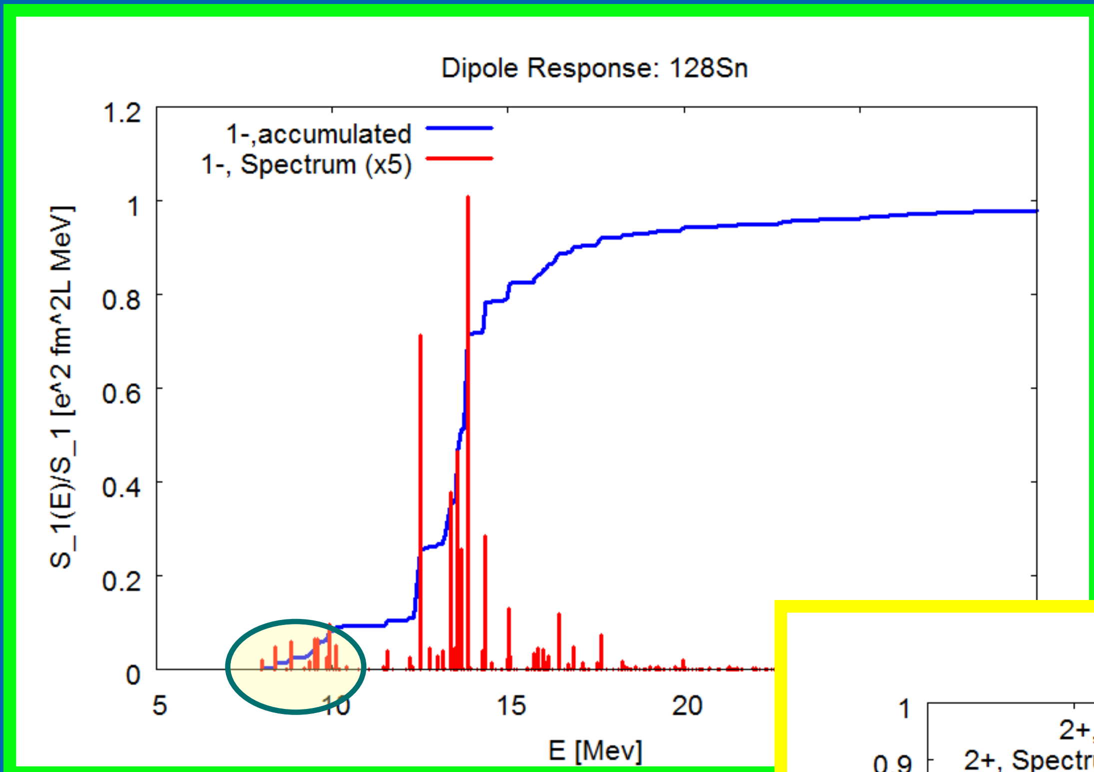
Electric Dipole Response of Exotic Nuclei

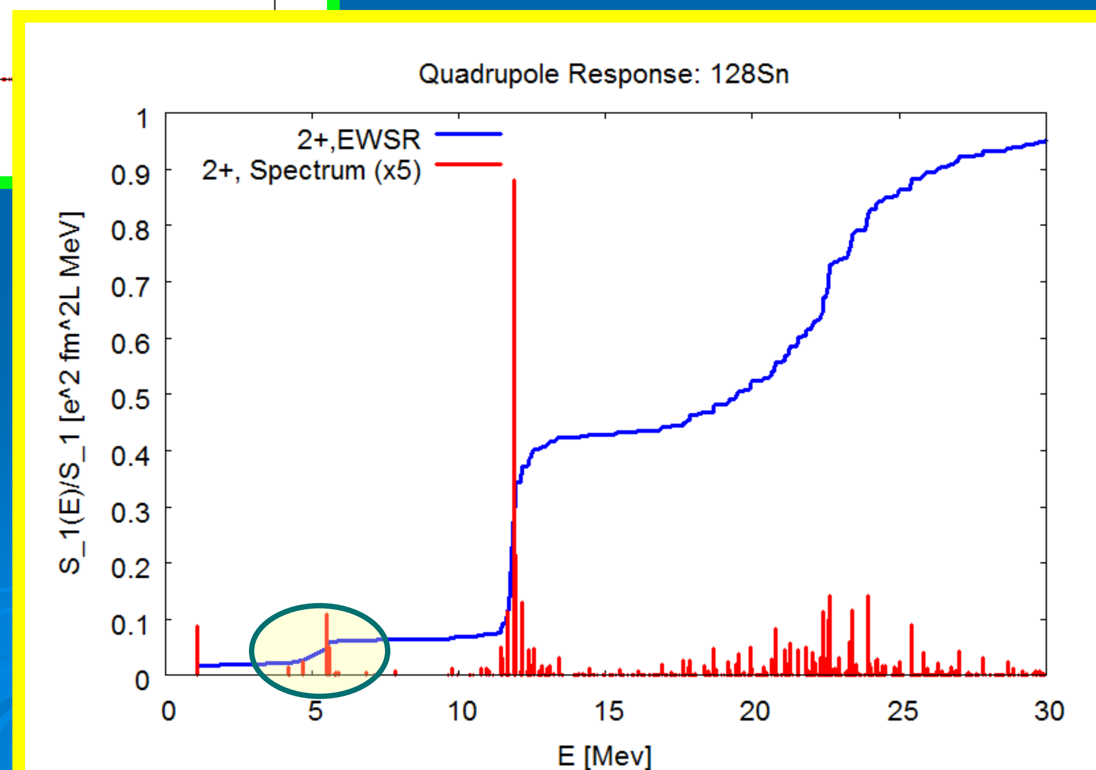


$$\vec{D} = \frac{1}{2}\sum_i \vec{\xi}_i (1 - \tau_{3i}) = -\frac{1}{2}\sum_i \vec{\xi}_i \tau_{3i} .$$

N. Tsoneva, H.L.

QRPA-Response ^{128}Sn Microscopic DD-QRPA (F_0, F'_0)

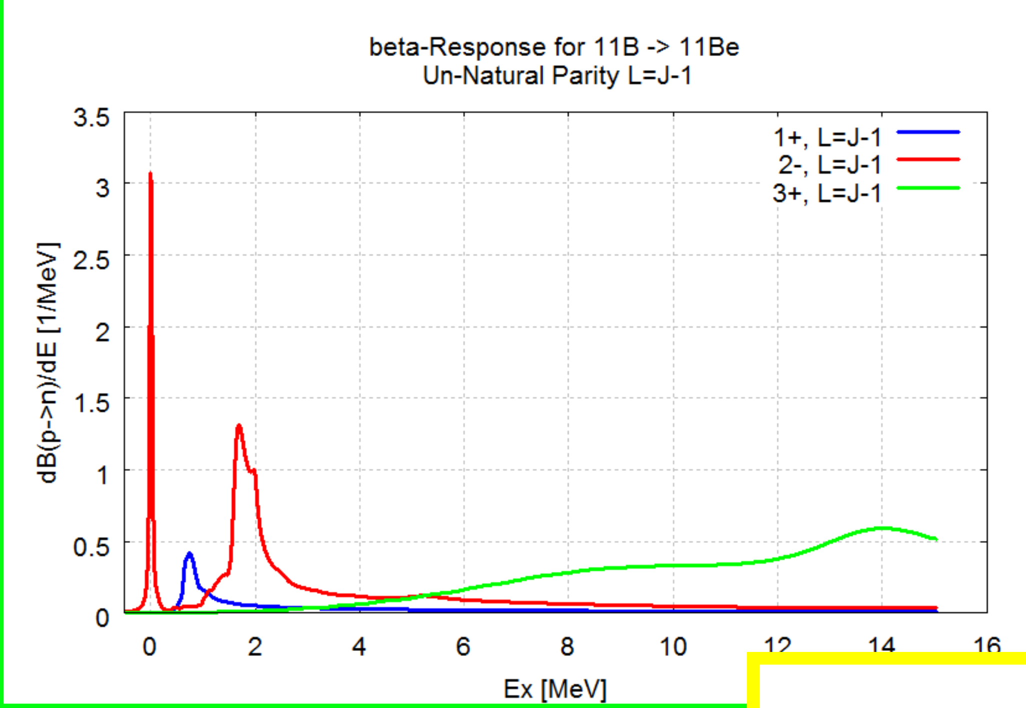




accumulated, normalized EWSR

$$R_1(E;E\lambda) = \frac{1}{S_1(E\lambda)} \sum_{E_c \le E} E_c B_c(E\lambda)$$

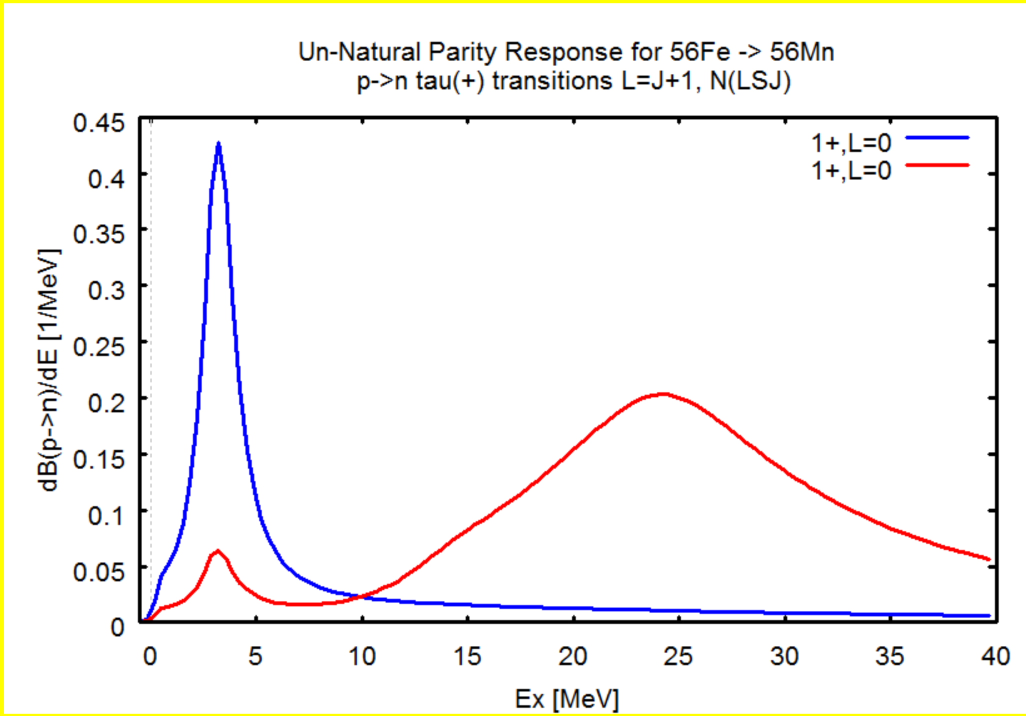
Phys. Lett. B695 (2011) 174



Response Functions for $^{11}B \rightarrow ^{11}Be$

Nucl.Phys. A 739 (2004) 30

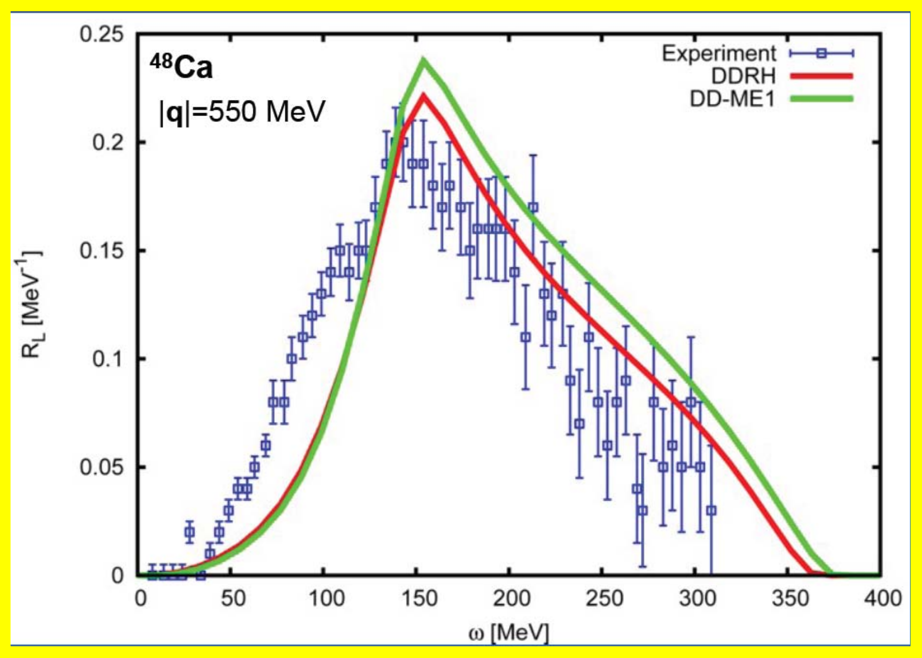
Probing G'$_0$, H'$_0$

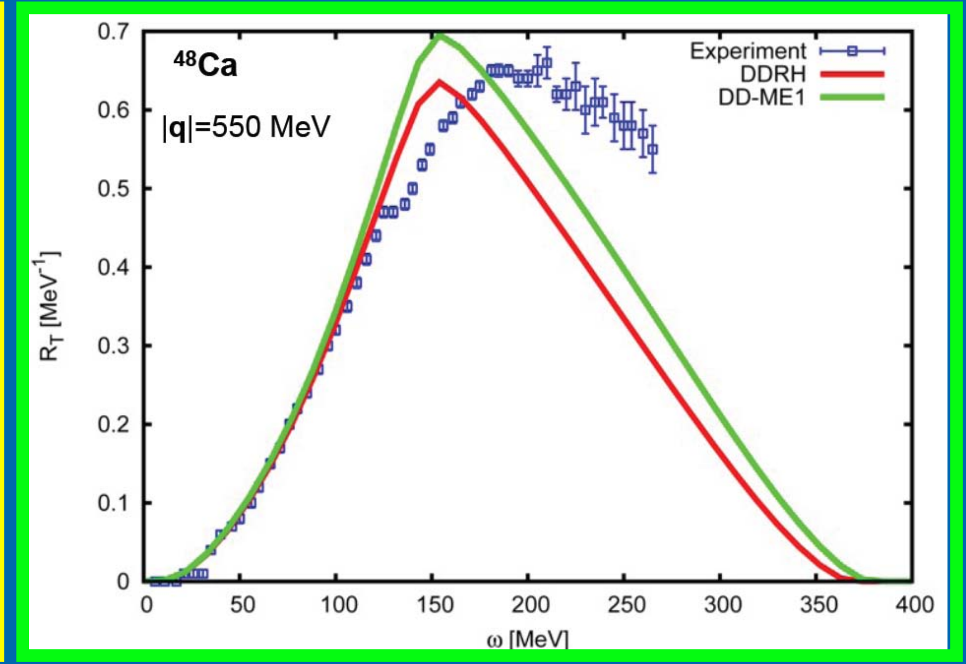


Response Functions for $^{56}Fe \rightarrow ^{56}Mn$

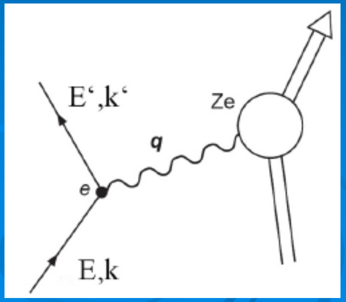
Nucl. Phys. A 744 (2004)108.
A. Ataie, H.L. 2011

(e,e') Response Functions: ^{48}Ca



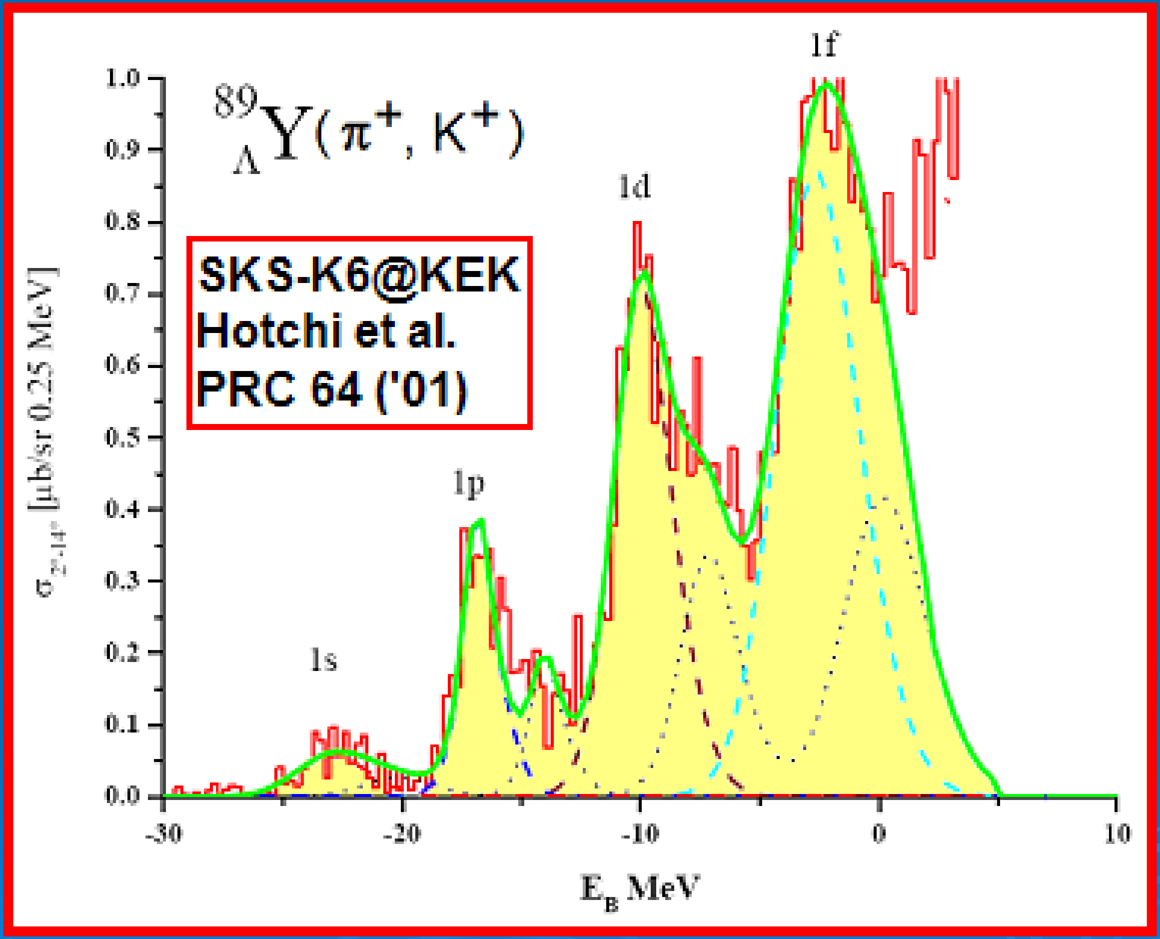


$$\frac{d^2\sigma}{d\omega d\Omega} = \sigma_{\mathrm{M}} \left[\left(\frac{Q^2}{q^2} \right) R_L(q,\omega) + \left(\frac{Q^2}{2q^2} + \tan^2\frac{\theta}{2} \right) R_T(q,\omega) \right]$$



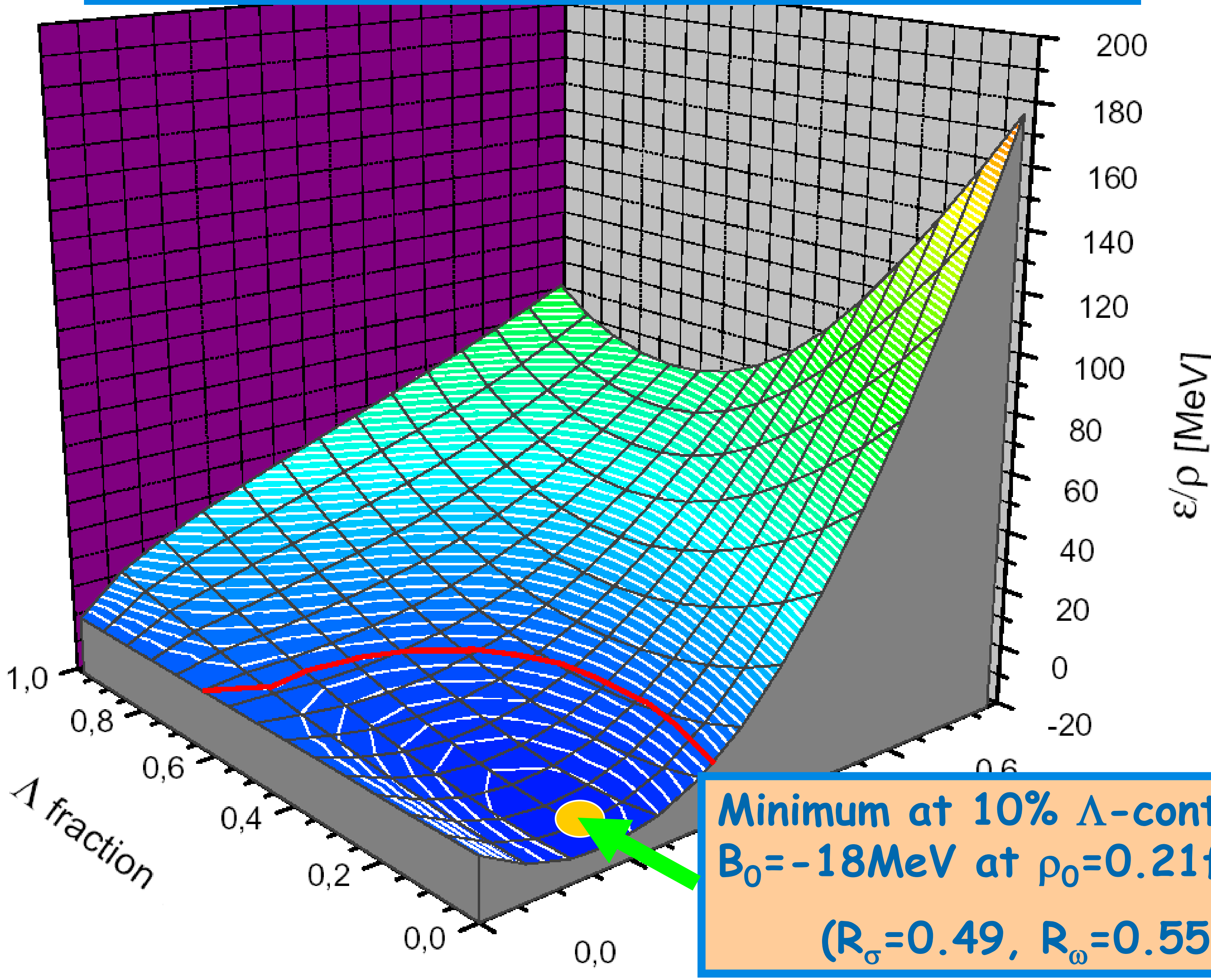
IV. Hypermatter and Neutron Stars

The BEST *experimental* Proof of Single Particle Motion in Nuclei:



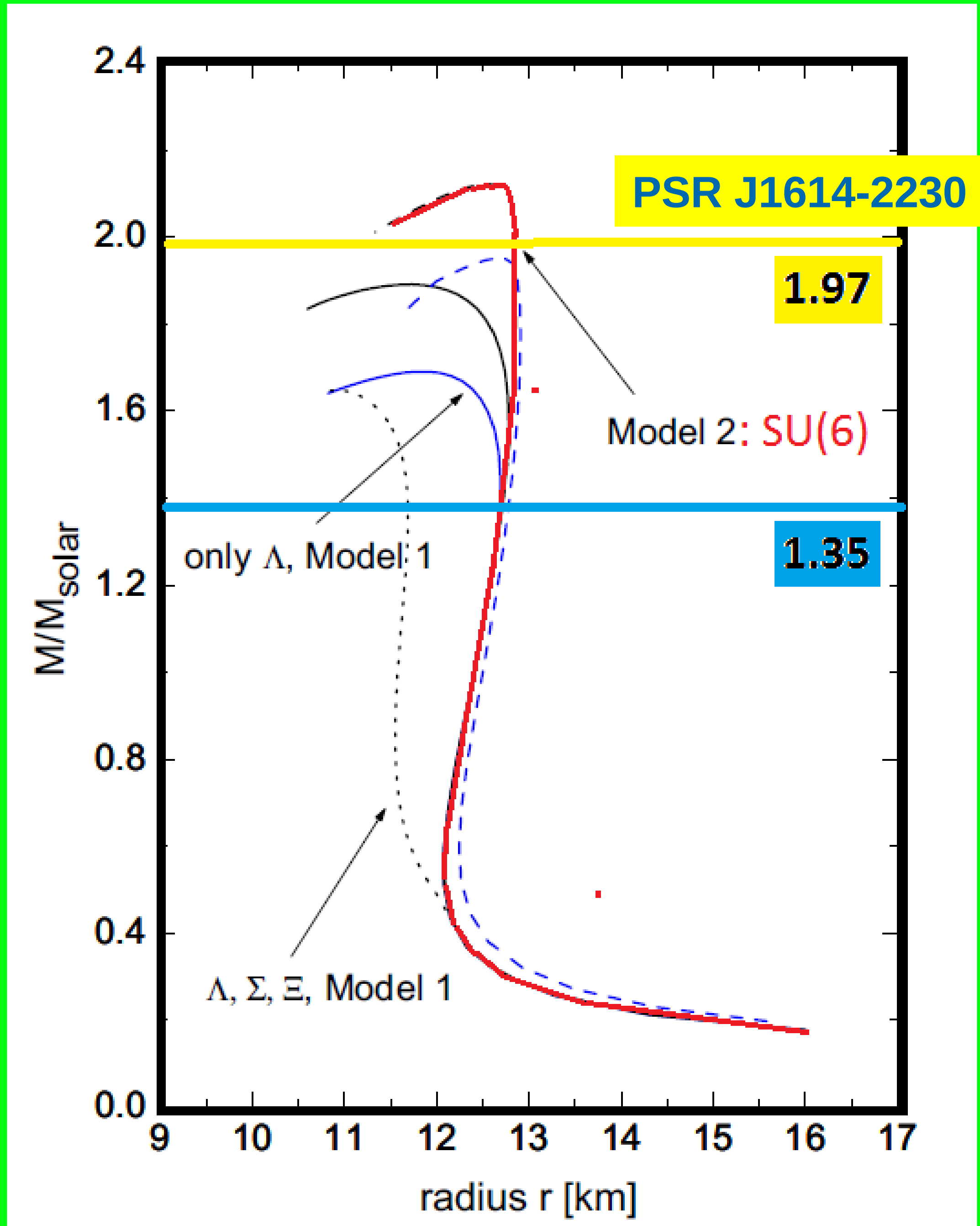
S. Bender, R. Shyam, HL, Nucl. Phys. A 839 (2010); P. Konrad, H.L.; Th. Gaitanos, U. Mosel, H.L.

DDRH Hypermatter Equation of State (Binding Energy per Baryon)

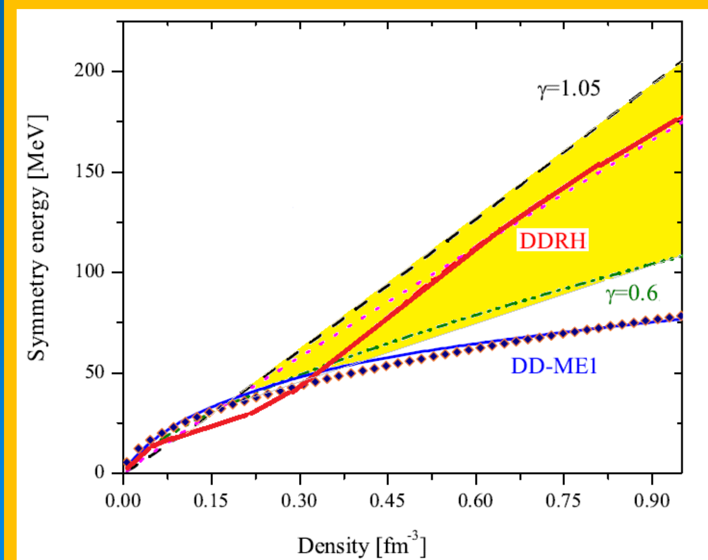


Minimum at 10% Λ-content:
B_0=-18MeV at ρ_0=0.21fm^{-3}
(R_σ=0.49, R_ω=0.55)

Neutron Star Mass-Radius Relation



- DB-Interactions
- baryon octet
- leptons: e, μ
- beta equilibrium
- TOV equation



Summary and Outlook

- Elements of DFT and Fermi-Liquid Theory
- Quasi-Particle Interaction and Landau-Migdal Parameters
- Nuclear Matter, Hypermatter, Neutron Stars
- Challenges:
  - Baryon Interactions from (L)QCD?!
  - Linking Many-body Dynamics to In-Medium Interactions
  - Flavour Dynamics in Matter

Credits to: Nadia Tsoneva, Urnaa Badarch, A. Ataie, A. Fedoseew, P. Konrad, Anika Obermann, Th. Gaitanos

Other Applications to Strong Interaction Physics:

• Dense Nuclear Matter: Landau Fermi-Liquid Theory and Chiral Lagrangian with Scaling, Phys.Rept. 347, C. Song

• Landau Theory of Relativistic Fermi Liquids, G. Baym, S.A. Chin, NPA 262 (1976)

• Symmetric and anti-symmetric Landau parameters and magnetic
properties of dense quark matter
K. Pal and A. K. Dutt-Mazumder, hep-ph:1001.1067v2

Composition of Neutron Star Matter