Response of Dense Relativistic Matter to a Magnetic Field

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Matter in magnetic field

- Relativistic matter & strong magnetic fields ($10^8 - 10^{15}$ G) are common inside compact stars
  - **Electrons** in white dwarfs

    $T \ll m \lesssim \mu$ (i.e., $T \lesssim 1$ keV & $\mu \lesssim 1$ MeV)
  - **Protons** in dense neutron matter

    $T \ll m \lesssim \mu$ (i.e., $T \lesssim 10$ MeV & $\mu \simeq 1$ GeV)
  - **Electrons** in dense neutron matter

    $m \lesssim T \ll \mu$ (i.e., $T \lesssim 10$ MeV & $\mu \lesssim 100$ MeV)
  - **Quark matter** in stellar cores (if formed)

    $T \lesssim m \ll \mu$ (i.e., $T \lesssim 10$ MeV & $\mu \lesssim 500$ MeV)

Magnetic catalysis (MC) scenario

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Catalysis of Dynamical Flavor Symmetry Breaking by a Magnetic Field in 2 + 1 Dimensions

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It is shown that in 2 + 1 dimensions, a constant magnetic field is a strong catalyst of dynamical flavor symmetry breaking, leading to generating a fermion dynamical mass even at the weakest attractive interaction between fermions. The effect is illustrated in the Nambu–Jona-Lasinio model in a magnetic field. The low-energy effective action in this model is derived, and the thermodynamic properties of the model are established.

$$E_n = \sqrt{2n|eB|} \;\Rightarrow\; E_n = \sqrt{2n|eB| + \Delta_0^2}$$

where $\Delta_0 \sim \begin{cases} \sqrt{|eB|} \\ \text{or} \\ |eB| \end{cases} \Longrightarrow \nu=0$

In relation to graphene:

Khveshchenko, Phys. Rev. Lett. 87, 206401 (2001);

Gorbar, Gusynin, V.M., Shovkovy, Phys. Rev. B 66, 045108 (2002)

Lesson from graphene

- Dynamics of Quantum Hall Effect in graphene

  (≈ 2-brane QED)

  - Parity and time-reversal odd Dirac (Chern-Simons) mass*

$$\Delta \quad \sim \quad \langle \bar{\Psi} \gamma^3 \gamma^5 \Psi \rangle$$

- Δ describes the 0^{th} plateau in Quantum Hall effect in graphene

$$iG^{-1}(u,u') \; = \; \Big[(i\partial_t + \mu)\gamma^0 - (\boldsymbol{\pi}\cdot\boldsymbol{\gamma}) - \pi^3\gamma^3 + \; i\tilde{\mu}\gamma^1\gamma^2 + \Delta\gamma^3\gamma^5 - m\Big]\delta^4(u-u')$$

*[Gorbar, Gusynin, V.M., Shovkovy, PRB 78, 085437 (2008)]

General idea

- Axial vector current in relativistic matter in a magnetic field (3+1 dimensions)

$$\langle j_5^3 \rangle_0 = \frac{-eB}{2\pi^2}\,\mu_0 \qquad \text{(free theory!)}$$

[Metlitski & Zhitnitsky, Phys Rev D **72**, 045011 (2005)]

- Is there a dynamical parameter Δ ("chiral shift") associated with this condensate?

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\Delta \quad \text{where} \quad \mathcal{L}_\Delta \simeq \Delta \bar{\psi}\gamma^3\gamma^5\psi$$

$$k^3 \to k^3 \mp s_\perp \Delta, \quad s_\perp \equiv \text{sign}(eB), \text{ for } \mathcal{P}^\mp = \frac{1}{2}(1 \mp s_\perp \gamma^5)$$

- Note: Δ=0 is not protected by any symmetry

Axial anomaly

- Does the chiral shift modify the axial anomaly relation?

- Using point splitting method, one derives

$$\langle \partial_\mu j_5^\mu(u) \rangle = -\frac{e^2 \epsilon^{\beta\mu\lambda\sigma} F_{\alpha\mu} F_{\lambda\sigma} \epsilon^\alpha \epsilon_\beta}{8\pi^2 \epsilon^2} \left(e^{-is_\perp \Delta \epsilon^3} + e^{is_\perp \Delta \epsilon^3} \right)$$

$$\rightarrow -\frac{e^2}{16\pi^2} \epsilon^{\beta\mu\lambda\sigma} F_{\beta\mu} F_{\lambda\sigma} \qquad \text{for} \quad \epsilon \rightarrow 0$$

[Gorbar, V.M., Shovkovy, Phys. Lett. B 695 (2011) 354]

- Therefore, the chiral shift does **not** affect the conventional axial anomaly relation

Axial current

- Does the chiral shift give any contribution to the axial current?

- In point splitting method, one derives

$$\langle j_5^\mu \rangle_{\mathrm{singular}} = -\frac{\Delta}{2\pi^2 \epsilon^2}\delta_3^\mu \sim \frac{\Lambda^2 \Delta}{2\pi^2}\delta_3^\mu$$

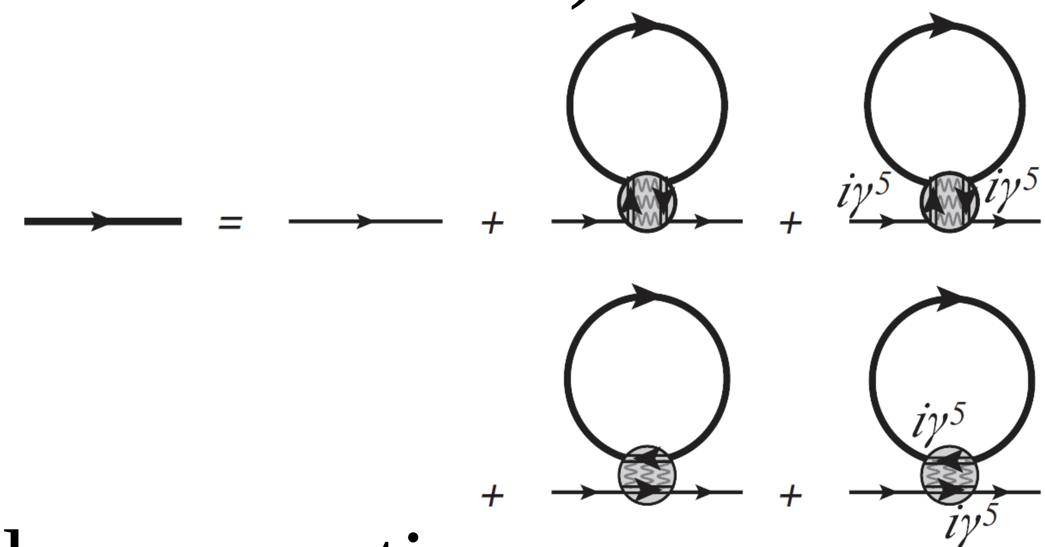
[Gorbar, V.M., Shovkovy, Phys. Lett. B 695 (2011) 354]

- This is consistent with the NJL calculations

- Since $\Delta \sim g\mu\, eB/\Lambda^2$, the axial current is finite

Chiral shift in NJL model

- NJL model (local interaction)



- This leads to three equations:

$$\mu = \mu_0 - \frac{1}{2} G_{\text{int}} \langle j^0 \rangle$$ ("effective" chemical potential)

$$m = m_0 - G_{\text{int}} \langle \bar{\psi}\psi \rangle$$ (dynamical mass)

$$\Delta = -\frac{1}{2} G_{\text{int}} \langle j_5^3 \rangle$$ (chiral shift parameter)

Solutions (T=0)

[Gorbar, V.M., Shovkovy, Phys. Rev. C 80, 032801(R) (2009)]

- Magnetic catalysis solution (vacuum state):

$$m^2 \simeq \frac{|eB|}{\pi} \exp\left(-\frac{4\pi^2}{G_{\text{int}}|eB|}\right) \qquad \left(|\mu_0| \lesssim \frac{m}{\sqrt{2}}\right)$$

$$\Delta = 0 \quad \& \quad \mu = \mu_0$$

- State with a chiral shift (nonzero density):

$$m = 0 \quad \& \quad \mu \simeq \frac{\mu_0}{1 + g/(\Lambda l)^2}$$

$$\Delta = \frac{g s_\perp \mu}{(\Lambda l)^2 + \frac{1}{2} g (\Lambda l)^2} \qquad \left(|\mu_0| \gtrsim \frac{m}{\sqrt{2}}\right)$$

Solutions (T≠0, m_0≠0)

[Gorbar, V.M., Shovkovy, Phys. Rev. D 83, 085003 (2011)]

- Magnetic catalysis solution (vacuum state):

$$m^2 \simeq \frac{|eB|}{\pi} \exp\left(-\frac{4\pi^2}{G_{\text{int}}|eB|}\right)$$

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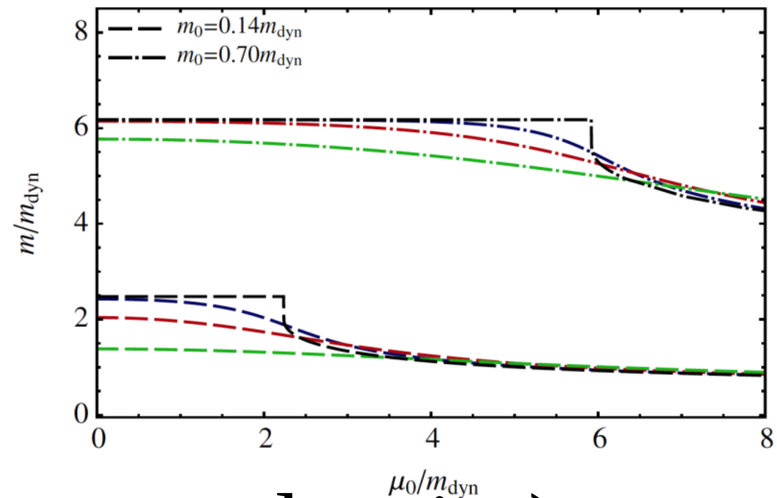

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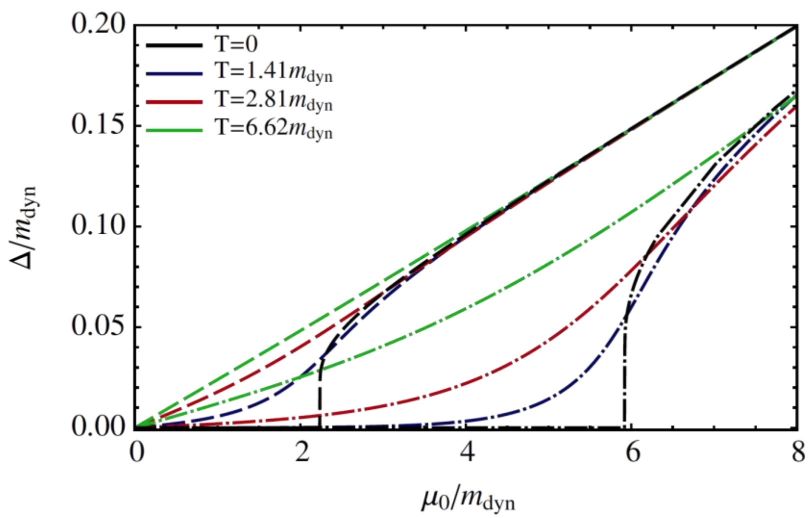


Chiral shift and Fermi surface

- Chirality is approx. well defined at Fermi surface ($|\boldsymbol{k^3}| \gg \boldsymbol{m}$)
- L-handed Fermi surface:

$$n = 0: \ k^3 = +\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

$$n > 0: \ k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

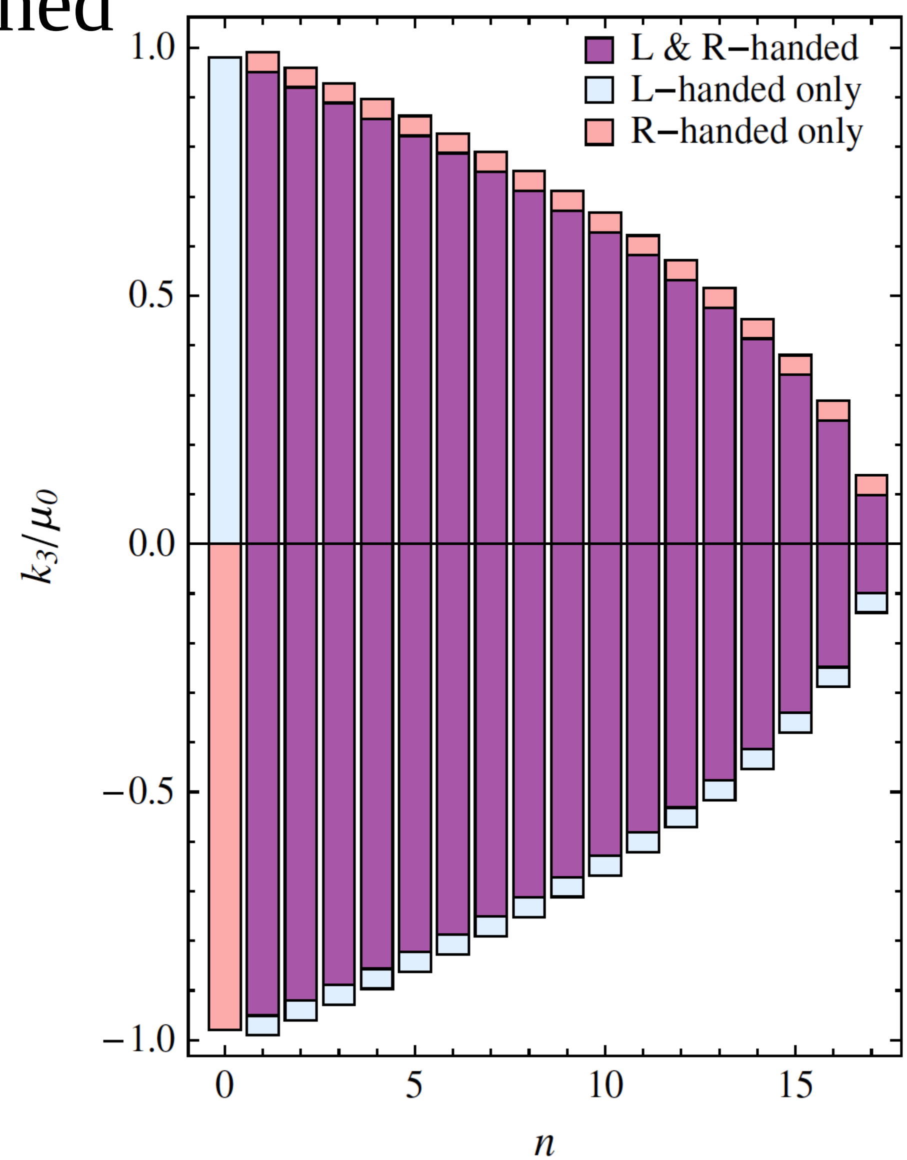
$$k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$

- R-handed Fermi surface:

$$n = 0: \ k^3 = -\sqrt{(\mu - s_\perp \Delta)^2 - m^2}$$

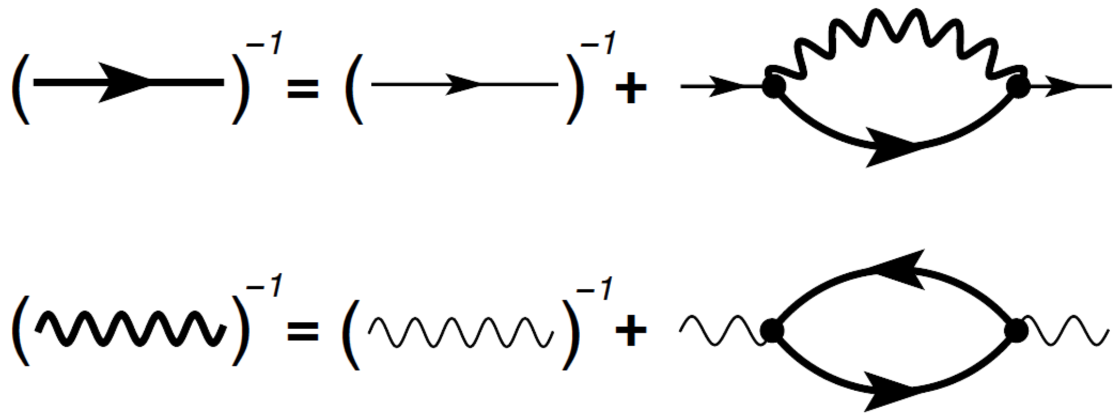
$$n > 0: \ k^3 = -\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} - s_\perp \Delta\right)^2 - m^2}$$

$$k^3 = +\sqrt{\left(\sqrt{\mu^2 - 2n|eB|} + s_\perp \Delta\right)^2 - m^2}$$



Preliminary results in QED

- Chiral shift is also generated in QED



- Screening effects are important
- Preliminary estimate for the chiral shift:

$$\Delta^{QED} \sim -\frac{\alpha}{\pi}\frac{eB}{M_D^2}\mu, \quad \text{where} \quad M_D^2 \sim \frac{\alpha\mu^2}{\pi}$$

Summary

- New dynamical parameter (chiral shift) is generated in magnetized dense matter
- Chiral shift induces a chiral asymmetry at the Fermi surface
- Potential applications:
  - Pulsar kicks (?)
    - Quark stars
  - Facilitation of supernova explosions (?)
  - Axial current in QGP
    - modified CME (chiral magnetic effect)