Nucleon structure from 2+1f dynamical DWF lattice QCD at nearly physical pion mass

Shigemi Ohta *†‡ for RBC and UKQCD Collaborations

Talk at "From quarks and gluons to hadrons and nuclei," Erice, September 22, 2011

RBC and UKQCD collaborations have been generating dynamical Domain-Wall Fermions (DWF) ensembles:

- good chiral and flavor symmetries,

that allowed us do a lot of good pion and kaon physics as well as nucleon.

We are now much closer to physical pion mass with large volume, than the previous sets of ensembles:

- light, $m_\pi \sim 170$ and 250 MeV, quarks ($m_{ud}a = 0.001$ and 0.0042, and $m_{\rm res}a \sim 0.002$),
- a large, $(4.6\text{fm})^3$, volume ($a^{-1} \sim 1.371(8)$ GeV),

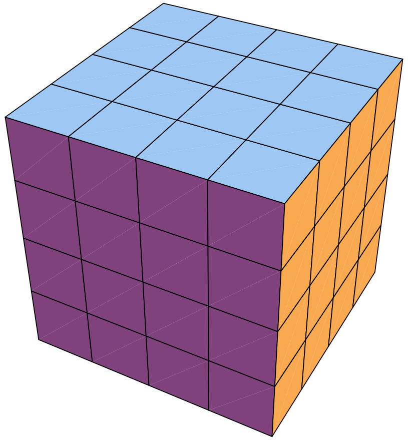
made possible by Iwasaki + dislocation suppressing determinant ratio (DSDR) gauge action.

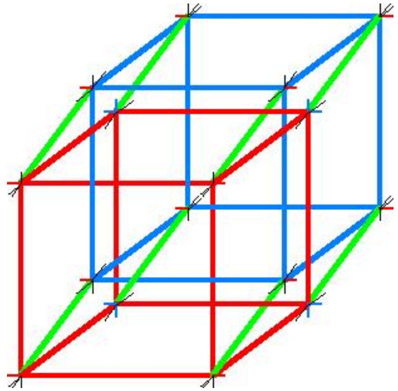
Here we report the current status of our nucleon calculations, by

- Meifeng Lin, Yasumichi Aoki, Tom Blum, Chris Dawson, Taku Izubuchi, Chulwoo Jung, SO, Shoichi Sasaki, Takeshi Yamazaki, ...

*Institute of Particle and Nuclear Studies, High-Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan
†Department of Particle and Nuclear Physics, Sokendai Graduate University of Advanced Studies, Hayama, Kanagawa 240-0193, Japan
‡RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA

Lattice: 4D simple hyper-cubic lattice, $L_0L_1L_2L_3$, Euclidean





site: $s = (n_0n_1n_2n_3)$, $0 \le n_i \le L_i - 1$ $(i = 0, 1, 2, 3)$.

link: $l = (s, \mu)$, $\mu \in \{0, 1, 2, 3\}$, connects s and $s + \hat{\mu}$.

constant separation (lattice constant) a between neighboring sites.

Taking $a \to 0$ through asymptotic scaling gives exact continuum physics.

Dynamical variables:

quark: $q(s)$, defined on site and forms basis of fundamental (3) representation of SU(3),

gluon: $U(s, \mu) = \exp(ig \int_s^{s+\hat{\mu}} A_\mu(y) \mathrm{d}y_\mu) \in \mathrm{SU}(3)$, now a group element defined on link.

There are many other ways to define lattice (eg. random lattice) with different advantages, but the way q, U and G are defined is basically the same.

Gauge transformation: $G(s) \in \mathrm{SU}(3)$, defined on site, maps quarks and gluons

$$q(s) \mapsto G(s)q(s) \text{ and } U(s,\mu) \mapsto G(s)U(s,\mu)G(s+\hat{\mu})^{-1}.$$

Gauge invariant objects (QCD action, observables):

- Quark: $\bar{\psi}(x)U(x,\mu)U(x+\hat{\mu},\nu)...U(y-\hat{\rho},\rho)\psi(y), \mapsto \bar{\psi}(x)\underline{G^{-1}(x)G(x)}U(x,\mu)\underline{G^{-1}(x+\hat{\mu})G(x+\hat{\mu})}U(x+\hat{\mu},\nu)...U(y-\hat{\rho},\rho)\underline{G^{-1}(y)G(y)}\psi(y)$.

- Gluon, $\mathrm{Tr}[U(x,\mu)U(x+\hat{\mu},\nu)...U(x-\hat{\rho},\rho)] \mapsto \mathrm{Tr}[\underline{G(x)}U(x,\mu)\underline{G^{-1}(x+\hat{\mu})G(x+\hat{\mu})}U(x+\hat{\mu},\nu)...U(x-\hat{\rho},\rho)\underline{G^{-1}(x)}]$.

Action: $S_{\mathrm{QCD}}[U,q,\bar{q}] = S_{\mathrm{gluon}}[U] + S_{\mathrm{quark}}[U,q,\bar{q}]$, must respect gauge invariance:

gluon part: such as $S_{\mathrm{gluon}}[U] = \frac{6}{g^2}\sum_s \sum_{\mu<\nu} \Box(s,\mu,\nu)$, gives $-\frac{1}{2}\mathrm{Tr}G^{\mu\nu}G_{\mu\nu}$ as $a \to 0$ and $g \to 0$,

- where the plaquette, $\Box(s,\mu,\nu) = 1 - \frac{1}{3}\mathrm{ReTr}U(s,\mu)U(s+\hat{\mu},\nu)U(s+\hat{\nu},\mu)^{-1}U(s,\nu)^{-1}$.

quark part: $S_{\mathrm{quark}}[U,q,\bar{q}] = \sum_{s,s'} \bar{q}(s)M[U](s,s')q(s')$, which should give $\bar{q}(i\gamma^\mu D_\mu - m)q$ as $a \to 0$ and $g \to 0$,

- with $M[U](s,s')$ describing quark propagation between sites s and s'.

Expectation values of any gauge-invariant observable: $\langle O \rangle = N^{-1} \int [\mathrm{d}U][\mathrm{d}q][\mathrm{d}\bar{q}] O[U, q, \bar{q}] \exp(-S_{\mathrm{QCD}}[U, q, \bar{q}])$,

or by integrating over the quark Grassmann variables: $N'^{-1} \int [\mathrm{d}U] (\det M[U]) \exp(-S_{\mathrm{gluon}}[U])$.
It is often convenient to use effective action: $\tilde{S}[U] = S_{\mathrm{gluon}}[U] - \mathrm{Tr} \log M[U]$.

Finite lattice and compact SU(3) assures finite $\langle O \rangle$.

Continuum limit is well defined because of the asymptotic freedom: consider an observable O with mass dimension,

- the expectation value is described as $\langle O \rangle = a^{-1} f(g)$ with some dimensionless function $f(g)$ of dimensionless coupling g.
- Renormalizability of the theory means the cutoff dependence should vanish $\frac{d\langle O \rangle}{da} \to 0$ as $a \to 0$, or

$$f(g) - f'(g) \left(a \frac{dg}{da} \right) = \beta(g) f'(g) + f(g) \to 0.$$

- This ($df/f = -dg/\beta$) is easily solved to give: $\langle O \rangle a \propto \exp\left(- \int^g \frac{dh}{\beta(h)} \right)$, or

$$\langle O \rangle a \propto (g^2 b_0)^{-\frac{b_1}{2b_0^2}} e^{-\frac{1}{2b_0 g^2}} [1 + O(g^2)],$$

where $\beta(g) \equiv -a \frac{dg}{da} = -b_0 g^3 - b_1 g^5 + O(g^7)$ is perturbatively well known.

Chiral symmetry:

- Invariance under global $U(N_f)$ transformations, $q \mapsto \exp(i\theta)q$, $\exp(i\theta'\gamma_5)q$, $\exp(i\alpha^a\frac{\lambda^a}{2})q$ and $\exp(i\beta^a\frac{\lambda^a}{2}\gamma_5)q$.

- Should be preserved in the absence of $m\bar{q}q$, like $U(N_f)_L \times U(N_f)_R = SU(N_f)_V \times SU(N_f)_A \times U(1)_V \times U(1)_A$.

- In fact spontaneously broken for light normal quarks, $m_u \sim m_d \sim 0$, $\langle \bar{u}u + \bar{d}d \rangle \neq 0$.
- Important for Nambu-Goldstone pion, PCAC, etc, $m_\pi^2 f_\pi^2 = m_q \langle \bar{q}q \rangle$.

However, difficult to maintain on regular lattices.

Naive lattice fermion action, with $M_{xy} = \frac{1}{2} a^{D-1} \sum_\mu \gamma_\mu [\delta_{x+\hat{\mu},y} - \delta_{x-\hat{\mu},y}]$, leads to a propagator $\Delta(p) = a[\gamma_\mu \sin(p_\mu a)]^{-1}$, which has 2^D poles at $p_\mu = 0$ or π/a: for $D = 4$, there are $2^4 = 16$ flavors/tastes instead of one.

Shifting of one component of p_μ, such as $\tilde{p}_\mu = p_\mu - \pi/a$, acts like

$$\gamma_\mu \sin(p_\mu a) = -\gamma_\mu \sin(\tilde{p}_\mu a)$$

so the chirality $\pm$ states are paired.

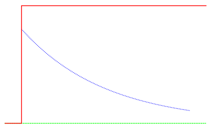
Nielsen and Ninomiya theorem: doubling inevitable (chirality $\pm$ states are paired) for a regular lattice and local, hermitian, and translationally invariant action.

Domain-wall fermions[1]: introduce a 5-th dimension, s, and define a 5D Dirac operator: $D = \gamma_\mu \partial_\mu + \gamma_5 \partial_s + m(s)$,

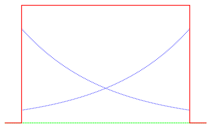
- With a monotonic $m(s)$ with $m(s=0)=0$, a 4D chiral modes emerge: $\psi_\pm(x,s) = u_p(x)\phi_\pm(s)\chi_\pm$.
- 4D Dirac plane wave u_p and γ_5 eigenstate, $\gamma_5\chi_\pm = \pm\chi_\pm$, indicate the s-dependence,

$$[\pm\partial_s + m(s)]\phi(s) = 0, \quad \text{or} \quad \phi(s) \propto \exp[\mp \int_0^s ds' m(s')],$$

pinned at the $s=0$ wall, and exponentially decay to $\pm s$ direction.



- On a finite lattice, two walls, with a pair of $\pm$ chiralities mix.



- No problem for a vector theory like QCD[2]: mixing exponentially suppressed, described by $m_{\rm res}$.

RIKEN-BNL-Columbia (RBC) Collaboration proved DWF works very well for QCD:

- light hadron mass spectrum,
- electroweak transitions among light hadrons (such as f_π, f_K, B_K and ϵ'/ϵ),

unlike conventional Wilson and staggered fermions.

[1] D.B. Kaplan, Phys. Lett. B288, 342 (1992), hep=lat/9206013.

[2] Y. Shamir, Nucl. Phys. B406, 90 (1993), hep-lat/9303005; V. Furman and Y. Shamir, Nucl. Phys. B439, 54 (1995), hep-lat/9405004; and references cited therein.

QCDSP and QCDOC computers: dedicated for lattice QCD calculations.

QCDSP: completed in 1998, 600 (RBRC) and 400 (Columbia) GFlops configurations

- based on commercial DSP
- assisted by custom designed 4D hypercubic nearest-neighbor communication
- 10$ per MFlops

Demonstrated the use of DWF in (quenched) lattice QCD

- Chiral and flavor symmetries and associated ease in non-perturbative renormalizations,
- hadron spectroscopy: masses and decay constants,
- hadron matrix elements: B_K, ϵ'/ϵ, K_{l3}, nucleon form factors and structure functions.

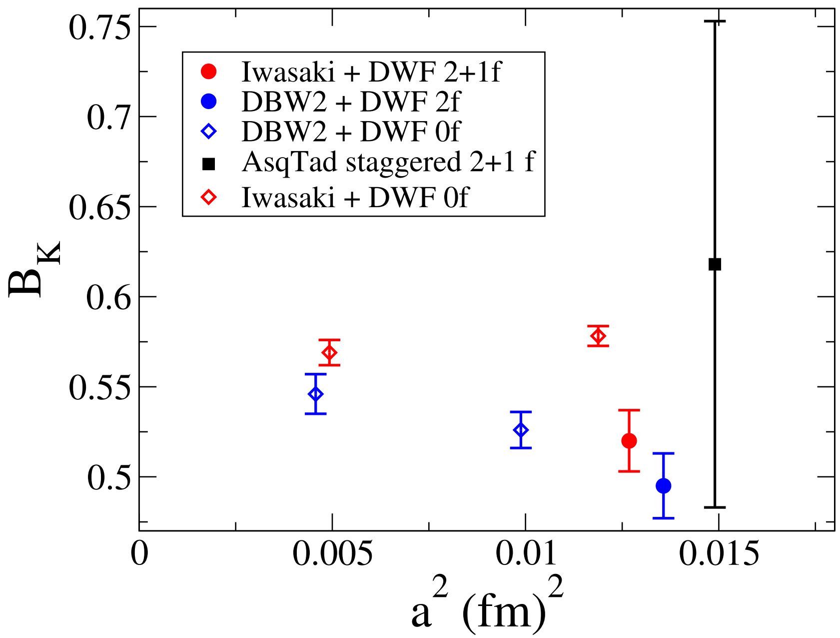
QCDOC: complete in 2005, 10 TFLops configurations in RBRC, BNL and Edinburgh.

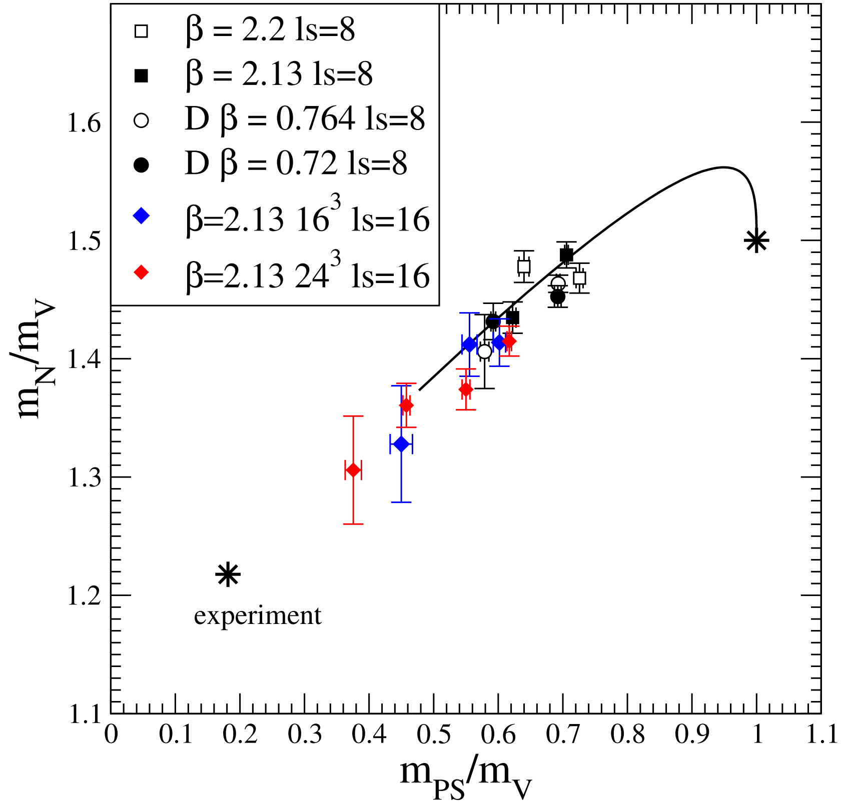
- based on system on a chip technology,
- a QCDSP card was shrunk to be a QCDOC chip, with custom-designed 6D hypercubic communicaitons,
- 1$ per MFlops.

Used for realistic (2+1)-flavor dynamical DWF lattice QCD.

Evolved into BG/L, P and Q $\sim$ QCDCQ.

RBC/UKQCD $N_f = 2+1$ dynamical DWF ensembles: $a^{-1} = 1.73(2)$ and $2.28(3)$ GeV with volumes larger than 2.7 fm across,





Chiral and continuum limit with good flavor and chiral symmetries:

- $f_\pi = 122(2)(5)$ MeV, $f_K/f_\pi = 1.21(3)$; $m_s^{\overline{\rm MS}(2{\rm GeV})} = 97(3)$ MeV, $m_{ud}^{\overline{\rm MS}(2{\rm GeV})} = 3.6(2)$ MeV,
- Very accurate constraints on CKM matrix: $B_K^{\overline{\rm MS}}(2{\rm GeV}) = 0.524(10)(28)$, K_{l3} $f_+(0) = 0.964(5)$, ...
- Chiral perturbation useless from our previous mass range, $m_\pi \sim 300$ MeV: e.g. NLO $\sim 0.5\times$LO.

Nucleon form factors, measured in elastic scatterings or β decay or muon capture:

$$\langle p|V_\mu^+(x)|n\rangle = \bar{u}_p\left[\gamma_\mu F_V(q^2) + \frac{\sigma_{\mu\lambda}q_\lambda}{2m_N}F_T(q^2)\right]u_n e^{iq\cdot x},$$

$$\langle p|A_\mu^+(x)|n\rangle = \bar{u}_p\left[\gamma_\mu\gamma_5 F_A(q^2) + iq_\mu\gamma_5 F_P(q^2)\right]u_n e^{iq\cdot x}.$$

$$F_V = F_1, F_T = F_2; G_E(q^2) = F_1 - \frac{q^2}{4m_N^2}F_2, G_M = F_1 + F_2.$$

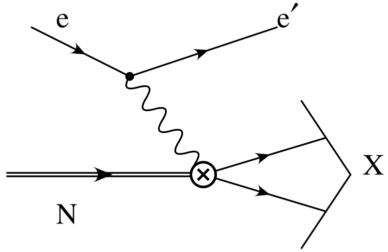
Related to mean-squared charge radius, magnetic moment, $g_V = F_V(0) = G_{\rm Fermi}\cos\theta_{\rm Cabibbo}$, $g_A = F_A(0) = 1.2701(25)g_V$, Goldberger-Treiman relation, $m_N g_A \propto f_\pi g_{\pi NN}$, ... determine much of nuclear physics.

On the lattice, with appropriate nucleon operator, for example, $N = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$, ratio of two- and three-point correlators such as $\frac{C_{\rm 3pt}^{\Gamma,O}(t_{\rm sink},t)}{C_{\rm 2pt}(t_{\rm sink})}$ with

$$C_{\rm 2pt}(t_{\rm sink}) = \sum_{\alpha,\beta}\left(\frac{1+\gamma_t}{2}\right)_{\alpha\beta}\langle N_\beta(t_{\rm sink})\bar{N}_\alpha(0)\rangle,$$

$$C_{\rm 3pt}^{\Gamma,O}(t_{\rm sink},t) = \sum_{\alpha,\beta}\Gamma_{\alpha\beta}\langle N_\beta(t_{\rm sink})O(t)\bar{N}_\alpha(0)\rangle,$$

give a plateau in t for a lattice bare value $\langle O\rangle$ for the relevant observable, with appropriate spin ($\Gamma = (1+\gamma_t)/2$ or $(1+\gamma_t)i\gamma_5\gamma_k/2$) or momentum-transfer (if any) projections.

Deep inelastic scatterings 

: $\left|\frac{\mathcal{A}}{4\pi}\right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu}$, $W^{\mu\nu} = W^{[\mu\nu]} + W^{\{\mu\nu\}}$

- unpolarized: $W^{\{\mu\nu\}}(x,Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x,Q^2) + \left(P^\mu - \frac{\nu}{q^2}q^\mu\right)\left(P^\nu - \frac{\nu}{q^2}q^\nu\right)\frac{F_2(x,Q^2)}{\nu}$,
- polarized: $W^{[\mu\nu]}(x,Q^2) = i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{S_\sigma}{\nu}(g_1(x,Q^2)+g_2(x,Q^2)) - \frac{q\cdot S P_\sigma}{\nu^2} g_2(x,Q^2)\right)$,

with $\nu = q\cdot P$, $S^2 = -M^2$, $x = Q^2/2\nu$.

Moments of the structure functions are accessible on the lattice:

$$
\begin{aligned}
2\int_0^1 dx x^{n-1} F_1(x,Q^2) &= \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2),\\
\int_0^1 dx x^{n-2} F_2(x,Q^2) &= \sum_{f=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2),\\
2\int_0^1 dx x^{n} g_1(x,Q^2) &= \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu) + \mathcal{O}(1/Q^2),\\
2\int_0^1 dx x^{n} g_2(x,Q^2) &= \frac{1}{2}\frac{n}{n+1}\sum_{q=u,d} [e_{2,n}^{q}(\mu^2/Q^2, g(\mu))\, d_n^q(\mu) - 2e_{1,n}^{q}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu)] + \mathcal{O}(1/Q^2)
\end{aligned}
$$

- c_1, c_2, e_1, and e_2 are the Wilson coefficients (perturbative),
- $\langle x^n\rangle_q(\mu)$, $\langle x^n\rangle_{\Delta q}(\mu)$ and $d_n(\mu)$ are forward nucleon matrix elements of certain local operators,
- so is $\langle 1\rangle_{\delta q}(\mu) = \langle P,S|\bar\psi i\gamma_5\sigma_{\mu\nu}\psi|P,S\rangle$ which may be measured by polarized Drell-Yan and RHIC Spin.

Unpolarized (F_1/F_2): on the lattice we can measure: $\langle x\rangle_q$, $\langle x^2\rangle_q$ and $\langle x^3\rangle_q$.

$$\frac{1}{2}\sum_s \langle P,S|\mathcal{O}^q_{\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = 2\langle x^{n-1}\rangle_q(\mu)[P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{trace})]$$

$$\mathcal{O}^q_{\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{trace})\right]q$$

Polarized (g_1/g_2): on the lattice we can measure: $\langle 1\rangle_{\Delta q}$ (g_A), $\langle x\rangle_{\Delta q}$, $\langle x^2\rangle_{\Delta q}$, d_1, d_2, $\langle 1\rangle_{\delta q}$ and $\langle x\rangle_{\delta q}$.

$$-\langle P,S|\mathcal{O}^{5q}_{\{\sigma\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{n+1}\langle x^n\rangle_{\Delta q}(\mu)[S_\sigma P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{traces})]$$

$$\mathcal{O}^{5q}_{\sigma\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_\sigma\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})\right]q$$

$$\langle P,S|\mathcal{O}^{[5]q}_{[\sigma\{\mu_1]\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{1}{n+1}d^q_n(\mu)[(S_\sigma P_{\mu_1}-S_{\mu_1}P_\sigma)P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{traces})]$$

$$\mathcal{O}^{[5]q}_{[\sigma\mu_1]\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_{[\sigma}\overleftrightarrow{D}_{\mu_1]}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})\right]q$$

and transversity (h_1):

$$\langle P,S|\mathcal{O}^{\sigma q}_{\rho\nu\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{m_N}\langle x^n\rangle_{\delta q}[(S_\rho P_\nu - S_\nu P_\rho)P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{traces})]$$

$$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \bar{q}[\left(\frac{i}{2}\right)^{n}\gamma_5\sigma_{\rho\nu}\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})]q$$

Higher moment operators mix with lower dimensional ones: Only $\langle x\rangle_q$, $\langle 1\rangle_{\Delta q}$, $\langle x\rangle_{\Delta q}$, d_1, and $\langle 1\rangle_{\delta q}$ can be measured with $\vec{P}=0$.

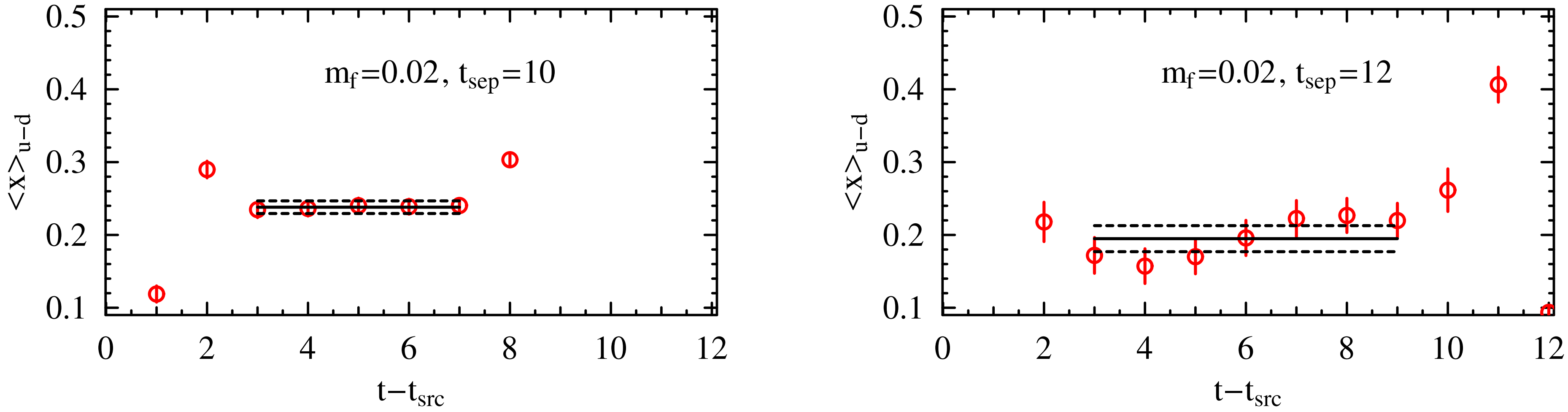
Previous RBC and RBC+UKQCD calculations addressed two important sources of systematics:

- Time separation between nucleon source and sink,
- Spatial volume.

And though not explicitly addressed yet, a better understanding of quark mass dependence is necessary.

Source/sink time separation:

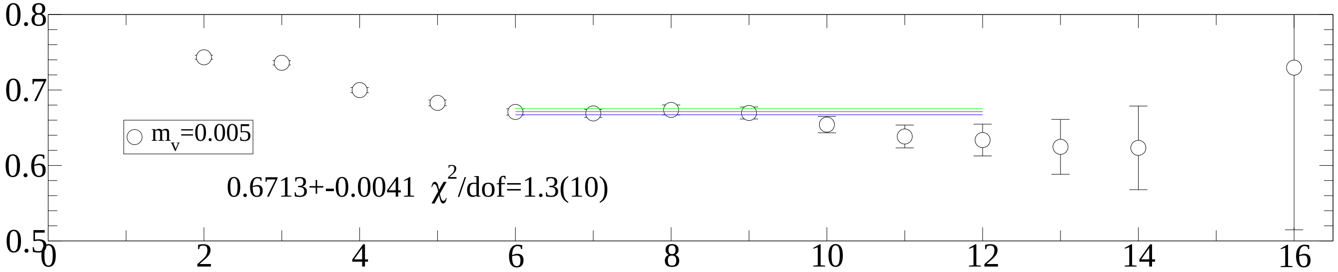
- If too short, too much contamination from excited states, but if too long, the signal is lost.



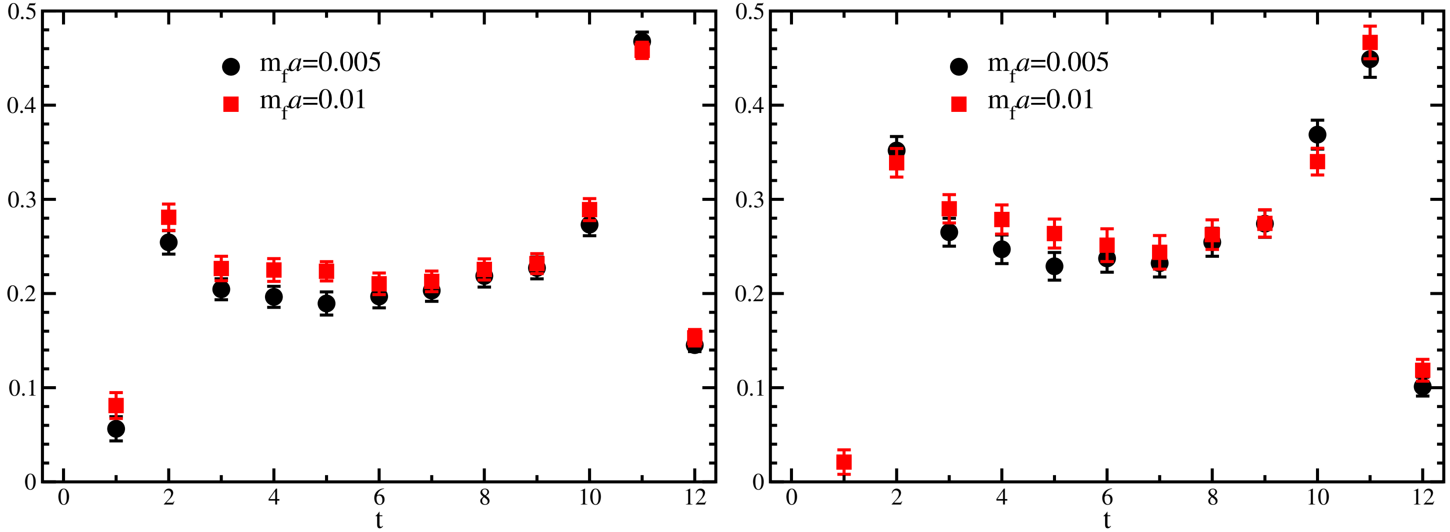
- In an earlier RBC 2-flavor DWF study at $a^{-1} \sim 1.7$ GeV, separation of 10 or 1.1 fm appeared too short.

In the previous (2+1)-flavor study we choose separation 12 or 13 , ∼1.4 fm:

Mass signal ($m_f = 0.005$):

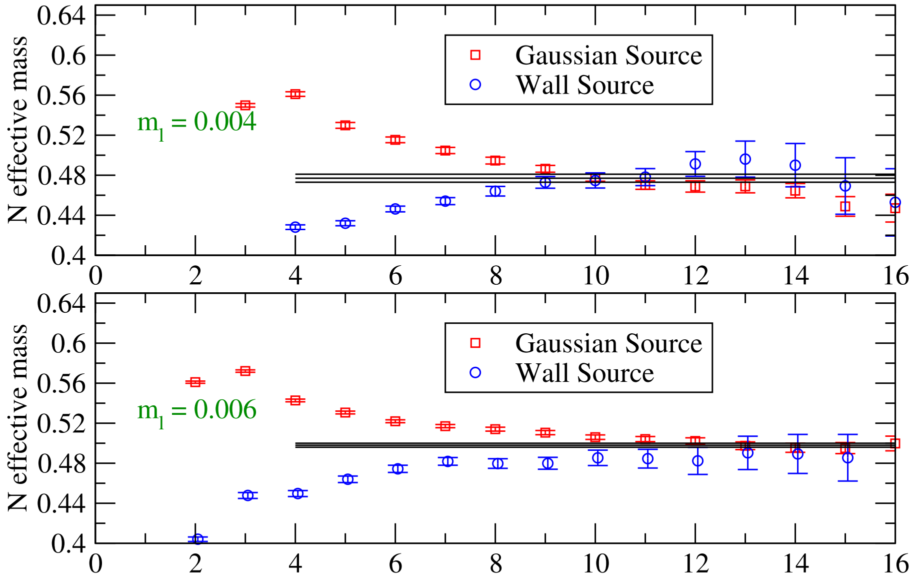


Bare three-point functions: $\langle x \rangle_{u-d}$ (left) and $\langle x \rangle_{\Delta u - \Delta d}$ (right), for $m_f = 0.005$ (red +) and 0.01 (blue ×):



In the present study we like to do at least as good, hopefully better: separation of 9 lattice units or longer.

On the other hand, with RBC+UKQCD 2.2-GeV (2+1)-flavor dynamical DWF ensemble:



2-state fits suggest excited-state survives $t_{\text{sink}} \geq 9$.

LHP analysis of vector form factors with $t_{\text{sep}} = 12$ or 1 fm agree with RBC+UKQCD 1.7-GeV results.

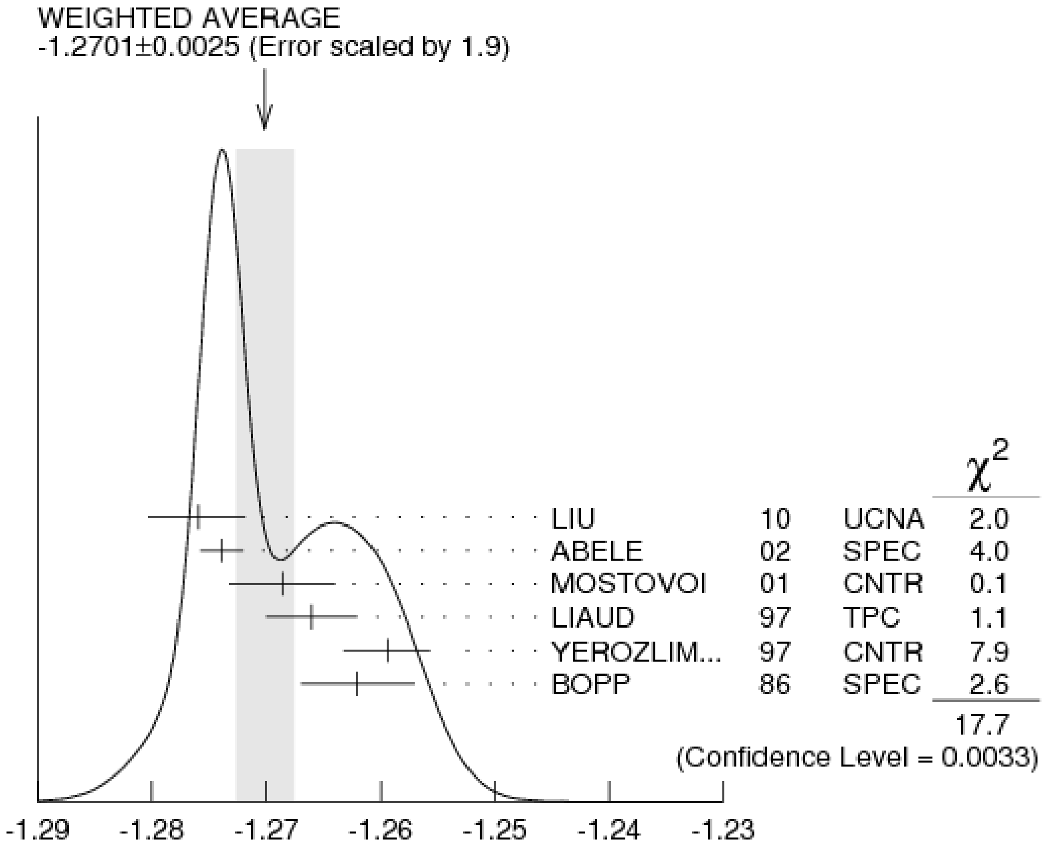
Vector current is less sensitive: conserved charge cannot tell excited-state contamination, for example.

Can we go shorter, ∼1 fm, separation, in spite of our lighter masses?

- Perhaps with better tuned source and sink smearing?
- Would be good as we have to fight growing error, $\sim \exp(-3m_\pi t)$.

LHP now seem to agree with us that their choice was too short.

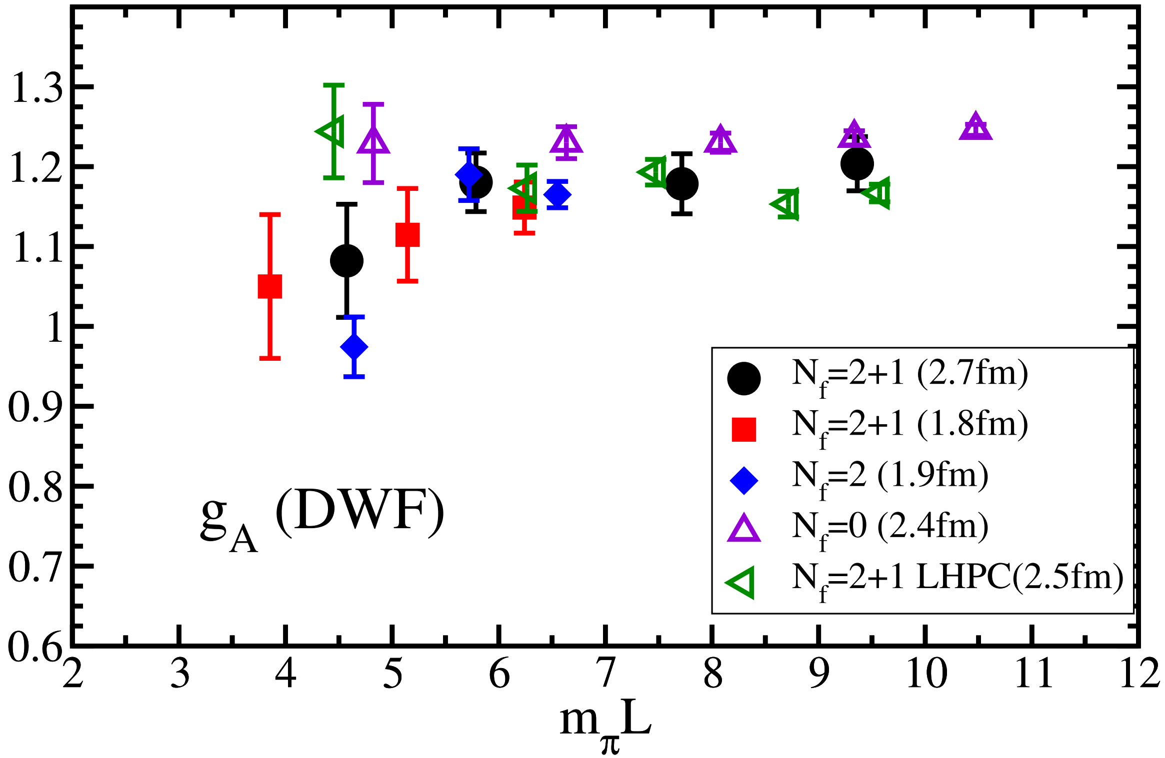
Spatial volume: let's look at nucleon isovector axial charge, g_A/g_V=1.2701(25),



Experimental value has been almost monotonically increasing since Maurice Goldhaber's first measurement.

Lattice calculations appeared to follow the same path, but, Spatial volume. In Lattice 2007 Takeshi Yamazaki reported unexpectedly large finite-size effect:

- in axial charge, $g_A/g_V = 1.2701(25)$, measured in neutron β decay, decides neutron life.



Our DWF on quenched and LHPC DWF on MILC calculations are presented for comparison.

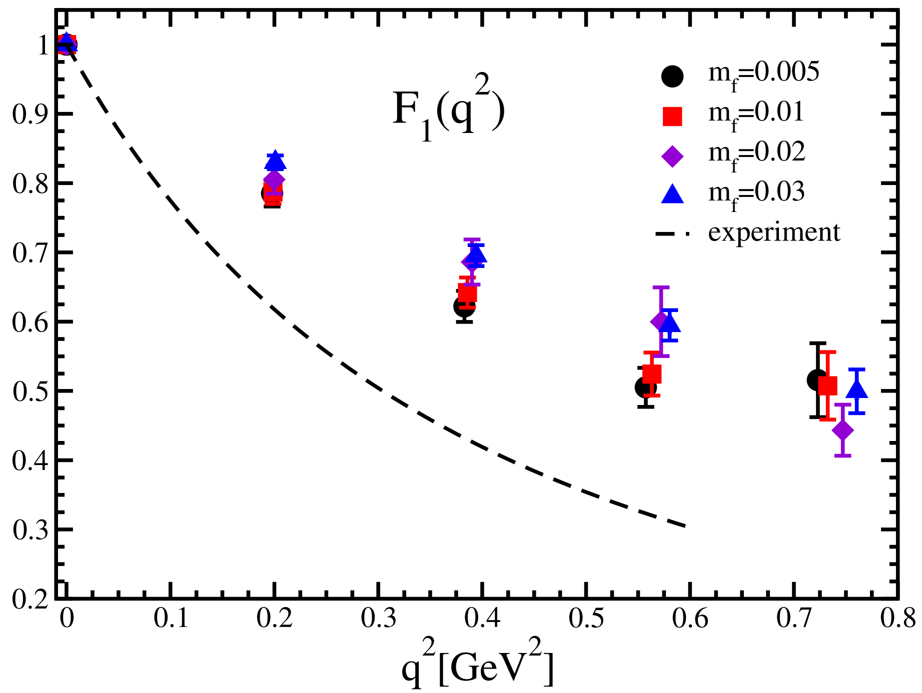
- Heavier quarks: almost consistent with experiment, no discernible quark-mass dependence.
- Lighter quarks: finite-size sets in as early as $m_\pi L \sim 5$, appear to scale in $m_\pi L$:
- If confirmed, first concrete evidence of pion cloud surrounding nucleons.

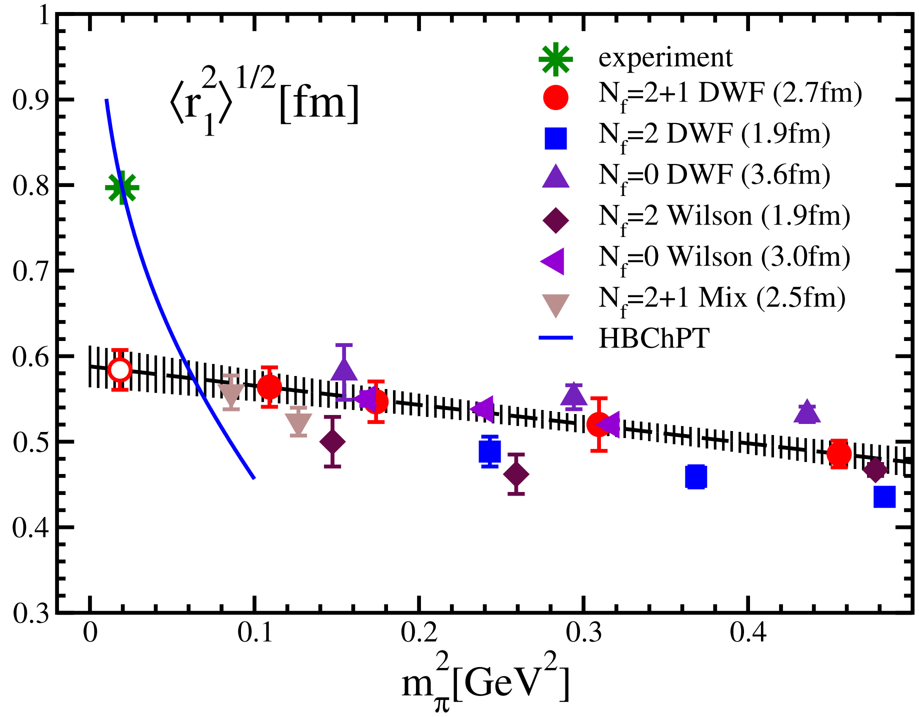
Structure function moments do not seem to suffer so badly, but we need large volume at least for form factors, such important quantities as g_A or $g_{\pi NN}$: present $(\sim 4.6\text{fm})^3$ volume is a good start.

RBC/UKQCD(2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Dirac form factor of the isovector vector current,



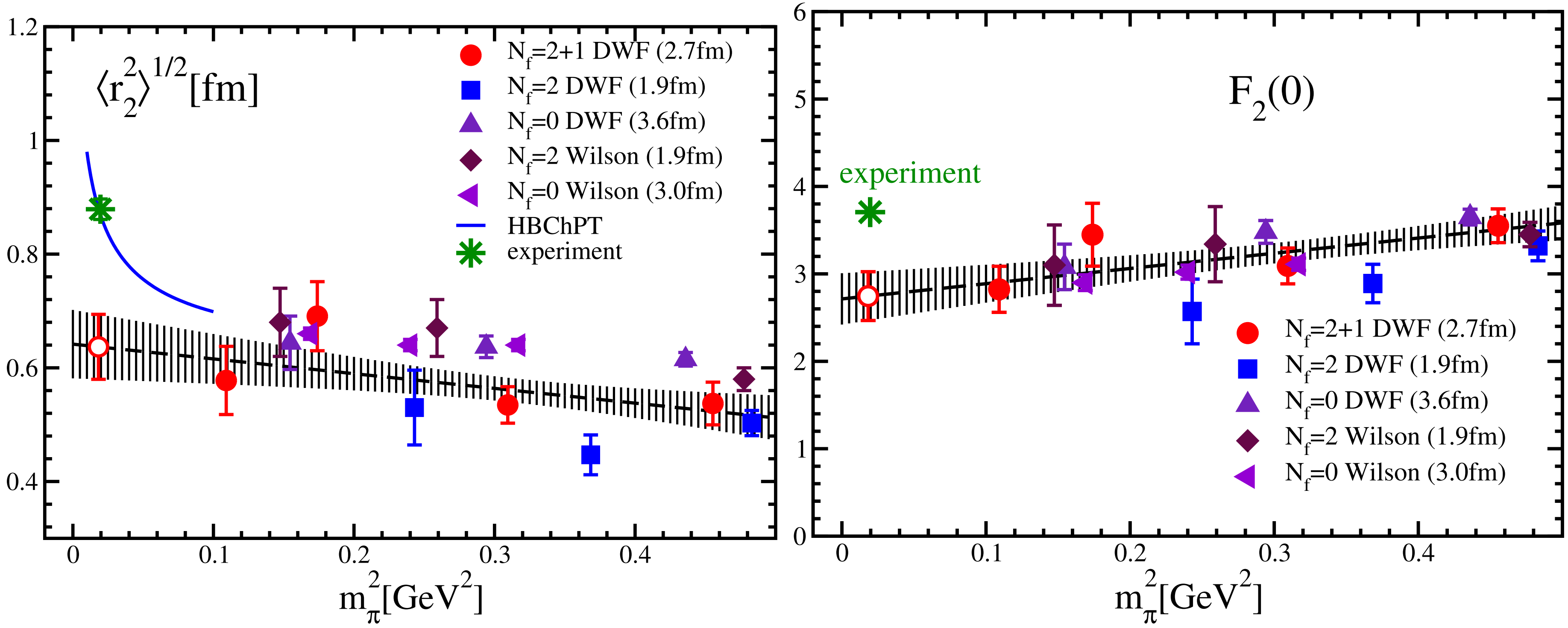


much too small rms radius,
no sign for logarithmic divergence anticipated from HBχPT.

RBC/UKQCD(2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Dirac form factor of the isovector vector current,

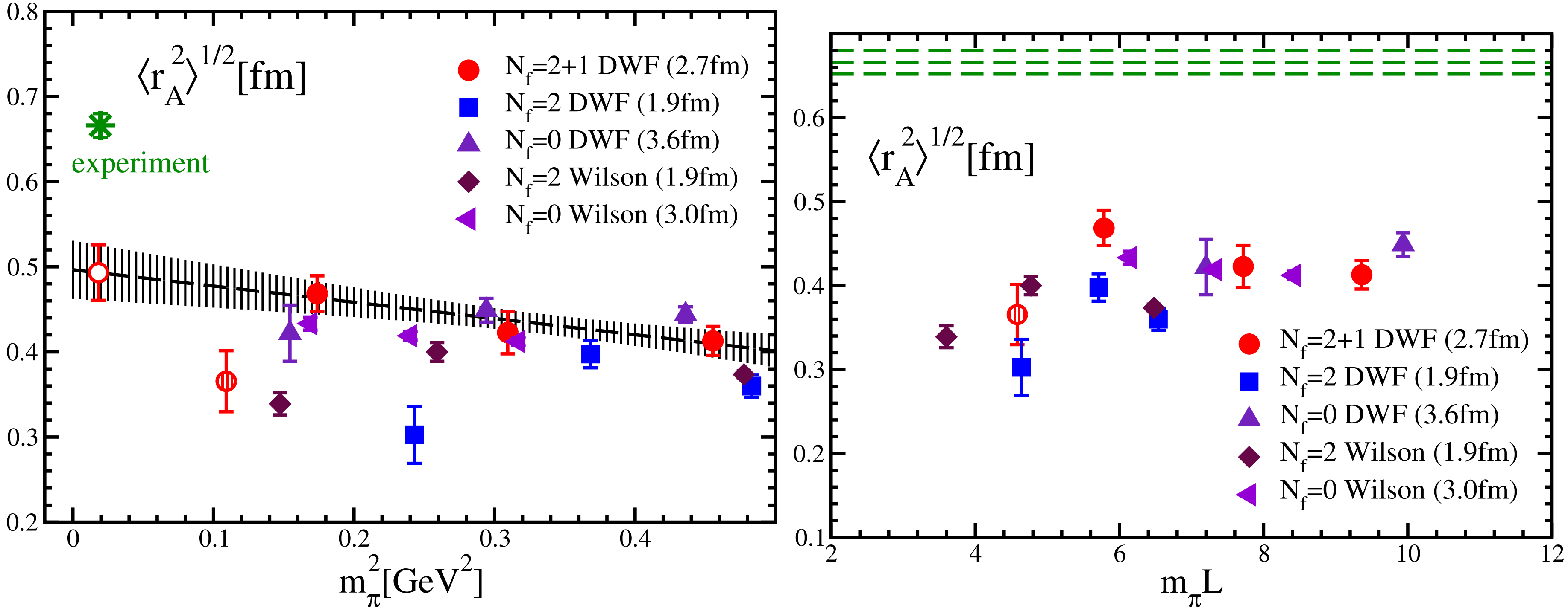


much too small rms radius,
no sign for logarithmic divergence anticipated from HBχPT,
perhaps better agreement with experiment for magnetic moment.

RBC/UKQCD(2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Isovector axialvector form factor from the axial-vector current,

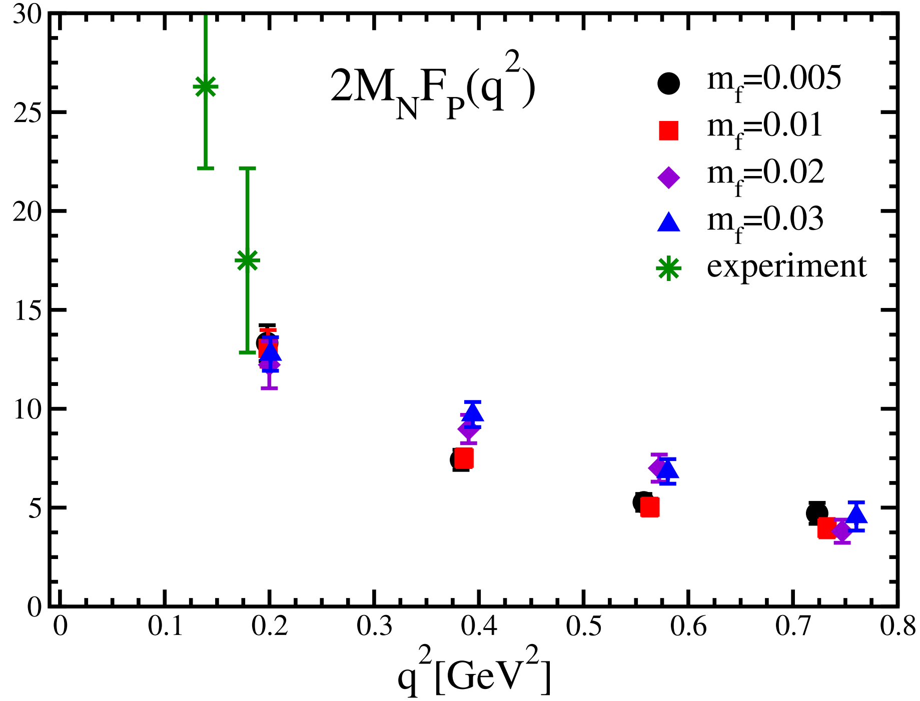


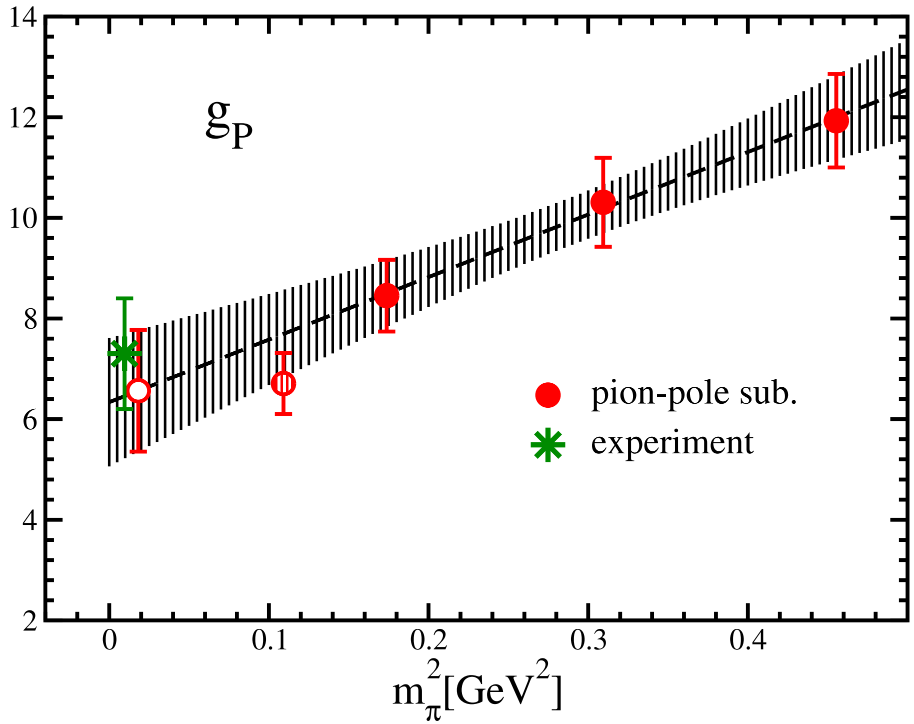
much too small rms radius,
similar dependence on $m_\pi L$ as g_A/g_V.

RBC/UKQCD(2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Isovector pseudo scalar form factor from the axial-vector current,



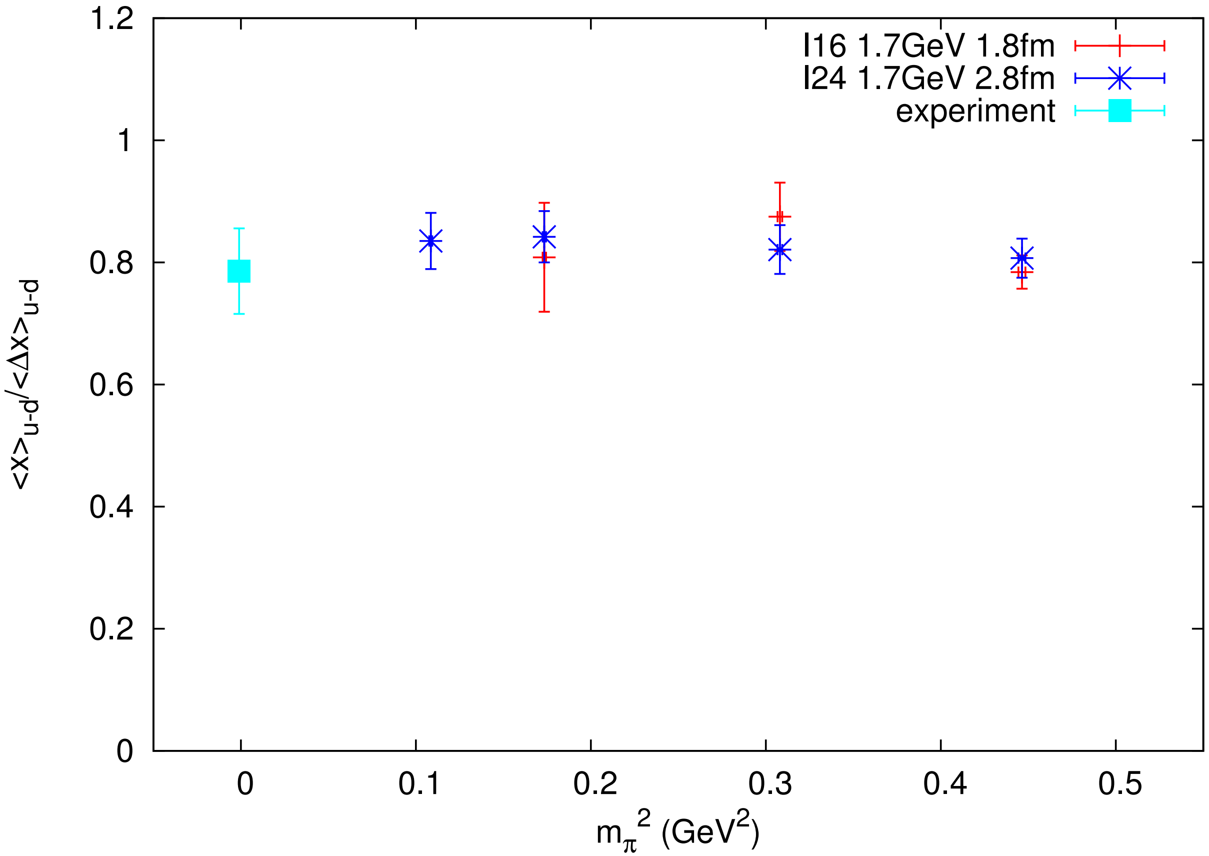


perhaps better agreement with experiments.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Ratio, $\langle x \rangle_{u-d}/\langle x \rangle_{\Delta u - \Delta d}$, of momentum and helicity fractions (naturally renormalized on the lattice),



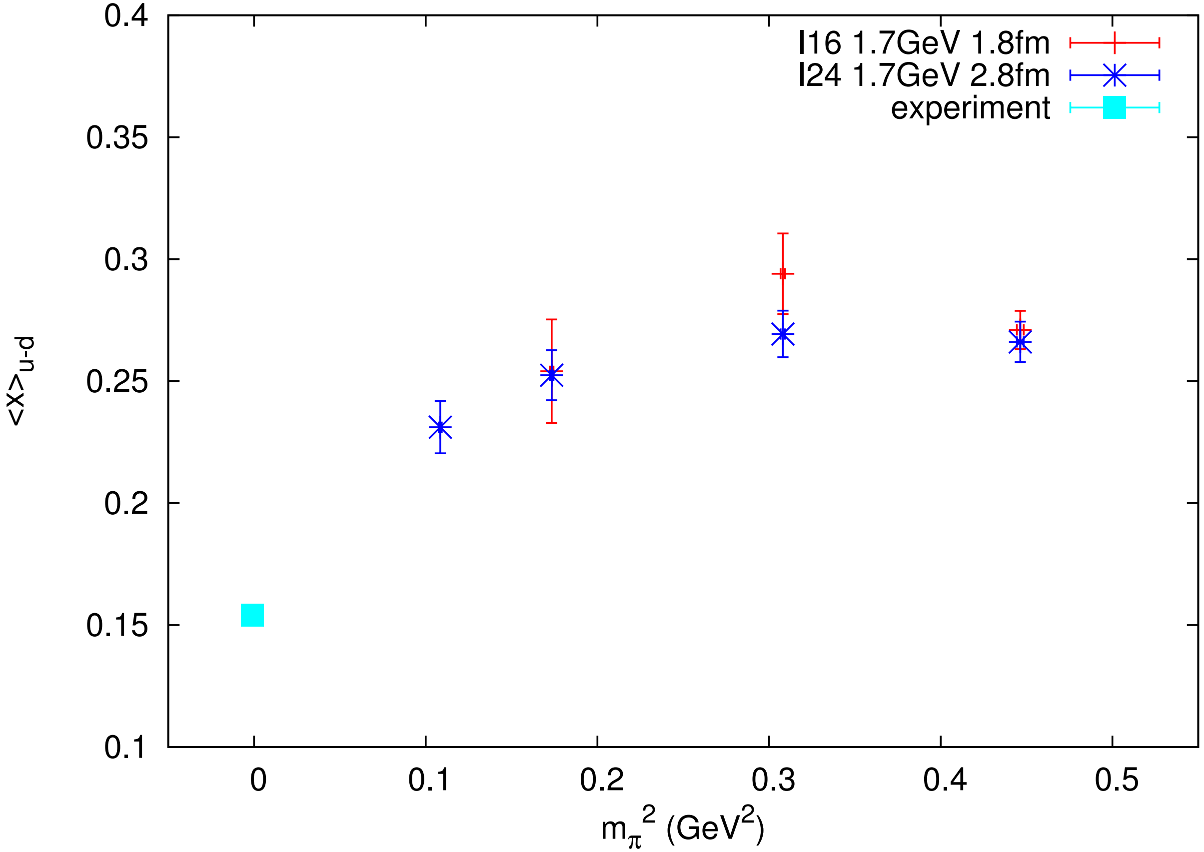
consistent with experiment, no discernible quark-mass dependence.

No finite-size effect seen, in contrast to g_A/g_V which is also naturally renormalized on the lattice.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67, 0.56, 0.42$ and 0.33 GeV; $m_N = 1.55, 1.39, 1.22$ and 1.15 GeV,

Momentum fraction, $\langle x \rangle_{u-d}$, with NPR, $Z^{\overline{\rm MS}}(2{\rm GeV}) = 1.15(4)$, plotted against m_π^2,



Absolute values have improved, trending to the experimental values, with NPR, $Z^{\overline{\rm MS}}(2{\rm GeV}) = 1.15(4)$.

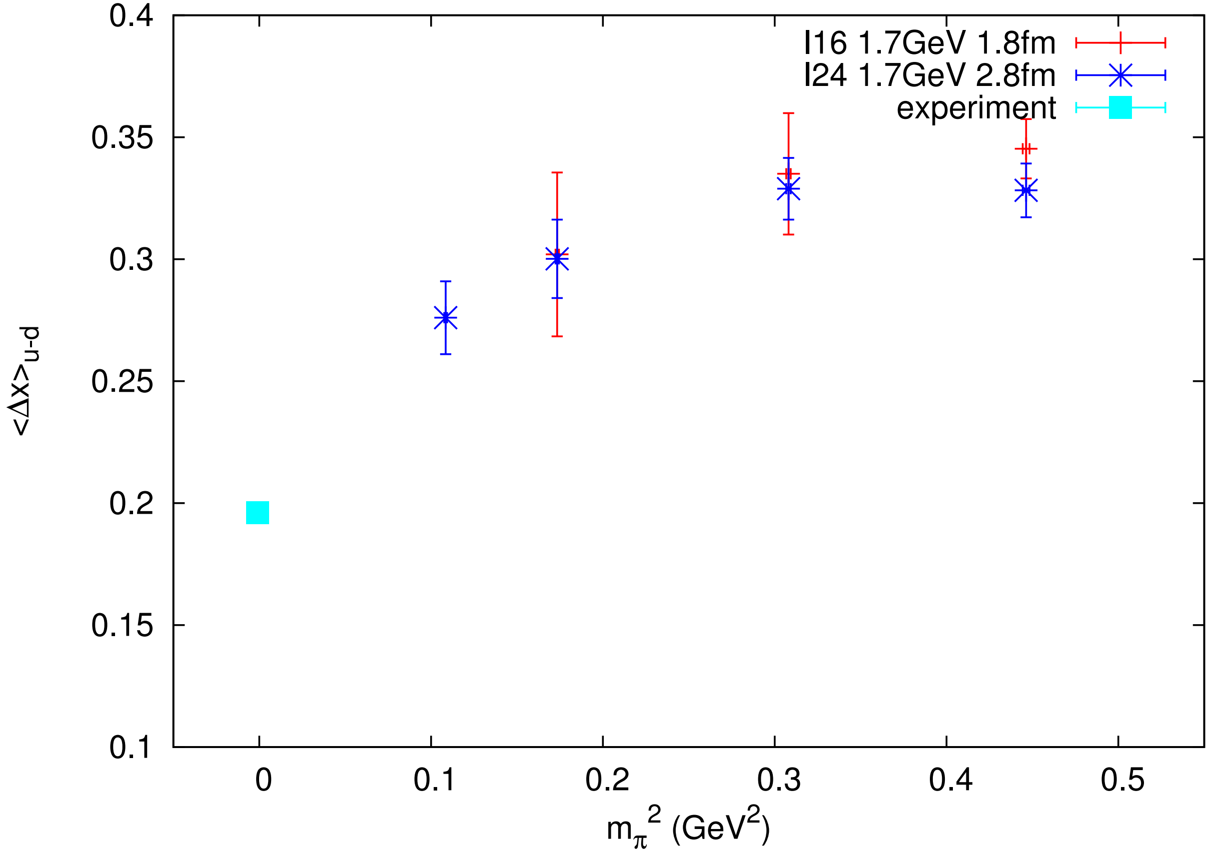
No finite size effect seen (16^3 (+) and 24^3 (×) results agree): Likely physical light-quark effect.

A better understanding of quark mass dependence is necessary.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.73(2)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Helicity fraction, $\langle x \rangle_{\Delta u - \Delta d}$, with NPR, $Z^{\overline{\rm MS}(2{\rm GeV})} = 1.15(3)$, plotted against m_π^2,



Absolute values have improved, trending to the experimental values, with NPR, $Z^{\overline{\rm MS}}(2{\rm GeV}) = 1.15(3)$.

No finite size effect seen (16^3 (+) and 24^3 ($\times$) results agree): Likely physical light-quark effect.

A better understanding of quark mass dependence is necessary.

(2+1)-flavor dynamical lattice-QCD calculations of nucleon structure so far ($m_\pi \sim$300 MeV) give

- much too small radii for vector-current form factors,
- while axial-current form factors seem to overflow,
- but structure function moments may be starting to behave.

Now RBC and UKQCD are jointly generating new DWF ensembles using FNAL ALCF, a BG/P facility:

- with Iwasaki and dislocation-suppressing-determinant-ratio (DSDR) gauge action, $\beta = 1.75$,
- and DWF fermion action, $L_s = 32$ and $M_5 = 1.8$, with $m_{\rm strange} = 0.045$, $m_{\rm ud} = 0.0042$ and 0.001,

We have reasonable topology distribution while maintaining small residual mass, $m_{\rm res}a \sim 0.002$:

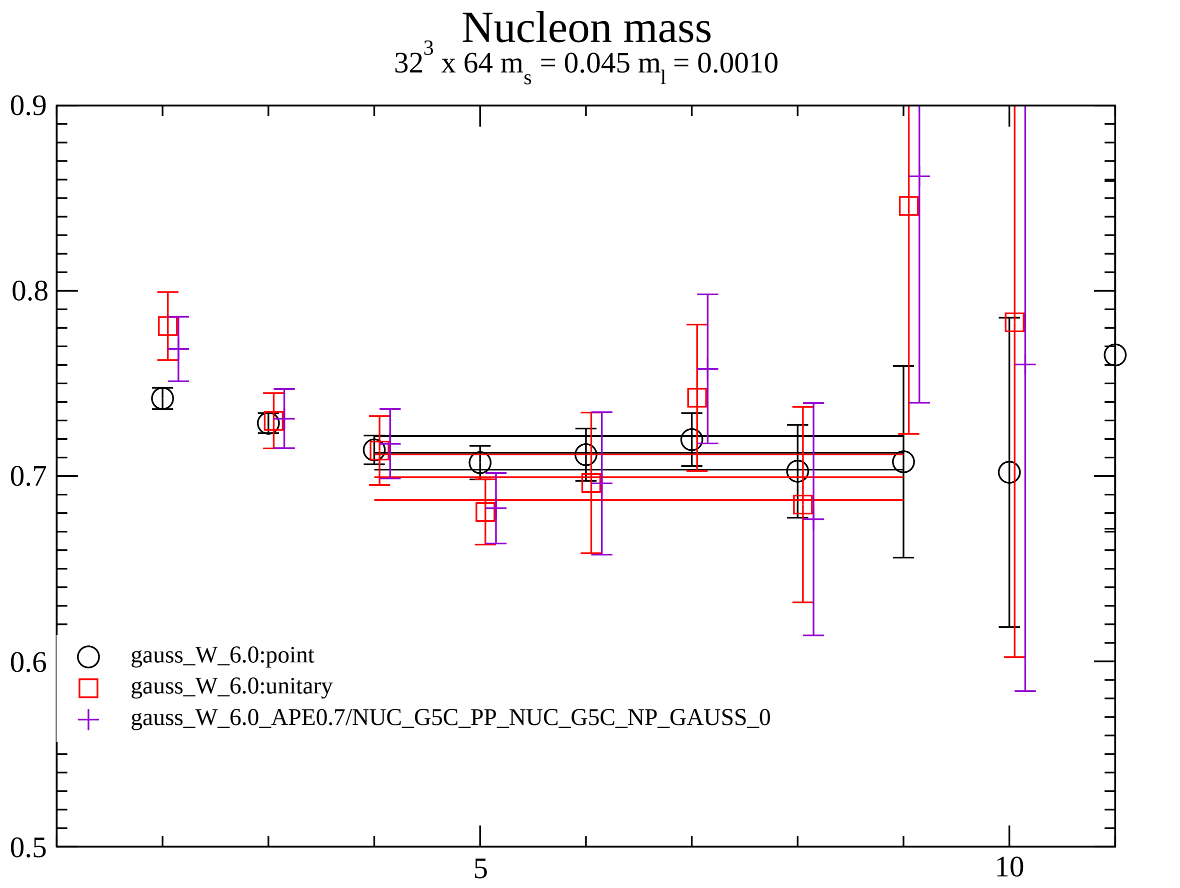
- lattice scale from Ω^-: $a^{-1} = 1.371(8)$ GeV,
- $m_\pi = 0.1816(8)$ and 0.1267(8), or $\sim$ 250 and 170 MeV.

$32^3 \times 64$ volume is about 4.6 fm across in space, 9.2 fm in time. We started nucleon structure calculations:

- finished tuing Gaussian smearing, width 6 favored over 4.
- sink separation at 9, four source positions per configuarion,
- 608–$\sim$1000/8 for 250-MeV, 500–$\sim$1000/8 for 170-MeV so far partially analyzed for 3pt,

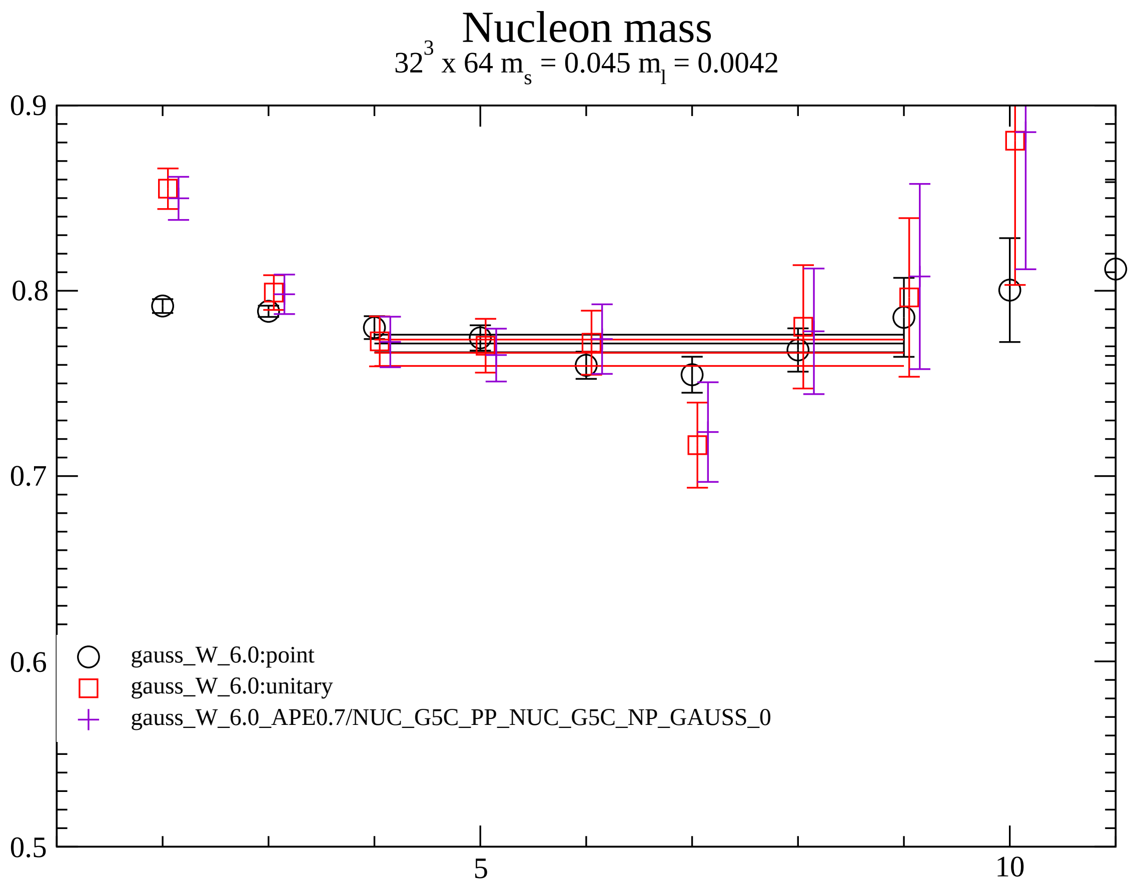
using RICC/RIKEN and Lonestar/Teragrid clusters.

RBC/UKQCD (2+1)-flavor, ID+DWF dynamical, $a^{-1} = 1.371(8)$ GeV, $m_\pi \sim 170$MeV,



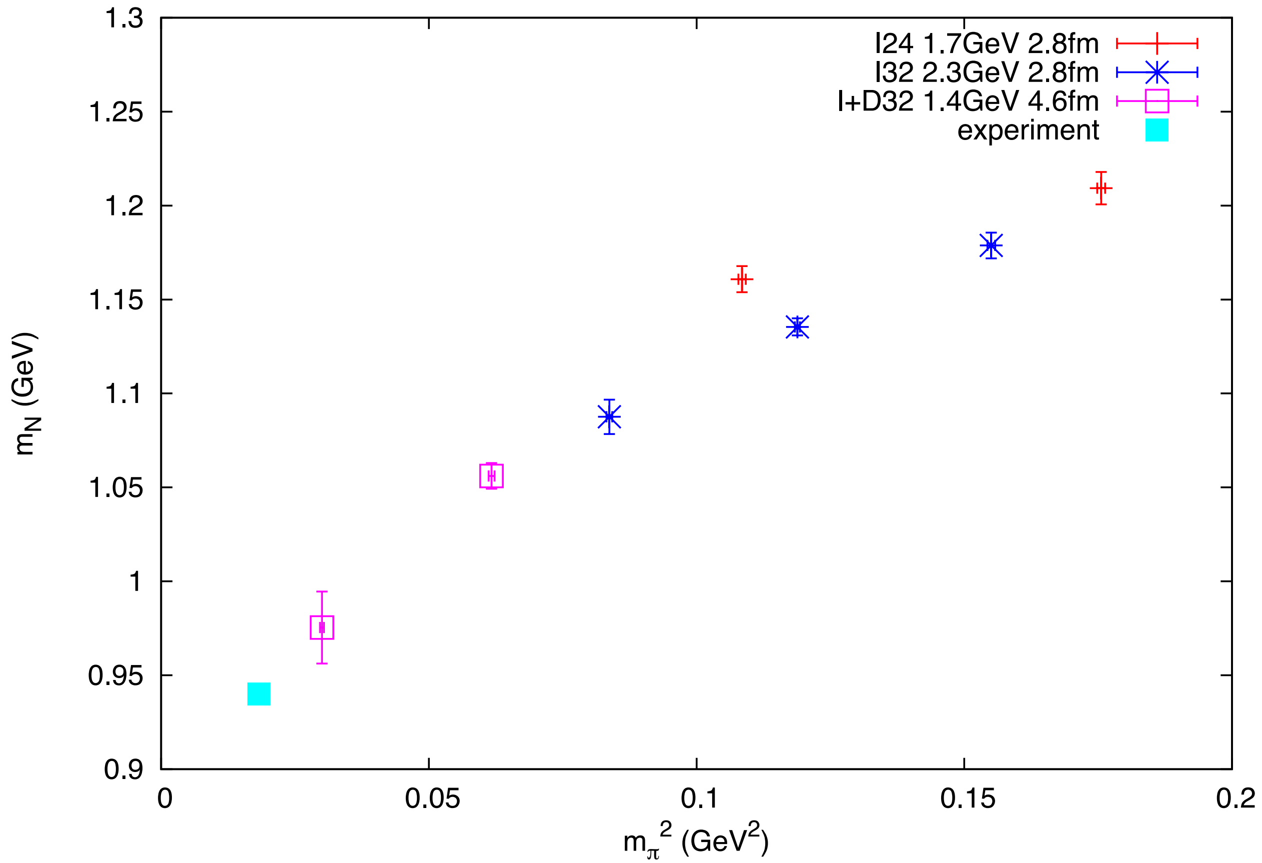
$m_N = 0.713(14)$ or ~ 0.98 GeV,
presently increasing the statistics.

RBC/UKQCD (2+1)-flavor, ID+DWF dynamical, $a^{-1} = 1.371(8)$ GeV, $m_\pi \sim 250$MeV,



$m_N = 0.772(5)$ or ~ 1.06 GeV,
presently increasing the statistics.

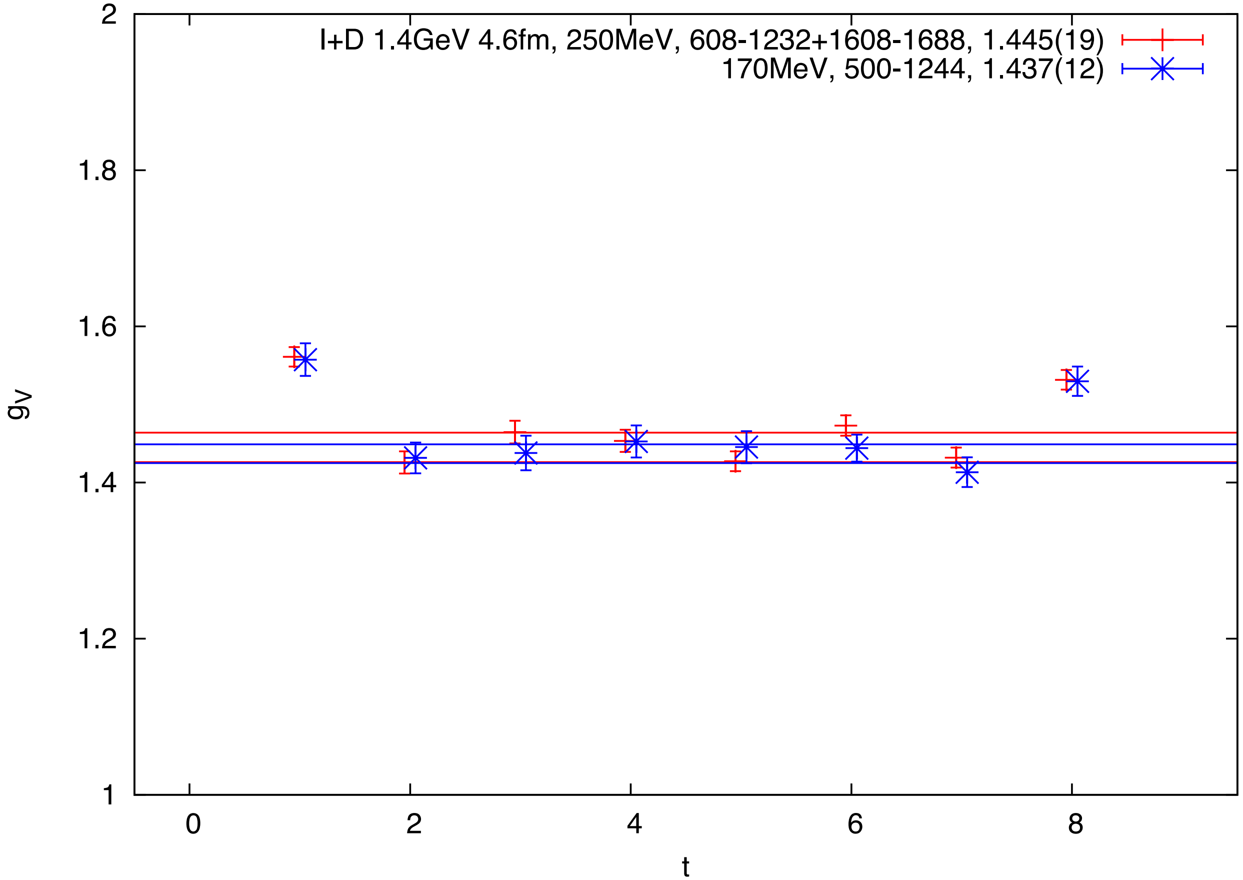
Nucleon mass: RBC/UKQCD (2+1)-flavor, ID+DWF ensembles are being analyzed for nucleon physics.



with $a^{-1} = 1.371(8)$ GeV, $(\sim 4.6\text{fm})^3$ spatial volume.
Closer to physical mass, $m_\pi = 170$ and 250 MeV, $m_N < 1.0$ GeV,

.

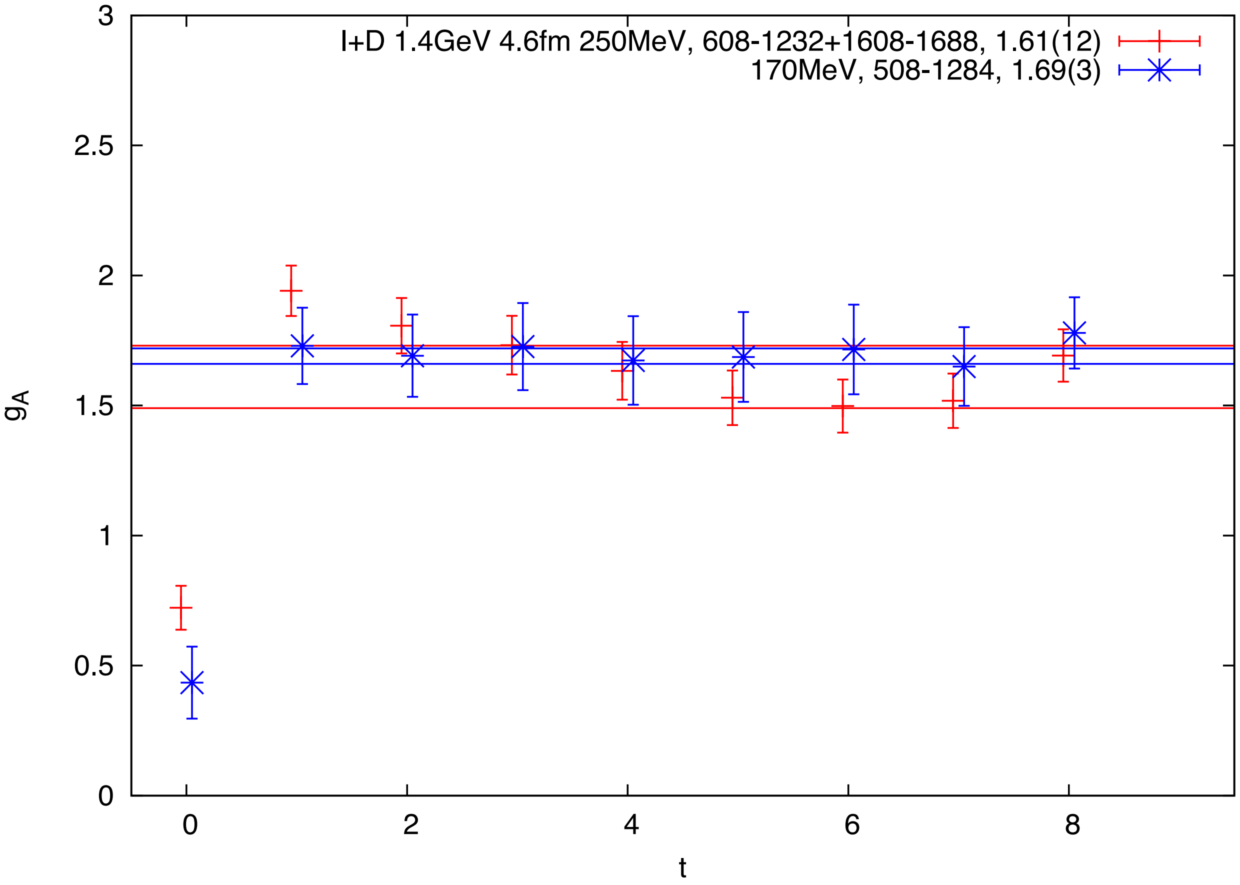
Nucleon isovector 3-pt functions are being obtained: for 89 configurations for 250-MeV, 99 for 170-MeV.



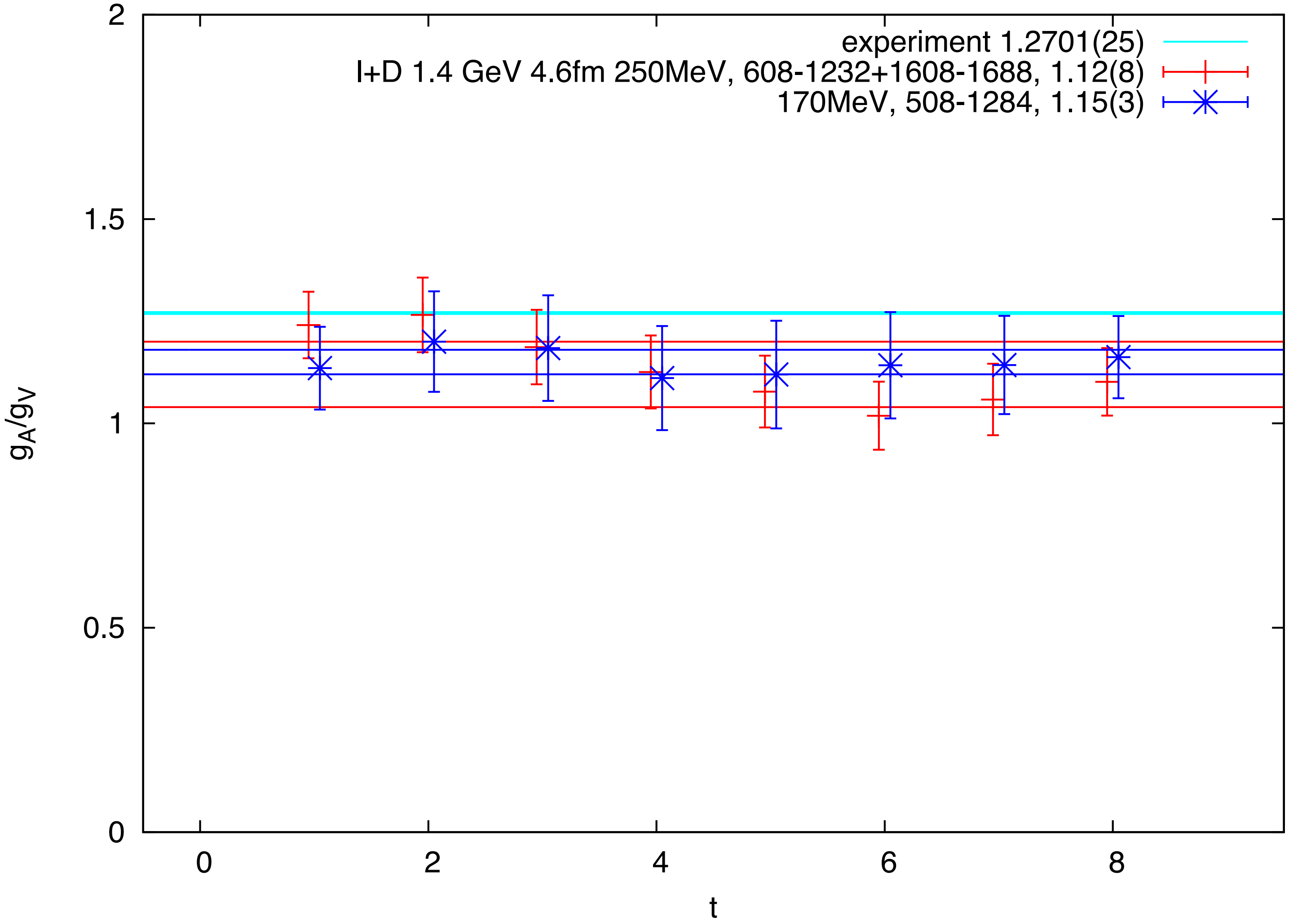
Local-current isovector vector charge, $g_V = 1.445(19)$ or $1.437(12)$, corresponds to $Z_V = 0.700(9)$,

- in good agreement with Z_A obtained in the meson sector,
- yet again proving good chiral and flavor symmetries up to $O(a^2)$.

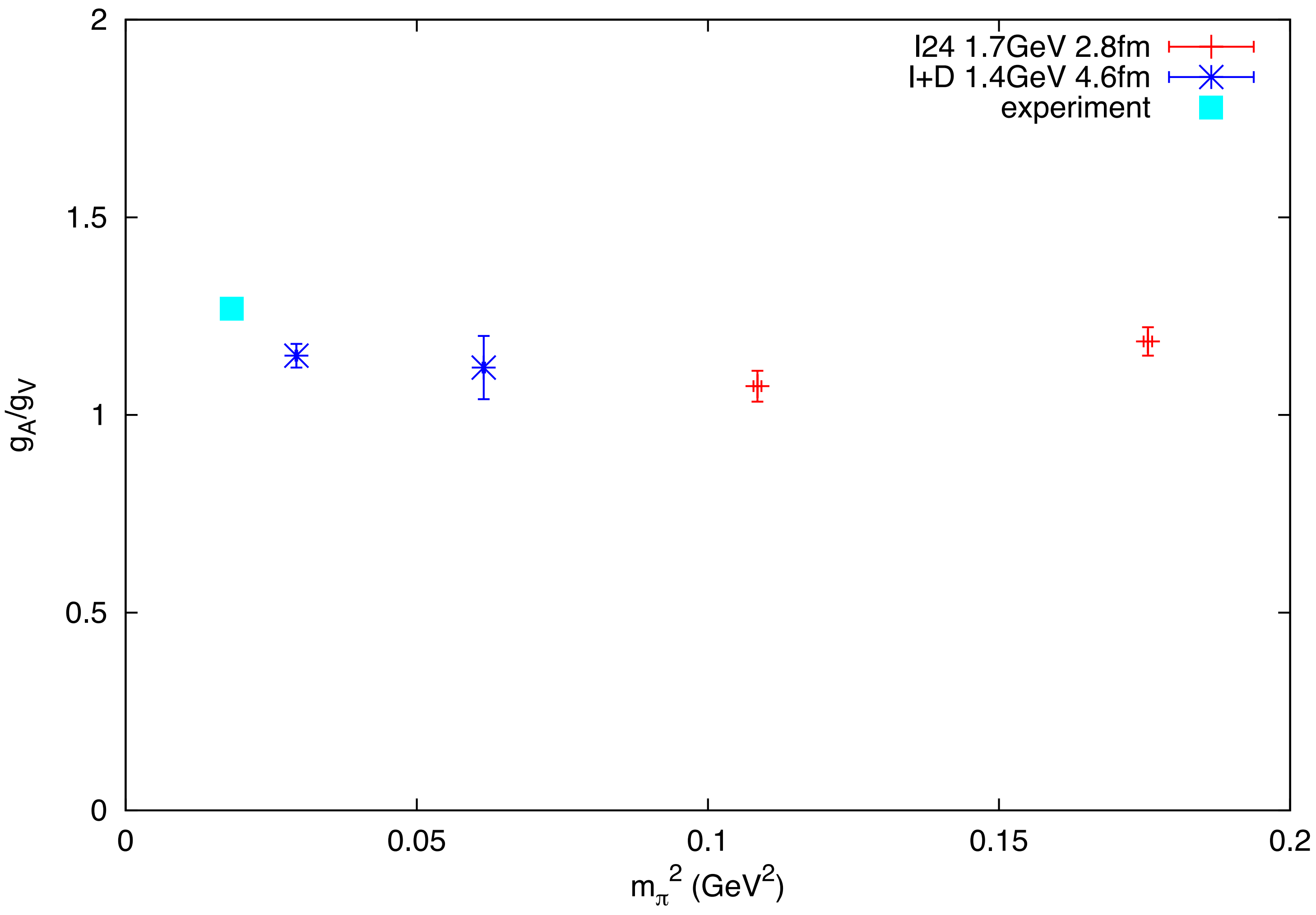
Axialvector current: Noisier than vector current, as expected,



g_A/g_V, ratio of isovector axial and vector charges, is less noisy, again as expected,

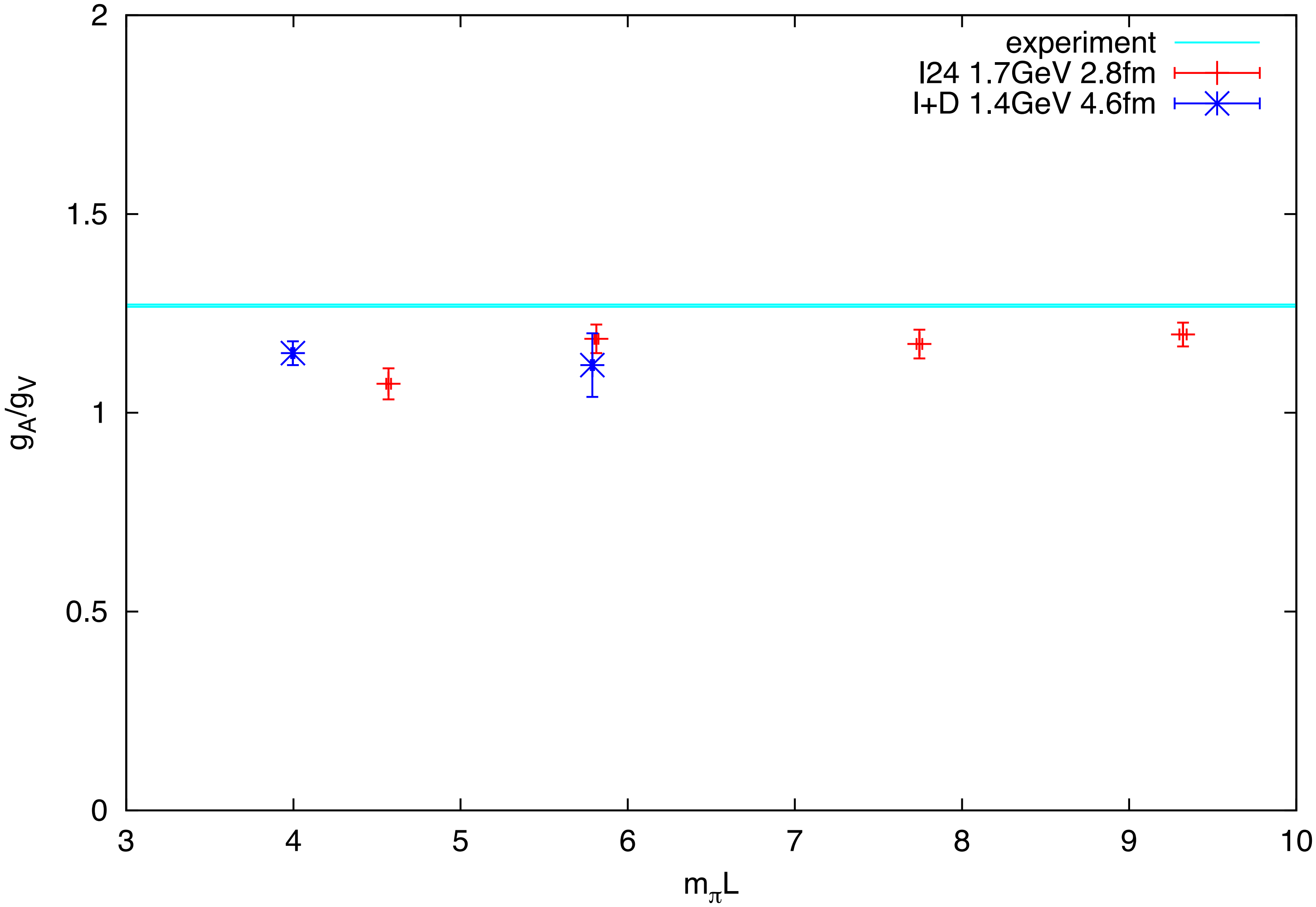


g_A/g_V: seems to stay away from the experiment as we set the pion mass lighter.



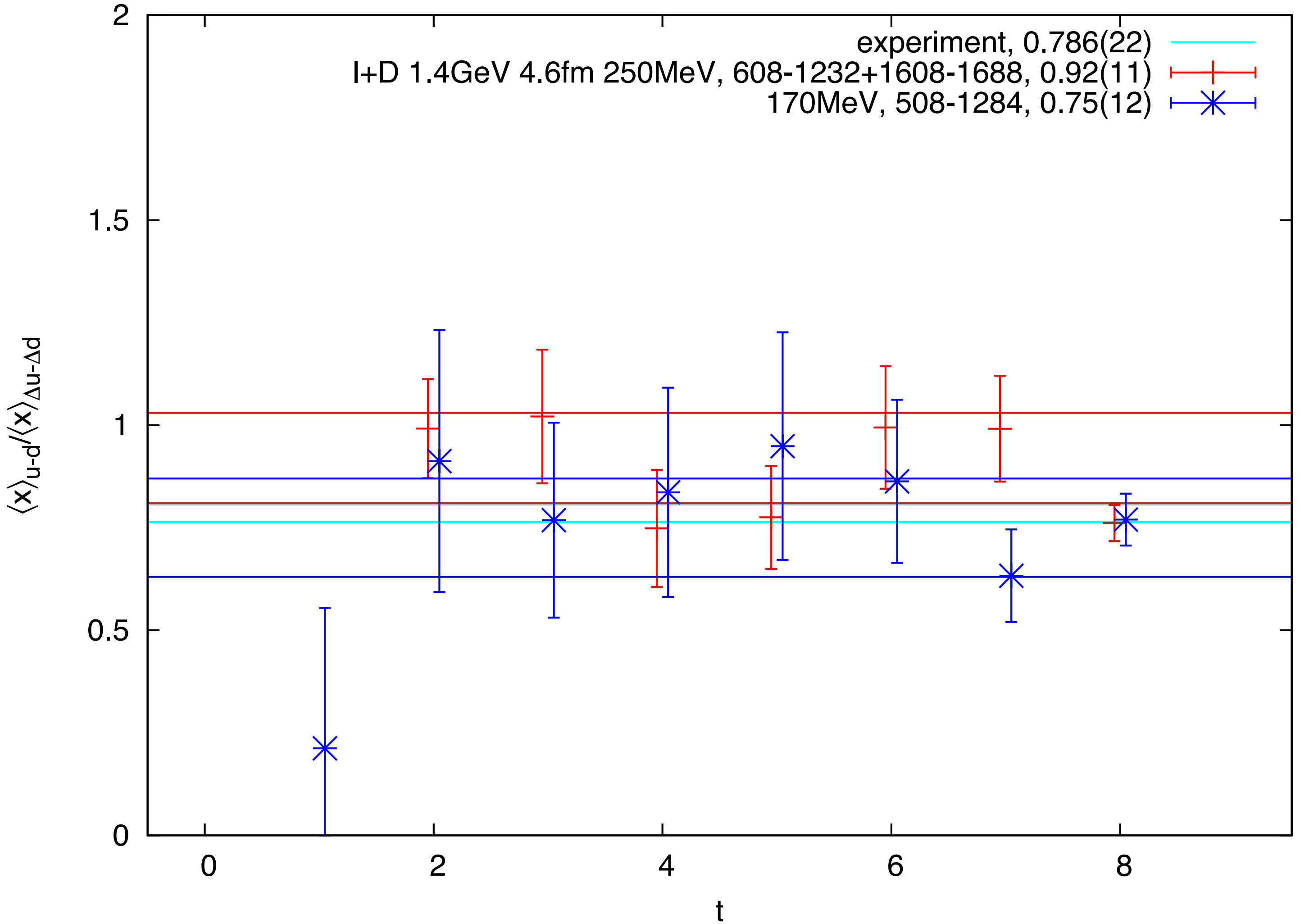
appears to be a finite-size effect.

g_A/g_V: appears to show finite-size effect as pion mass becomes lighter,



and is consistent with scaling in $m_\pi L$ that has been observed.

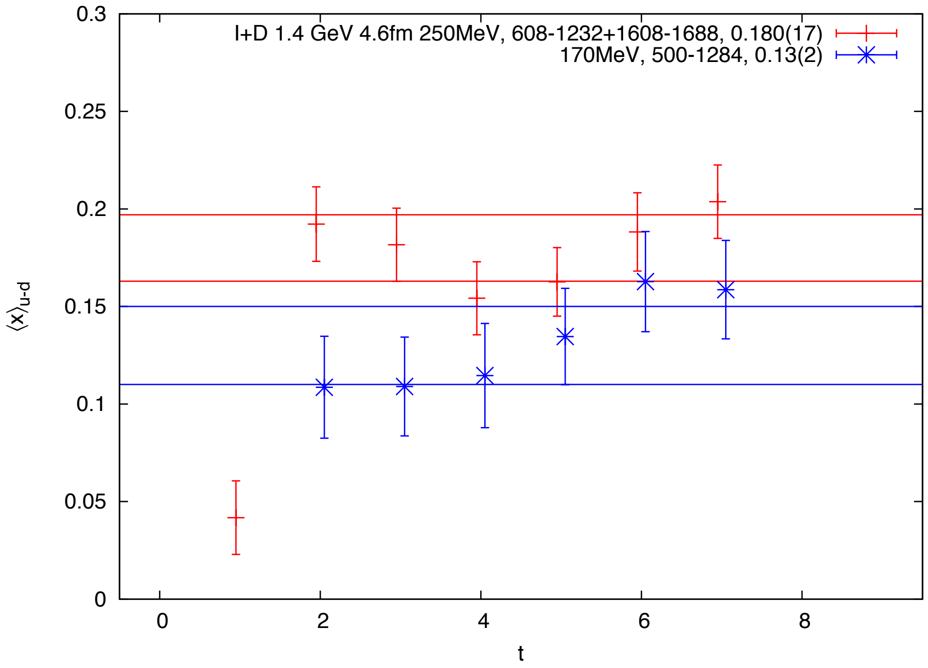
Ratio, $\langle x \rangle_{u-d}/\langle x \rangle_{\Delta u - \Delta d}$, of moments of structure functions, naturally renormalized like g_A/g_V:

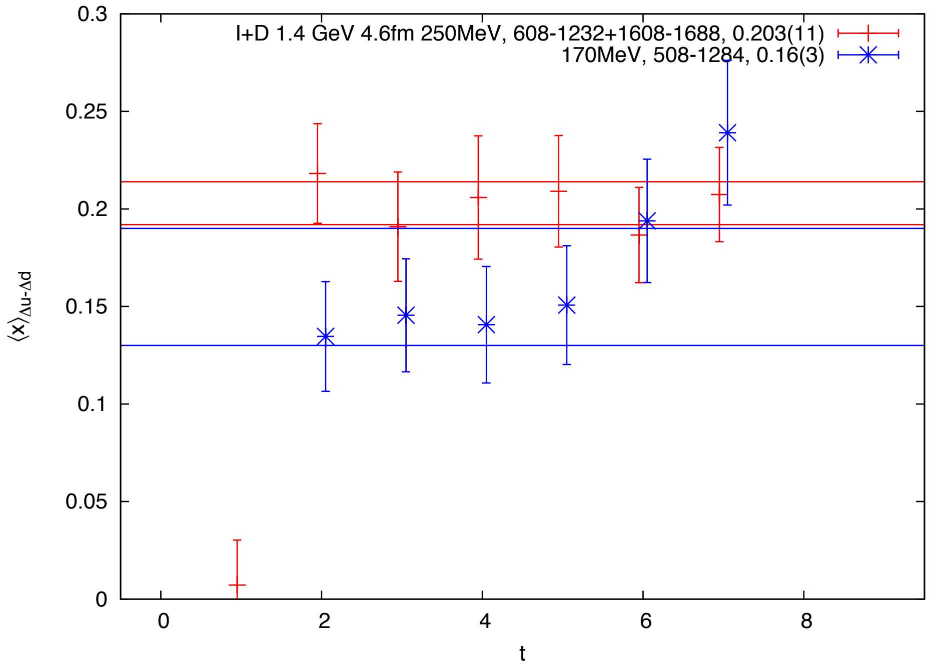


much too noisy yet,

but broadly consistent with experiment, 0.786(22).

Individual moments of structure functions, though yet to be renormalized: signals are seen,





possibly decreasing with mass.

Conclusions

RBC+UKQCD continue to work on nucleon structure using the 2+1f dynamical DWF ensembles,

- lattice cutoff $\sim$ 1.4 GeV, $(4.6\text{fm})^3$ spatial volume,
- good chiral and flavor symmetries, $m_{\text{res}}a \sim 0.002$,
- $m_\pi \sim 170$ and 250 MeV, $m_N \sim 0.98$ and 1.05 GeV.

Even with the current preliminary low statistics, isovector vector-current form factor are well under control,

- local-current charge, g_V, agrees well with Z_A^{-1} from the meson sector.

Axialvector-current form factors are noisier, yet

- consistent with finite-size effect in g_A/g_V that was previously observed, and
- is consistent with broad scaling in $m_\pi L$ also observed earlier,
- suggesting the first concrete evidence for the pion cloud surrounding nucleon.

Moments of structure functions are even noisier, but calculations are well under way:

- $\langle x \rangle_{u-d}/\langle x \rangle_{\Delta u - \Delta d}$ consistent with experiment,
- individual fractions possibly showing desired mass dependence.

Our statistics will more than double at least, with the current 8-trajectory interval, and can double further.