On separation of variables in SU(2) Gluodynamics

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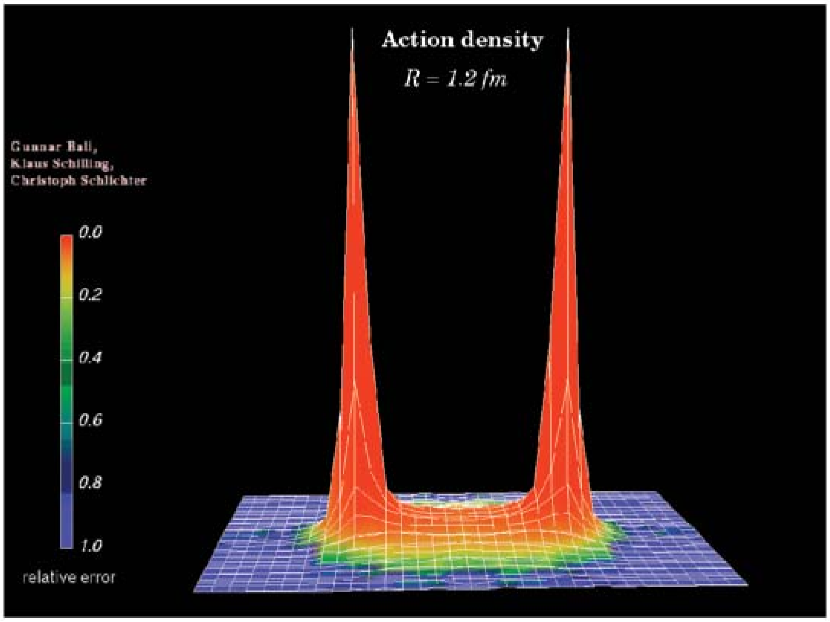
Erice-2011

Abstract: We discuss the possibilty of soliton existence in 2D and 3D SU(2) gluodynamics in Lorentz gauge. Hamiltonians in terms of radial functions are presented. We are looking for localized in space YM field distributions which provide local minima to these hamiltonians. Such nontopological solitons if exist may be relevant to extended gluonic strings in mesons (in 2D) and glueball states (in 3D). Finally separation of variables and Hamiltonian density are presented for 3D SU2-Higgs EW model.

Quark-antiquark with gluonic string

The famous action density distribution between two static colour sources [G.S. Bali, K. Schilling, C. Schlichter '95].



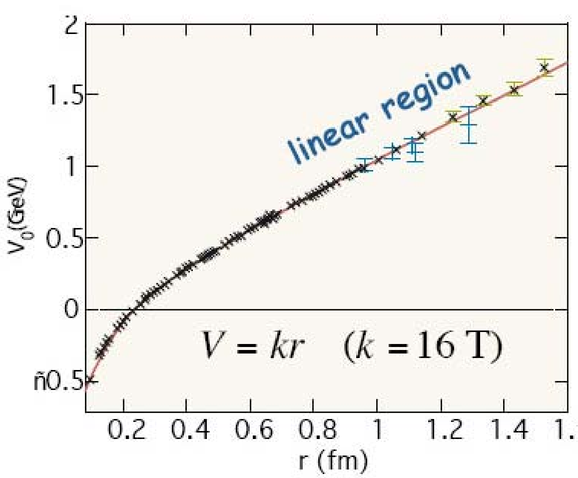


Figure 1: Structure of mesons

Introduction

- Until now there is no satisfactory theoretical description of extended string connecting quark and antiquark in mesons.

- Study of 2D solitons can clarify this issue.

- For now nobody proposed adequate ansatz for description of $2D$ Yang-Mills solitons.

- For 3D case only the simplest one-term ansatz has been studied, for it $\partial_\mu A_\mu = 0$ and $\partial_k A_k = 0$ is valid.

- Generic 3-term ansatz requires detailed study, for it $\partial_\mu A_\mu = 0$ (and $\partial_k A_k = 0$) is not automatically satisfied.

- 3D YM solitons if exist could be viewed as classical glueballs.

- In previous studies of Yang-Mills solitons specifics of Yang-Mills fields as gauge ones has been never taken into account.

- Non-perturbative effects in Salam-Weinberg EW theory are not sufficiently taken into account for now.

Ansatz for Yang-Mills in D=2

- Consider the vector $SU(2)$ Yang-Mills field $A_\mu^a(x^\nu)$,

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^a)^2,$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\varepsilon^{abc}A_\mu^b A_\nu^c,$$

$$D = 2, \quad \mu,\nu = 0,1,2, \quad a,b,c = 1,2,3, \quad g{-}const.$$

- We look for solitonic solutions and use the following ansatz:

$$A_0^a = 0,$$

$$gA_i^a = \delta_{a3}\,\varepsilon_{iak}\,x_k\,\frac{1}{R^2}s(R) +$$

$$+ (\delta_{a1}+\delta_{a2})\left[(\delta_{ia}R^2 - x_i x_a)\,\frac{b(R)}{R^3} + \frac{p(R)x_i x_a}{R^4}\right],$$

$$i,k = 1,2 \qquad R^2 = x^2 + y^2.$$

Hamiltonian density for D=2

No gauge fixing here.

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> H_YM_2D;
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$$\frac{1}{2}\frac{s(R)^2\,p(R)^2}{R^2 g^2}-\frac{b(R)\,p(R)}{R^3 g^2}-\frac{p(R)\left(\frac{d}{dR}b(R)\right)}{R^2 g^2}+\frac{1}{2}\frac{\left(\frac{d}{dR}b(R)\right)^2}{g^2}$$
$$+\frac{p(R)^2\,s(R)}{R^3 g^2}+\frac{b(R)\left(\frac{d}{dR}b(R)\right)}{R\,g^2}+\frac{1}{2}\frac{b(R)^2}{R^2 g^2}+\frac{1}{2}\frac{\left(\frac{d}{dR}s(R)\right)^2}{g^2}$$
$$+\frac{1}{2}\frac{b(R)^2\,p(R)^2}{R^2 g^2}+\frac{1}{2}\frac{s(R)^2}{R^2 g^2}+\frac{s(R)\left(\frac{d}{dR}s(R)\right)}{R\,g^2}$$
$$+\frac{\left(\frac{d}{dR}s(R)\right)b(R)\,p(R)}{R\,g^2}-\frac{\left(\frac{d}{dR}b(R)\right)s(R)\,p(R)}{R\,g^2}+\frac{1}{2}\frac{p(R)^2}{R^4 g^2}$$

Maple output 1: Hamiltonian density, D=2.

Yang-Mills in $D = 3$ (1)

- Consider the vector $SU(2)$ Yang-Mills field $A^a_\mu(x^\nu)$,

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2,$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^{abc} A^b_\mu A^c_\nu,$$

$$D = 3, \quad \mu,\nu = 0,1,2,3 \quad a,b,c = 1,2,3, \quad g{-}const.$$

- Generic ansatz for $D = 3$ YM solitons:

$$A^a_0 = \frac{x^a}{R} q(R),$$

$$gA^a_i = \varepsilon_{iak}\frac{x_k}{R^2} s(R) +$$

$$+ \frac{b(R)}{R^3}(\delta_{ia}R^2 - x_i x_a) + \frac{p(R) x_i x_a}{R^4};$$

$$i,k = 1,2,3, \qquad R^2 = x^2 + y^2 + z^2.$$

Yang-Mills in $D = 3$ (2)

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> H_YM_3D;
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$$\frac{\left(\frac{d}{dR}\mathrm{b}(R)\right)^2}{R^2} + \frac{\left(\frac{d}{dR}\mathrm{s}(R)\right)^2}{R^2} + \frac{2\,\mathrm{s}(R)^3}{R^4} + \frac{2\,\mathrm{s}(R)\,\mathrm{b}(R)^2}{R^4}$$
$$- \frac{2\left(\frac{d}{dR}\mathrm{b}(R)\right)\mathrm{s}(R)\,\mathrm{p}(R)}{R^4} + \frac{1}{2}\frac{\mathrm{b}(R)^4}{R^4} + \frac{2\,\mathrm{b}(R)\,\mathrm{p}(R)\left(\frac{d}{dR}\mathrm{s}(R)\right)}{R^4}$$
$$+ \frac{2\,\mathrm{s}(R)^2}{R^4} - \frac{2\,\mathrm{p}(R)\left(\frac{d}{dR}\mathrm{b}(R)\right)}{R^4} + \frac{\mathrm{b}(R)^2\,\mathrm{s}(R)^2}{R^4} + \frac{1}{2}\frac{\mathrm{s}(R)^4}{R^4} + \frac{\mathrm{p}(R)^2}{R^6}$$
$$+ \frac{\mathrm{s}(R)^2\,\mathrm{p}(R)^2}{R^6} + \frac{\mathrm{b}(R)^2\,\mathrm{p}(R)^2}{R^6} + \frac{2\,\mathrm{p}(R)^2\,\mathrm{s}(R)}{R^6} + \frac{3}{4}\frac{\mathrm{q}(R)^2\left(\frac{d}{dR}\mathrm{p}(R)\right)^2}{R^2}$$
$$+ \frac{1}{4}\mathrm{q}(R)^2 + \frac{3}{2}\left(\frac{d}{dR}\mathrm{q}(R)\right)^2 + \frac{3\,(\mathrm{s}(R)+1)^2\,\mathrm{q}(R)^2}{R^2}$$

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Maple output 2: Hamiltonian density, D=3, no gauge fixing.

Apply Coulomb/Lorentz gauge

Now apply Coulomb (Lorentz) gauge

$$\partial_k A_k = 0, \qquad k = 1, ..., D.$$

For D=2 Hamiltonian density takes the form:

$$\mathcal{H}_{sol} = \frac{1}{2g^2}\Big[\Big(\frac{ds}{dR}+\frac{s}{R}+\frac{p}{R^3}\frac{dp}{dR}\Big)^2+\frac{1}{R^2}\Big(\frac{d^2p}{dR^2}-\frac{p}{R}(s+\frac{1}{R})\Big)^2\Big]. \quad (1)$$

For D=3 Hamiltonian density reads:

$$\mathcal{H}_{sol} = \frac{1}{g^2}\Big\{\frac{1}{32\,R^4}\Big[\Big(\frac{dp}{dR}\Big)^2 + 8s + 4s^2\Big]^2 +$$

$$\Big[\frac{p\,(s+1)}{R^3} - \frac{1}{2R}\frac{d^2p}{dR^2}\Big]^2 + \Big[\frac{1}{R}\frac{ds}{dR} + \frac{1}{2R^3}\frac{dp)}{dR}p\Big]^2 +$$

$$\Big[\frac{3}{4}\frac{\Big(\frac{dp}{dR}\Big)^2}{R^2} + \frac{1}{4} + \frac{3(s+1)^2}{R^2}\Big]q^2 + \frac{3}{2}\Big(\frac{dq}{dR}\Big)^2\Big\}. \quad (2)$$

- Stationary solitons do not exist (see Jaffe, Taubes, Vorices & Monopoles, 1980).

⇒ Numerical search for a) time-dependent localized solutions and b) "quantum solitons" is in progress. We start with Monte-Carlo simulations.

No-Go Theorems, Coleman & Co. (1)

Coleman's study: let $A^a_\mu(x)$ - classical localized solution. Make transformatons

$$\begin{aligned} A^a_0(x_k;\sigma,\lambda) &= \sigma\lambda A_0{}^a(\lambda x_k), \\ A^a_i(x_k;\sigma,\lambda) &= \lambda A_i{}^a(\lambda x_k). \end{aligned} \tag{1}$$

Denote

$$\begin{aligned} H_1 &= \frac{1}{2}\int d^{\mathcal{D}}x(F^a_{0i})^2 \\ &= \frac{1}{2}\int d^{\mathcal{D}}x(\partial_i A^a_0 + ec^{abc}A^b_0 A^c_i)^2, \end{aligned} \tag{2}$$

$$\begin{aligned} H_2 &= \frac{1}{2}\int d^{\mathcal{D}}x(F^a_{ij})^2 \\ &= \frac{1}{4}\int d^{\mathcal{D}}x(\partial_j A^a_i + ec^{abc}A^b_i A^c_j)^2. \end{aligned} \tag{3}$$

Then under transformation (1)

$$H(\sigma,\lambda) = \sigma^2\lambda^{(4-\mathcal{D})}H_1 + \lambda^{(4-\mathcal{D})}H_2 \,.$$

No-Go Theorems, Coleman & Co. (2)

Requiring stationarity:

$$\frac{\partial H}{\partial \sigma} = 0, \quad \frac{\partial H}{\partial \lambda} = 0 \quad \text{at} \quad \sigma = 1, \quad \lambda = 1,$$

Coleman has found for $D \neq 4:$ $\quad H_1 = H_2 = 0$.

For $\mathcal{D} \neq 4$ from here: $F^a_{\mu\nu} = 0$, $Q.E.D.$

Coleman's conclusion was:

"There are no classical glueballs".

⇒ Thus, Coleman has shown that there are no minima of Hamilonian in extended space of variables, corresponding to non-fixed gauge fields and including nonphysical degrees of freedom. E.g. fixing the Lorentz gauge, we get the physical space of dynamical variables, whose dimensionality is less then that of extended space of gauge field without gauge fixing.

EW SU2-Higgs model (1)

- Lagrangian density:

$$\mathcal{L} = |\mathcal{D}_\mu \varphi|^2 - \frac{1}{4}(F^a_{\mu\nu})^2,$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^{abc} A^b_\mu A^c_\nu,$$

$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}, \quad \mathcal{D}_\mu \varphi = (\partial_\mu - \frac{ig}{2}\tau_a A^a_\mu)\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}.$$

Here φ is isospinor of SU(2) group,

$D = 3, \quad \mu, \nu = 0, 1, 2, 3; \quad a, b, c = 1, 2, 3; \quad g - const.$

- Isospinor φ is represented by four real values $\phi_\alpha, (\alpha = 0, 1, 2, 3)$:

$$\varphi_1 = \frac{\phi_1 + i\phi_2}{\sqrt{2}},$$

$$\varphi_2 = \frac{\phi_0 + i\phi_3}{\sqrt{2}},$$

such that $\phi_0^2 + \phi_1^2 + \phi_2^2 + \phi_3^2 = 1.$

i.e. unit 4-vector ϕ_α takes values on unit sphere S^3.

EW SU2-Higgs model (2)

- Localized 3D configurations of quasi-Higgs field φ, $\varphi(\infty) = \varphi_0$,

  define maps $R^3_{comp} \to S^3$ or equivalently $S^3 \to S^3$.

  Hence φ-configurations with nontrivial topological indices (mapping degrees Q_{top}) are possible.

- Consider the case $Q_{top} = 1$ and try the following ansatz for EW SU2-Higgs model:

$$A_0^a = \frac{x^a}{R}\, q(R),$$

$$gA_i^a = \varepsilon_{iak}\frac{x_k}{R^2}\, s(R) + \frac{1}{2}\frac{dp(R)}{dR}\frac{1}{R^3}\left(\delta_{ia}R^2 - x_i x_a\right) + \frac{p(R)x_i x_a}{R^4},$$

$$\varphi = \begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix} = \left[\sin\theta(R)\frac{\tau_a x_a}{R} + i\cos\theta(R)\right]\begin{bmatrix}1\\0\end{bmatrix},$$

$$\tau_1 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \quad \tau_2 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \quad \tau_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},$$

$$\cos\theta(0) = -1, \quad \cos\theta(\infty) = 1.$$

EW SU2-Higgs model (3)

Also apply Lorentz gauge $\partial_\mu A_\mu = 0$.

For D=3 Hamiltonian density reads:

$$\mathcal{H}_{sol} = \frac{1}{g^2}\Big\{ \frac{1}{32\,R^4}\Big[\Big(\frac{dp}{dR}\Big)^2 + 8s + 4s^2\Big]^2 +$$

$$\Big[\frac{p\,(s+1)}{R^3} - \frac{1}{2R}\frac{d^2p}{dR^2}\Big]^2 + \Big[\frac{1}{R}\frac{ds}{dR} + \frac{1}{2R^3}\frac{dp}{dR}\,p\Big]^2 +$$

$$\Big[\frac{3}{4}\frac{1}{R^2}\Big(\frac{dp}{dR}\Big)^2 + \frac{1}{4} + \frac{3(s+1)^2}{R^2}\Big]q^2 + \frac{3}{2}\Big(\frac{dq}{dR}\Big)^2 +$$

$$\Big(\frac{d\theta}{dR}\Big)^2 + \frac{1}{R^2}\Big[2\sin^2(\theta)(s+1) + \frac{1}{2}\frac{dp}{dR}\sin(2\theta) + p\frac{d\theta}{dR}\Big]\Big\}.$$

$\Rightarrow$ Numerical search for localized solutions is in progress. We plan to start with Monte-Carlo simulations.

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