Chiral crossover effects on the shear viscosity of a pion gas

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Michael Buballa

International School of Nuclear Physics
33rd Course
"From Quarks and Gluons to Hadrons and Nuclei"
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Collaborators

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- based on
  - Klaus Heckmann, Ph.D. thesis, TU Darmstadt, 2011
  - K. Heckmann, M.B., J. Wambach, work in progress

Motivation

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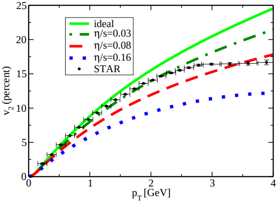
- flow data at RHIC



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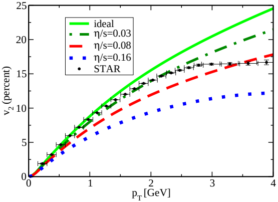

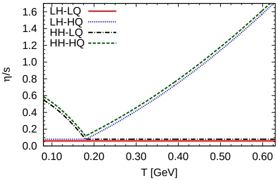

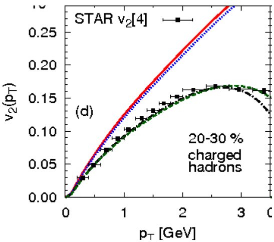

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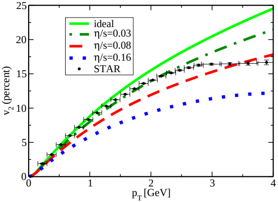

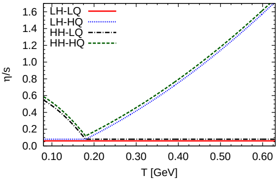

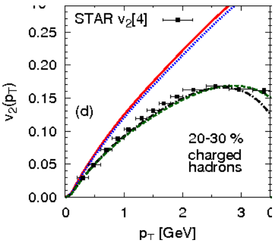

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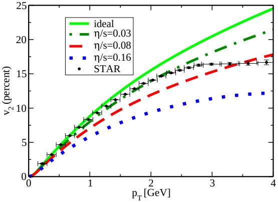

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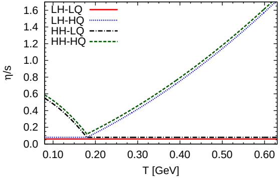

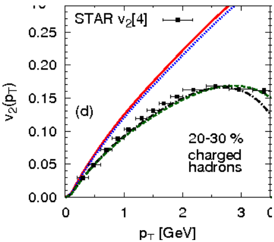

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- here: BUU approach to $\pi\pi$-scattering in the NJL model
  - correct low-temperature limit
  - imprints of the chiral crossover and the compositeness of the pions

Viscous relativistic hydrodynamics

- basic ingredients and conservation laws:
  - fluid 4-velocity $u^\mu(x)$, $u^\mu(x)u_\mu(x) = 1$
  - energy-momentum tensor $T^{\mu\nu}(x)$, $\partial_\mu T^{\mu\nu}(x) = 0$
  - particle current $J^\mu(x) = n(x)u^\mu(x)$, $\partial_\mu J^\mu(x) = 0$
- additional assumption:

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- gradient expansion:

  $T^{\mu\nu} = T^{(0)\mu\nu} + T^{(1)\mu\nu} + \ldots$ (and similar for J^μ)
  - ideal fluid:

    $$T^{(0)\mu\nu} = (\epsilon + p)\, u^\mu u^\nu - p\, g^{\mu\nu}$$
  - 1st-order viscous correction:

    $$T^{(1)\mu\nu} = \eta \left(\partial^\mu u^\nu + \partial^\nu u^\mu + u^\mu u^\lambda \partial_\lambda u^\nu + u^\nu u^\lambda \partial_\lambda u^\mu\right) + \left(\zeta - \tfrac{2}{3}\eta\right)(g^{\mu\nu} - u^\mu u^\nu)\, \partial_\lambda u^\lambda$$

Quantum relativistic kinetic theory

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- underlying assumption: mean free path $\lambda \gg$ interaction range *r*

Quantum relativistic kinetic theory

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➜ $T^{\mu\nu}(x) = \sum_a g_a \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E_p} f_a(\vec{x}, \vec{p}, t)$

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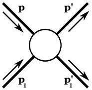

- Boltzmann-Uehling-Uhlenbeck (BUU) equation $(2 \to 2)$

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$$\times \left[f_a' f_{1b}' (1 + f_a)(1 + f_{1b}) - f_a f_{1b} (1 + f_a')(1 + f_{1b}') \right] \Big\}$$

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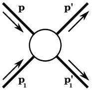

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- linearization (2nd-order Chapman-Enskog expansion)

  $f_a = f_a^{(0)} + f_a^{(1)} + \ldots$, local equilibrium: $f_a^{(0)}(x,p) = \frac{1}{\exp\left[\left(p^\mu u_\mu(x) - \mu_\pi(x)\right)/T(x) - 1\right]}$

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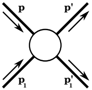

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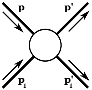

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- physics input: scattering matrix element $\mathcal{M}_{ab}$ here: $\pi\pi \to \pi\pi$ in NJL

Mesons in the NJL model

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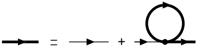
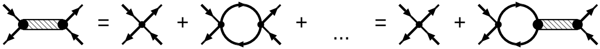
- Lagrangian: $\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + g\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right]$
- gap equation:  ➜ dynamical quark masses
- mesons (RPA): 

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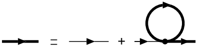
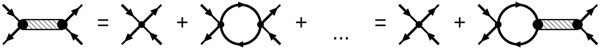
- gap equation:  ➜ dynamical quark masses
- mesons (RPA): 
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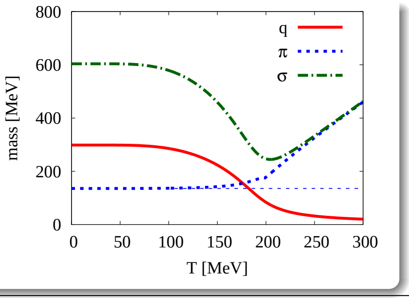

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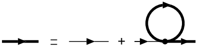
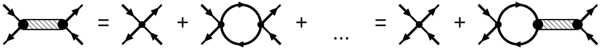
- gap equation:  ➜ dynamical quark masses
- mesons (RPA): 
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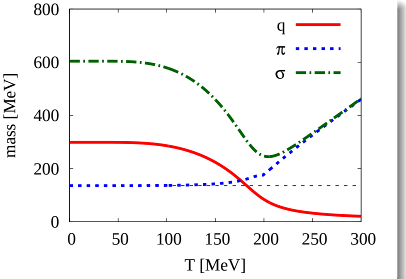

- characteristic temperatures:

  [Quack et al., PLB (1995)]

  - σ-dissociation temperature:

    $m_\sigma(T_{diss}) = 2m_\pi(T_{diss})$

    here: $T_{diss} = 180$ MeV

  - Mott temperature:

    $m_\pi(T_{Mott}) = 2m_q(T_{Mott})$

    here: $T_{Mott} = 199$ MeV

In-medium $\pi\pi$-scattering

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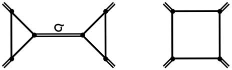
- scattering amplitude

  [Bernard et al., PLB (1991), Schulze, JPG (1995)]

  - leading order $1/N_c$
  - to be taken in *s*-, *t*-, and *u*-channel
  - consistent with chiral low-energy theorems

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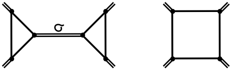

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- scattering length

$$a^I = \frac{1}{32\pi m_\pi} \mathcal{M}^I_{\pi\pi}(s = 4m_\pi^2, t = u = 0)$$

- chiral expansion [Weinberg, PRL (1966)]

$$a^0_W = \frac{7m_\pi}{32\pi f_\pi^2}, \quad a^2_W = -\frac{2m_\pi}{32\pi f_\pi^2}$$

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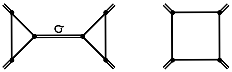

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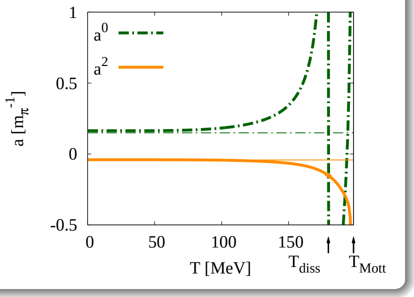
- numerical results

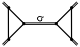
  cf. [Quack et al. PLB (1995)]



  - Weinberg values at low T
  - "Feshbach resonances" at T_{diss} and T_{Mott}

In-medium cross section

- isospin averaged cross section: $\left(\frac{d\sigma}{d\Omega}\right)_{cm} = \frac{1}{9}\sum_{I=0}^{2}(2I+1)\frac{\left|\mathcal{M}^{I}_{\pi\pi}\right|^2}{64\pi^2 s}$

- $i\mathcal{M}_{\pi\pi}$ = 

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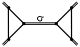
- $i\mathcal{M}_{\pi\pi}$ = 

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- approximations:
  1. Weinberg amplitude

     $\mathcal{M}^{I}_{\pi\pi} = 32\pi\ m_\pi\ a^{I}_{W}$

     (T and momentum independent)
  2. evaluate $\mathcal{M}$ at threshold

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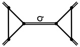
- $i\mathcal{M}_{\pi\pi}$ =  + 

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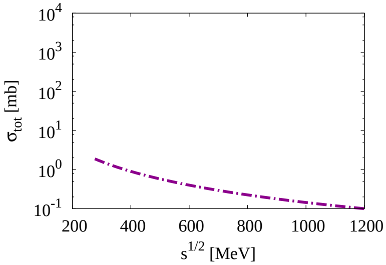
total cross section ($T = 0$)



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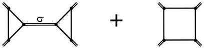
- $i\mathcal{M}_{\pi\pi}$ = 

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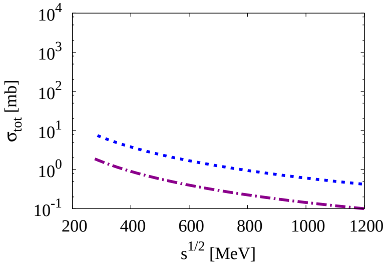
total cross section (T = 150 MeV)



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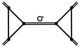
- $i\mathcal{M}_{\pi\pi}$ = 

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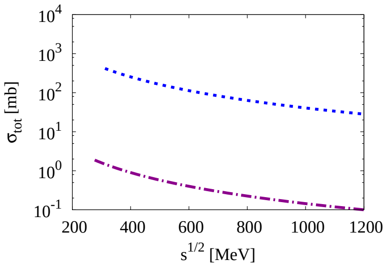
total cross section (T = 177 MeV)



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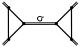
- $i\mathcal{M}_{\pi\pi}$ =  + 

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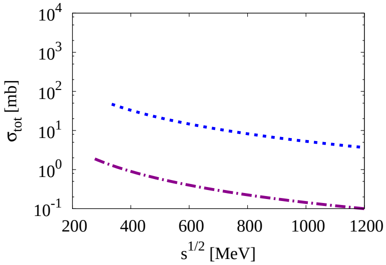

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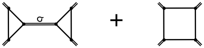

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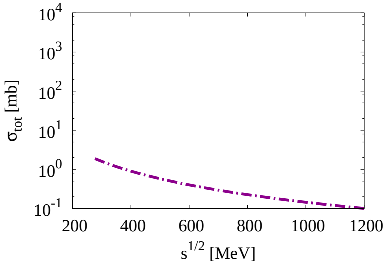

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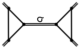
- $i\mathcal{M}_{\pi\pi}$ =  + 

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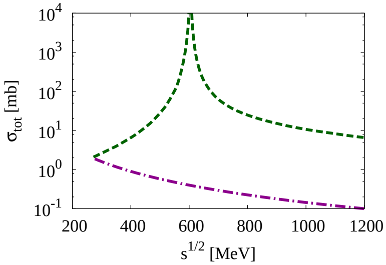

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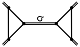
- $i\mathcal{M}_{\pi\pi}$ =  + 

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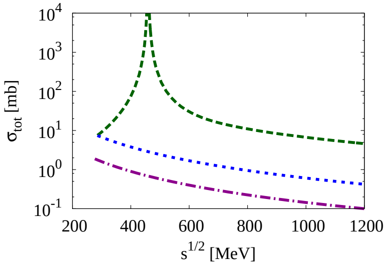

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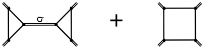

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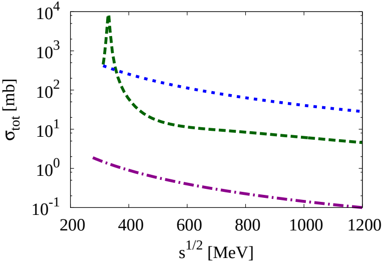

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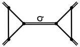
- $i\mathcal{M}_{\pi\pi}$ = 

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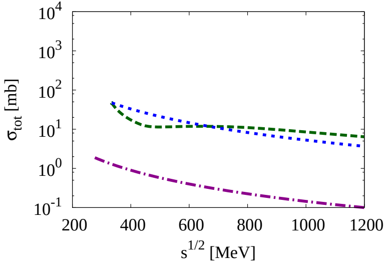

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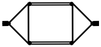
➜ include 

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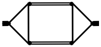
➜ include 

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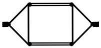
➜ include 

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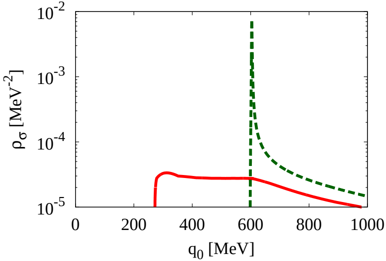

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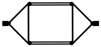
➜ include 

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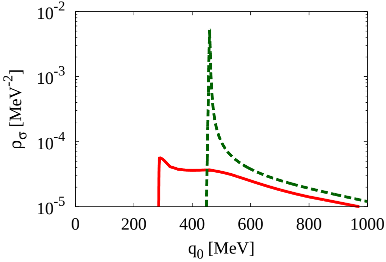

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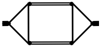
➜ include 

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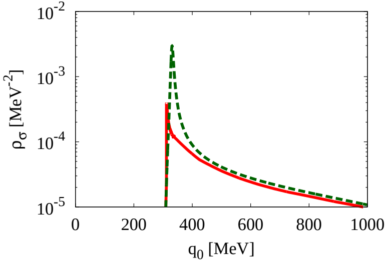

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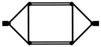
➜ include 

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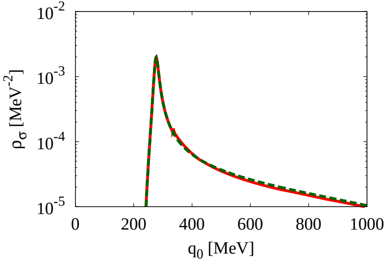
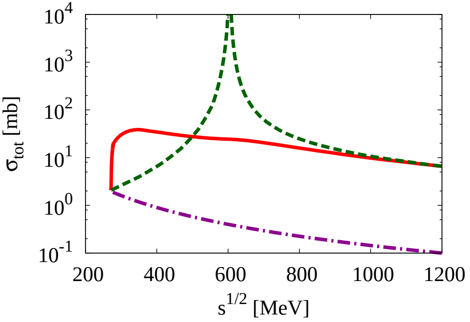


Cross section with sigma-decay width

total cross section $(T = 0)$



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Cross section with sigma-decay width

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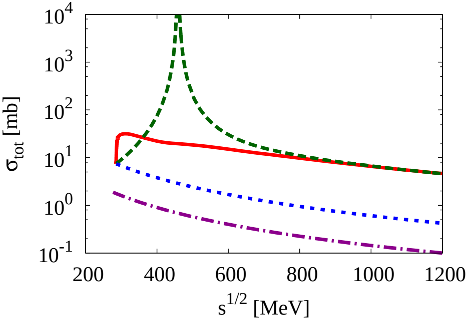
total cross section (T = 150 MeV)



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Cross section with sigma-decay width

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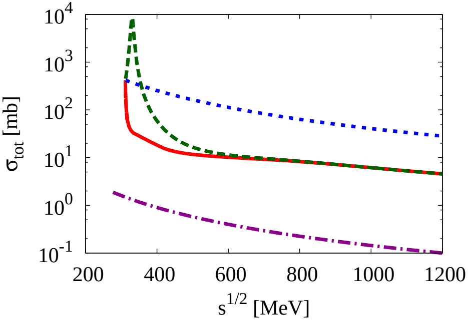

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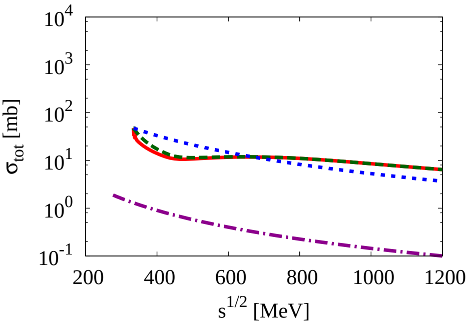

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- $T > T_{diss}$:

  $\sigma \to \pi\pi$ irrelevant

Shear viscosity: numerical results

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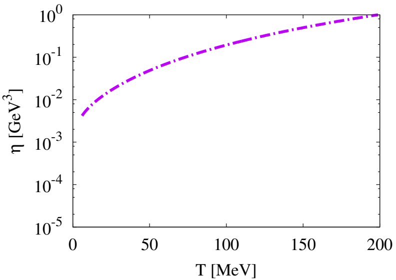

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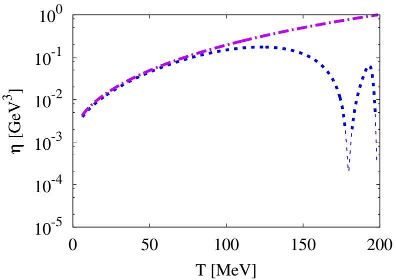

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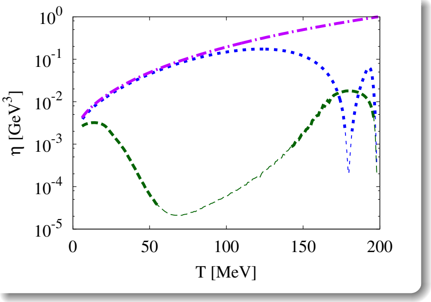

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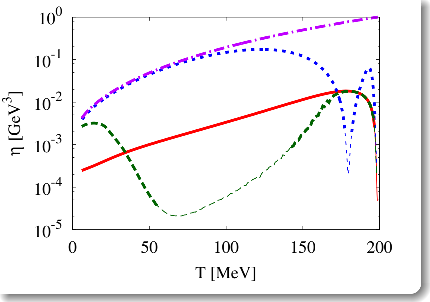

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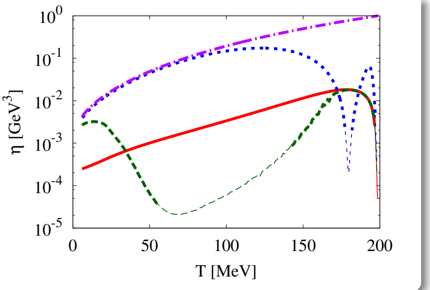

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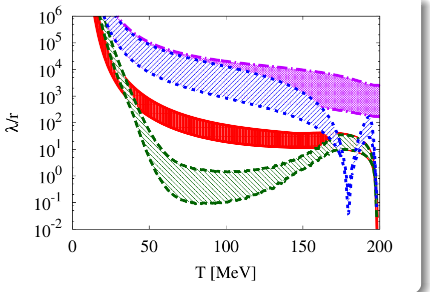
- validity of the kinetic aproach:
  - criterion: $\frac{\lambda}{r} \gg 1$ (dilute gas)
  - $\lambda = \frac{1}{n\sigma}$ mean free path
  - r = interaction range
    (e.g., $1/m_\sigma$, $1/m_\pi$, hard sphere: $\sqrt{\frac{\sigma}{\pi}}$)



Fluidity

- most popular measure: η/s
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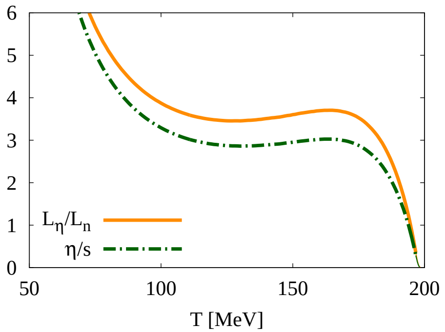
  - entropy density of an ideal pion gas
- alternative measure: L_η/L_n
  [Liao & Koch, PRC (2010)]
  - $L_\eta = \frac{\eta}{hc_s}, \quad L_n = n^{-1/3}$



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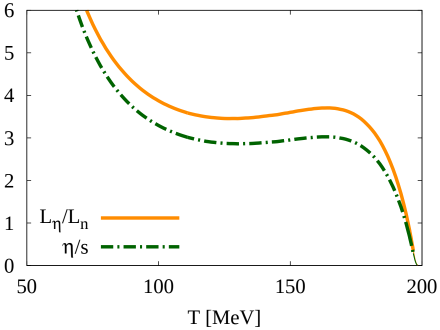

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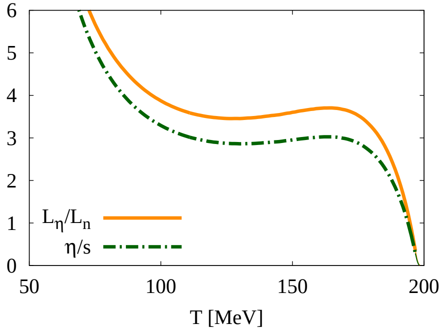

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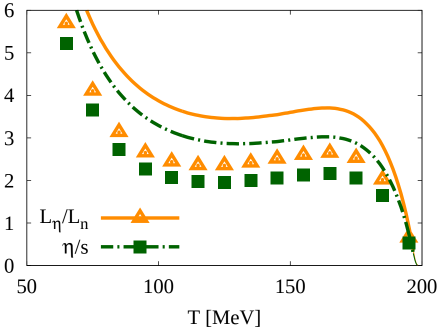

- qualitative agreement!
- simple estimate: $\eta \approx \frac{1}{3} n\bar{p}\lambda = \frac{\bar{p}}{3\sigma(\bar{p})}$ works quite well ...

Conclusions

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  - shear viscosity from $\pi\pi$-scattering in the NJL model in kinetic theory
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  - quantitative results very sensitive to details of the model
- outlook:
  - better description of *p*-wave $\pi\pi$ scattering (include ρ-meson)
  - further scattering channels, e.g., kaons
  - ...