Non-Gaussian Wave Functionals in Coulomb gauge Yang–Mills theory

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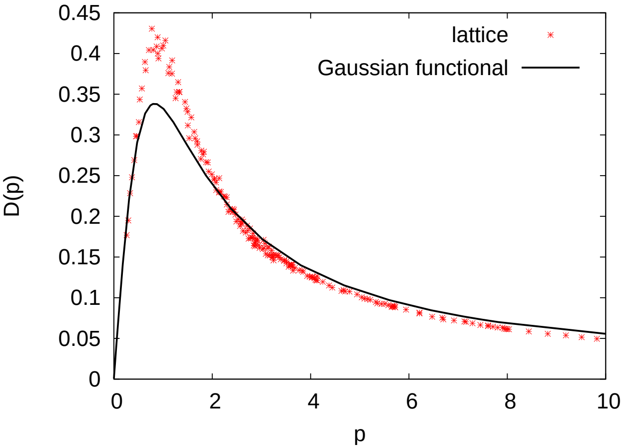
Outline

1. Motivation
2. Ansatz for the vacuum wave functional
   - Hamiltonian Green's functions
   - Dyson–Schwinger-like equations
3. Variational solution of the Yang–Mills Schrödinger equation
   - Variational equations
   - Preliminary results
4. Conclusions

Motivation

Coulomb-gauge gluon propagator in the Hamiltonian approach with Gaussian functional



D. Epple, H. Reinhardt and W. Schleifenbaum, PR**D75**, 045011 (2007)

G. Burgio, M. Quandt and H. Reinhardt, PRL**102**, 032002 (2009)

Hamiltonian Green's functions

V.e.v. of an operator

$$\langle K[A] \rangle = \int_\Omega \mathcal{D}A \, \mathcal{J}_A \, |\psi[A]|^2 \, K[A]$$

- $\mathcal{J}_A = \mathrm{Det}(G_A)$ is the Faddeev–Popov determinant of Coulomb gauge
- integration over transverse field configurations
- integration restricted to the first Gribov region

Ansatz for the vacuum functional

Gaussian ansatz used so far

$$\psi[A] = \mathcal{J}_A^{-1/2} \, \exp\left\{-\frac{1}{2}\,\omega\,A^2\right\}$$

- reproduces correctly IR and UV behaviour
- mismatch in the mid-momentum regime
- no three-gluon vertex

Generalized ansatz

$$\psi[A] = \exp\left\{-\frac{1}{2}\left[\omega\,A^2 + \frac{1}{3!}\,\gamma_3\,A^3 + \frac{1}{4!}\,\gamma_4\,A^4\right]\right\}$$

Dyson–Schwinger-like equations

Formal equivalence to Lagrangian approach

Writing the vacuum wave functional as

$$\psi[A] =: \exp\left\{-\frac{1}{2}S[A]\right\}$$

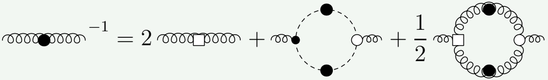
we have an Euclidean QFT defined by an "action" $S[A]$.

DSEs derived from the identity

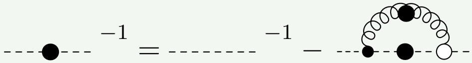
$$0 = \int_\Omega \mathcal{D}A \, \frac{\delta}{\delta A(1)} \left\{\mathcal{J}_A \, \mathrm{e}^{-S[A]} \, K[A]\right\}$$

Propagator DSEs

Gluon propagator $\langle AA \rangle \equiv 1/2\Omega(\mathbf{p})$



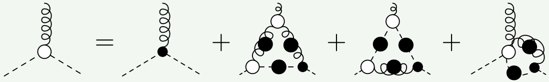
Ghost propagator $\langle G_A \rangle$



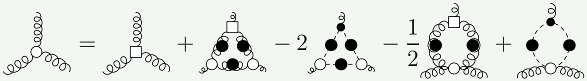
Not quite equations of motion, rather relations between the Green functions and the so far undetermined variational kernels.

Vertex functions DSEs

Ghost-gluon vertex



Three-gluon vertex



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Yang–Mills Hamiltonian in Coulomb gauge

$$H = \frac{1}{2}\int\Big[\mathcal{J}_A^{-1}\,\Pi\,\mathcal{J}_A\,\Pi + \ B^2\Big] + \frac{g^2}{2}\int \mathcal{J}_A^{-1}\,(\hat{A}\Pi)\,\mathcal{J}_A\,F_A\,(\hat{A}\Pi)$$

N. H. Christ and T. D. Lee, PR**D22**, 939 (1980)

- $\Pi = -\mathrm{i}\delta/\delta A$ is the canonical momentum ($\equiv$ electric field)
- B is the non-abelian magnetic field
- $F_A = G_A(-\partial^2)G_A$ is the Coulomb kernel

Energy density

Minimization of the v.e.v. of the Hamiltonian

$$\langle H \rangle \to \text{min.}$$

Truncation scheme

Skeleton expansion: use the DSEs for the vertex functions iteratively to express the energy density as a function of the variational kernels.

The three-gluon kernel

Variation of the energy density fixes the three-gluon kernel to

$$\gamma_{ijk}^{abc}(\mathbf{p},\mathbf{q},\mathbf{k}) = \frac{2\,g\,T_{ijk}^{abc}(\mathbf{p},\mathbf{q},\mathbf{k})}{\Omega(\mathbf{p})+\Omega(\mathbf{q})+\Omega(\mathbf{k})}$$

where

$$T_{ijk}^{abc}(\mathbf{p},\mathbf{q},\mathbf{k}) = \mathrm{i}\,f^{abc}\big[\delta_{ij}(p-q)_k + \delta_{jk}(q-k)_i + \delta_{ki}(k-p)_j\big]$$

is the colour/Lorentz structure of the cubic term in the Hamiltonian.

Lowest-order perturbative vertex

$$\Gamma_{ijk}^{abc(0)}(\mathbf{p},\mathbf{q},\mathbf{k}) = \frac{2\,g\,T_{ijk}^{abc}(\mathbf{p},\mathbf{q},\mathbf{k})}{|\mathbf{p}|+|\mathbf{q}|+|\mathbf{k}|}$$

Two-gluon kernel ↔ Gluon propagator

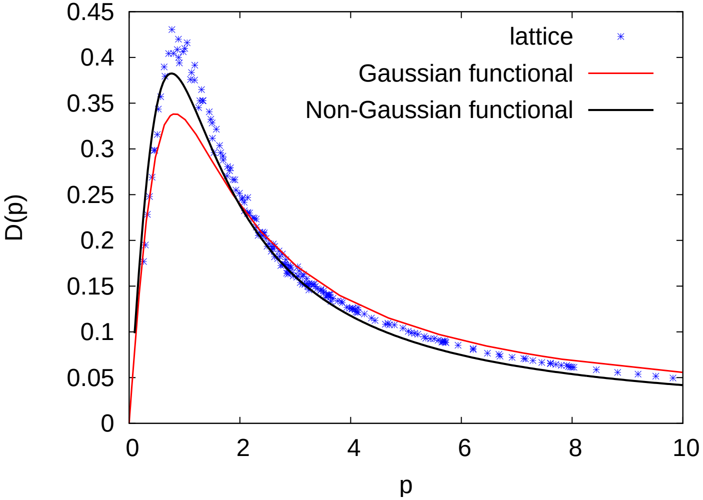
Since the gluon propagator DSE contains the two-gluon kernel ω (the "bare" propagator) only once, we can rewrite the variational equation for ω as an equation for the (inverse) propagator Ω.

Gap equation (Variational equation + Gluon DSE)

$$\Omega(\mathbf{p})^2 = \mathbf{p}^2 + \chi(\mathbf{p})^2 + I_{\mathrm{C}}(\mathbf{p}) - I_{\mathrm{G}}(\mathbf{p})$$

- $\chi(\mathbf{p})$ ghost loop,
- $I_{\mathrm{C}}(\mathbf{p})$ contribution of the Coulomb kernel,
- $I_{\mathrm{G}}(\mathbf{p})$ contribution of the gluon loop.

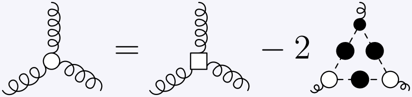
Corrections to the gluon propagator



Three-gluon vertex

Three-gluon vertex DSE

Consider the DSE for the three-gluon vertex under the assumption of ghost dominance



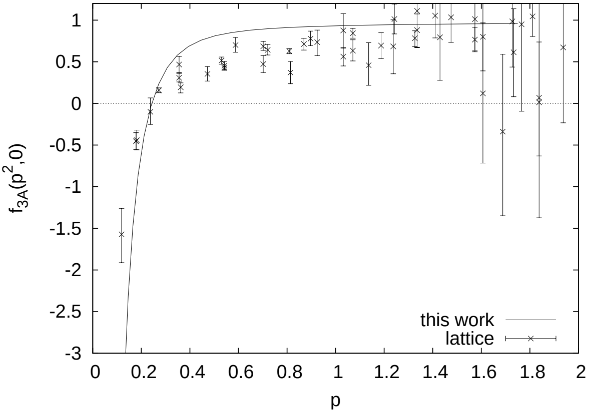
and use the determined three-gluon kernel as bare vertex with the ghost and gluon propagators obtained with a Gaussian functional.

Restricted kinematic configuration

$$f_{3A} = \frac{\Gamma_3^{(0)} \cdot \Gamma_3}{\Gamma_3^{(0)} \cdot \Gamma_3^{(0)}} , \quad \mathbf{q}^2 = \mathbf{p}^2 = p^2 , \quad \mathbf{q} \cdot \mathbf{p} = c\, p^2$$

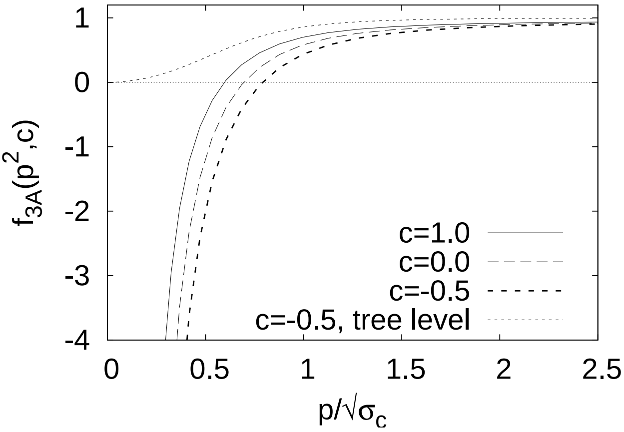
Three-gluon vertex — comparison with lattice data

Qualitative agreement between these results and 3-dimensional Yang–Mills theory in Landau gauge. (Two orthogonal momenta.)



A. Cucchieri, A. Maas and T. Mendes, PR**D77**, 094510 (2008)

Three-gluon vertex — results for different configurations



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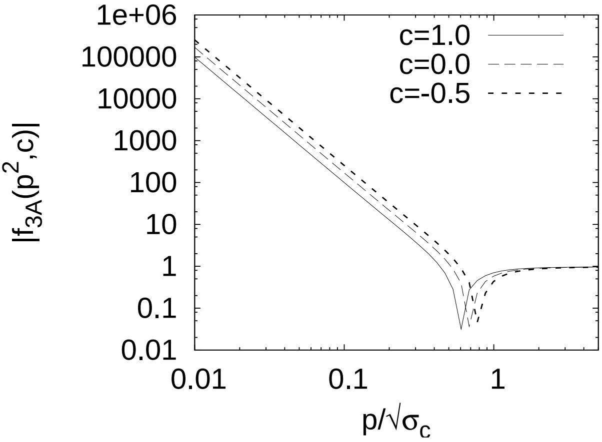
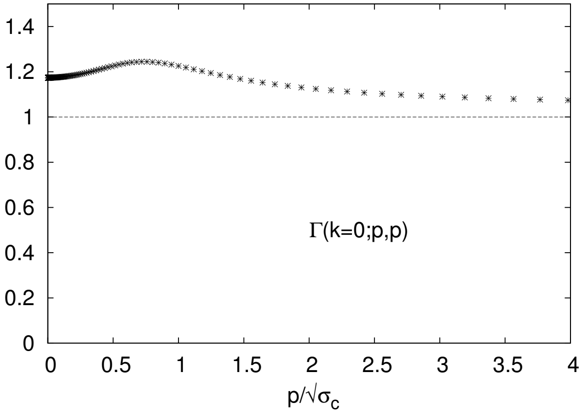


Ghost-gluon vertex — vanishing gluon momentum



Conclusions

Summary

- standard DSE techniques can be used to treat non-Gaussian wave functionals
- three-gluon kernel and gluon loop contribution to the gap equation have been determined
- effects of the terms have been estimated

Outlook

- include quarks
- try other types of ansatzes
- include non-trivial ghost-gluon vertex?