Phenomenology of and nonperturbative approaches to heavy mesons

Bruno El-Bennich, São Paulo

ERICE INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS

33rd course

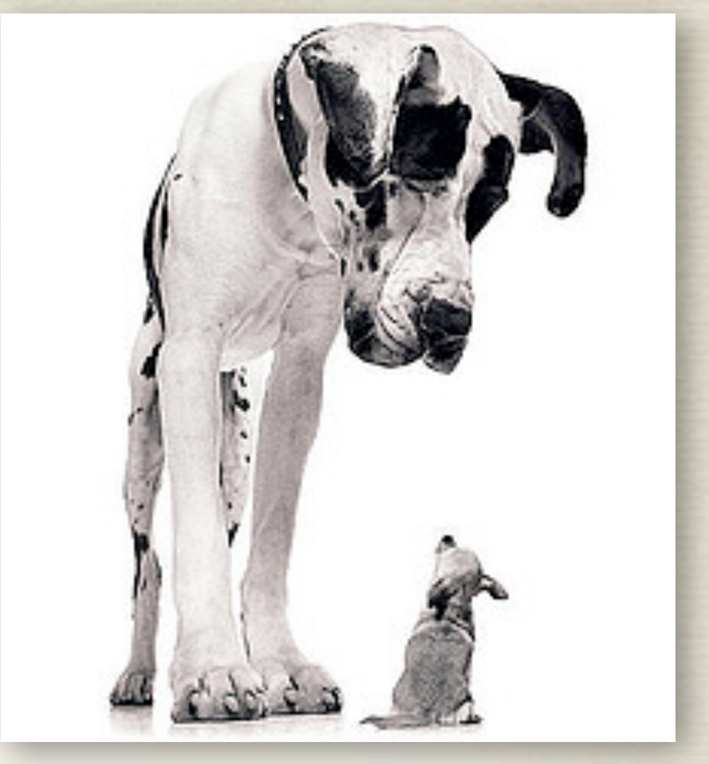
From Quarks and Gluons to Hadrons and Nuclei

Universidade Cruzeiro do Sul

unesp UNIVERSIDADE ESTADUAL PAULISTA "JÚLIO DE MESQUITA FILHO" Campus de São Paulo

FAPESP Fundação de Amparo à Pesquisa do Estado de São Paulo

What about heavy flavours?

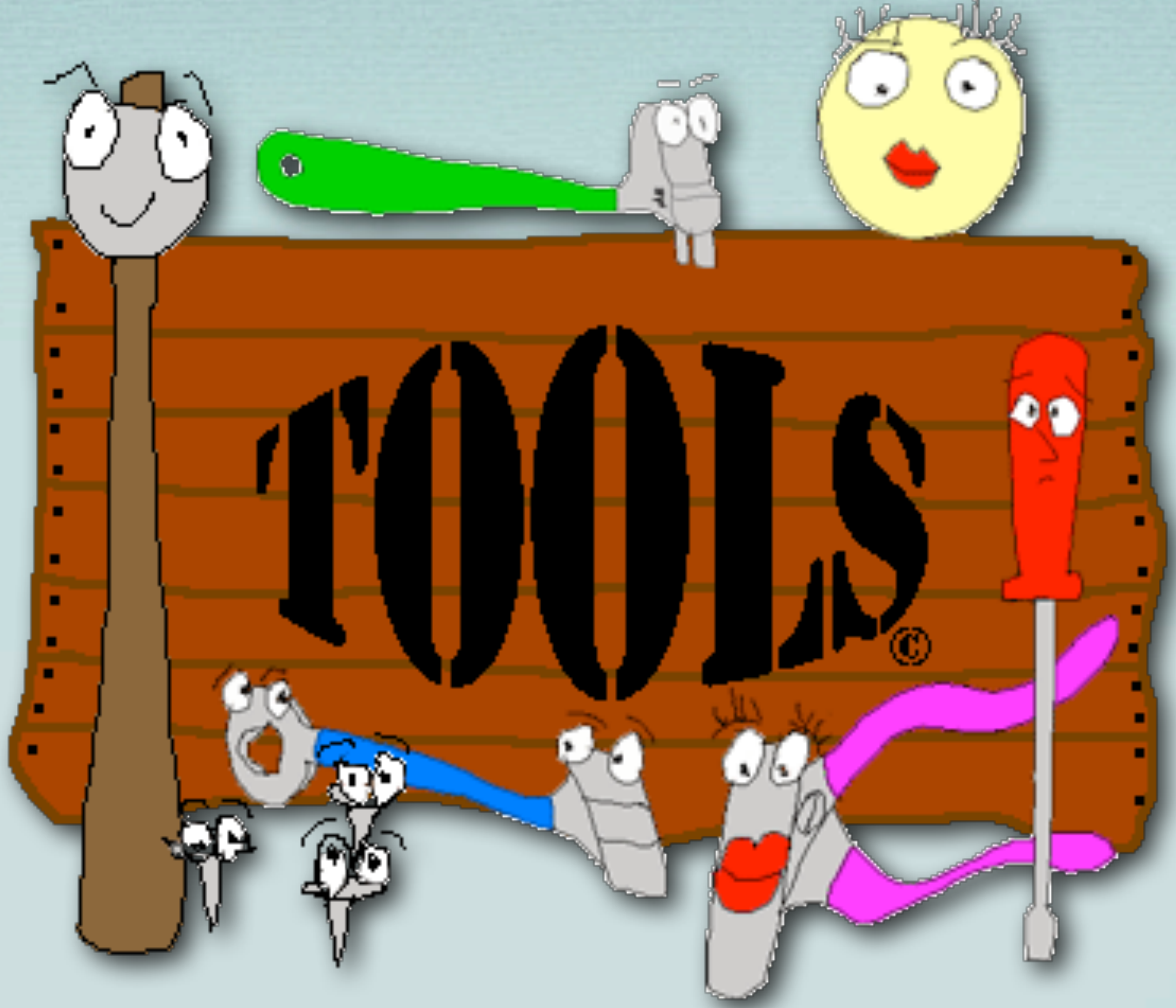


- Nonperturbative QCD contributions to strong *and* weak beauty and charm meson decays are commonly described by form factors which codify part of the strong phases.

- These phases can significantly alter *CP* violating amplitudes in $B_{(s)}$ or $D_{(s)}$ decays, whether in oscillation or direct; their precise knowledge is crucial for LHC*b* and SuperKEKB.

- Charm decays are particularly afflicted by hadronic uncertainties since corrections to the decay amplitudes of order $\Lambda_{\mathrm{QCD}}\,/\,m_c$ are far from being under control.

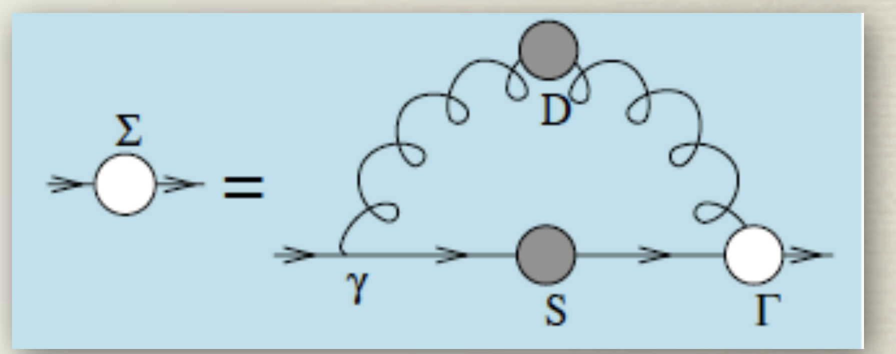
- Charmonium production rate at RHIC is sensitive to existence and properties of the intermediate "quark-gluon plasma" **and** to final-state interactions (J/ψ suppression).



The Tools of Field Theory

Flavoured mass gap

The propagator can be obtained from QCD's gap equation; namely the Dyson-Schwinger Equation for the dressed-fermion self-energy:



Gap Equation

$$
\begin{aligned}
S^{-1}(p) &= Z_2(i\gamma\cdot p + m^{\mathrm{bm}}) + \Sigma(p) := i\gamma\cdot p\, A(p^2) + B(p^2) \\
\Sigma(p) &= Z_1 \int^{\Lambda} \frac{d^4q}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma^a_\nu(q,p)
\end{aligned}
$$

with the *running* mass function $M(p^2) = B(p^2)/A(p^2)$.

$$S^{-1}(p)|_{p^2=\zeta^2} = i\gamma\cdot p + m(\zeta)$$

where ζ is the renormalization point.

$D_{\mu\nu}$: dressed-gluon propagator

$\Gamma^a_\nu(q,p)$: dressed quark-gluon vertex

Z_2 : quark wave function renormalization constant

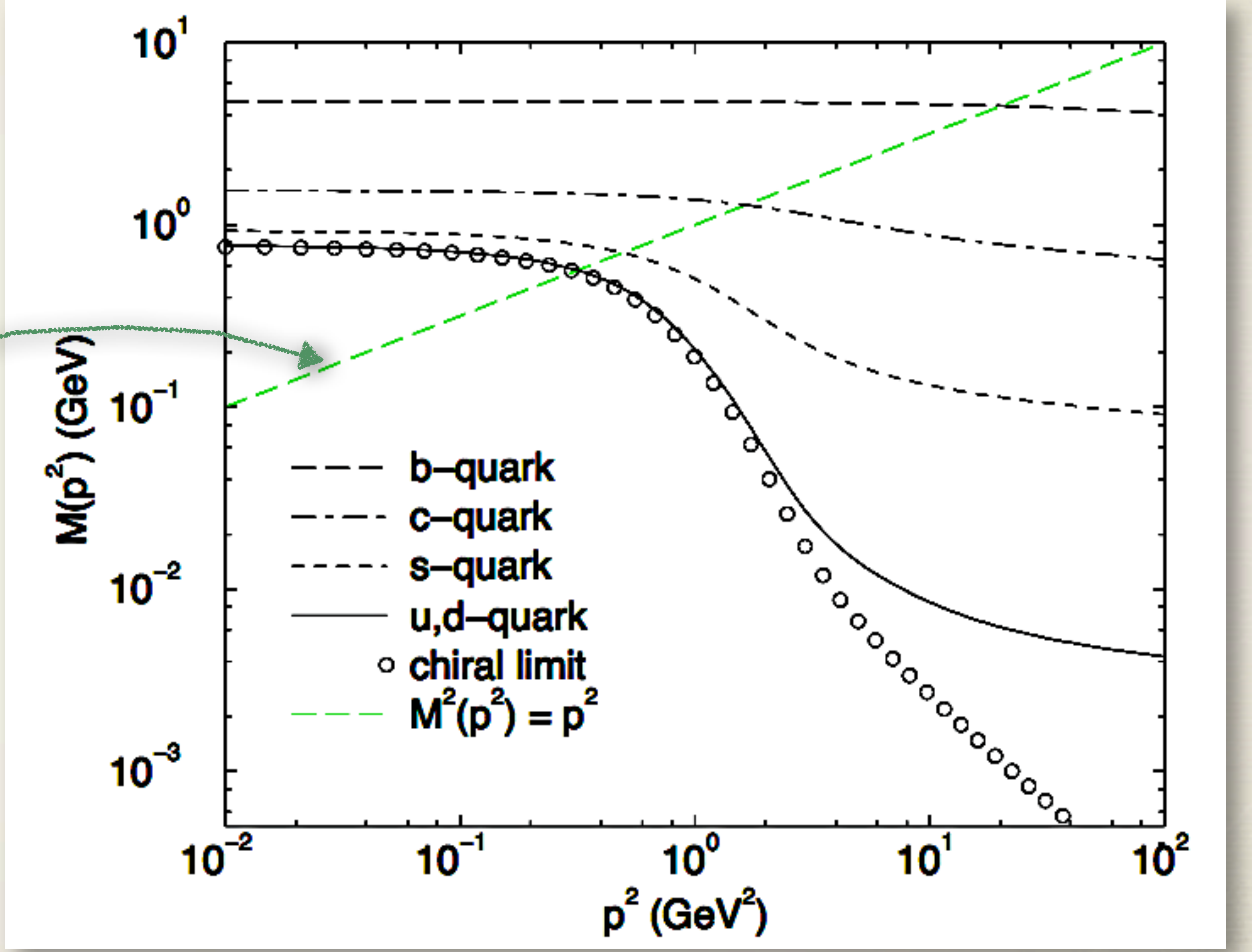
Z_1 : quark-gluon vertex renormalization constant

$\Rightarrow$ see talks by Christian Fischer, Tobias Göcke, Gernot Eichmann and Stefan Strauss

The large current-quark mass of the *b* quark almost entirely suppresses momentum-dependent dressing, so that $M_b(p^2)$ is nearly constant on a substantial domain. This is true to a lesser extent for the *c* quark.

$$(\hat{M}^E)^2 = \{s | s + M^2(s) = 0\}$$

Euclidean mass definition



Euclidean mass, sigma term

Define a single quantitative measure, namely the renormalization-point invariant ratio $\boxed{\zeta_f := \frac{\sigma_f}{M_f^E}}$

with the constituent-quark $\boldsymbol{\sigma}$ term: $\boxed{\sigma_f := m_f(\zeta)\frac{\partial M_f^E}{\partial m_f(\zeta)}}$ $\quad \sigma_f \xrightarrow{\hat{m}\to 0} 0$

The solutions of $(\hat{M}^E)^2 = \{s|s + M^2(s) = 0\}$ in GeV:

f	chiral	u,d	s	c	b
M_f^E	0.42	0.42	0.56	1.57	4.68

The ratio ζ_f thus quantifies the effect of explicit chiral symmetry breaking on the dressed-quark mass function compared with the sum of the effects of explicit and dynamical chiral symmetry breaking.

f	u,d	s	c	b
ζ_f	0.02	0.23	0.65	0.8

Reasonable approximation : $\quad S_{Q=c,b} \approx \dfrac{1}{i\gamma\cdot p + \hat{M}_Q}$

Bether-Salpeter amplitudes

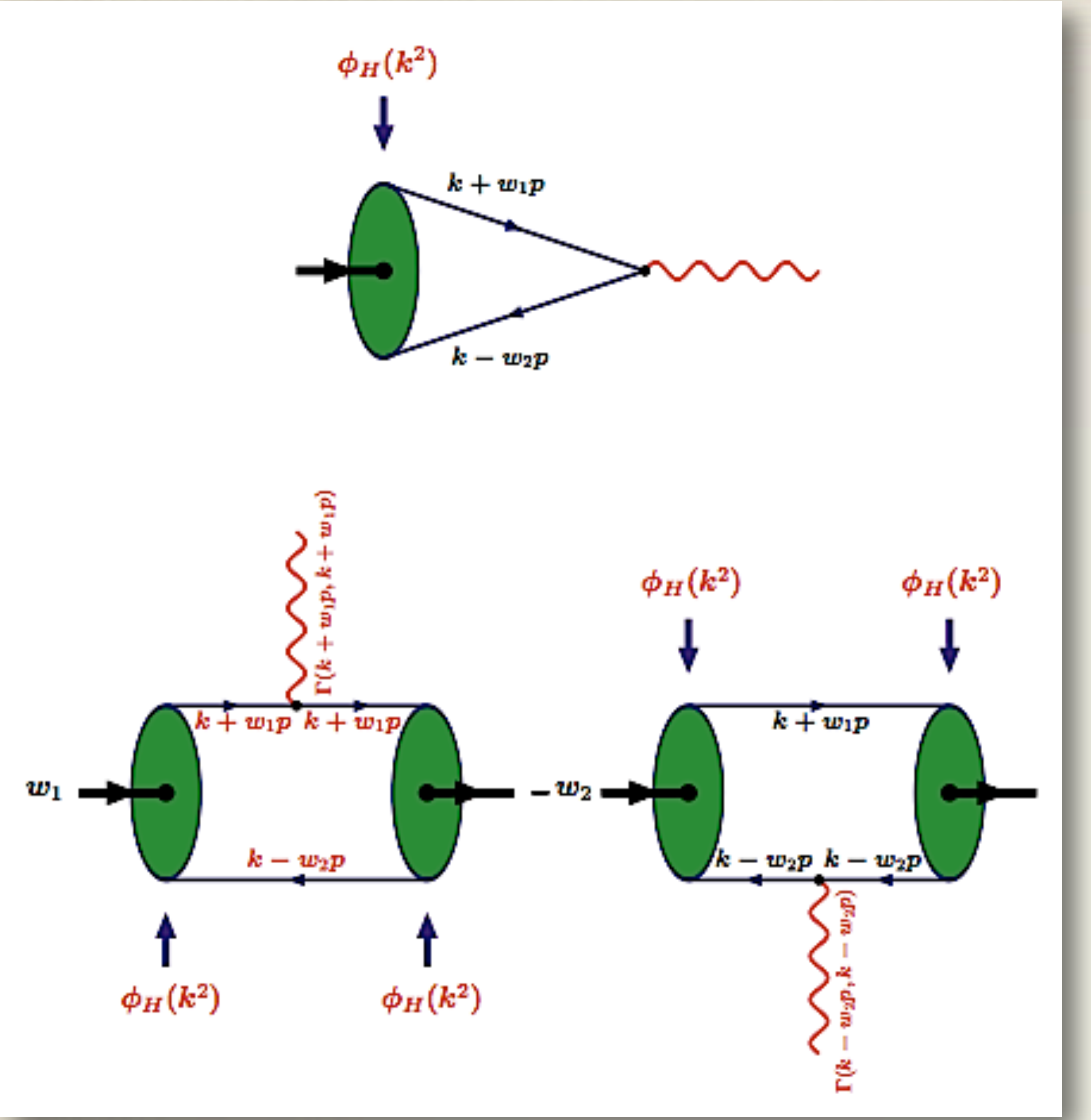
The Bethe-Salpeter Equation (BSE) provides a Poincaré covariant tool to generate bound states in quantum field theory; the inhomogeneous BSE for the pseudoscalar quark-antiquark vertex is:

$$\left[\Gamma_5^j(k;P)\right]_{rs} = Z_2\,\gamma_5\frac{\tau^j}{2} + \int_q^\Lambda S_r(q_+)\,\Gamma_5^j(q;P)\,S_s(q_-)K_{rs}^{tu}(q,k;P) \qquad q_\pm = q \pm \frac{P}{2}$$

The solution is generally:

$$i\Gamma_5^j(k;P) = \frac{\tau^j}{2}\gamma_5\left[iE_5(k;P) + \gamma\cdot P\,F_5(k;P) + \gamma\cdot k\;k\cdot P\,G_5(k;P) + \sigma_{\mu\nu}k_\mu P_\nu\,H_5(k;P)\right]$$

- Simultaneous solutions of the quarks's DSE and the heavy meson's BSA with renormalisation-group improved ladder truncations for heavy-light systems prove to be problematic.
- The treatment of the full BSA is costly; for vector mesons one has *eight* amplitudes with pole structure.



Pseudoscalar and vector decay constants

$$P_\mu f_{ps} = \sqrt{2}\,\mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\gamma_5\gamma_\mu S_1(k+w_1P)\Gamma^{ps}(k;P)S_2(k-w_2P)$$

$$M_V f_V = \frac{\sqrt{2}}{3}\,\mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\gamma_\mu S_1(k+w_1P)\Gamma^{V}(k;P)S_2(k-w_2P)$$

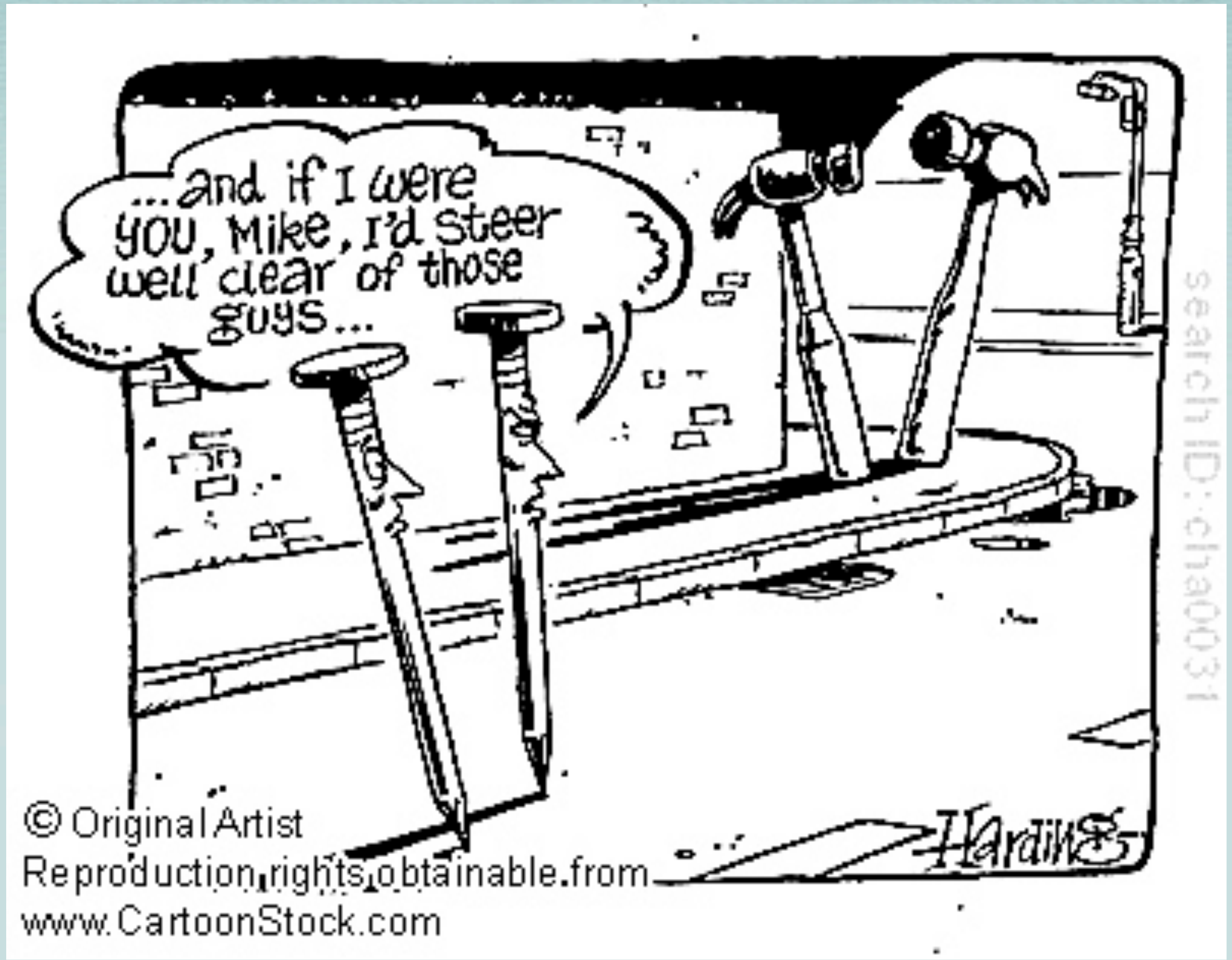
$$\begin{aligned} \Gamma^H(k;P) &:\ \text{Bethe-Salpeter amplitude} \\ S_i(k;P) &:\ \text{Dressed quark propagators} \\ w_1+w_2=1 &:\ \text{Momentum partition parameters} \end{aligned}$$

Bethe-Salpeter meson amplitude normalization

$$\begin{aligned} 2P_\mu &= \left[\frac{\partial}{\partial K_\mu}\Pi(P,K)\right]_{K=P}^{P^2=-m_{ps}^2} \\ \Pi(P,K) &= \mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\bar{\Gamma}_{ps}(k;-P)S_1(k+w_1K)\Gamma_{ps}(k;P)S_2(k-w_2K) \end{aligned}$$

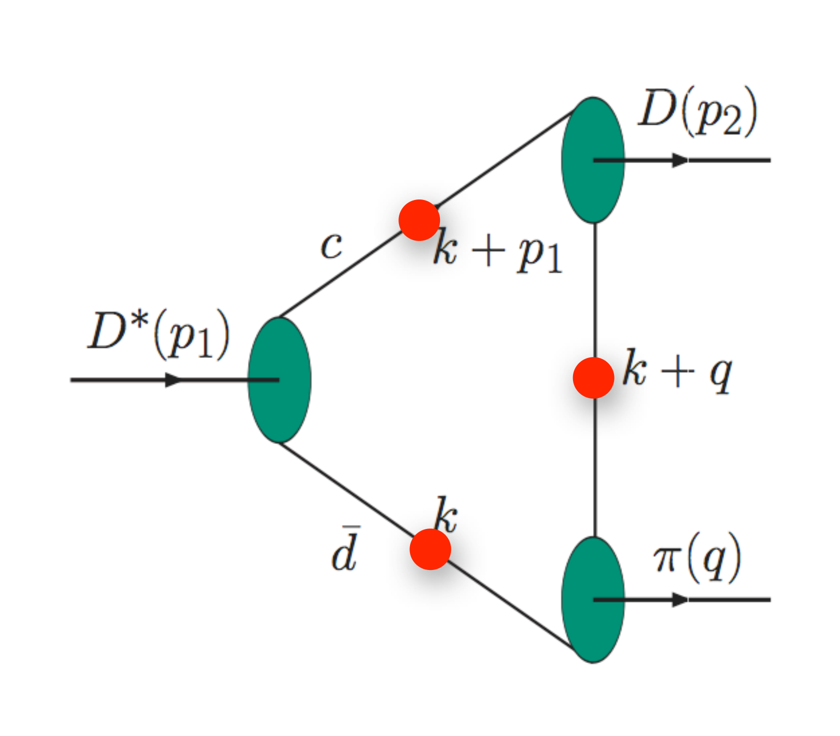
Caveat

- No assumption of heavy-quark symmetry; no static heavy-quark approximation.
- In particular, pseudoscalar and vector meson masses and decay constants are not degenerate.
- Although impulse approximation and truncations are employed, $\Lambda_{\rm QCD}/m_{c,b}$ contributions are included.

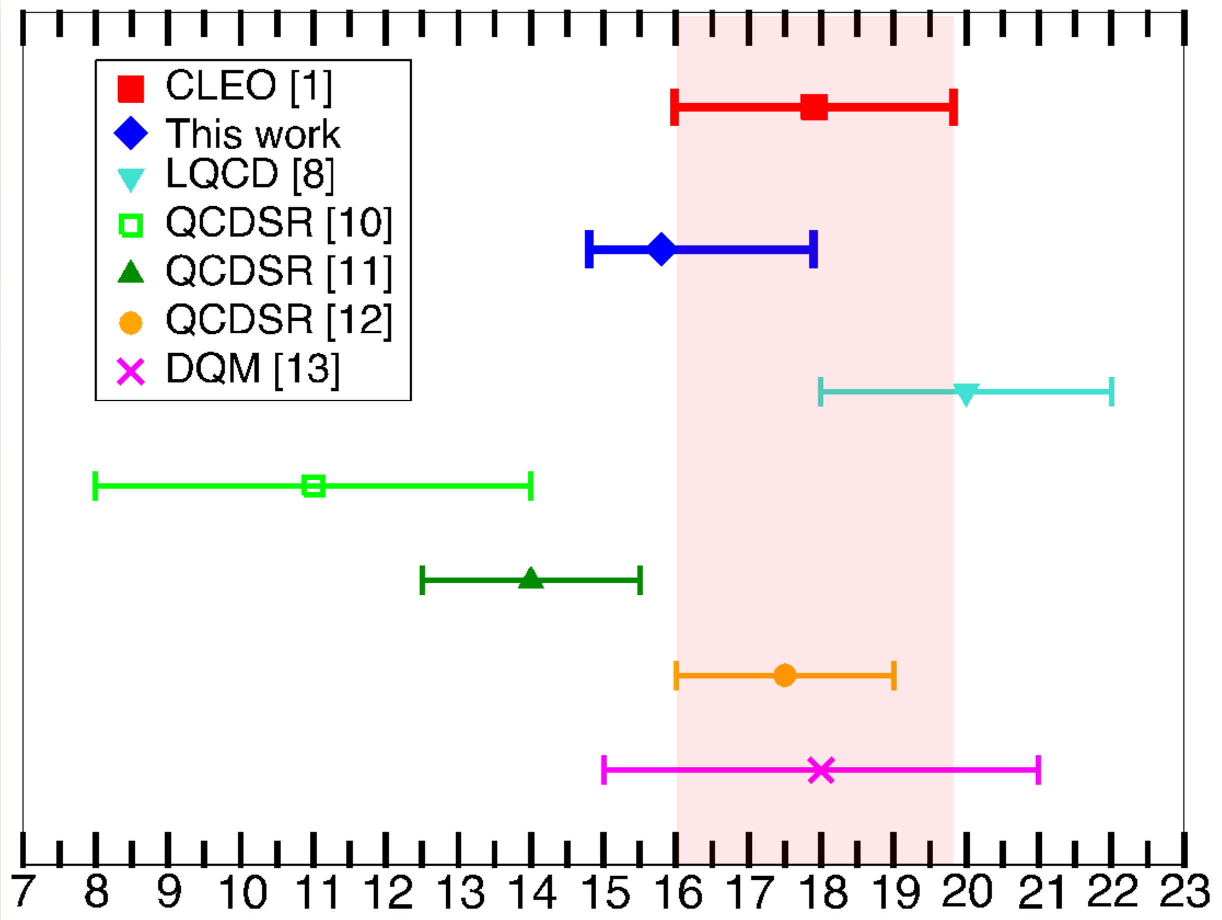


Using the Toolbox

Strong decays: $D^* \to D\pi$



B. El-Bennich, M.A. Ivanov and C.D. Roberts, Phys.Rev. C83 (2011)



$$A(D^* \to D\pi) = \epsilon_\mu^{\lambda_{D^*}}(p_{D^*}) M^\mu(p_D^2, p_{D^*}^2) := \epsilon_\mu^{\lambda_{D^*}}(p_{D^*}) p_D^\mu \, g_{D^* D\pi}$$

Coupling yields D^* width

$$M^\mu(p_D^2, p_{D^*}^2) = N_c \,\mathrm{tr} \int^\Lambda \frac{d^4k}{(2\pi)^4} \bar{\Gamma}_D(k; -P_D) S_c(k + P_{D^*}) i\Gamma_{D^*}^\mu(k; P_{D^*}) S_u(k) \bar{\Gamma}_\pi(k; -Q_\pi) S_u(k + Q_\pi)$$

The coupling $g_{H^*H\pi}$ can be calculated even if decay is kinematically forbidden, since for B^*: $m_{B^*} - m_B \approx 46$ MeV

Unphysical decay $B^* \to B\pi$ to extract effective heavy quark coupling

Heavy Quark Effective Theory

- Dynamics is constrained by heavy quark symmetry.
- Blind to the heavy quark flavor and spin.
- Heavy pseudoscalar and vector mesons are mass degenerate.
- Can be improved upon — take into account light degrees of freedom, chiral symmetry breaking $\Rightarrow$ HMChPT.

$$\mathcal{L}_{\text{heavy}} = -\mathrm{tr}_a \mathrm{Tr}[\bar{H}_a iv\cdot D_{ba} H_b] + \hat{g}\, \mathrm{tr}_a \mathrm{Tr}[\bar{H}_a H_b \gamma_\mu \mathbf{A}^\mu_{ba} \gamma_5]$$

$$D^\mu_{ba} H_b = \partial_\mu H_a - H_b \frac{1}{2}[\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger]_{ba} \;;$$

$$\mathbf{A}^{ab}_\mu = \frac{i}{2}\left[\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger\right]_{ab} ;$$

$$H_a(v) = \frac{1+\not{v}}{2}\left[P^{*a}_\mu(v)\gamma_\mu - P^a(v)\gamma_5\right];$$

$$\xi = \exp(i\Phi/f^0_\pi) \;;$$

Φ is matrix of $N_f^2 - 1$ pseudo-Goldstone boson.

R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Rept. **281**, 145 (1997)

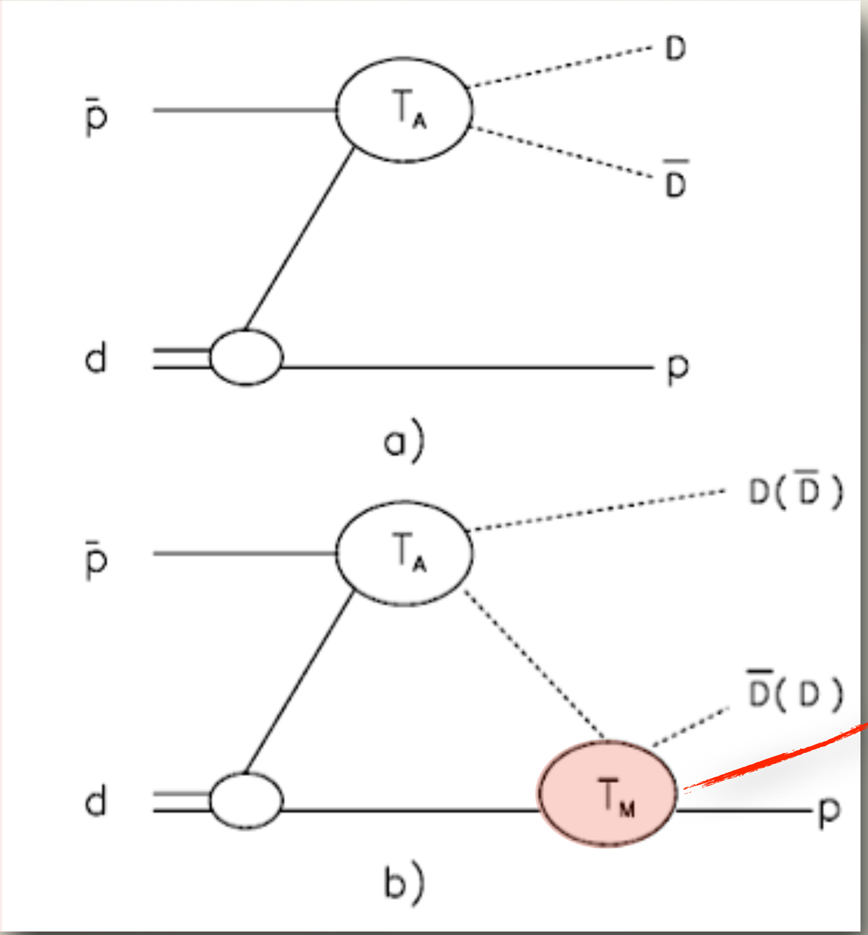
$$\hat{g} = \frac{g_{B^*B\pi}}{2\sqrt{m_B m_{B^*}}} f_\pi = 0.37^{+0.04}_{-0.02}$$

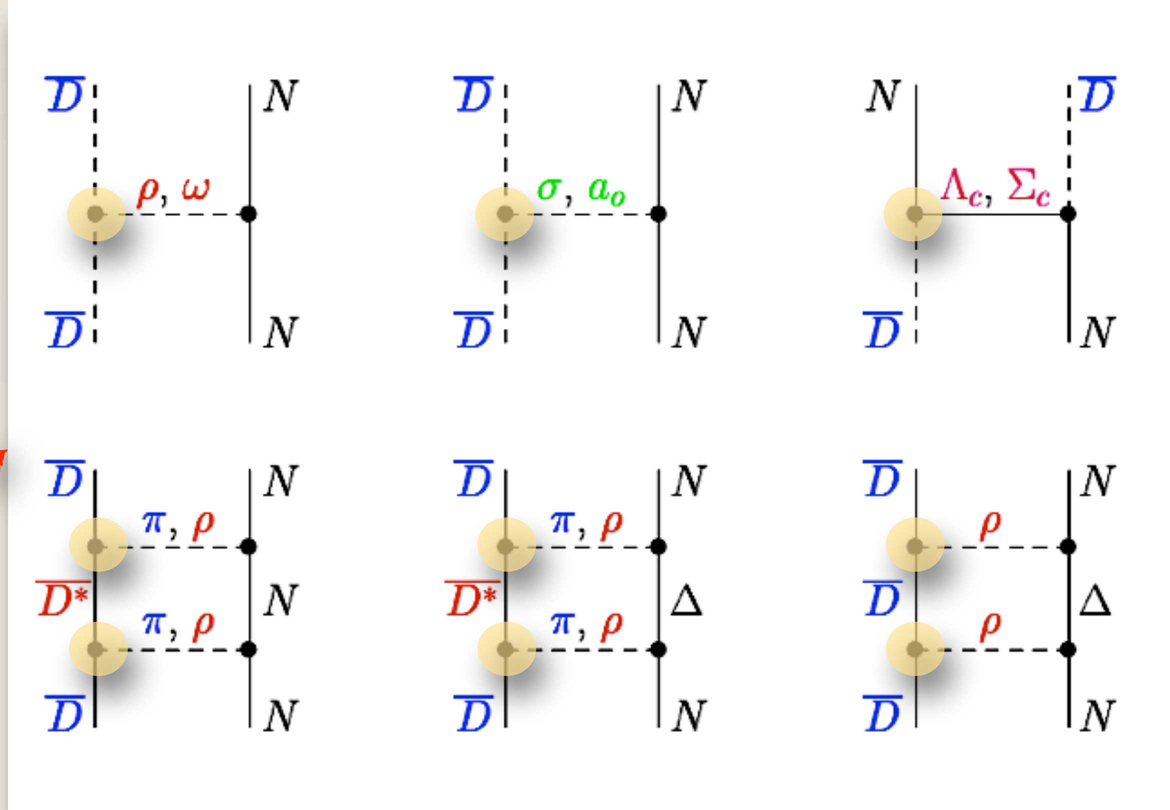
B. El-Bennich, M.A. Ivanov and C.D. Roberts, Phys.Rev. **C**83 (2011)

LQCD: $\hat{g} = \frac{g_{B^*B\pi}}{2\sqrt{m_B m_{B^*}}} f_\pi = 0.44 \pm 0.03 \pm 0.07,$

D. Bećirević et al. (2009)

Antiproton annihilation on the deuteron (PANDA @ FAIR)





Heavy Meson Effective Lagrangians

$$\mathcal{L}_{\pi PP^*} = 2\frac{g_\pi}{f_\pi}(P_a^\dagger P^*_{b\,\mu} + P^{*\,\dagger}_{a\,\mu} P_b)\partial^\mu \hat{\pi}_{ab}$$

$$\mathcal{L}_{\pi P^* P^*} = 2i\frac{g_\pi}{f_\pi}\epsilon^{\mu\nu\alpha\beta} v_\mu P^{*\,\dagger}_{a\,\beta} P^*_{b\,\nu}\partial_\alpha \hat{\pi}_{ab}$$

$$\mathcal{L}_{vPP} = -\sqrt{2}\beta g_V P_b P_a^\dagger v\cdot\hat{\rho}_{ba}\,,$$

$$\mathcal{L}_{vPP^*} = -2\sqrt{2}\lambda g_V v_\mu \epsilon^{\mu\nu\alpha\beta}\left(P_a^\dagger P^*_{b\,\beta} - P^{*\,\dagger}_{a\,\beta} P_b\right)\partial_\nu(\hat{\rho}_\alpha)_{ba}\,,$$

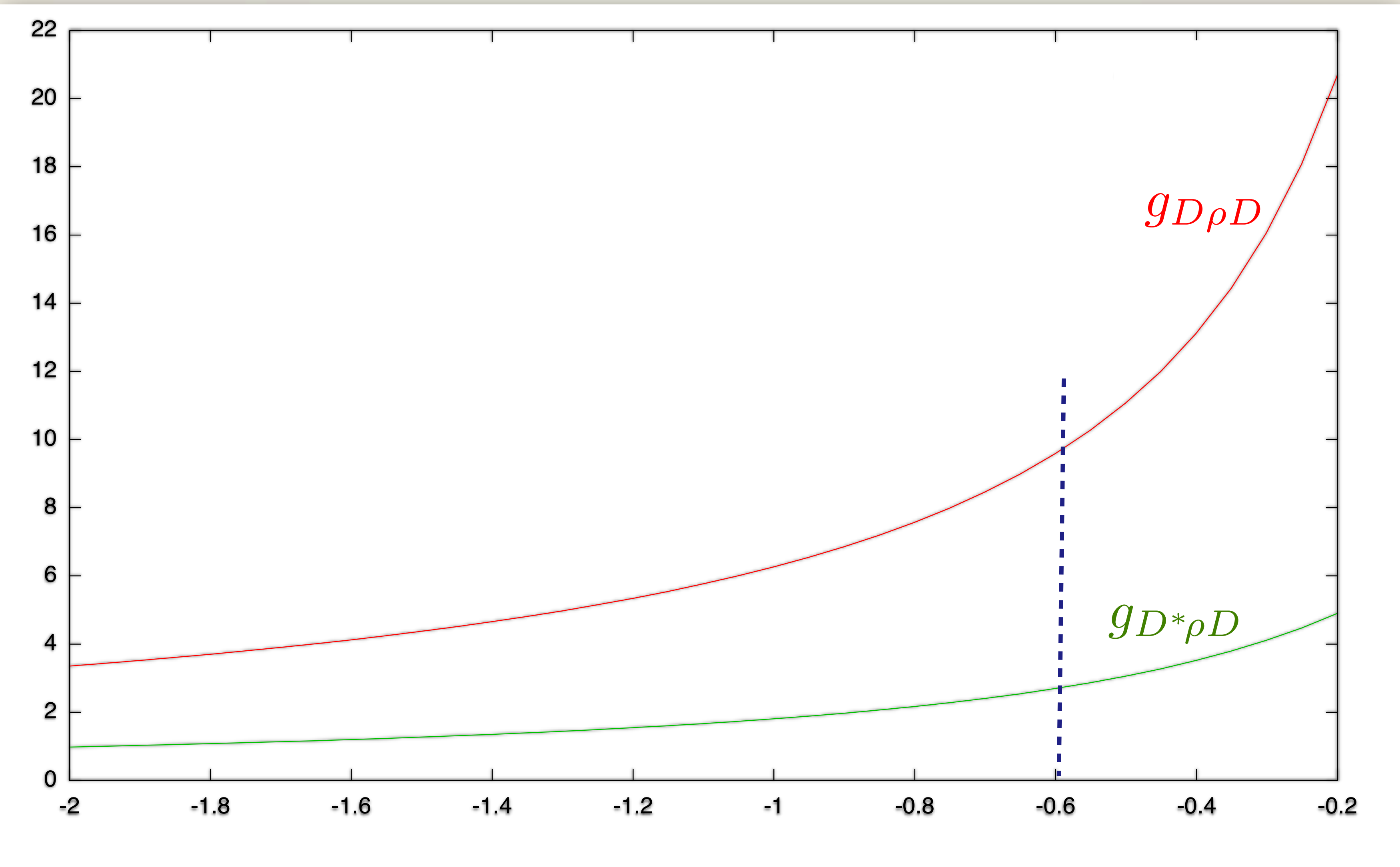
$$\begin{aligned}\mathcal{L}_{vP^*P^*} &= \sqrt{2}\beta g_V P^*_b P^{*\dagger}_a v\cdot\hat{\rho}_{ba} \\ &\quad + i2\sqrt{2}\lambda g_V P^{*\,\dagger}_{a\,\mu} P^*_{b\,\nu}(\partial^\mu(\hat{\rho}^\nu)_{ba} - \partial^\nu(\hat{\rho}^\mu)_{ba})\,.\end{aligned}$$

Y. Yamaguchi, S. Ohkoda, S. Yasui and A. Hosaka, Phys.Rev. D84 (2011);
G. Krein, A. Thomas, and K. Tsushima, Phys. Lett. B697, 136 (2011).

Generalization to higher spin structures

- $D^* \to D\pi$ can be generalized to three-point functions for vector mesons with off-shell momenta.
- Couplings $D^*\pi D$, $D^*\Phi D$, $D^*\rho D$, $D^*\rho D^*$ needed in final-state interactions with intermediate $D - D^*$ states.

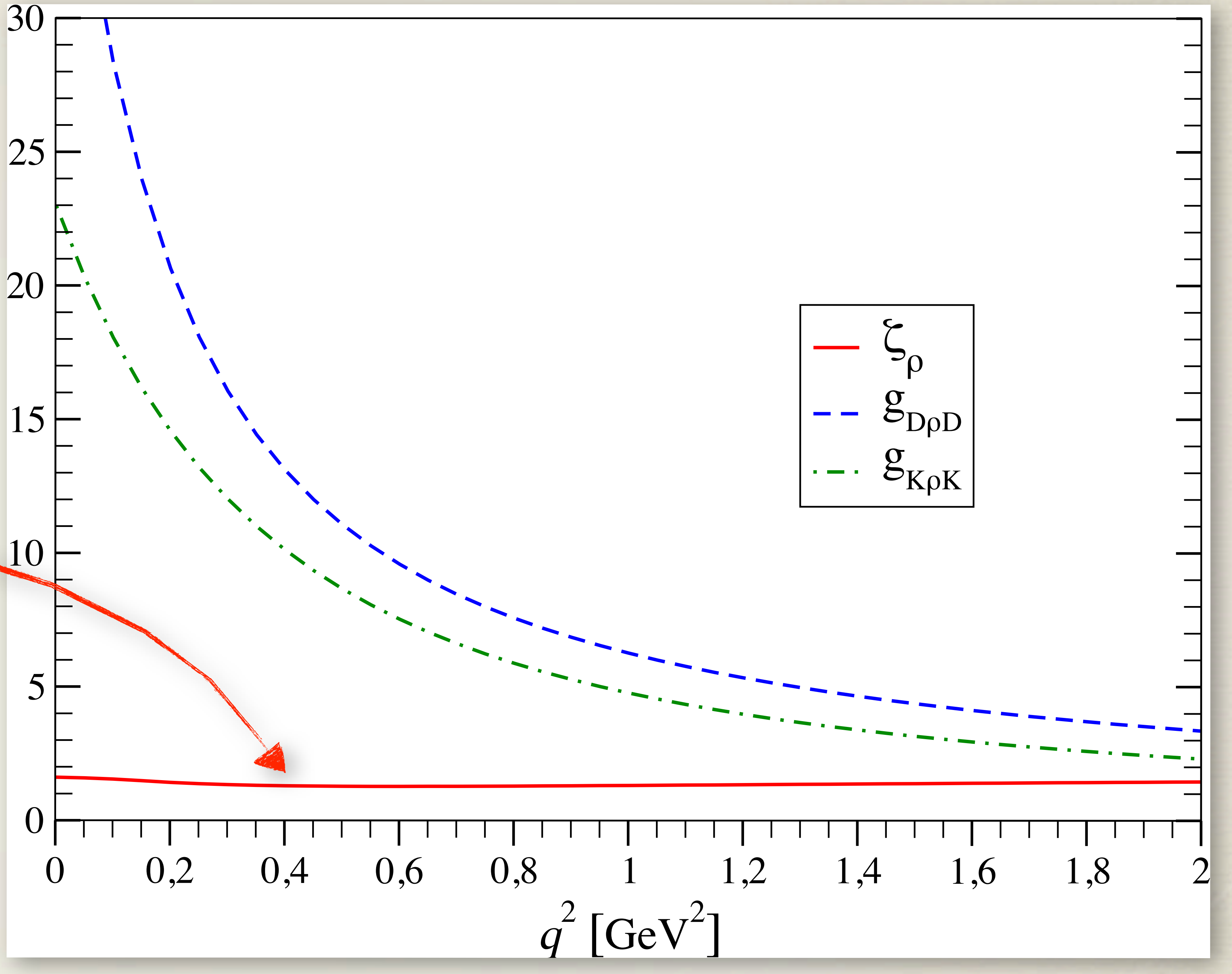
$D^*\rho D$ coupling : $\mathcal{M} \equiv \varepsilon^{\rho}_{\alpha}\varepsilon^{D^*}_{\beta} M^{\alpha\beta}(p^{D^*}, p^{\rho}) = \epsilon^{\alpha\beta\mu\nu}\varepsilon_{\alpha}(p^{D^*})\varepsilon_{\beta}(p^{\rho})\, p^{D^*}_{\mu} q^{\rho}_{\nu} \dfrac{g_{D^*\rho D}}{m_{D^*}}$



Ratios of couplings as measure of flavour symmetry breaking

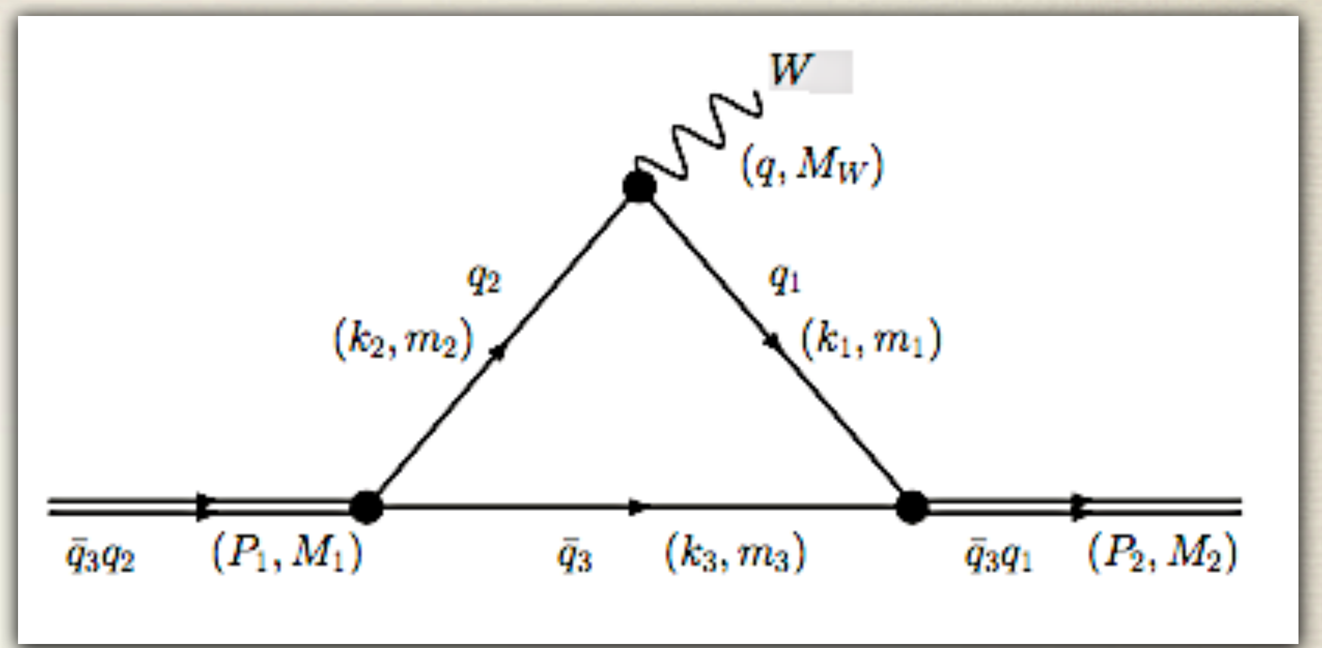
Define $\zeta_\rho := \dfrac{g_{D\rho D}(q^2)}{g_{K\rho K}(q^2)}$

Ratio almost constant over large domain, measures the effect of SU(4) breaking ≈ 30% – 50%



B. El-Bennich, G. Krein and C.D. Roberts (in preparation).

Heavy-to-light transition form factors



- IN principle obtained from the Poincaré- and gauge-invariant three-point correlation functions with fully dressed light-quark propagators and quark-gluon vertices.

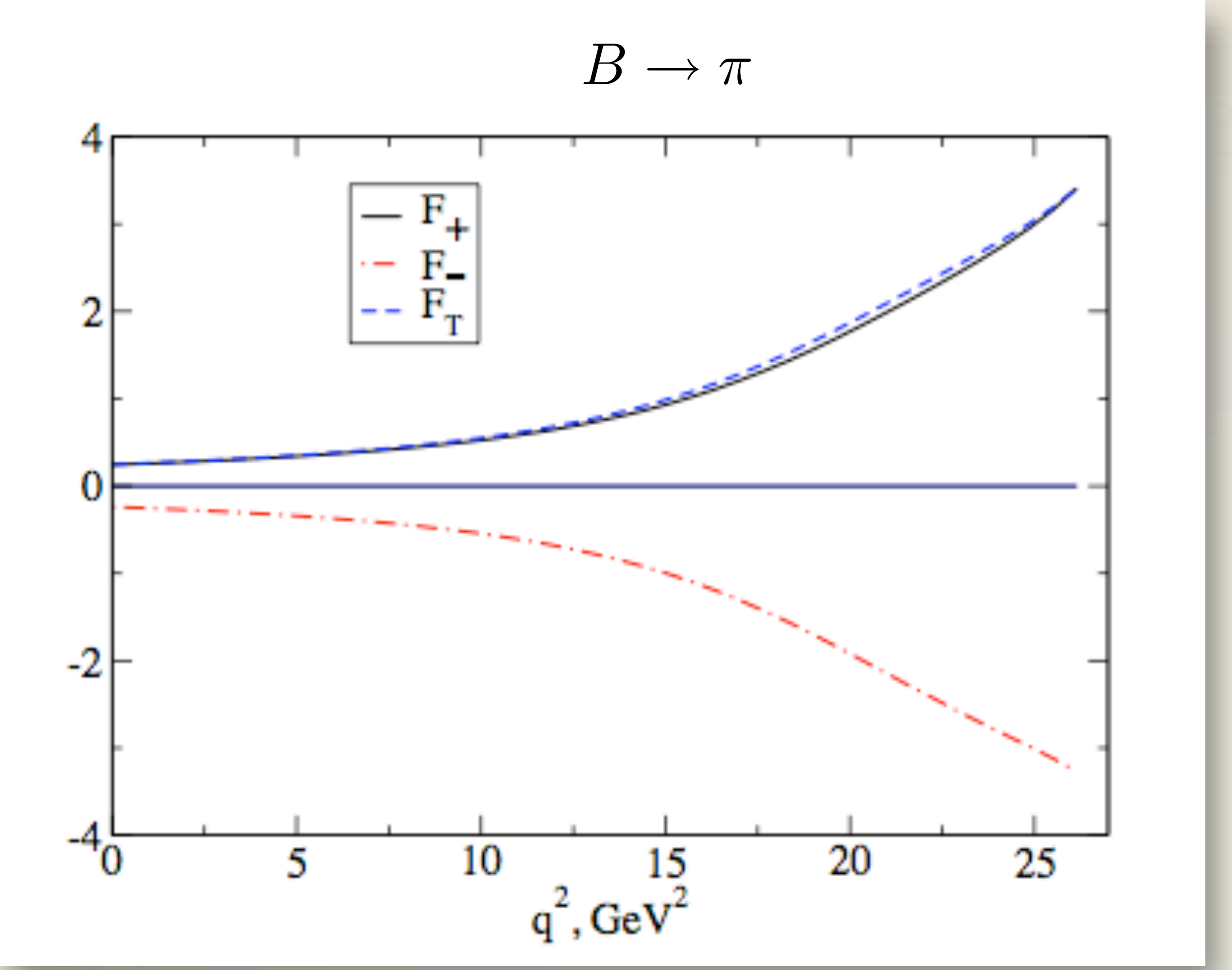
- IN practice these amplitudes must be truncated and modeling to a certain degree is unavoidable *now*.

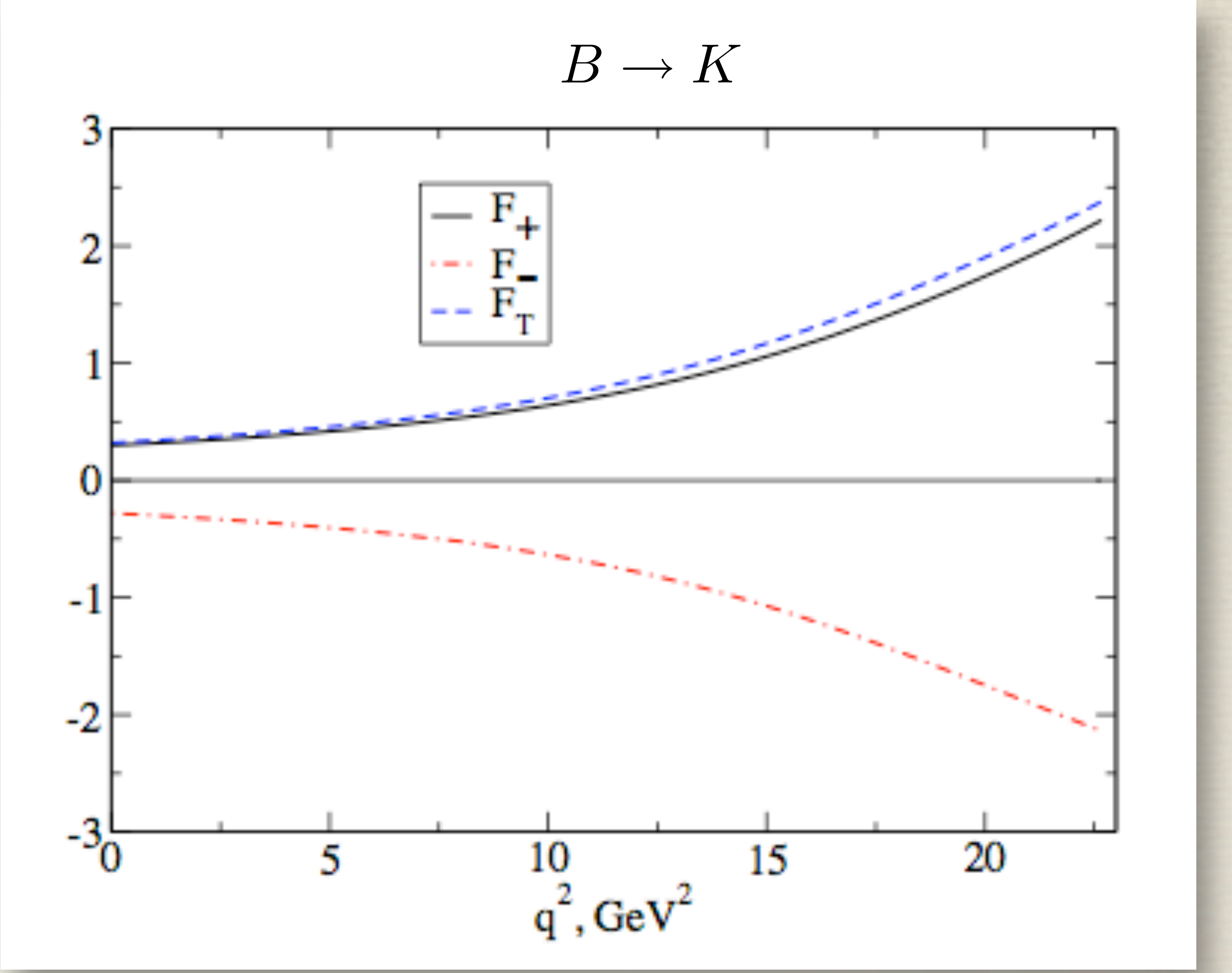
- IN Dyson-Schwinger equation models, transition amplitudes using the rainbow ladder truncation are calculated in impulse approximation; a problem is the treatment of the heavy-light BSA for heavy meson bound states. $B \to \pi, K, \rho, \phi$: M. A. Ivanov et al. (2007).

- IN Lattice QCD, form factors are obtained on ever fine lattices, dynamical light quarks *and* heavy quark mass are still a problem ⇒ chiral extrapolation of light quarks, heavy quarks sometimes treated as static; calculation only at large recoil $q^2 > 15$ GeV^2, extrapolated with pole models to $q^2 = 0$.

$B \to \pi, K$: E. Dalgic *et al.* (2006); C. Aubin *et al.* [Fermilab, MILC & HPQCD Collab.] (2005); A. Abada *et al.* (Orsay-Rome), (2001).

Example of applications of DSE to flavour physics: transition form factors





$$\langle M(p_M)|\bar{q}_i\gamma_\mu(1-\gamma_5)b|B(p_B)\rangle := F_+(q^2)(p_B+p_M)_\mu + F_-(q^2)(p_B-p_M)_\mu \;; \quad q = p_B - p_M$$

$$\langle M(p_M)|\bar{q}_i\,\gamma_\mu(1-\gamma_5)b|B(p_B)\rangle = \left(p_B + p_M - \frac{m_B^2 - m_M^2}{q^2}q\right)_\mu F_1^{B\to M}(q^2) + \frac{m_B^2 - m_M^2}{q^2}q_\mu\; F_0^{B\to M}(q^2)$$

$$q = p_B - p_M; \quad q_i = u, d, s$$

Epilogue

- Form factors are the single important source of uncertainties and much work is left to pin down theoretical uncertainties to a few percent in beauty and charmonium physics.
- Despite important theoretical progress in perturbative QCD and in particular of factorization theorems, uncertainties of hadronic nature still blur this success:
  how well are the strong phases known in these decays?
- One aspect learned over and over again is: heavy-quark symmetry is not correct in charmed mesons and $\Lambda_{\rm QCD}/m_{c,b}$ corrections are not negligible.
- DSE/BSA treatments of heavy mesons must go beyond the rainbow-ladder approximation.

Beyond rainbow ladder: clear indications

T. Nguyen, N. A. Souchlas and P. Tandy, arXiv:1005.3315 [nucl-th]

TABLE 2. Calculated masses and electroweak decay constants for ground state pseudoscalar and vector heavy-light mesons, together with experimental data [11], all in GeV. In the rows labelled *calc M*, the heavy quark is described by a constituent mass fit to the lightest pseudoscalar (marked by †). In the rows labelled $k_{\rm min}$, the heavy quark is dressed through the DSE with an infrared suppression of the gluon momentum as described in the text. No such infrared suppression was applied to the dressing of the light quark and the binding kernel. The bracketed values for B_s and B_c indicate that the entire $\alpha^{\rm IR}(k^2)$ term of the kernel was eliminated, as in earlier work, and this corresponds to a $k_{\rm min}$ value that is about 20% larger than the value used for B.

	D	D*	D_s	D_s^*	B	B*	B_s	B_s^*	B_c	B_c^*
expt M	1.86	2.01	1.97	2.11	5.28	5.33	5.37	5.41	6.29	?
calc M	1.85†	2.04	1.97	2.17	5.27†	5.32	5.38	5.42	6.36	6.44
with $k_{\rm min}$	1.88		1.90		5.15		(4.75)		(5.83)	
expt f	0.222	?	0.294	?	0.176	?	?	?	?	?
calc f	0.154	0.160	0.197	0.180	0.105	0.182	0.144	0.20	0.210	0.18
with $k_{\rm min}$	0.260		0.275		0.265		(0.164)		(0.453)	