Chiral symmetry breaking and the spin content of hadrons

L. Ya. Glozman

Institut für Physik, FB Theoretische Physik, Universität Graz

With Christian Lang and Markus Limmer .

PRL 103 (2009) 121601

PRD 82 (2010) 097501

1106.1010 - submitted to PLB

Contents of the Talk

- The "spin crisis".
- How to define the chiral and the spin content? Mesons.
- Chiral and spin content from lattice simulations.
- Conclusions.

The "spin crisis"

From the parton distributions in the infinite momentum frame one obtains that only about 30% of the nucleon spin is carried by the spins of quarks.

Similar results within the same definition are obtained on the lattice both for nucleon and mesons.

Where is the rest of the nucleon spin?

Is it a true physical result or a consequence of an inadequate definition of the spin content?

Chiral content of mesons

One needs a complete and orthogonal basis for the $\bar{q}q$ component in mesons. Representations of $SU(2)_L \times SU(2)_R$ is such a basis.

$$J = 0$$

$$\begin{aligned} (1/2,1/2)_a &: \quad 1,0^{-+} \longleftrightarrow 0,0^{++} \\ (1/2,1/2)_b &: \quad 1,0^{++} \longleftrightarrow 0,0^{-+}, \end{aligned}$$

Even $J > 0$

$$\begin{aligned} (0,0) &: \quad 0,J^{--} \longleftrightarrow 0,J^{++} \\ (1/2,1/2)_a &: \quad 1,J^{-+} \longleftrightarrow 0,J^{++} \\ (1/2,1/2)_b &: \quad 1,J^{++} \longleftrightarrow 0,J^{-+} \\ (0,1)\oplus(1,0) &: \quad 1,J^{++} \longleftrightarrow 1,J^{--} \end{aligned}$$

Odd $J > 0$

$$\begin{aligned} (0,0) &: \quad 0,J^{++} \longleftrightarrow 0,J^{--} \\ (1/2,1/2)_a &: \quad 1,J^{+-} \longleftrightarrow 0,J^{--} \\ (1/2,1/2)_b &: \quad 1,J^{--} \longleftrightarrow 0,J^{+-} \\ (0,1)\oplus(1,0) &: \quad 1,J^{--} \longleftrightarrow 1,J^{++} \end{aligned}$$

L.Ya.G., Phys. Lett. B 587 (2004) 69

Chiral content of mesons

Example: $\rho(I, J^{PC} = 1, 1^{--})$

$$(1,0)+(0,1): \quad \mathcal{O}_V = \bar{q}\tau\gamma^i q = \bar{R}\tau\gamma^i R + \bar{L}\tau\gamma^i L$$

$$(1/2,1/2)_b: \quad \mathcal{O}_T = \bar{q}\tau\sigma^{0i} q = \bar{R}\tau\sigma^{0i} L + \bar{L}\tau\sigma^{0i} R$$

Chiral partners:

$$(1,0)+(0,1): \quad \rho(1,1^{--}) \longleftrightarrow a_1(1,1^{++})$$

$$(1/2,1/2)_b: \quad \rho(1,1^{--}) \longleftrightarrow h_1(0,1^{+-})$$

${}^{2S+1}L_J$ content of mesons in the rest frame

A unitary transformation exists from the chiral basis to the $\{I; {}^{2S+1}L_J\}$ basis (L.Ya.G. and A. V. Nefediev, PRD 76 (2007) 096004; PRD 80 (2009) 057901):

$$|R; IJ^{PC}\rangle = \sum_{LS} \sum_{\lambda_q \lambda_{\bar{q}}} \chi^{RPI}_{\lambda_q \lambda_{\bar{q}}} \times \sqrt{\frac{2L+1}{2J+1}} C^{S\Lambda}_{\frac{1}{2}\lambda_q \frac{1}{2} -\lambda_{\bar{q}}} C^{J\Lambda}_{L0S\Lambda} |I; {}^{2S+1}L_J\rangle.$$

$$\rho: \; |(0,1)+(1,0); 1\; 1^{--}\rangle \;=\; \sqrt{\frac{2}{3}}|1; {}^3S_1\rangle + \sqrt{\frac{1}{3}}|1; {}^3D_1\rangle,$$

$$\rho: \; |(1/2,1/2)_b; 1\; 1^{--}\rangle \;=\; \sqrt{\frac{1}{3}}|1; {}^3S_1\rangle - \sqrt{\frac{2}{3}}|1; {}^3D_1\rangle.$$

If we know the mixture of the two allowed chiral representations in a physical state, we are also able to obtain the angular momentum content in the rest frame.

Mixture of the chiral representations in a physical state.

In the continuum the corresponding amplitudes are given as

$$\langle 0|\bar{q}(0)\gamma^\mu q(0)|V(p;\lambda)\rangle = m_\rho f_\rho^V e_\lambda^\mu \,, \quad (1)$$

$$\langle 0|\left(\bar{q}(0)\sigma^{\alpha\beta} q(0)\right)(\mu)|V(p;\lambda)\rangle = i f_\rho^T(\mu) e_\lambda^\mu (e_\lambda^\alpha p^\beta - e_\lambda^\beta p^\alpha) \,, \quad (2)$$

where $V(p;\lambda)$is the ρ-state .

The vector current is conserved, consequently the vector coupling constant f_ρ^V is scale-independent.

The "tensor current" is not conserved and is subject to a nonzero anomalous dimension. Consequently the tensor coupling $f_\rho^T(\mu)$manifestly depends on the scale μ .

In the rest frame the ratio

$$\frac{f_\rho^V}{f_\rho^T(\mu)} = \frac{\langle 0|\bar{q}(0)\gamma^i q(0)|V(\lambda)\rangle}{\langle 0|\left(\bar{q}(0)\sigma^{0i} q(0)\right)(\mu)|V(\lambda)\rangle} \quad (3)$$

defines a mixture of the chiral representations in the physical state at the scale μ .

Some elements of lattice technology.

Assume we have a hadron with excitation energies $n = 1, 2, 3, ...$

$$C(t)_{ij} = \langle \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \rangle = \sum_n a_i^{(n)} a_j^{(n)*} \mathrm{e}^{-E^{(n)} t} \tag{4}$$

where

$$a_i^{(n)} = \langle 0 | \mathcal{O}_i | n \rangle \ .$$

The generalized eigenvalue problem:

$$\widehat{C}(t)_{ij} u_j^{(n)} = \lambda^{(n)}(t, t_0) \widehat{C}(t_0)_{ij} u_j^{(n)}. \tag{5}$$

Each eigenvalue and eigenvector corresponds to a given state. If a basis $\mathcal{O}_i$ is complete enough, one extracts energies and "wave functions" of all states.

$$\frac{C(t)_{ij} u_j^{(n)}}{C(t)_{kj} u_j^{(n)}} = \frac{a_i^{(n)}}{a_k^{(n)}} \ . \tag{6}$$

Chiral and ${}^{2S+1}L_J$ content of mesons on lattice

We want to study $\rho = \rho(770)$ and its first excitation $\rho' = \rho(1450)$.We need energies, chiral as well as the angular momentum decomposition of the states.
Then a sufficient basis of interpolators:

$$(1,0)+(0,1): \quad \mathcal{O}_V = \bar{q}(x)\tau\gamma^i q(x) = \bar{R}(x)\tau\gamma^i R(x) + \bar{L}(x)\tau\gamma^i L(x)$$

$$(1/2,1/2)_b: \quad \mathcal{O}_T = \bar{q}(x)\tau\sigma^{0i} q(x) = \bar{R}(x)\tau\sigma^{0i} L(x) + \bar{L}(x)\tau\sigma^{0i} R(x)$$

If local interpolators are used, then we extract the "wave functions" at the origin (more exactly, at the scale $\mu \sim (lattice\ spacing)^{-1}$ fixed by the lattice spacing):

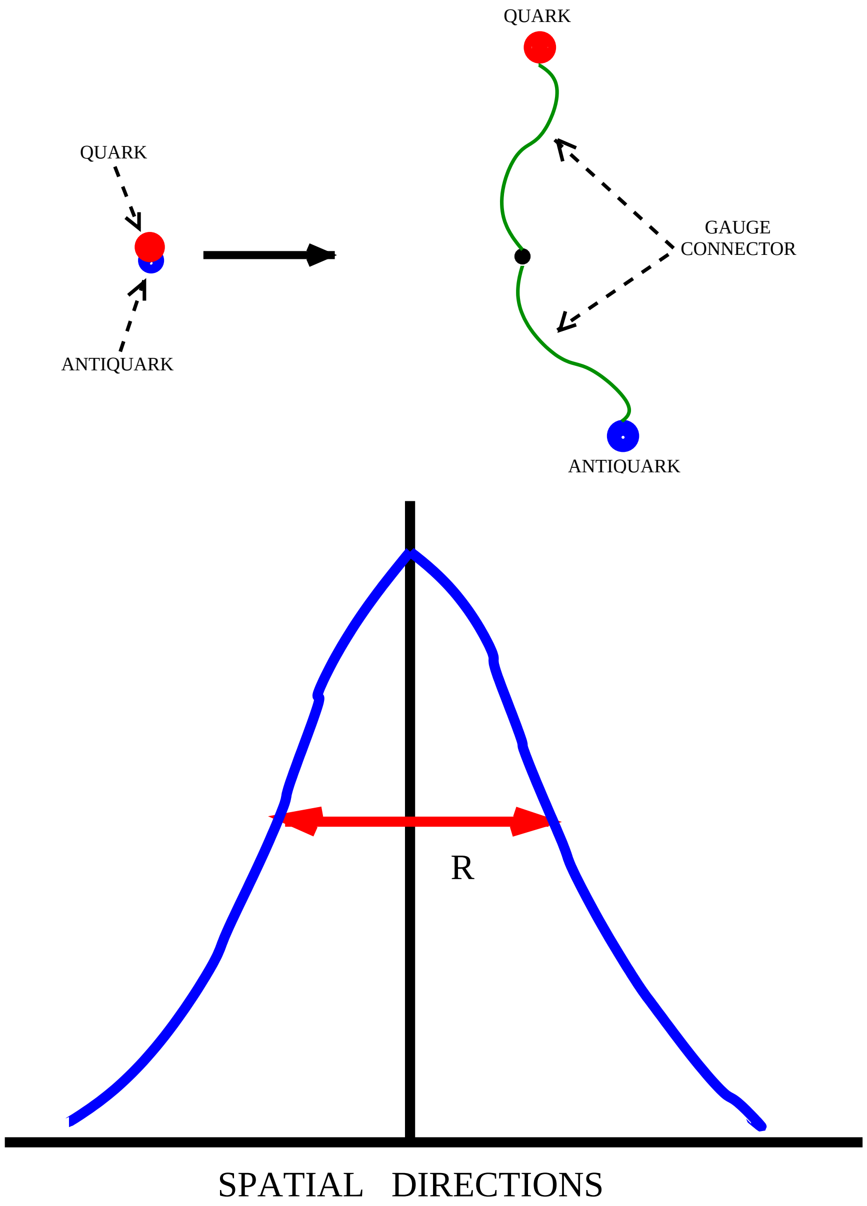
$$a_i^{(n)} = \langle 0 \mid \mathcal{O}_i(\mu) \mid n \rangle \ .$$

But we want to know the "wave functions" at the infrared scales, where mass is generated! What to do?

Smear the local interpolators in spatial directions over the range R in a gauge-invariant way. Then you will study a hadron "wave function" at the resolution scale fixed by R.

Chiral and ${}^{2S+1}L_J$ content of mesons on lattice

Gauge-invariant Gaussian smearing and resolution scale R definition.



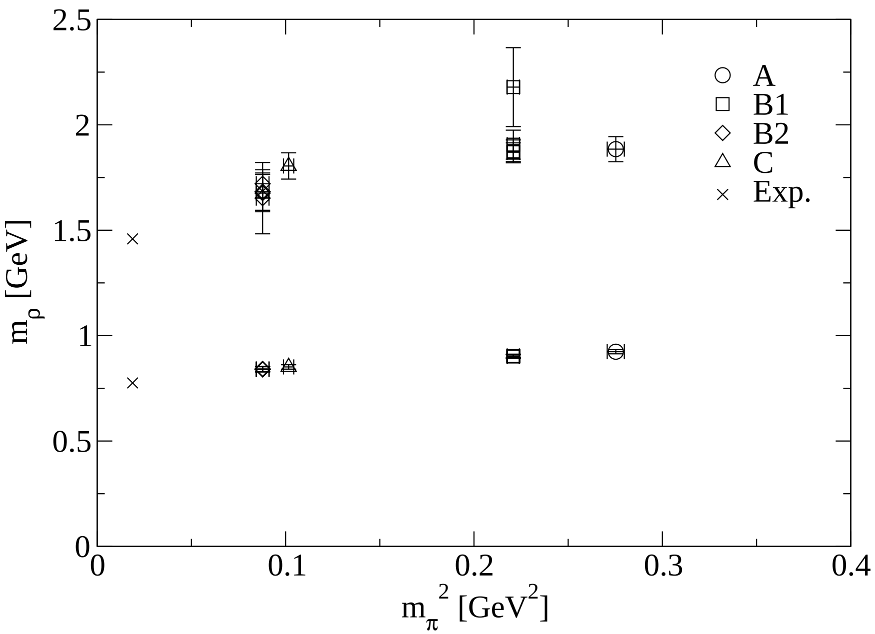
Chiral and $^{2S+1}L_J$ content of mesons on lattice

Lattice size 2.4 fm. Two light sea flavors. Chirally Improved Dirac operator. Three smearings: $(1) = 0.34$ fm, $(2) = 0.67$ fm and $(3) = 0.67$ fm. Six operators:

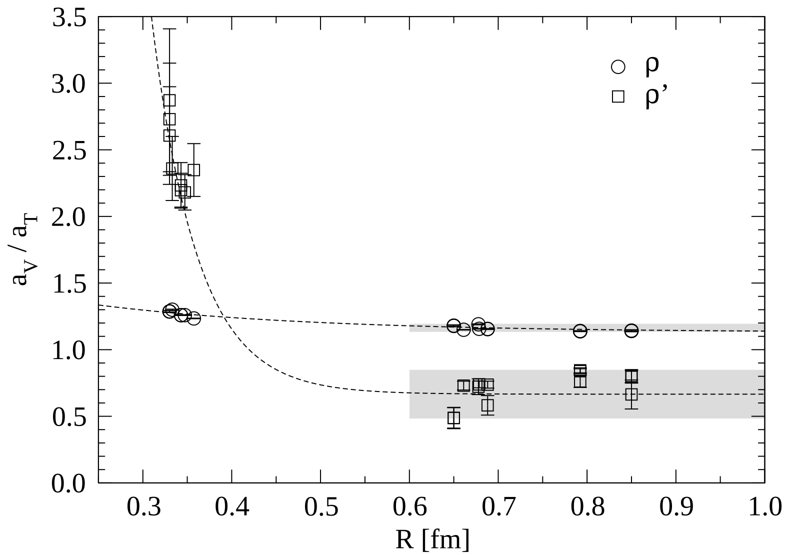
$$\mathcal{O}_V^{(1)} = \overline{u}_{(1)}\gamma^i d_{(1)} \,, \qquad \mathcal{O}_V^{(2)} = \overline{u}_{(2)}\gamma^i d_{(2)} \,..., \tag{7}$$

$$\mathcal{O}_T^{(1)} = \overline{u}_{(1)}\gamma^t\gamma^i d_{(1)} \,, \qquad \mathcal{O}_T^{(2)} = \overline{u}_{(2)}\gamma^t\gamma^i d_{(2)} \,... \tag{8}$$

6*6 and 4*4 matrices:



Chiral and $^{2S+1}L_J$ content of mesons on lattice



ρ: at the scale of the ρ size - strong chiral symmetry breaking: $\frac{(0,1)+(1,0)}{(1/2,1/2)_b} \sim 1.1 - 1.2$

$\rho \simeq 0.99|^3S_1\rangle - 0.1|^3D_1\rangle \approx |^3S_1\rangle$

ρ': at the scale of 1 fm - weaker chiral symmetry breaking: $\frac{(0,1)+(1,0)}{(1/2,1/2)_b} \to 0.5 - 0.8$

$\rho' \simeq 0.9|^3S_1\rangle - 0.4|^3D_1\rangle$

ρ' is not a pure radial excitation of ρ.

Conclusions

(i) It is possible to define and measure in a gauge invariant manner a degree of chiral symmetry breakling as well as the angular momentum content of mesons in the rest frame at different resolution scales.

(ii) The angular momentum content of hadrons is deeply connected with their chiral content.

(iii) The ρ meson is the 55% - 45% mixture of $(0,1)+(1,0)$ and $(1/2,1/2)_b$ chiral representations. Its angular momentum content in the rest frame is approximately the $^3S_1$ state. No trace of the "spin crisis" !

(iv) Chiral symmetry breaking in ρ' is weaker. There is a significant contribution of the $^3D_1$ wave.

(v) Is the "spin crisis" a consequence of a nonadequate definition of the spin content of hadrons as extracted from the parton distributions in the infinite momentum frame?

(vi) Is it possible to separate in a sensible manner the contributions of quarks and gluons in the confining regime with manifest gauge invariance? The gauge invariance means that quarks and gluons are connected. Then how can we separate them?