New QCD Sum Rules Based on Canonical Commutation Relations

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Introduction I

○ Dipole Sum Rules in Nuclear Physics (Giant Dipole Resonance)

$$\sigma_{\text{tot}} = \frac{4\pi^2 e^2}{\hbar c} \sum_\nu (E_\nu - E_0) |\langle \nu | D | 0 \rangle|^2$$

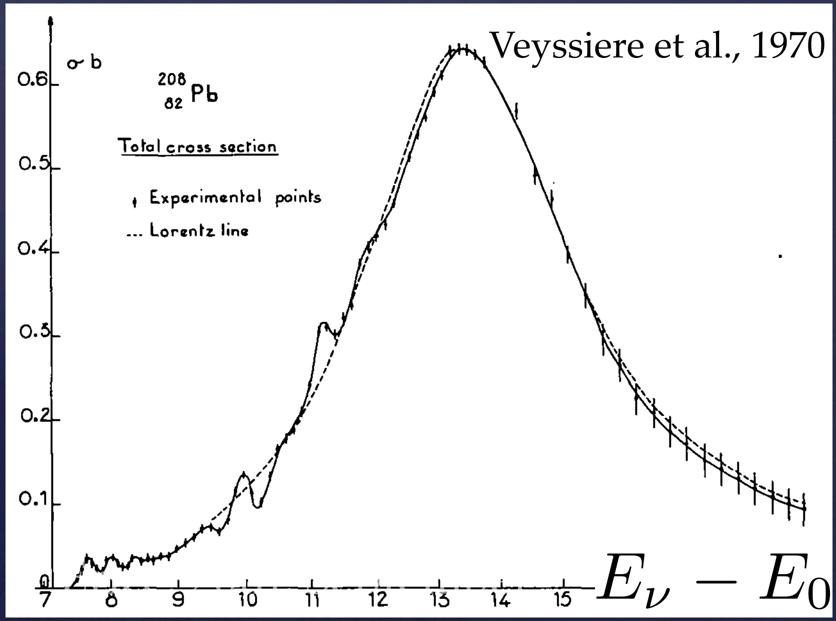
$$= \frac{4\pi^2 e^2}{\hbar c} \langle 0 | [D, [H, D]] | 0 \rangle$$

$$= \frac{2\pi^2 e^2 \hbar}{mc} \frac{NZ}{A} (1 + K)$$

Energy weighted sum

Double commutator

Universal constant



What about sum rules in QFT such as QCD ?

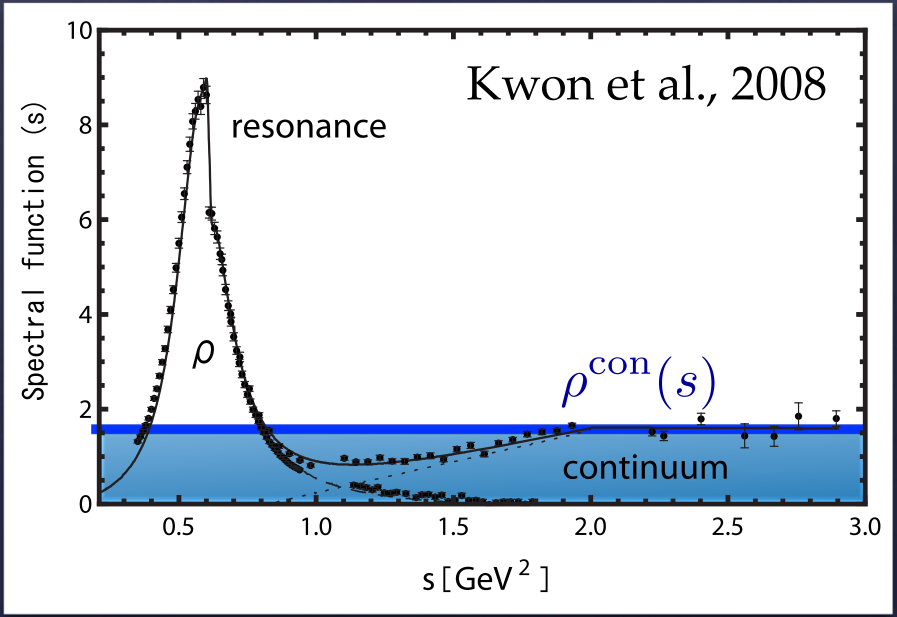
Introduction II

○ QCD sum rules (SVZ sum rules) from OPE+dispersion relation

[Shifman, Vainshtein, Zhakharov , 1978]

$$\int_0^\infty \frac{ds}{2\pi} s^n(\rho(s) - \rho^{\rm con}(s)) \quad = \quad \text{condensations}$$

LHS (dominated by resonances) ⇔ RHS ($\bar{q}q$, G^2 etc)



Can we derive/generalize QCD sum rules
from commutation relations alone without using OPE?

Canonical Quantization of QCD

[Kugo and Ojima, 1978]

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu} \\ &- \partial_\mu B^a A^{a\mu} + \frac{\alpha}{2}(B^a)^2 - i\partial^\mu \bar{c}^a D^{\text{ad}}_\mu c^a\end{aligned}$$

QCD Hamiltonian

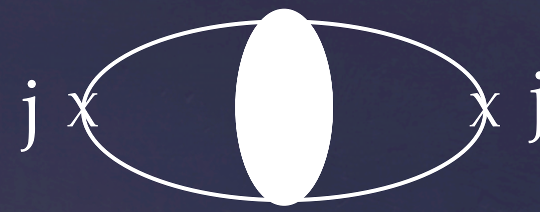
$$\begin{aligned}\mathcal{H}_{\text{eff}} &= -gA^a_0\bar{q}_f\gamma^0 t^a q_f + \bar{q}_f(-i\gamma^k D_k - m_f)q_f + \frac{1}{2}((\vec{E}^a)^2 + (\vec{H}^a)^2) \\ &+\vec{E}^a\cdot(\nabla A^a_0 - gf_{abc}\vec{A}^b A^c_0) + \partial_k B^a A^{ak} - \frac{\alpha}{2}(B^a)^2 \\ &+i\Pi^a_c\Pi^a_{\bar{c}} + gf_{abc}\Pi^a_c A^b_0 c^c - i\partial^k\bar{c}^a D^{\text{ad}}_k c^a\end{aligned}$$

- Heisenberg fields $q_f,\ \bar{q}_f,\ A^a_\mu,\ B^a,\ E^a_k,\ c^a,\ \bar{c}^a,\ \Pi^a_c,\ \Pi^a_{\bar{c}}$

  quarks ($q_f,\ \bar{q}_f$) — gluons ($A^a_\mu,\ B^a,\ E^a_k$) — ghosts ($c^a,\ \bar{c}^a,\ \Pi^a_c,\ \Pi^a_{\bar{c}}$)

- Canonical commutators: $\{q,\bar{q}\},\ [A^a, B^b],\ [A^a_i, E^{bj}],\ \{c^a, \Pi^b_c\}$
- Physical states are BRST invariant: $Q_B|\text{phys}\rangle = 0$

Sum Rules for QCD Current Correlator



○ Spectral function

$$\rho(q^2) \quad = \quad -\frac{1}{3q^2}\sum_p (2\pi)^4\delta^{(4)}(q-p)\langle 0|j_\mu(0)|p\rangle\langle p|j^\mu(0)|0\rangle$$

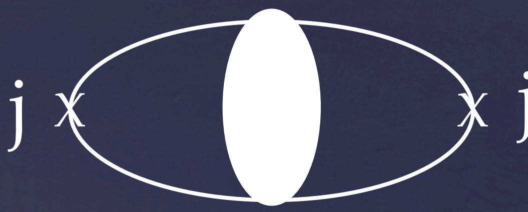
○ Energy weighted sum rules (n-th moment) at q=0

$$\boxed{\int_0^\infty \frac{\mathrm{d}s}{2\pi}\, s^n\rho(s) \quad = \quad -\frac{1}{3}\int \mathrm{d}^3x\, \langle 0|[\cdots[j_\mu(0,\vec{x})\underline{,\mathrm{H}]\cdots,\mathrm{H}]}_{2n-1}, j^\mu(0)]|0\rangle}$$

○ Renormalization of perturbative UV divergence

$$\int_0^\infty \frac{\mathrm{d}s}{2\pi}\, s^n(\rho(s)-\rho^{\mathrm{con}}(s)) = \\ -\frac{1}{3}\int \mathrm{d}^3x\, \langle 0|[\cdots[j_\mu(0,\vec{x})\underline{,\mathrm{H}]\cdots,\mathrm{H}]}_{2n-1}, j^\mu(0)]|0\rangle_{\mathrm{NP}}$$

Commutators for QCD Current Correlator



○ Basic commutator

$$[\bar{u}\gamma^\mu u, \mathrm{H}] = -i\bar{u}\gamma^\mu\gamma^0\gamma^k D_k u$$
$$-iD_k^*\bar{u}\gamma^k\gamma^0\gamma^\mu + m_u\bar{u}[\gamma^\mu, \gamma^0]u$$

○ Double commutator

$$[[\bar{u}\gamma^\mu u, \mathrm{H}], \mathrm{H}] = \Big(\bar{u}\gamma^\mu\gamma^k D_k\gamma^{k'} D_{k'} u - D_k^*\bar{u}\gamma^k\gamma^0\gamma^\mu\gamma^0\gamma^k D_k u$$
$$+ \quad 2im_u\bar{u}\gamma^0\gamma^\mu\gamma^0\gamma^k D_k u + ig\bar{u}\gamma^\mu\gamma^0\gamma^k E_k^a t^a u$$
$$+ \quad m_u^2\bar{u}(\gamma^\mu - \gamma^{\mu\dagger})u\Big) + \text{ h. c.}$$

Canonical commutation relations alone, no OPE at all.

Weinberg's Sum Rules

difference between vector and axial-vector currents

$$j_\mu^{(\rho)} = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) \ \ / \ \ j_\mu^{(A_1)} = \frac{1}{2}(\bar{u}\gamma_\mu \gamma^5 u - \bar{d}\gamma_\mu \gamma^5 d)$$

- 1st moment

$$\int_0^\infty \frac{\mathrm{d}s}{2\pi}\, s(\rho_A(s) - \rho_V(s)) \quad = \quad \langle 0| \frac{4m_u}{3}\bar{u}u + \frac{4m_d}{3}\bar{d}d|0\rangle$$

- 2nd moment

$$\int_0^\infty \frac{\mathrm{d}s}{2\pi}\, s^2(\rho_A(s) - \rho_V(s)) \quad = \quad \langle 0|2im_q^2\bar{q}\overleftrightarrow{\not{D}}q - \frac{16}{3}m_q^3\bar{q}q$$
$$+\frac{\pi}{2}\alpha_s(\bar{u}_L\gamma_\mu t^a u_L - \bar{d}_L\gamma_\mu t^a d_L)(\bar{u}_R\gamma^\mu t^a u_R - \bar{d}_R\gamma^\mu t^a d_R)|0\rangle$$

Non-OPE derivation of Weinberg's sum rules

Sum Rules for the vector correlation

○ Bare sum rule

$$\int_0^\infty \frac{\mathrm{d}s}{2\pi}\, s\rho_V(s) = \langle 0|\frac{i}{3}\bar{u}\overleftrightarrow{D}_k\gamma^k u - m_u\bar{u}u|0\rangle + (u \Leftrightarrow d)$$

○ UV regulated (bare) Eq. of motion [Fujikawa, 1979]

$$\langle 0|\bar{u}(i\not{D} - m)u]|0\rangle = \lim_{\epsilon\to 0}\mathrm{Tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-ik(x+\epsilon)} f\left(\frac{\not{D}^2}{M^2}\right)\mathrm{e}^{ikx}$$

Pauli-Villars type regulator $f(\frac{\not{D}^2}{M^2}) = \frac{M^2}{\not{D}^2 + M^2}$

$$\langle 0|\bar{u}(i\not{D} - m)u]|0\rangle = \mathrm{Tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{e}^{-ik^0\epsilon}\left(\frac{M^2}{-k^2+M^2}\right) - \frac{g^2}{48\pi^2}G^a_{\mu\nu}G^{a\mu\nu}$$

$$\int_0^\infty \frac{\mathrm{d}s}{2\pi}\, s\left(\rho_V(s) - \rho_V^{\mathrm{con}}(s)\right) = \langle 0| - \frac{m_q}{2}\bar{q}q - \frac{\alpha_s}{24\pi}G^a_{\mu\nu}G^{a\mu\nu}|0\rangle_{\mathrm{NP}}$$

Consistent result with SVZ sum rules from OPE

New types of sum rules with fractional weight

$$\int_0^\infty \frac{\mathrm{d}s}{2\pi}\, s^{\frac{n}{2}} \rho(s) \quad = \quad -\frac{1}{3V} \langle 0 | \{ [\cdots [Q_\mu, \underbrace{\mathrm{H}] \cdots\ , \mathrm{H}]}_{n-1}, Q^\mu \} | 0 \rangle$$

$$;Q^\mu \quad = \quad \int \mathrm{d}^3x\; j^\mu(0, \vec{x})$$

e.g., ½ -th moment

$$\int_0^\infty \frac{\mathrm{d}s}{2\pi} \sqrt{s}\rho(s) \quad = \quad -\frac{2}{3V} \langle 0 | Q_\mu Q^\mu | 0 \rangle$$

$$= \quad -\frac{8}{3V} \langle 0 | Q_0^2 | 0 \rangle$$

Need further investigation for their implication

Summary

1. New derivation of QCD sum rules by canonical commutators
   ⇔ simple & straightforward generalization of dipole sum rule
   on the basis of { Kugo-Ojima operator formalism; suitable subtraction of UV divergences }

2. "exact" Weinberg's sum rules are derived

3. Vector -current sum rules are derived
   ⇔ consistent with the SVZ sum rules from OPE

4. New fractional power sum rules are derived
   → need further study

Future

- In-medium QCD sum rules from the commutators
- Other mesonic and baryonic currents
- Regularization scheme dependence

Thank you for your attention!!