T. Kawanai and S. Sasaki, Phys. Rev. Lett. 107, 091601 (2011)
T. Kawanai and S. Sasaki, in preparation.

INTERQUARK POTENTIAL FOR THE CHARMONIUM SYSTEM WITH ALMOST PHYSICAL QUARK MASSES

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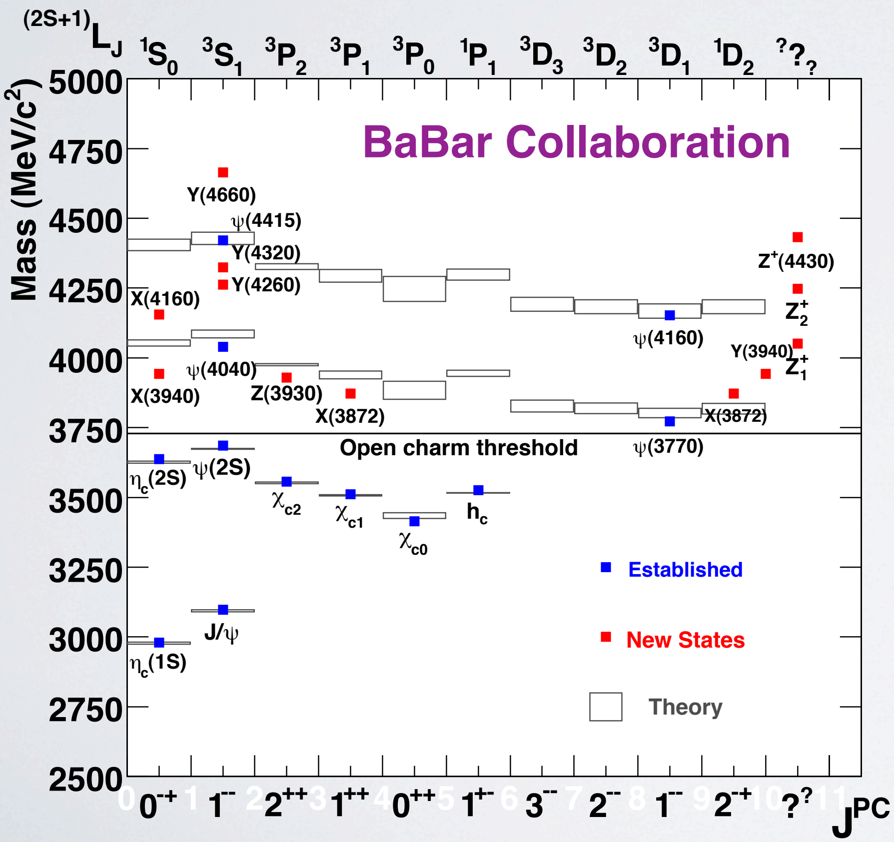
INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS
33rd Course
From Quarks and Gluons to Hadrons and Nuclei
Erice-Sicily: 16 - 24 September 2011

Why cc^{bar} potential？

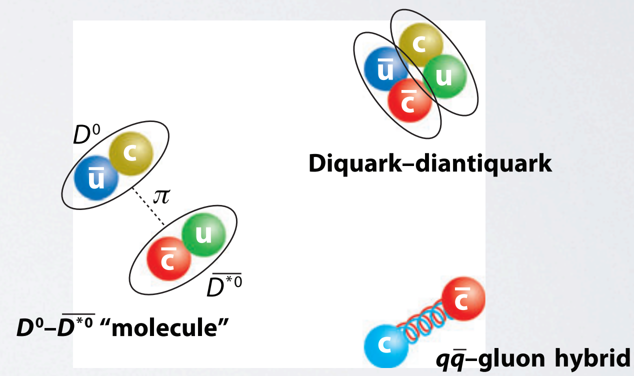
- ✦ Exotic XYZ charmonium-like mesons

"Standard" states can be defined in potential models



The XYZ mesons are expected to be good candidates for non-standard quarkonium mesons

S. Godfrey and S. L. Olsen, Ann. Rev. Nucl. Part. Sci. 58, 51 (2008)



"Exotic" = "Non-standard"?

Why cc^{bar} potential？

✦ qq^{bar} interquark potential in quark models

S. Godfrey and N. Isgur, PRD 32, 189 (1985).
T. Barnes, S. Godfrey and E. S. Swanson, PRD 72, 054026 (2005)

$$V_{c\bar{c}} = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r + \frac{32\pi\alpha_s}{9m_q^2}\delta(r)\mathbf{S}_q\cdot\mathbf{S}_{\bar{q}} + \frac{1}{m_q^2}\left[\left(\frac{2\alpha_s}{r^3} - \frac{b}{2r}\right)\mathbf{L}\cdot\mathbf{S} + \frac{4\alpha_s}{r^3}T\right]$$

Cornell potential — spin-dependent potential

- Spin-spin, tensor and spin-orbit terms appear as corrections in the $1/m_q$ expansion.
- Functional forms of the spin-dependent terms are determined by one-gluon exchange.

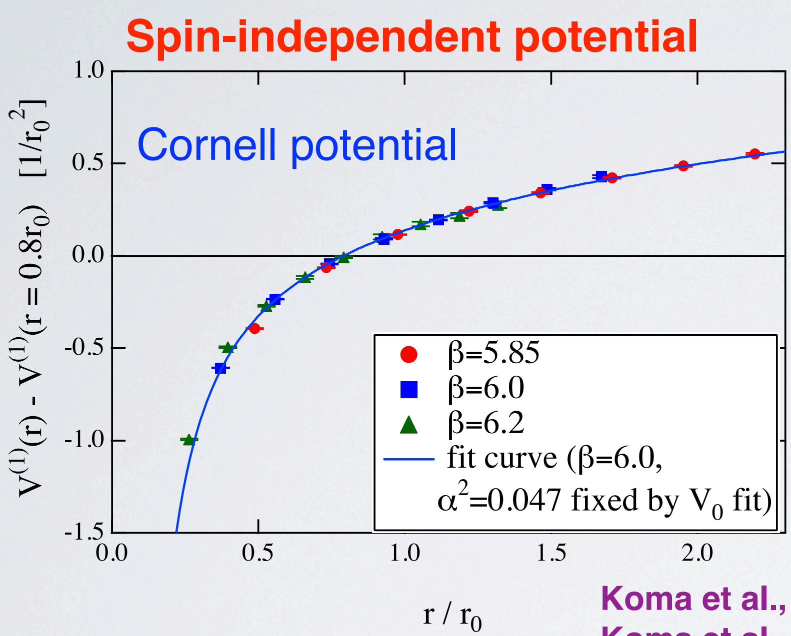
→ Properties of higher charmonium states predicated in potential models may suffer from large uncertainties.

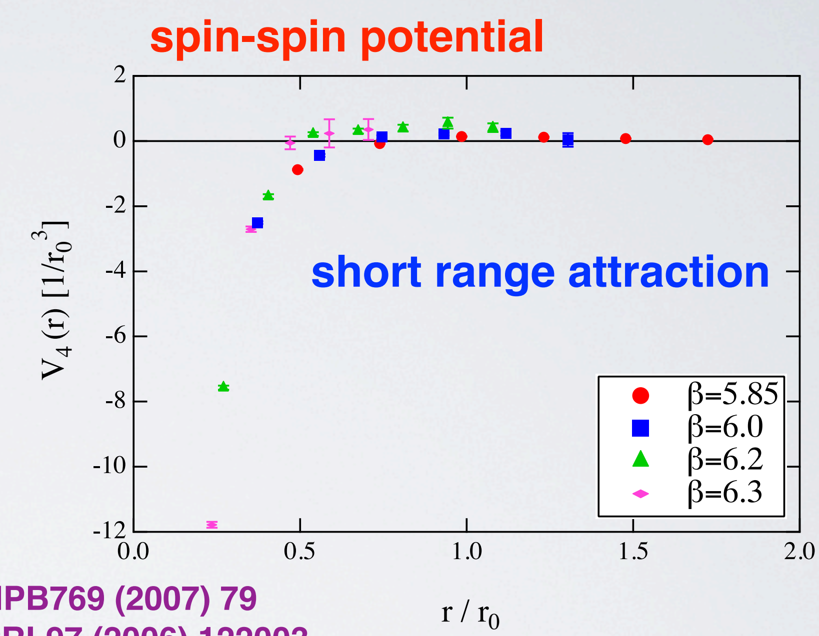
A reliable charmonium potential directly derived from first principles QCD is very important.

Why cc^{bar} potential？

G. S. Bali, Phys. Rept. 343, 1 (2001).

✦ Static interquark potential from Wilson loop





Koma et al., NPB769 (2007) 79
Koma et al., PRL97 (2006) 122003

- The static potential obtained from Wilson loops have been precisely calculated from lattice.
- Relativistic corrections are classified in powers of $1/m_q$ within framework of pNRQCD.

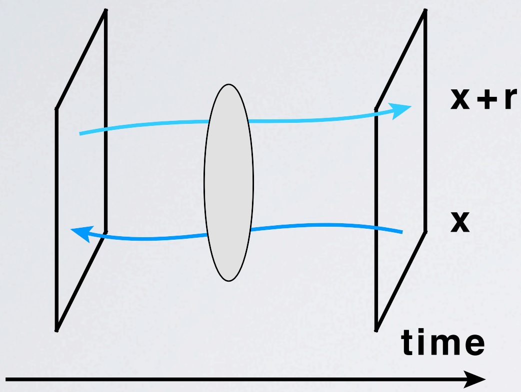
→ spin-spin potential induced by $1/m_q^2$ correction exhibits short range attraction.
cf. short range repulsion is required in phenomenology.

How to calculate cc^{bar} potential？

S. Aoki, T. Hatsuda and N. Ishii, Prog. Theor. Phys. 123 (2010) 89.
Y. Ikeda and H. Iida, arXiv:1102.2097 [hep-lat].

1. Equal-time BS wavefunction

$$\phi_\Gamma(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|q\bar{q}; J^{PC}\rangle$$



$$\sum_{\mathbf{x},\mathbf{x}',\mathbf{y}'} \langle 0|\bar{q}(\mathbf{x},t)\Gamma q(\mathbf{x}+\mathbf{r},t)\left(\bar{q}(\mathbf{x}',t_{\mathrm{src}})\Gamma q(\mathbf{y}',t_{\mathrm{src}})\right)^\dagger|0\rangle$$
$$= \sum_n A_n \langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|n\rangle e^{-M_n^\Gamma(t-t_{\mathrm{src}})}$$
$$\xrightarrow{t \gg t_0} A_0 \phi_\Gamma(\mathbf{r}) e^{-M_0^\Gamma(t-t_{\mathrm{src}})}$$

2. Schrödinger equation with non-local potential

$$-\frac{\nabla^2}{2\mu}\phi_\Gamma(\mathbf{r}) + \int dr' U(\mathbf{r},\mathbf{r}')\phi_\Gamma(\mathbf{r}') = E_\Gamma \phi_\Gamma(\mathbf{r})$$

3. Velocity expansion

$$U(\mathbf{r}',\mathbf{r}) = \{V(r) + V_{\mathrm{S}}(r)\mathbf{S}_Q\cdot\mathbf{S}_{\overline{Q}} + V_{\mathrm{T}}(r)S_{12} + V_{\mathrm{LS}}(r)\mathbf{L}\cdot\mathbf{S} + \mathcal{O}(\nabla^2)\}\delta(\mathbf{r}'-\mathbf{r})$$

How to calculate cc^{bar} potential？

S. Aoki, T. Hatsuda and N. Ishii, Prog. Theor. Phys. 123 (2010) 89.
Y. Ikeda and H. Iida, arXiv:1102.2097 [hep-lat].

5. Projection to "S-wave" $\phi_\Gamma(\mathbf{r}) \rightarrow \phi_\Gamma(r; A_1^+)$

$$\left\{-\frac{\nabla^2}{m_q} + V(r) + \mathbf{S}_q \cdot \mathbf{S}_{\overline{q}} V_{\mathrm{S}}(r)\right\} \phi_\Gamma(r) = E_\Gamma \phi_\Gamma(r)$$

6. Linear combination

$$\begin{aligned} V(r) &= E_{\mathrm{ave}} + \frac{1}{m_q}\left\{\frac{1}{4}\frac{\nabla^2\phi_{\mathrm{PS}}(r)}{\phi_{\mathrm{PS}}(r)} + \frac{3}{4}\frac{\nabla^2\phi_{\mathrm{V}}(r)}{\phi_{\mathrm{V}}(r)}\right\} \\ V_{\mathrm{S}}(r) &= E_{\mathrm{hyp}} + \frac{1}{m_q}\left\{-\frac{\nabla^2\phi_{\mathrm{PS}}(r)}{\phi_{\mathrm{PS}}(r)} + \frac{\nabla^2\phi_{\mathrm{V}}(r)}{\phi_{\mathrm{V}}(r)}\right\} \end{aligned}$$

The quark kinetic mass m_q is essentially involved in the definition of the potentials.
Under a simple, but reasonable assumption of $\lim_{r\to\infty} V_S(r) = 0$

T. Kawanai and S. Sasaki, arXiv:1102.3246 [hep-lat].

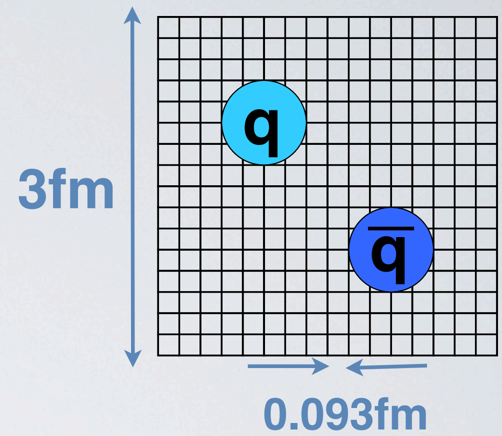
$$m_q = \lim_{r\to\infty} \frac{-1}{\Delta E_{\mathrm{hyp}}}\left(\frac{\nabla^2\phi_{\mathrm{V}}(r)}{\phi_{\mathrm{V}}(r)} - \frac{\nabla^2\phi_{\mathrm{PS}}(r)}{\phi_{\mathrm{PS}}(r)}\right)$$

$$\Delta E_{\mathrm{hyp}} = M_{\mathrm{V}} - M_{\mathrm{PS}}$$

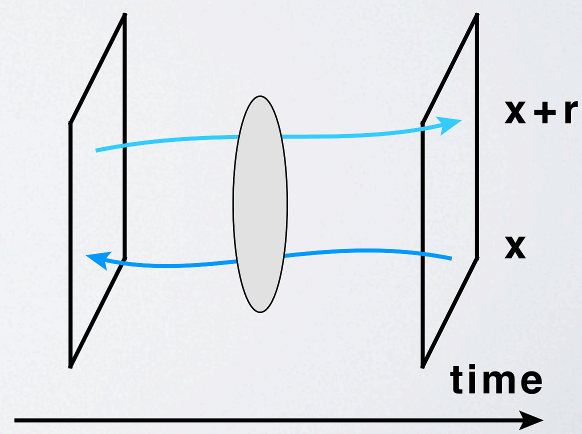
1. Quenched lattice QCD simulation

2. N_f =2+1 dynamical QCD simulation

Lattice Set up



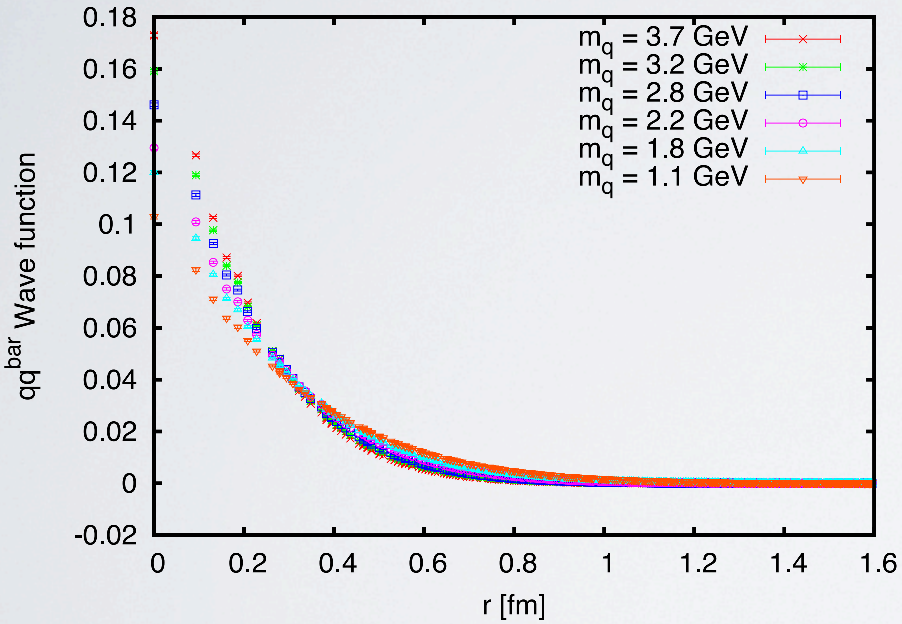
- Quenched QCD simulation
- Lattice size : $L^3 \times T = 32^3 \times 48$ (~$3fm^3$)
- plaquette gauge action $\beta=6.0$ (a=0.093 fm, a^{-1}=2.1GeV)
  + RHQ action with tad-pole improved one-loop PT coefficients
  Y. Kayaba et al. [CP-PACS Collaboration], JHEP 0702, 019 (2007).
- 6 hopping parameters : $0.06667 \le \kappa_Q \le 0.11456$
  $1.87 \text{ GeV} \le m_{pseudo} \le 5.83 \text{ GeV}$
- Statistics : 150 configs
- Wall source
- Coulomb gauge fixing



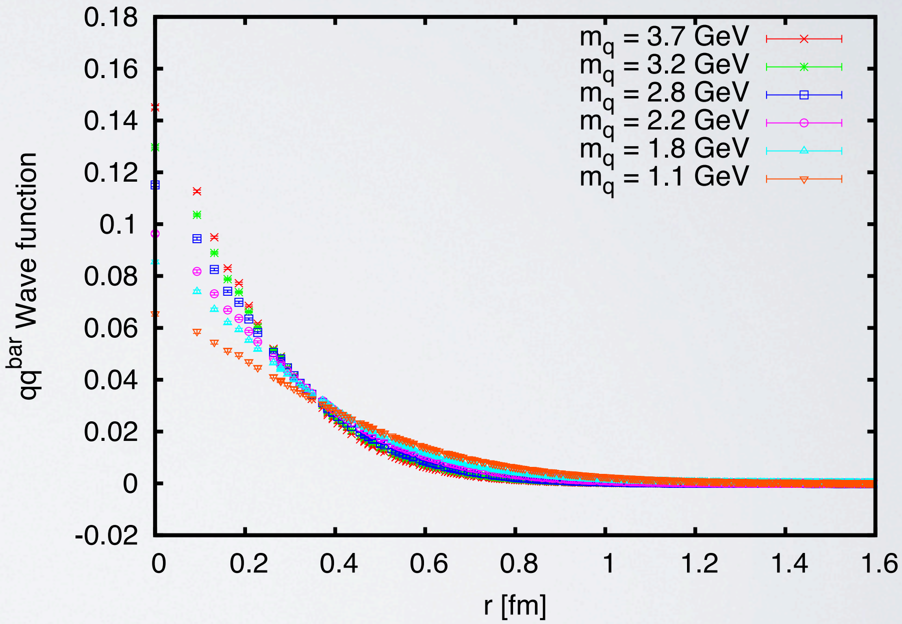
Result; qq^{bar} wave function

$$\phi_\Gamma(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|q\bar{q}; J^{PC}\rangle$$

Pseudo scalar $J^P = 0^-$



Vector $J^P = 1^-$

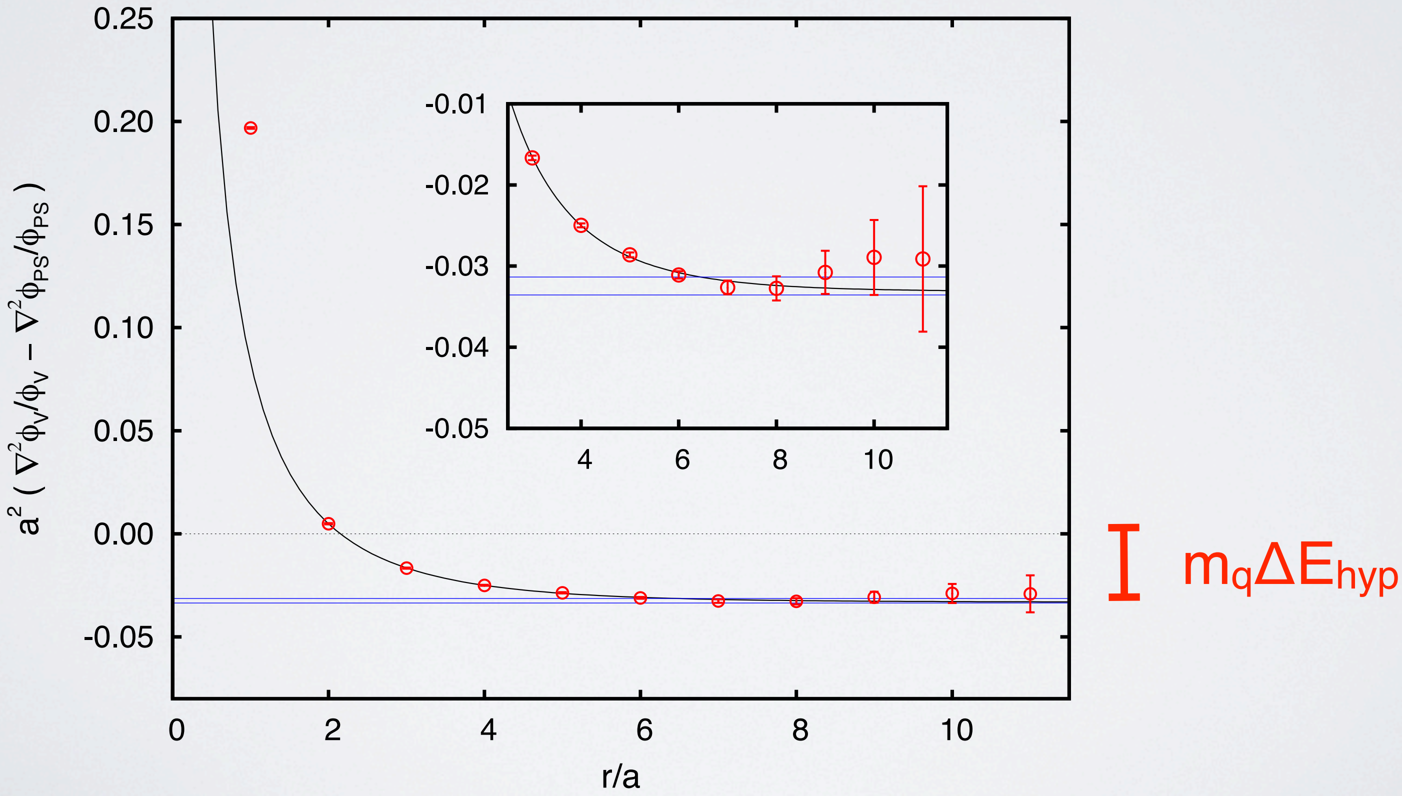


- Normalization $\int dr^3\, \psi^2(r) = 1$
- BS wave functions vanish at r ~ 1fm
- Size of wave function with heavier quark mass become smaller.

Determination of kinetic quark mass

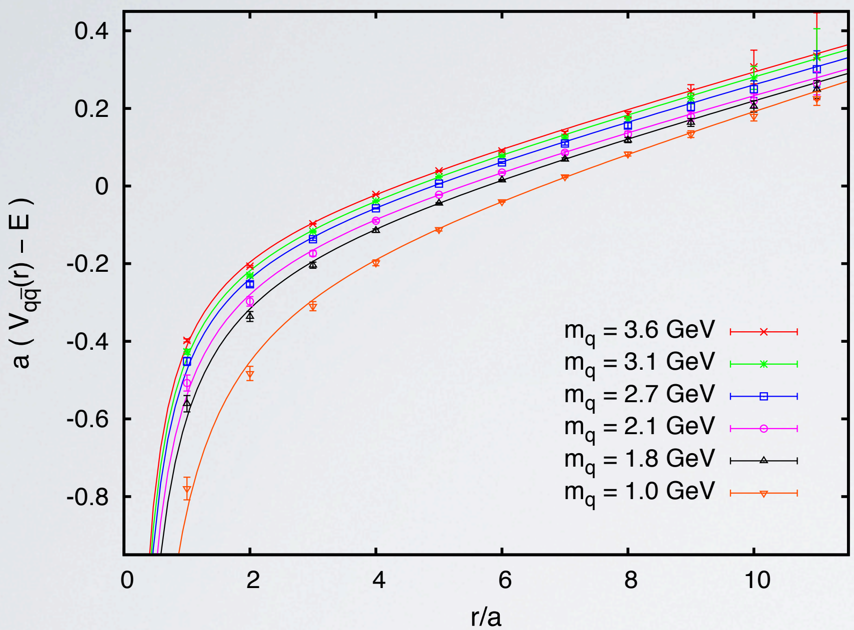
T. Kawanai and S. Sasaki, arXiv:1102.3246 [hep-lat].

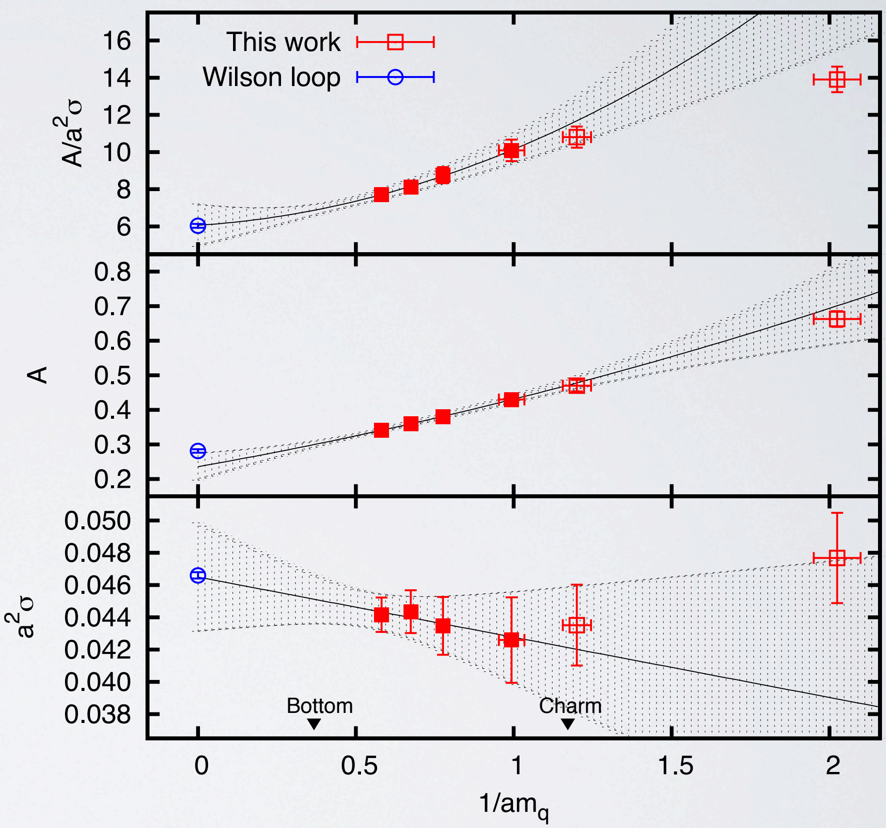
$$m_q = \lim_{r\to\infty} \frac{-1}{\Delta E_{\rm hyp}} \left(\frac{\nabla^2 \phi_{\rm V}(r)}{\phi_{\rm V}(r)} - \frac{\nabla^2 \phi_{\rm PS}(r)}{\phi_{\rm PS}(r)} \right)$$



Result; spin-independent $q\bar{q}$ potential

T. Kawanai and S. Sasaki, arXiv:1102.3246 [hep-lat].





Cornell parameterization

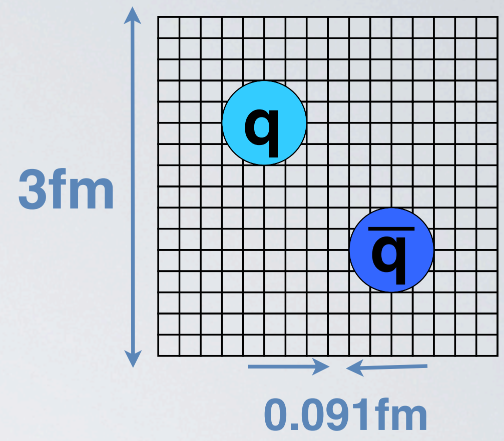
$$V(r) = -\frac{A}{r} + \sigma r + V_0$$

Consistent with the Wilson loops in the $m_q \to \infty$ limit

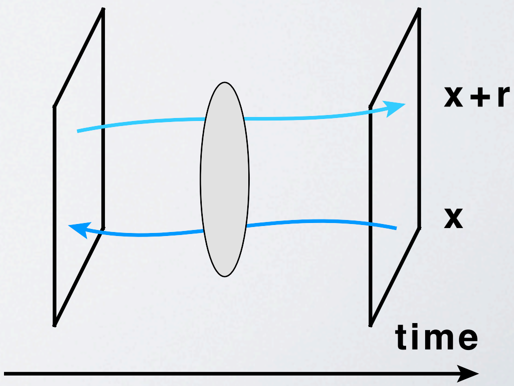
1. Quenched lattice QCD simulation

2. $N_f = 2+1$ dynamical QCD simulation

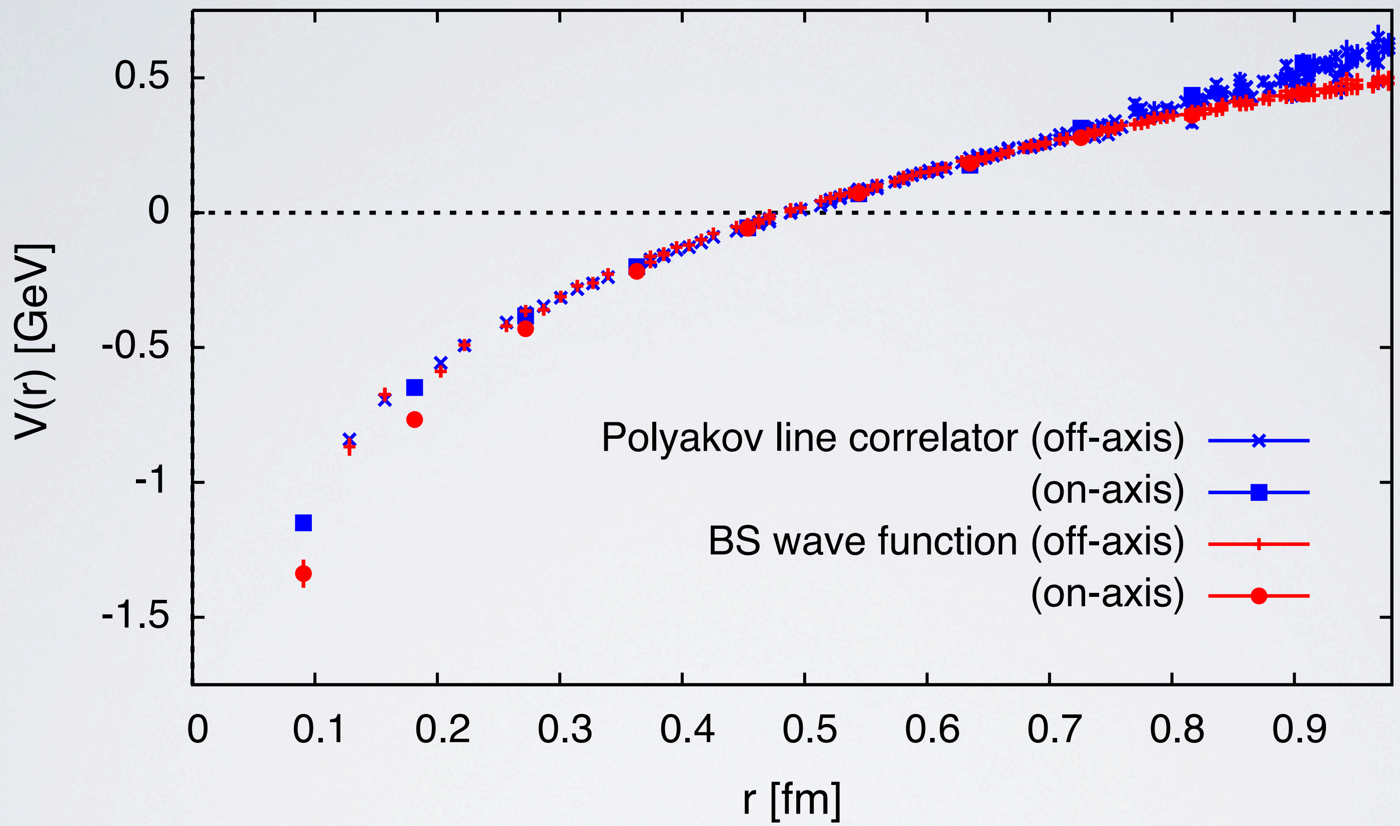
Lattice Set up



- 2+1 flavor dynamical gauge configurations generated by PACS-CS collaboration.
- Lattice size : $L^3 \times T = 32^3 \times 64$ (~$3fm^3$)
- Iwasaki gauge action β=1.9 ($a \approx 0.091$ fm, $a^{-1} \approx 2.3$GeV) + RHQ action with partially non-perturbative RHQ parameters.
- Light quark mass : m_π = 156(7) MeV, m_K = 553(2)MeV
  Charm quark mass : m_{ave}(1S) =3.069(2) GeV, m_{hyp}(1S)=111(2) MeV
- Statistics : 198 configs
- Wall source
- Coulomb gauge fixing

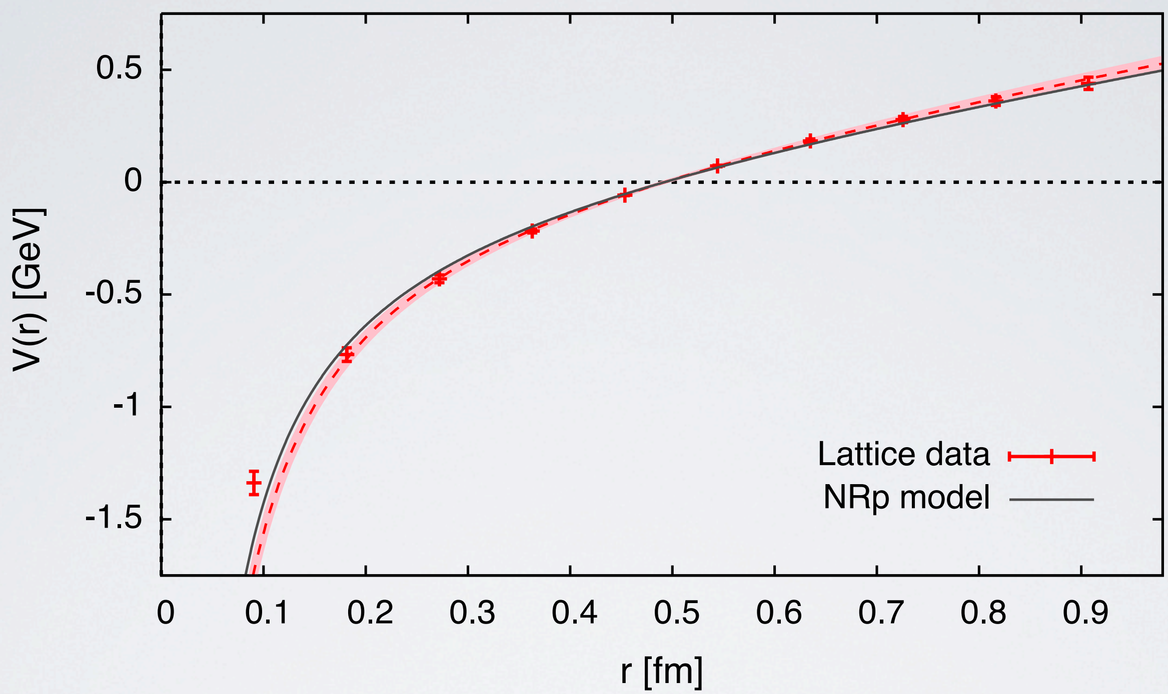


Result; spin-independent cc^{bar} potential



- we take a weighted average of data points in the wide range of $(t-t_{src})/a$ = 34-44
- A discretization error appears especially near the origin.

Result; spin-independent cc^{bar} potential



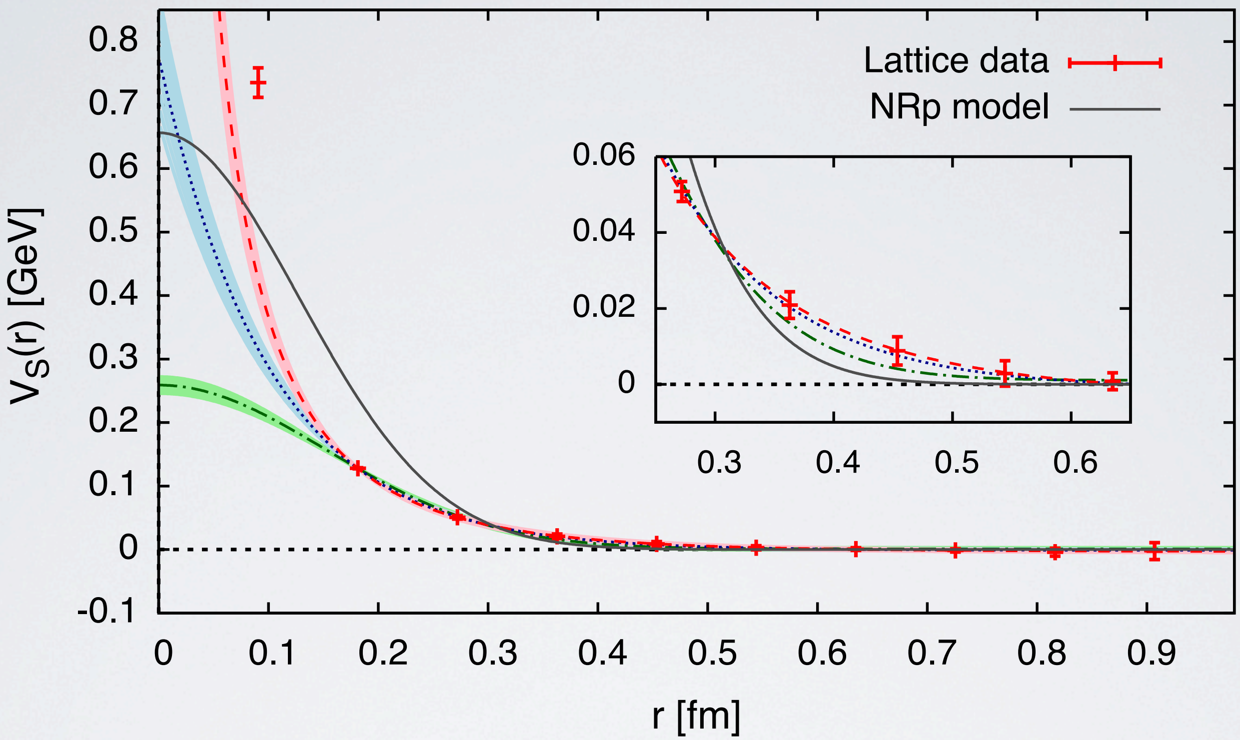
	This work	Static	NRp model
A	0.714(30)	0.515(2)	0.7281
$\sqrt{\sigma}$ [GeV]	0.434(11)	0.430(1)	0.3775
m_q [GeV]	1.81(7)	∞	1.4794

▶ The charmonium potential obtained from the BS wave function resembles the NRp model.

▶ String breaking is not observed

Non-relativistic potential (NRp) model
T.Barnes, S. Godfrey, E.S. Swanson, PRD72 (2005) 054026

Result; spin-spin $c\bar{c}$ potential



- Short range, but non-point like, repulsive interaction
- A difference appears in the spin-spin potential

Fitting function

$$V_S(r) = \begin{cases} \alpha \exp(-\beta r)/r \\ \alpha \exp(-\beta r) \\ \alpha \exp(-\beta r^2) \end{cases}$$

	α	β	χ/ d.o.f
Yukawa	0.297(12)	0.982(47) GeV	0.89
exponential	0.866(29) GeV	2.067(37) GeV	0.45
Gaussian	0.309(7) GeV	1.069(17) GeV^2	12.40

Summary

- ✦ We have derived both the spin-independent and -dependent part of the central qq^{bar} interquark potential from the BS wave function in Quenched QCD simulation and 2+1 flavor dynamical lattice QCD simulation with almost physical quark masses.
  - ✓ spin-independent qq^{bar} potential from BS wave function smoothly approaches the static qq^{bar} potential from Wilson loop.
  - ✓ The spin-independent charmonium potential obtained from the BS wave function resembles the one used in the NRp model.
  - ✓ Spin-spin charmonium potential from lattice QCD has new and valuable information to the NRp models.
- ✦ Future perspective
  - ✓ Other spin-dependent potential: tensor and LS force.
  - ✓ Taking the Continuum limit.
  - ✓ More precise prediction for higher charmonium state
  - ✓ Three body force, cs^{bar} system, string breaking ....

Thank you for your attention!
Grazie!