Dimensionally reduced QCD at high temperature

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Overview

Goals

- What do we want to know?
  - The structure of the QCD phase diagram: Transition line, critical point, etc.
  - Equation of state of quark-gluon plasma (QGP): e.g. pressure $p \longrightarrow$ heavy ion collisions, early universe,...
- How?
  - Lattice simulations
  - Here: Perturbative expansion in small coupling $\longrightarrow$ Ok at high T, but far away from T_C!
- But:
  - Naive loop-expansion breaks down at higher order! $\longrightarrow$ Infrared divergencies

QCD equilibrium-thermodynamics

- Thermodynamic oberservables via partition function in path integral representation and Euclidean space-time:

$$\mathcal{Z}(T) \equiv \mathrm{Tr}[\exp(-\beta H)] \rightarrow \int \mathcal{D}A_\mu^a \mathcal{D}\bar{c}^a \mathcal{D}c^a \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\{-S_E\}$$

- with $S_E = \int_0^\beta d\tau \int d^d\mathbf{x} \mathcal{L}_E$ and $\beta = 1/T$, T: temperature
- Observ.: e.g. pressure p determined by $p = \lim_{V\to\infty} \frac{T}{V} \ln \mathcal{Z}$, etc.

Thermodynamic pressure p of massless QCD

- Small coupling expansion in $g = \sqrt{4\pi\alpha_s}$ is nontrivial:

$$p = T^4 \left[1 + \underset{'78}{g^2} + \underset{'79}{g^3} + \underset{'83}{g^4 \ln g^{-1}} + \underset{'94}{g^4} + \underset{'95}{g^5} + \underset{'02}{g^6 \ln g^{-1}} + \underset{'13?}{g^6} + \dots \right]$$

- Already at g^3, IR problems arise $\longrightarrow$ possible workaround:
  - Resummation of so-called plasmon-diagrams $\longrightarrow$ complicated!
  - Here: Computation within effective theory framework $\longrightarrow$ conceptually clean!
- Necessary: Identification of different momentum scales in hot QCD!

Momentum scales of hot QCD

- Where do the IR divergencies come from?
  - $T > 0 : \int d^4k \rightarrow T \sum_{k_0} \int d^3k$
  - Propagator for massless field:

$$\frac{1}{\omega_n^2 + \mathbf{k}^2} \text{ with } \omega_{n,b} = 2n\pi T, \omega_{n,f} = (2n+1)\pi T$$

  - Only the bos. $n = 0$ modes can propagate over dist. $\gg 1/T$.
- Momentum scales T, gT und g^2T:
  - Scale T: Typical momentum of a particle in plasma.
  - Scale gT: Associated with color-electric screening.
  - Scale g^2T: Associated with color-magnetic screening.
- Scale hierarchy $\rightarrow$ integrate out massive modes $(n \neq 0)$ [Appelquist,Pisarski]

Electrostatic QCD (EQCD)

- Result: 3-dimensional effective theory over distances $\gtrsim 1/gT$: [Braaten,Nieto]

$$\mathcal{L}_{EQCD} = \frac{1}{4}F^a_{ij}F^a_{ij} + \mathrm{Tr}[D_i, A_0]^2 + m_E^2 \mathrm{Tr}[A_0^2] + \lambda_E^{(1)}(\mathrm{Tr}[A_0^2])^2 + \lambda_E^{(2)}\mathrm{Tr}[A_0^4] + \cdots$$

where $F^a_{ij} = \partial_i A^a_j - \partial_j A^a_i + g_E f^{abc} A^b_i A^c_j$ and $D_i = \partial_i - i g_E A_i$.

- Higher order operators do not (yet) contribute:

$$\frac{\delta p_{\mathrm{QCD}}(T)}{T} \sim g^2 \frac{D_k D_l}{(2\pi T)^2}\mathcal{L}_{EQCD} \sim g^2 \frac{(gT)^2}{(2\pi T)^2}(gT)^3 \sim g^7 T^3 \,.$$

Eff. gauge coupling g_E^2 und mass m_E^2

- Four matching coefficients have to be determined:

$$\begin{aligned} m_E^2 &= T^2\left[\#g^2 + \#g^4 + \#g^6 + \dots\right], \\ g_E^2 &= T\left[\#g^2 + \#g^4 + \#g^6 + \#g^8 + \dots\right], \\ \lambda_E^{(1/2)} &= T\left[\#g^4 + \dots\right]. \end{aligned}$$

- 2-loop correction [Laine,Schröder]'05

- Coefficients can be determined by matching: require the same result in QCD and EQCD.

- Many possibilities, Here: Computation of self-energies $\Pi_{\mu\nu}$ on both sides.

Some details of the computation

- bg QCD self-energies $\to \sim 500$ diagrams $\to$ autom. necessary.
  - generate graphs: QGRAF [Nogueira]
  - Feynman-R.,Gamma-T., color- and lorentz contract., Taylor, Tensordecomp. $\to$ Implementation in FORM [Vermaseren]
  - Reduction by Integration-by-parts relations [Laporta]
- $\to$ Reduction of ~ 10 Million to ~ 35 master-integrals.
- Cross-check: Longitudinal part $\Pi_\mathrm{L} = 0$ und gauge-terms $\xi, \ldots, \xi^6$ vanish.

Class of Master Integrals

- We end up with Basketball-like sum-integrals with one or two propagators raised to some power and an irreducible scalar product in the numerator.
- We have shown that

$$B_{N,M} \equiv \sum\!\!\!\!\!\!\!\int_{PQR} \frac{Q_0^M}{[Q^2]^N \,(P-Q)^2 \, R^2 \,(P-R)^2}$$

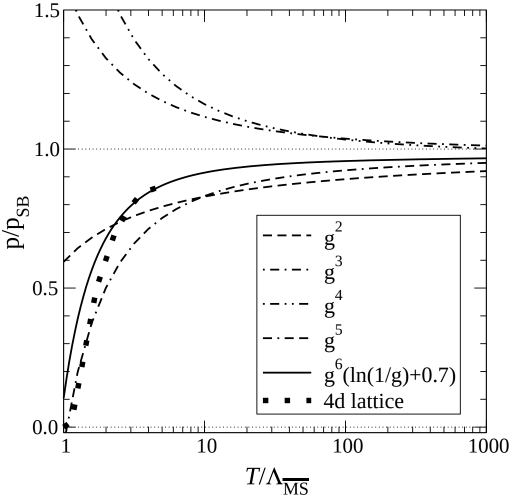
  can be computed for arbitrary $N, M/2 \in \mathbb{N}$ up to $\mathcal{O}(\epsilon^0)$. [JM, Schröder]'10
- However, some are necessary up to $\mathcal{O}(\epsilon^2) \rightarrow$ turned out to be a serious problem.
- Change of basis can help, but integrals introduced are more complicated.

Conclusion

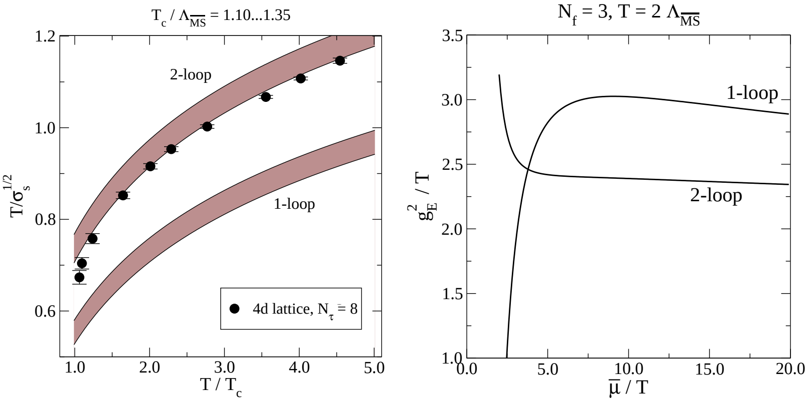
- IR problem solved by separation of momentum scales.
  - Construction of two effective field theories, gT 3d pert.,g^2T Latt.
  - Stat. observables computable up to arbitrary accuracy, systematically!
- 3-loop-correction necessary for $\mathcal{O}(g^7)$ pressure, free E., etc.
- Outlook e.g. correction to spatial string tension σ_s

$$\frac{\sqrt{\sigma_s}}{g_M^2} = 0.553(1)\,.$$

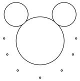
Backup: Thermodynamic pressure p of massless QCD



Backup: Spatial String Tension



Backup: Resummation in ϕ^4 theory

- The dominant IR contribution:
  -  N loops $\sim \beta V \Pi_1^N T \, \mathrm{d}p \, p^{-2(N-1)}$
- Summation over N yields:

$$-\frac{1}{2}\beta V T \sum_n \int \frac{d^3p}{(2\pi)^3}\left[\ln\left(1+\frac{\alpha T^2}{\omega_n^2+\mathbf{p}^2}\right)-\frac{\alpha T^2}{\omega_n^2+\mathbf{p}^2}\right]$$

$$=\frac{\beta V}{12\pi}\alpha^{3/2}T^4+\dots$$