Two, three, many body systems involving mesons

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Systems of two mesons and a baryon
Three body systems with strangeness
Three body with hidden strangeness
Nuclei with mesons? Multirho states
K*-multirho states
Pseudotensor mesons, $J^{PC}=2^{-+}$

Dynamically generated resonances in three-body systems

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in collaboration with

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$\Sigma(1480)$ Bumps

$I(J^P) = 1(?^?)$ Status: *

$\Sigma(1620)$ S_{11}

$I(J^P) = 1(\frac{1}{2}^-)$ Status: **

$\Sigma(1620)$ Production Experiments

$I(J^P) = 1(?^?)$

OMITTED FROM SUMMARY TABLE

Production experiments: Partial-wave analyses of course separate partial waves, whereas a peak in a cross section or an invariant mass distribution usually cannot be disentangled from background and analyzed for its quantum numbers; and more than one resonance may be contributing to the peak. Results from partial-wave analyses and from production experiments are generally kept separate in the Listings, and in

$\Sigma(1770)$ P_{11}

$I(J^P) = 1(\frac{1}{2}^+)$ Status: *

$\Sigma(1560)$ Bumps

$I(J^P) = 1(?^?)$ Status: **

$N(1710)\ P_{11}$

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ Status: ***

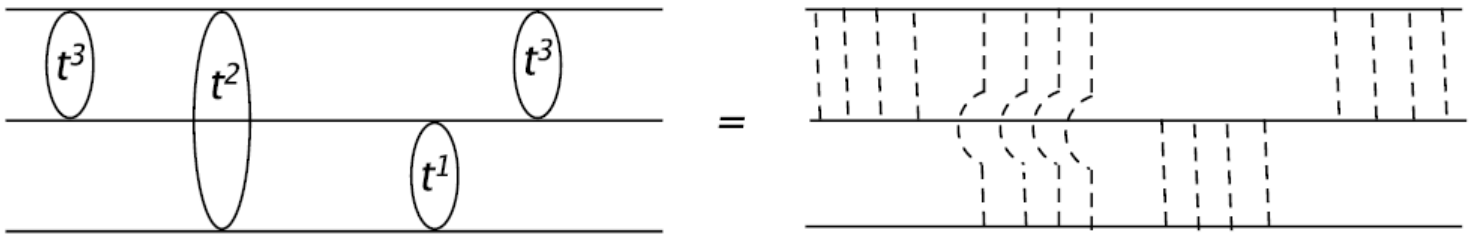
N(1710) DECAY MODES

The following branching fractions are our estimates, not fits or averages.

	Mode	Fraction (Γ_i/Γ)
Γ_1	$N\pi$	10–20 %
Γ_2	$N\eta$	(6.2±1.0) %
Γ_3	$N\omega$	(13.0±2.0) %
Γ_4	ΛK	5–25 %
Γ_5	ΣK	
Γ_6	$N\pi\pi$	40–90 %
Γ_7	$\Delta\pi$	15–40 %
Γ_8	$\Delta(1232)\pi$, P-wave	
Γ_9	$N\rho$	5–25 %
Γ_{10}	$N\rho$, S=1/2, P-wave	
Γ_{11}	$N\rho$, S=3/2, P-wave	
Γ_{12}	$N(\pi\pi)^{I=0}_{S\text{-wave}}$	10–40 %
Γ_{13}	$p\gamma$	0.002–0.05%
Γ_{14}	$p\gamma$, helicity=1/2	0.002–0.05%
Γ_{15}	$n\gamma$	0.0–0.02%
Γ_{16}	$n\gamma$, helicity=1/2	0.0–0.02%

Three body Faddeev equations

One starts from a two nucleon potential and draws all possible diagrams with any amount of interactions between two particles

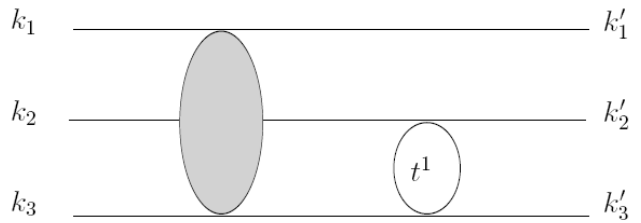


The iterative potential exchanges between two same lines sum up to give the two nucleon t-matrix

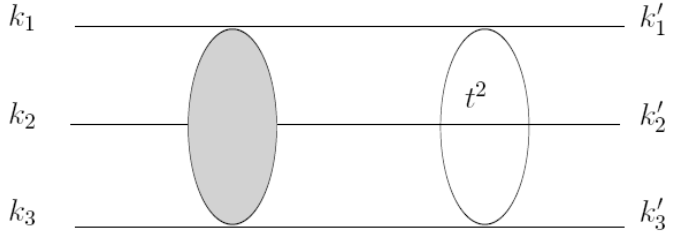
Formalism

$$T = T^1 + T^2 + T^3$$

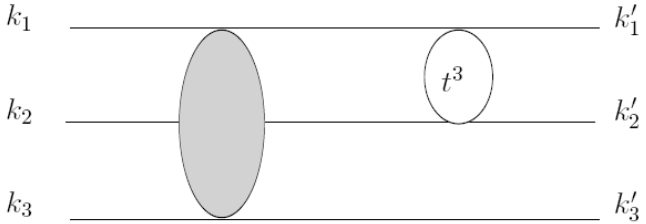
$T^1 =$



$T^2 =$



$T^3 =$



- The input of the Faddeev equations,

$$
\begin{aligned}
T^1 &= t^1 + t^1 G \left[T^2 + T^3 \right] \\
T^2 &= t^2 + t^2 G \left[T^1 + T^3 \right] \\
T^3 &= t^3 + t^3 G \left[T^1 + T^2 \right]
\end{aligned}
$$

i.e., the two-body t-matrices are calculated by solving the Bethe-Salpeter equation → potential from Unitary Chiral Dynamics [9,10,11].

- In general, one has to solve the Faddeev equations with t *off-shell*.

Kaiser, Siegel and Weise, Nucl. Phys A594 (1995); Kaiser , Waas and Weise Nucl. Phys A612 (1997)

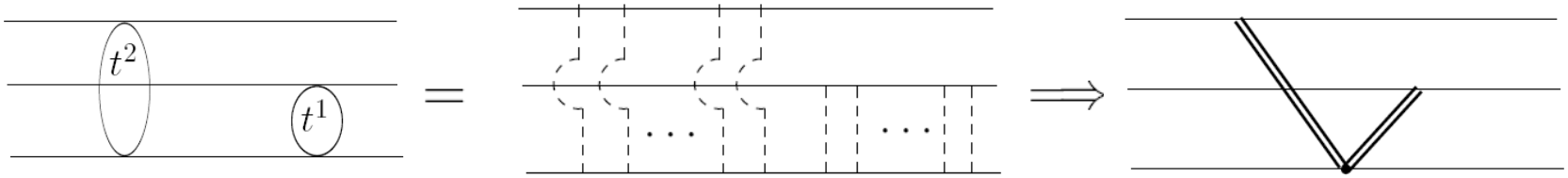
[9] *E. Oset, A. Ramos, Nucl. Phys. A 635 (1998) 99, J. A. Oller, E. Oset, Nucl. Phys. A 620 (1997) 438*

[10] *T. Inoue, E. Oset, M. J. Vicente Vacas, Phys. Rev. C 65 035204*

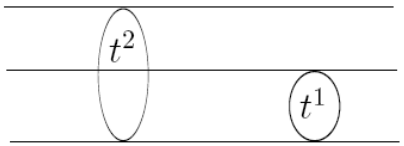
[11] *J.A. Oller, Ulf-G. Meissner, Phys. Lett. B 500 (2001) 263-272, J.A. Oller, Ulf-G. Meissner, Phys. Rev. D,1999.*

- Chiral amplitudes → "on-shell" part (c.m energy)
  → "off-shell" part $\propto$ 1 / propagator

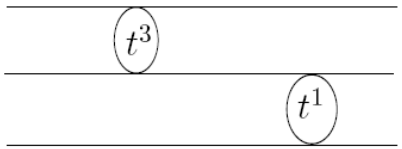
$t=t_{on}+\alpha(q^2-m^2)$ → three-body forces



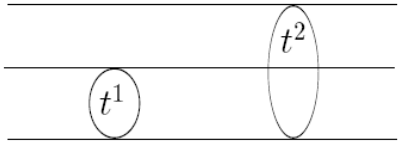
- There are six diagrams with two successive interactions.



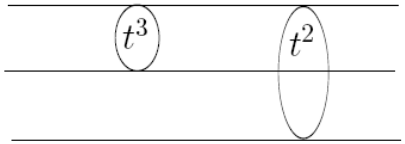
(a)



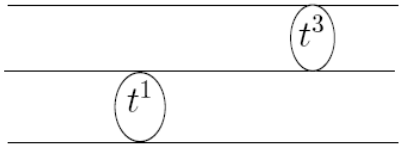
(b)



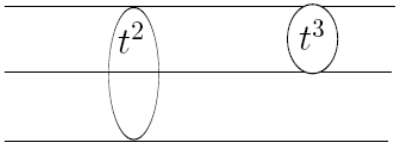
(c)



(d)

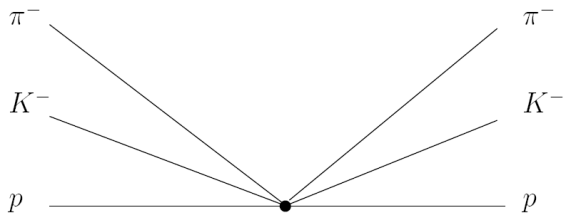


(e)



(f)

- This is not the only source of three body forces.
- They also arise directly from the chiral Lagrangian[12].



Σ (off-shell part of the t-matrices)

+

three-body forces

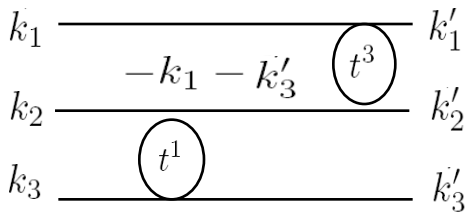
Exact cancellation in the SU(3) limit for s-waves.

For a realistic case → < 5% of the total on-shell contribution of the t-matrices.

In order to solve the Faddeev equations, only the on-shell part of the two-body (chiral) t- matrices is significant.

[12] *F. J. Llanes-Estrada, E. Oset and V. Mateu, Phys. Rev. C 69, 055203*

- Hence, the diagrams containing two t-matrices, like



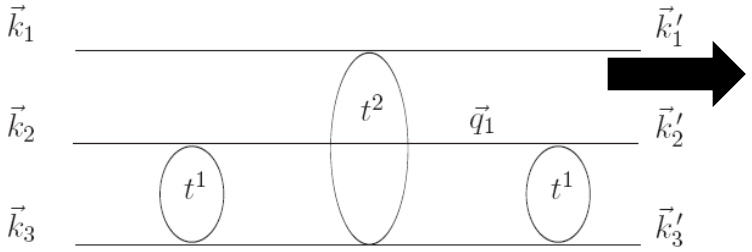
can be written mathematically as $t^i g^{(ij)} t^j$ where,

$$g^{(ij)} = \frac{N_k}{2E_k} \frac{1}{\sqrt{s} - E_i(\vec{k}'_i) - E_j(\vec{k}_j) - E_k(\vec{k}'_i + k_j) + i\epsilon}$$

$$N_l = \begin{cases} 1 & \text{meson-meson interaction} \\ 2M_l & \text{meson-baryon interaction.} \end{cases}$$

- If we add another interaction to this diagram, we have a loop of three particles.

- Let us consider the diagram shown below.



$t^1g^{12}t^2g^{21}$

- We write its contribution as

$$t^1G^{121}t^2g^{21}t^1 = t^1(\sqrt{s_{23}})G^{121}t^2(\sqrt{s_{13}})g^{21}(\vec{k}'_2,\vec{k}_1)t^1(\sqrt{s_{23}})$$

where

$$G^{121} = \int \frac{d\vec{q}_1}{(2\pi)^3}\frac{1}{2E_2(\vec{q}_1)}\frac{M_3}{E_3(\vec{q}_1)}\frac{1}{\sqrt{s_{23}}-E_2(\vec{q}_1)-E_3(\vec{q}_1)+i\epsilon}\times F^{121}(\vec{q}_1,\vec{k}'_2,\vec{k}_1,s_{13})$$

$$F^{121}(\vec{q}_1,\vec{k}'_2,\vec{k}_1,s_{13}) = t^2(s_{13}^{q_1})\times g^{21}(\vec{q}_1,\vec{k}_1)\times[g^{21}(\vec{k}'_2,\vec{k}_1)]^{-1}\times[t^2(\sqrt{s_{13}})]^{-1}$$

- Note that s_{23} is defined in the diagram from the external variables, the argument s_{13} of the t^2 t-matrix in F^{121} depend on the loop variable

$$s_{13}^{q_1} = s - m_2^2 - 2\sqrt{s}\,\frac{E_2(\vec{q}_1)(\sqrt{s}-E_3(\vec{k}'_3))}{\sqrt{s_{12}}}$$

Reformulation of the Faddeev Equations

- We re-write the Faddeev equations in terms of the G and g functions as

$$
\begin{aligned}
T_R^{(12)} &= t^1 g^{(12)} t^2 + t^1 \left[G^{(121)} T_R^{(21)} + G^{(123)} T_R^{(23)} \right] \\
T_R^{(13)} &= t^1 g^{(13)} t^3 + t^1 \left[G^{(131)} T_R^{(31)} + G^{(132)} T_R^{(32)} \right] \\
T_R^{(21)} &= t^2 g^{(21)} t^1 + t^2 \left[G^{(212)} T_R^{(12)} + G^{(213)} T_R^{(13)} \right] \\
T_R^{(23)} &= t^2 g^{(23)} t^3 + t^2 \left[G^{(231)} T_R^{(31)} + G^{(232)} T_R^{(32)} \right] \\
T_R^{(31)} &= t^3 g^{(31)} t^1 + t^3 \left[G^{(312)} T_R^{(12)} + G^{(313)} T_R^{(13)} \right] \\
T_R^{(32)} &= t^3 g^{(32)} t^2 + t^3 \left[G^{(321)} T_R^{(21)} + G^{(323)} T_R^{(23)} \right]
\end{aligned}
$$

which are algebraic equations.

Note that the Faddeev partitions have been redefined as $T^i = \left[(2\pi)^3 \tilde{N}_i \delta^3(\vec{k}_i - \vec{k}_i') \right] t^i + T_R^{(ij)} + T_R^{(ik)}$

The terms denoted as T_R contain only connected diagrams.

The $\pi\bar{K}N$ system and its coupled channels

A. Martinez Torres, K. Khemchandani , E. O, PRC77 (2008)

- We study the $\pi\bar{K}N$ system by solving the Faddeev equations in the coupled channel approach.

- The $\bar{K}N$ system couples strongly to the $\Lambda(1405)$

correlation of the $\bar{K}N$ and its coupled channels should be largely kept during the three body scattering

We begin with a given invariant mass for the $\bar{K}N$ system, s_{23}, for a fixed total energy. Later we vary these two variables.

- The three pairs (πN, KN, $\bar{K}\pi$) couple strongly to many other channels.

⟹ **We need to solve the Faddeev equations taking coupled channels into account.**

π + { pseudoscalar meson of the SU(3) octet + baryon of the 1/2 $^+$ octet } ⟹ 22 coupled channels

S = -1

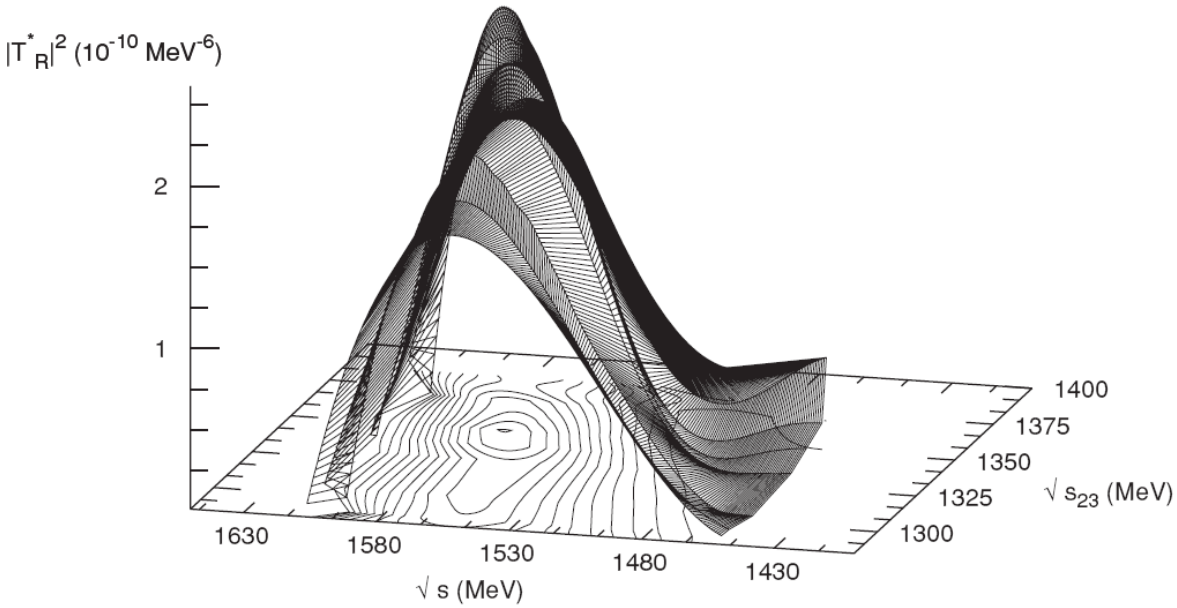
$$\pi^0 K^- p,\ \pi^0 \bar{K}^0 n,\ \pi^0\pi^0\Sigma^0,\ \pi^0\pi^+\Sigma^-,\ \pi^0\pi^-\Sigma^+,\ \pi^0\pi^0\Lambda,\ \pi^0\eta\Sigma^0,\ \pi^0\eta\Lambda,\ \pi^0 K^+\Xi^-,\ \pi^0 K^0\Xi^0$$
$$\pi^+ K^- n,\ \pi^+\pi^0\Sigma^-,\ \pi^+\pi^-\Sigma^0,\ \pi^+\pi^-\Lambda,\ \pi^+\eta\Sigma^-,\ \pi^+ K^0\Xi^-$$
$$\pi^- \bar{K}^0 p,\ \pi^-\pi^0\Sigma^+,\ \pi^-\pi^+\Sigma^0,\ \pi^-\pi^+\Lambda,\ \pi^-\eta\Sigma^+,\ \pi^- K^+\Xi^0$$

$\Lambda(1600)\ P_{01}$

$I(J^P) = 0(\frac{1}{2}^+)$ Status: ***

There are quite possibly two P_{01} states in this region

I = 0, $I_{\pi\bar{K}} = 1/2$



- We find two peaks at 1568 MeV (width 70 MeV) and 1700 MeV (width 136 MeV) in the $\pi\bar{K}N$ amplitude .

- Solving these equations for the $\pi\bar{K}N$ system and its coupled channels, we find four Σ's and two Λ's as dynamically generated resonances in this system.

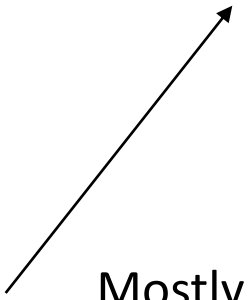
	Γ (PDG) (MeV)	Peak position (this work) (MeV)	Γ (this work) (MeV)
Isospin = 1			
$\Sigma(1560)$	10 - 100	1590	70
$\Sigma(1620)$	10 - 100	1630	39
$\Sigma(1660)$	40 - 200	1656	30
$\Sigma(1770)$	60 - 100	1790	24
Isospin = 0			
$\Lambda(1600)$	50 - 250	1568, 1700	60, 136
$\Lambda(1810)$	50 - 250	1740	20

- It is rewarding that the widths for all these resonances are smaller than the total ones listed by the PDG.

It should be emphasized that all the Σ's and all the Λ's $1/2^+$ up to 1810 MeV get dynamically generated as three body resonances.

Other states obtained with π π N and coupled channels:
The π N amplitudes at energies bigger than 1600 MeV have been taken from experiment,
K. Khemchandani, A. Martinez Torres , E. O,EPJA37 (2008)

$I(J^P)$	Theory			PDG data		
	channels	mass (MeV)	width (MeV)	name	mass (MeV)	width (MeV)
$1/2\,(1/2^+)$	only $\pi\pi N$	1704	375	$N^*(1710)$	1680-1740	90-500
	$\pi\pi N$, $\pi K\Sigma$, $\pi K\Lambda$, $\pi\eta N$	$\sim$ no change	$\sim$ no change			
$1/2\,(1/2^+)$	only $\pi\pi N$	2100	250	$N^*(2100)$	1885-2270	80-400
	$\pi\pi N$, $\pi K\Sigma$, $\pi K\Lambda$, $\pi\eta N$	2080	54			
$3/2\,(1/2^+)$	$\pi\pi N$, $\pi K\Sigma$, $\pi K\Lambda$, $\pi\eta N$	2126	42	$\Delta(1910)$	1870-2152	190-270
$1/2\,(1/2^+)$	$N\pi\pi$, $N\pi\eta$, NKK	1924	20	$N^*(?)$	?	?



Mostly N $f_0(980)$, N $a_0(980)$

First predicted by Jido and Eny'o, PRC78, 2008

Martinez Torres, Khemchandani, Meissner, Oset, EPJA 2009, claim this state is responsible for peak of γ p → K^+ Λ around 1920 MeV

Mart and Bennhold PRC 2000

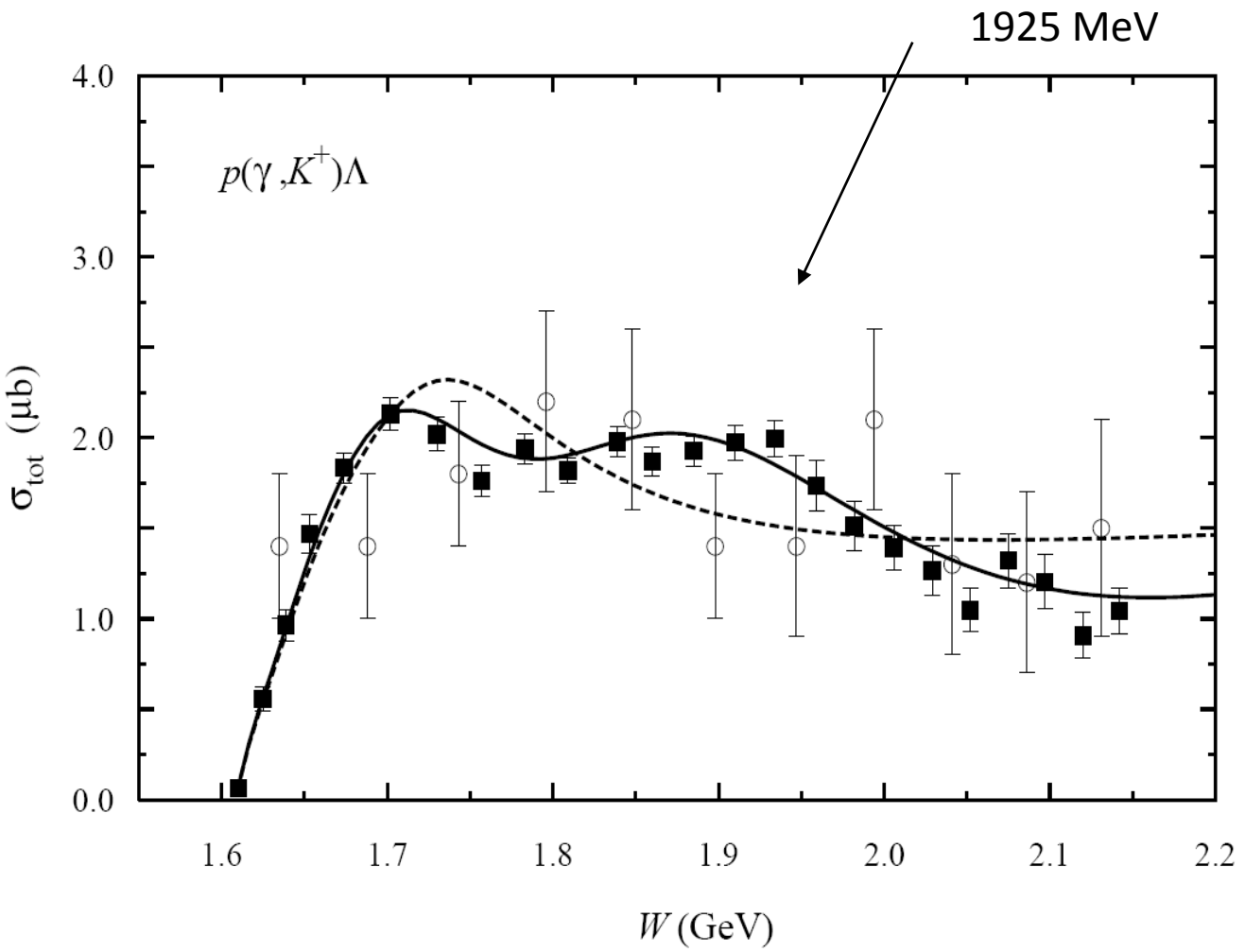


FIG. 1. Total cross section for $K^+\Lambda$ photoproduction on the proton. The dashed line shows the model without the $D_{13}(1960)$ resonance, while the solid line is obtained by including the $D_{13}(1960)$ state. The new SAPHIR data [6] are denoted by the solid squares, old data [22] are shown by the

Experimental tests for the signature this resonance

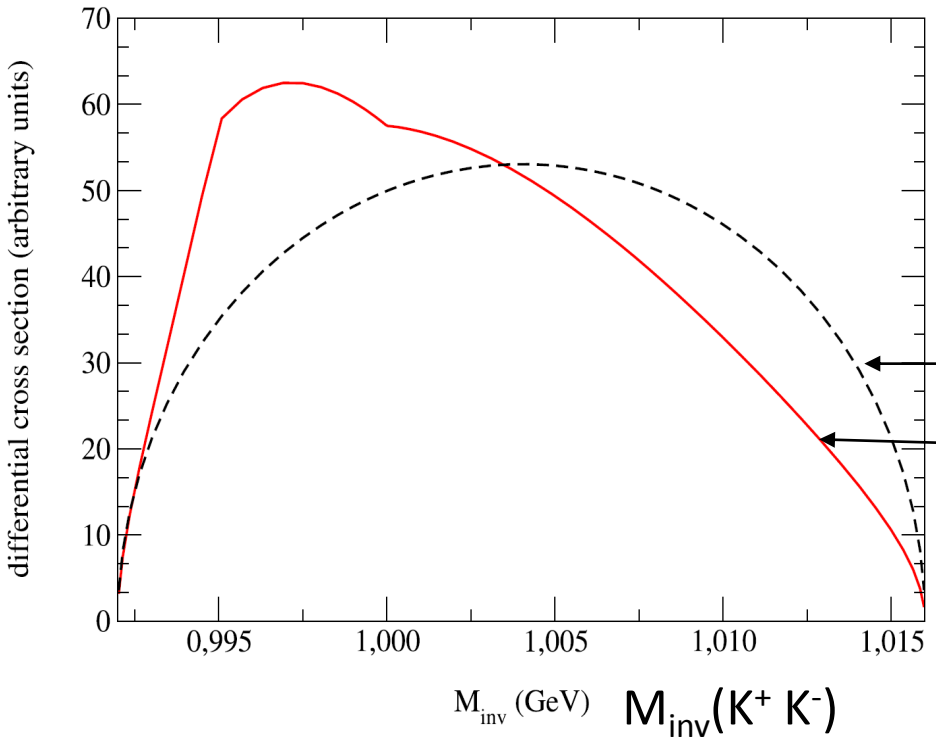
In $\gamma\, p \rightarrow K^+ \Lambda$ use polarized beam and target and look for

$S_z=3/2$ and $S_z=1/2$ (standard technique since a few years)

Take $S_z=3/2$: if the peak remains one cannot have J=1/2
if the peak disappears one does not have J=3/2

Schumacher has the key to that at Jeff Lab

$\gamma\, p \rightarrow K^+ K^- p$



$M_{inv}(K^+ K^-)$

$d\sigma / d M_{inv}(K^+ K^-)$

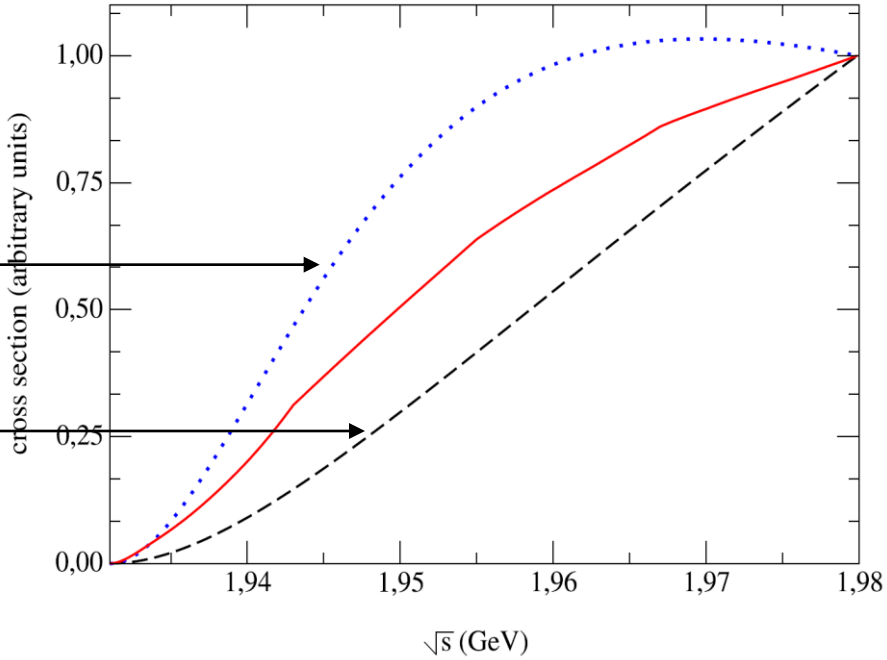
Phase space

K Kbar N resonance below threshold

Integrated cross section

K Kbar N resonance below threshold

Phase space



Preliminary experimental results available from T. Nakano

Multirho states:

The vector vector interaction can be studied using the local hidden gauge formalism, Bando et al.

$$\mathcal{L}^{(4V)} = \frac{g^2}{2}\langle V_\mu V_\nu V^\mu V^\nu - V_\nu V_\mu V^\mu V^\nu \rangle \ , \qquad g=M_V/2f_\pi$$

$$\mathcal{L}^{(3V)} = ig\langle (V^\mu \partial_\nu V_\mu - \partial_\nu V_\mu V^\mu) V^\nu \rangle \ ,$$

$$V^{(I=0,S=2)}(s) = -4g^2 - 8g^2\left(\frac{3s}{4m_\rho^2} - 1\right) \sim -20g^2$$

$$V^{(I=2,S=2)}(s) = 2g^2 + 4g^2\left(\frac{3s}{4m_\rho^2} - 1\right) \sim 10g^2$$

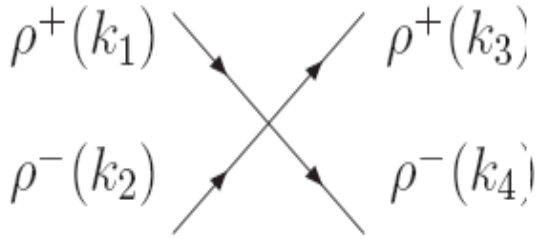
$$T = \frac{V}{1 - VG},$$

$$G(s) = i\int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2 - m_\rho^2 + i\epsilon}\frac{1}{(Q-p)^2 - m_\rho^2 + i\epsilon} \ ,$$

Rho-rho interaction in the hidden gauge approach

R.Molina, D. Nicmorus, E. O. PRD (08)

V=



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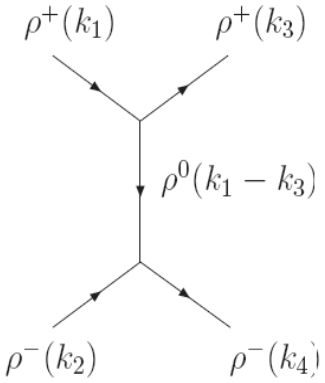

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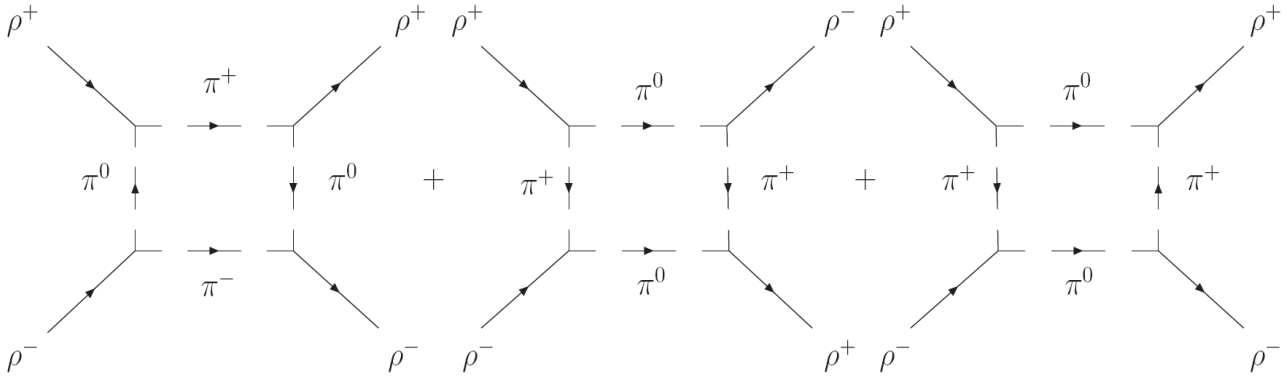


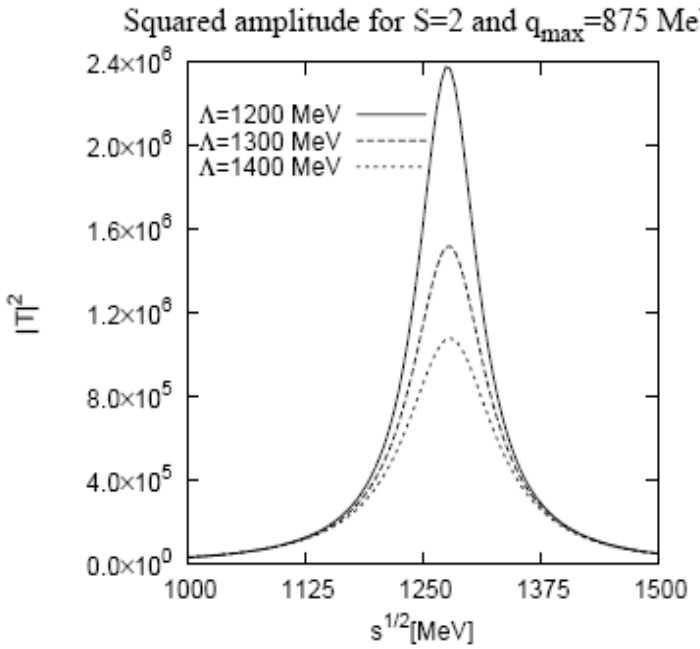
Spin projectors neglecting q/M_V, in L=0

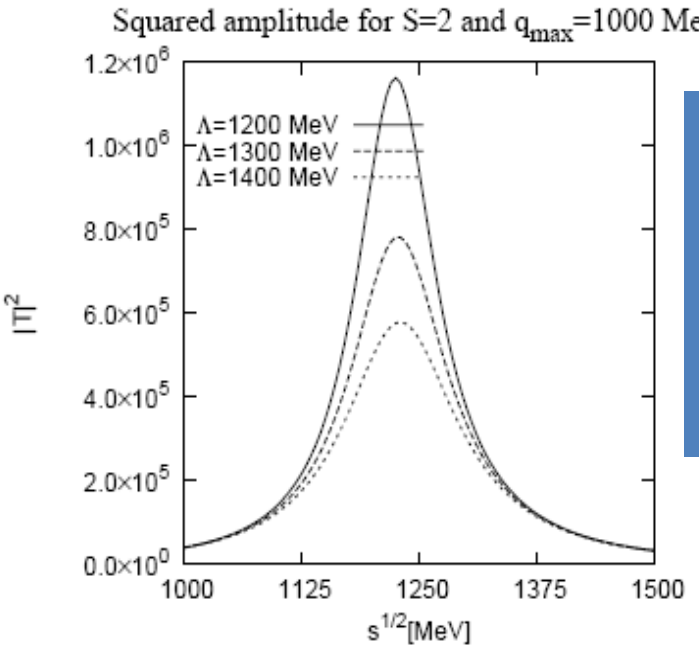
$$\mathcal{P}^{(0)} = \frac{1}{3}\epsilon_\mu\epsilon^\mu\epsilon_\nu\epsilon^\nu$$

$$\mathcal{P}^{(1)} = \frac{1}{2}(\epsilon_\mu\epsilon_\nu\epsilon^\mu\epsilon^\nu - \epsilon_\mu\epsilon_\nu\epsilon^\nu\epsilon^\mu)$$

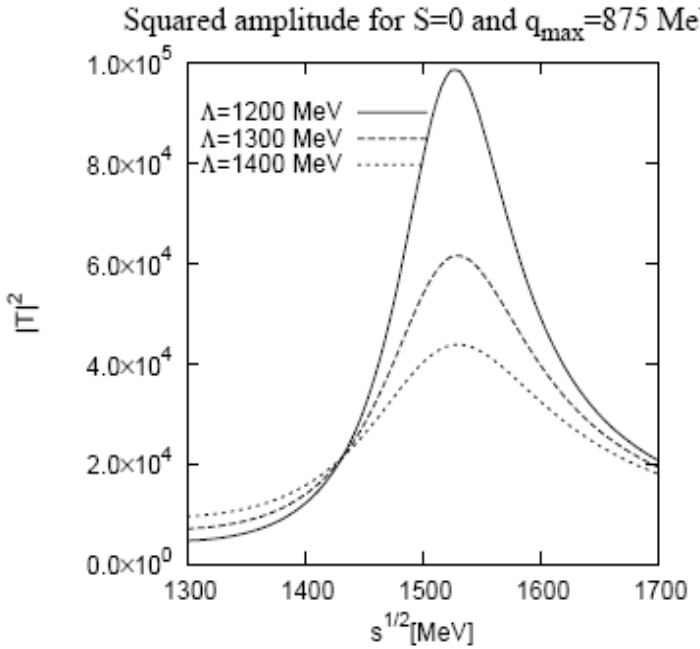
$$\mathcal{P}^{(2)} = \{\frac{1}{2}(\epsilon_\mu\epsilon_\nu\epsilon^\mu\epsilon^\nu + \epsilon_\mu\epsilon_\nu\epsilon^\nu\epsilon^\mu) - \frac{1}{3}\epsilon_\alpha\epsilon^\alpha\epsilon_\beta\epsilon^\beta\}$$

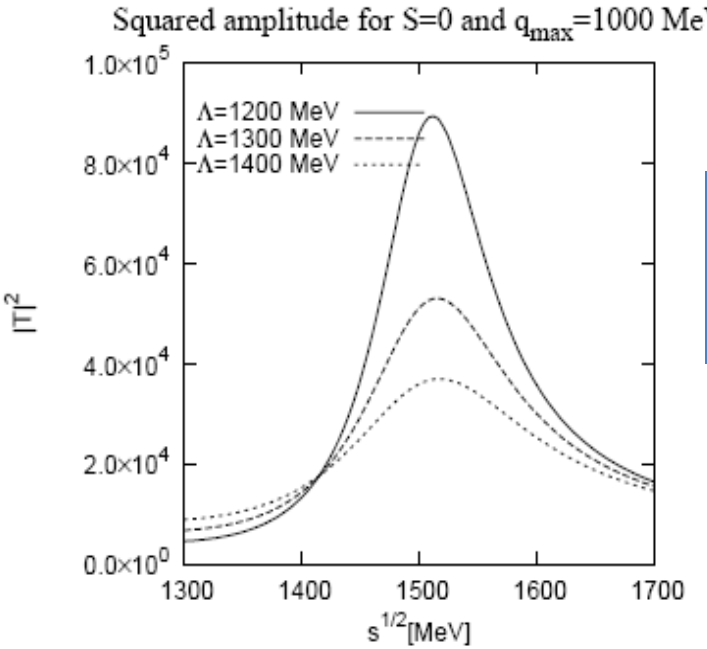
Bethe Salpeter eqn. $$T = \frac{V}{1 - VG}$$ G is the ρρ propagator





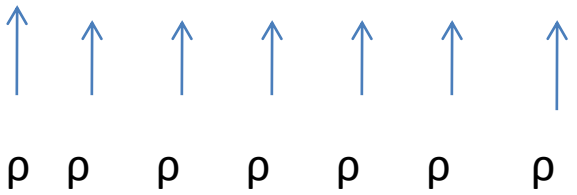
Two I=0 states generated f_0, f_2 that we associate to $f_0(1370)$ and $f_2(1270)$





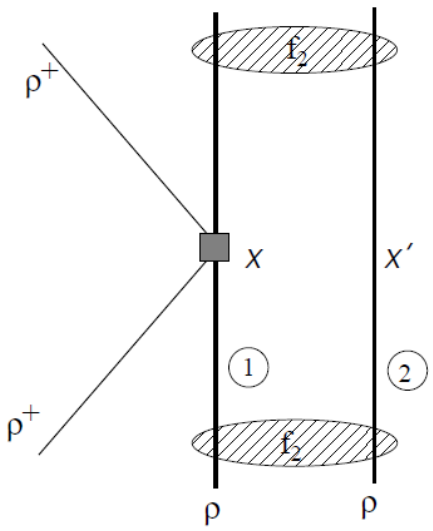
Belle finds the f0(1370) around 1470 MeV

We would like to construct states of many ρ with parallel spins, so as to have maximum binding for any pair



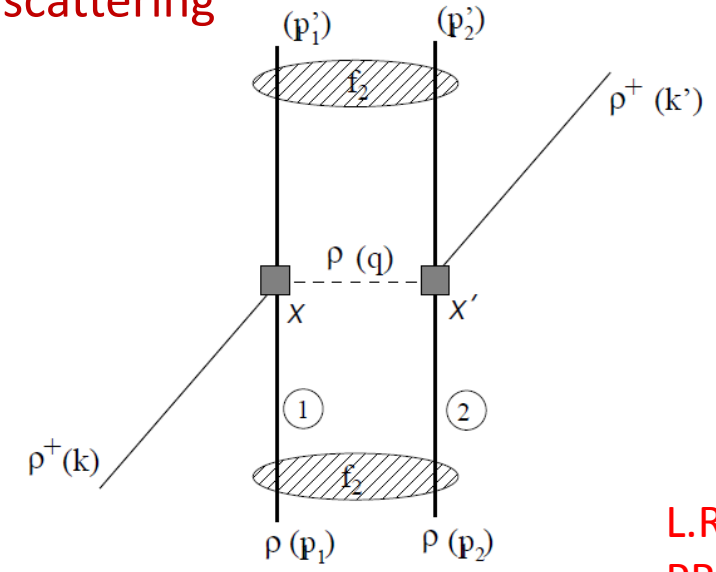
This is like a ferromagnet of ρ mesons

Fixed center approximation to ρ f_2 scattering



a)

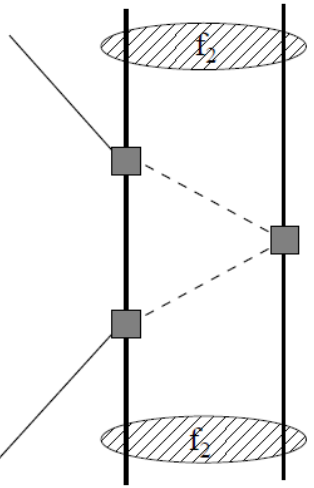
+



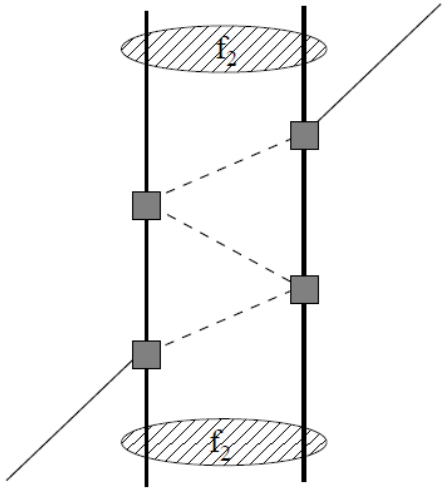
b)

L.Roca, E.O.
PRD82(2010)

+



+



+ . . .

d)

This interaction generates the ρ_3

$$T_1 = t_1 + t_1 G_0 T_2$$
$$T_2 = t_2 + t_2 G_0 T_1$$
$$T = T_1 + T_2$$

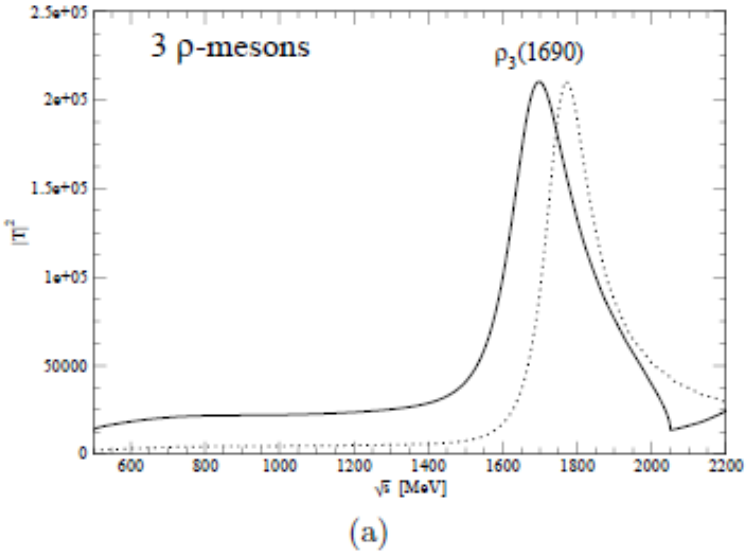
$$G_0 \equiv \frac{1}{M_{f_2}} \int \frac{d^3q}{(2\pi)^3} F_{f_2}(q) \frac{1}{q^{0\,2} - \vec{q}^{\,2} - m_\rho^2 + i\epsilon}$$

$$F_{f_2}(q) = \frac{1}{\mathcal{N}} \int_{\substack{p<\Lambda \\ |\vec{p}-\vec{q}|<\Lambda}} d^3p \, \frac{1}{M_{f_2} - 2\omega_\rho(\vec{p})} \, \frac{1}{M_{f_2} - 2\omega_\rho(\vec{p}-\vec{q})}$$

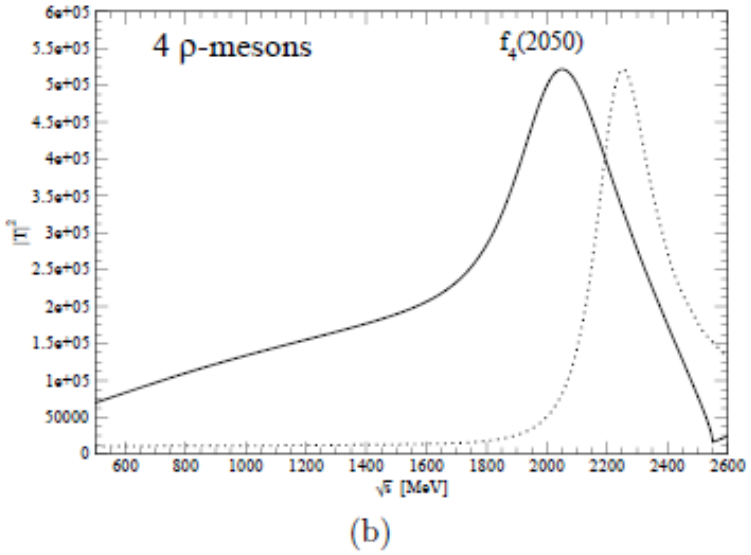
where the normalization factor $\mathcal{N}$ is

$$\mathcal{N} = \int_{p<\Lambda} d^3p \frac{1}{\left(M_{f_2} - 2\omega_\rho(\vec{p})\right)^2}$$

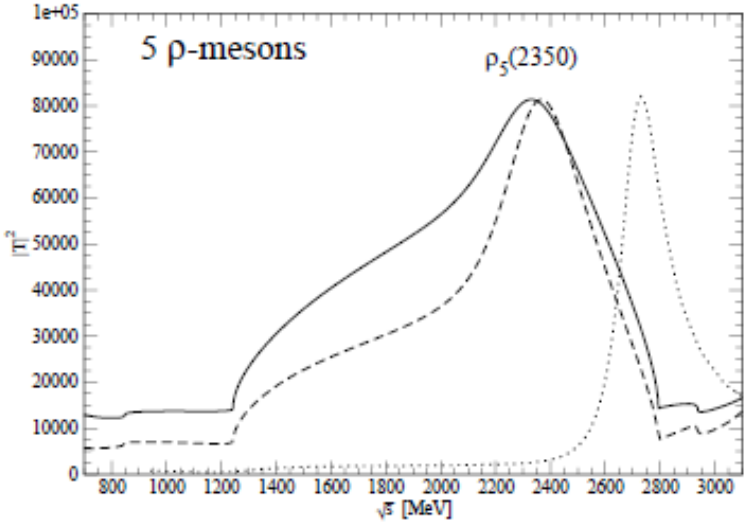
One then continues and makes scattering of f_2 with f_2 to get the f_4
Then ρ interaction with f_4 to give ρ_5 and finally f_2 with f_4 to give f_6



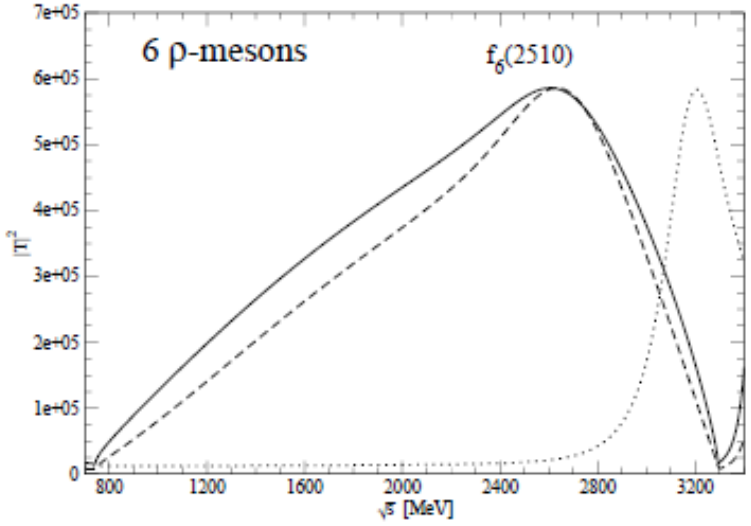
(a)



(b)



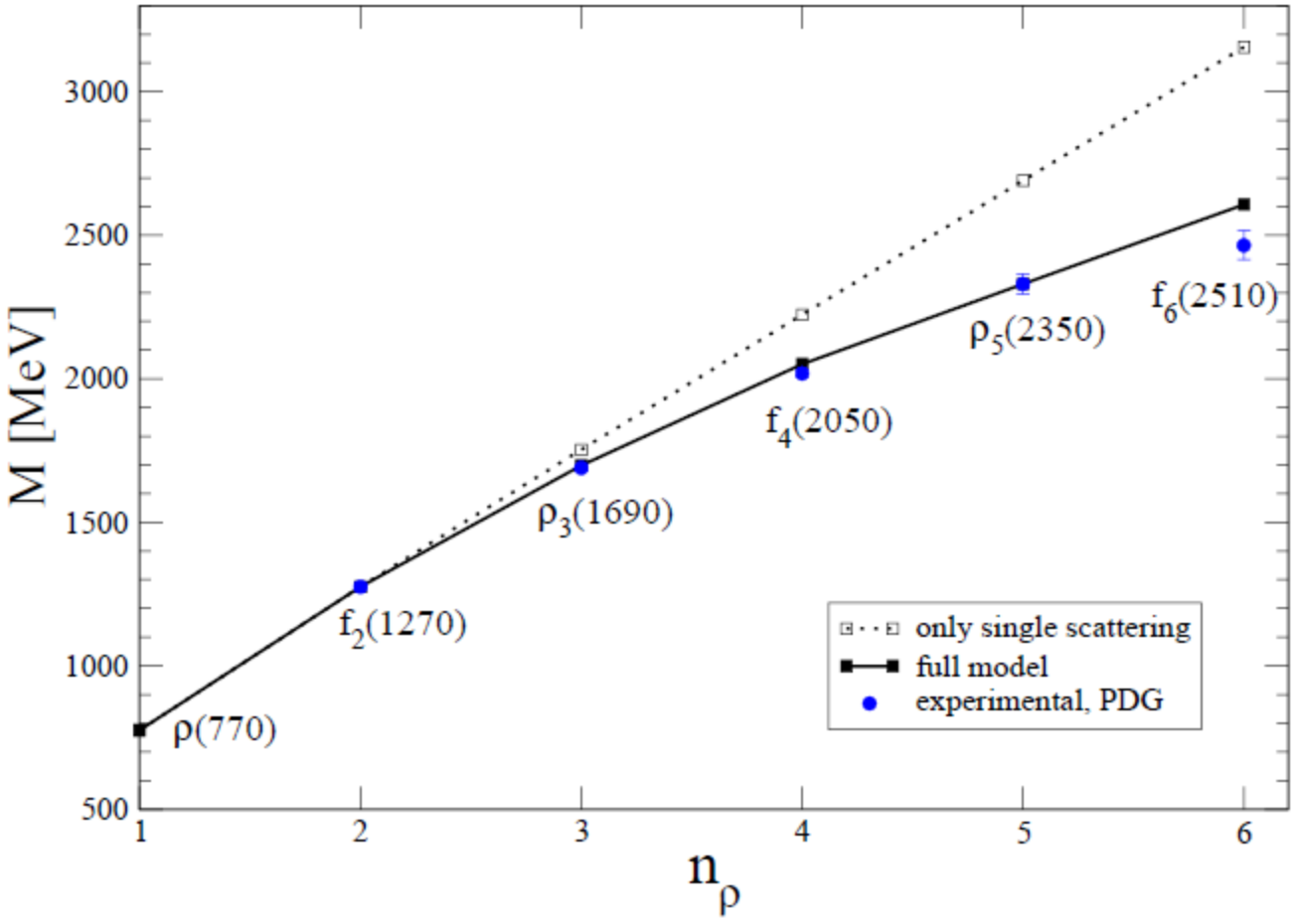
(c)



(d)

Luis Roca
E.O. 2010

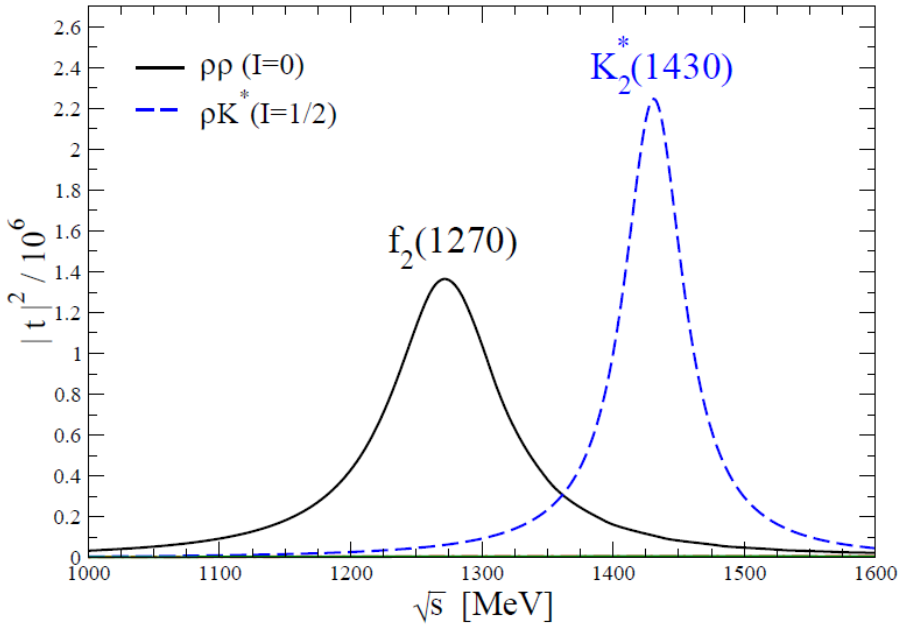
See details in L. Roca Talk


M [MeV]
3000
2500
2000
1500
1000
500
1
2
3
4
5
6
n_ρ
ρ(770)
f_2(1270)
ρ_3(1690)
f_4(2050)
ρ_5(2350)
f_6(2510)
only single scattering
full model
experimental, PDG

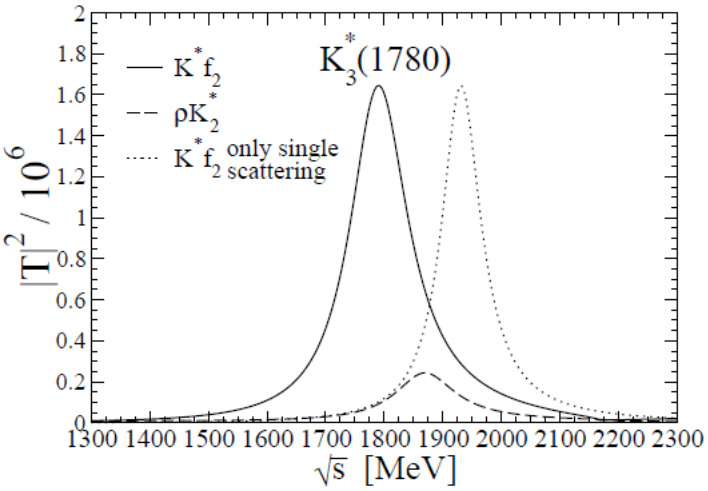
On the nature of the $K_2^*(1430)$, $K_3^*(1780)$, $K_4^*(2045)$, $K_5^*(2380)$ and K_6^* as K^*–multi-ρ states

J. Yamagata-Sekihara[1] L. Roca[2] and E. Oset[1]

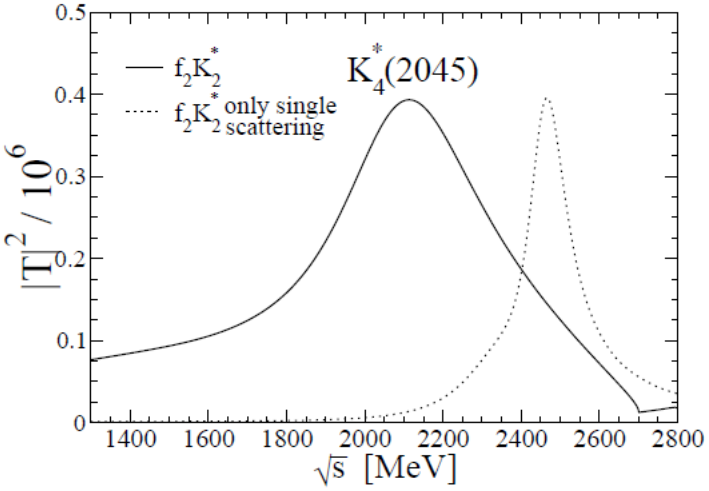
Phys.Rev. D82 (2010) 094017



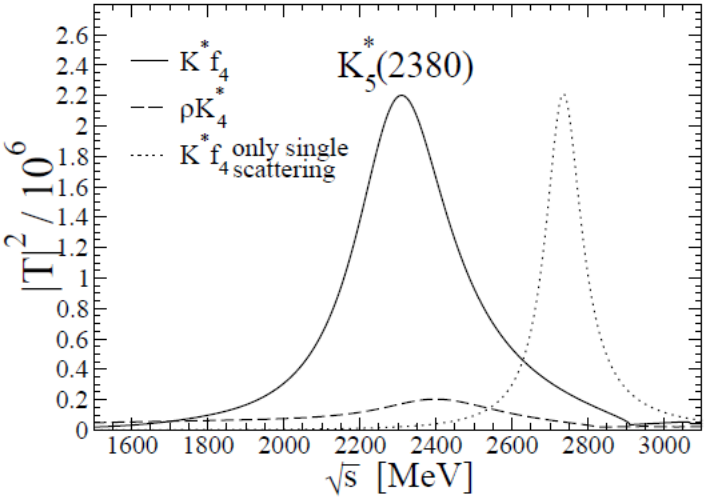
	A	B $(b_1 b_2)$
two-body	ρ	K^*
three-body	K^*	f_2 $(\rho\rho)$
	ρ	K_2^* (ρK^*)
four-body	f_2	K_2^* (ρK^*)
five-body	K^*	f_4 $(f_2 f_2)$
	ρ	K_4^* $(f_2 K_2^*)$
six-body	K_2^*	f_4 $(f_2 f_2)$
	f_2	K_4^* $(f_2 K_2^*)$


$K_3^*(1780)$
$K^* f_2$
ρK_2^*
$K^* f_2$ only single scattering
$|T|^2 / 10^6$
$\sqrt{s}$ [MeV]

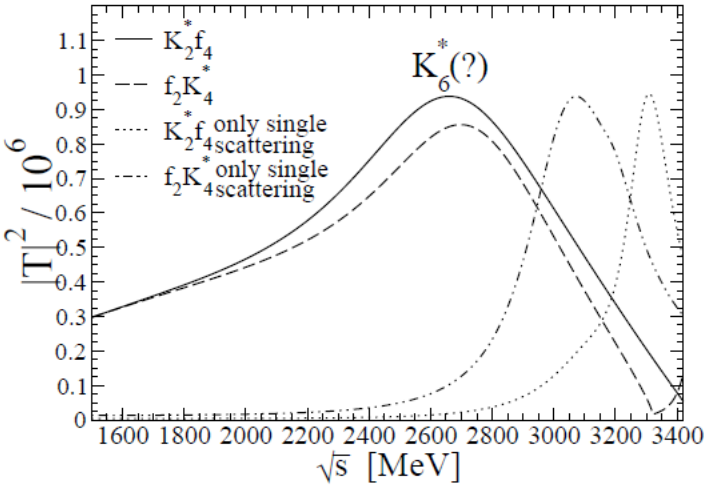
(a)


$K_4^*(2045)$
$f_2 K_2^*$
$f_2 K_2^*$ only single scattering
$|T|^2 / 10^6$
$\sqrt{s}$ [MeV]

(b)


$K_5^*(2380)$
$K^* f_4$
ρK_4^*
$K^* f_4$ only single scattering
$|T|^2 / 10^6$
$\sqrt{s}$ [MeV]

(c)


$K_6^*(?)$
$K_2^* f_4$
$f_2 K_4^*$
$K_2^* f_4$ only single scattering
$f_2 K_4^*$ only single scattering
$|T|^2 / 10^6$
$\sqrt{s}$ [MeV]

(d)

generated resonance	amplitude	mass, PDG [21]	mass only single scatt.	mass full model
$K_2^*(1430)$	ρK^*	1429 ± 1.4	–	1430
$K_3^*(1780)$	$K^* f_2$	1776 ± 7	1930	1790
$K_4^*(2045)$	$f_2 K_2^*$	2045 ± 9	2466	2114
$K_5^*(2380)$	$K^* f_4$	$2382 \pm 14 \pm 19$	2736	2310
K_6^*	$K_2^* f_4$-$f_2 K_4^*$	–	3073-3310	2661-2698

Luis Roca, ArXiv 1107.5489

Systems with $J^{PC} = 2^{-+}$ can be regarded as molecules made of a pseudoscalar (P) 0^{-+} and a tensor 2^{++} meson

with the 2^{++} state made out of two vector mesons

assigned resonance	dominant channel	mass PDG [47]	mass, only single scatt.	mass full model
$\pi_2(1670$	$\eta a_2(1320)$	1672 ± 3	1800	1660
$\eta_2(1645)$	$\eta f_2(1270)$	1617 ± 5	1795	1695
$K_2^*(1770)$	$K a_2(1320)$	1773 ± 8	1775	1775

Conclusions

--The combination of chiral unitary dynamics with Faddeev equations has proved very useful.
--Conceptually it removes the ambiguities tied to the off shell 2 body t-matrix.
--One finds that all $1/2^+$ low lying states (except the Roper) qualify largely as states of two mesons and one baryon.

--Multimeson states are also emerging:
--$\Phi(2170)$ appears as resonance of $\phi(1020)$ K Kbar
--Multirho states could be identified with meson states of increasing spin
--K*-multirho states can also be identified with K* states of increasing spin
--Low lying pseudotensor states also can be constructed from a pseudoscalar and two vectors coupled to a tensor.
--Many other states are under investigation by several groups , Jido, Martinez Torres, Roca, Oller, Bayar, Sun, Xie …….