Transport Coefficients of a Unitarized Pion Gas

International Erice School on Nuclear Physics 2011

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What are the TC?

They describe how the system tends to equilibrium when a external perturbation is applied, producing a gradient of some hydrodynamical field. At first order:

$$\text{TC} \times \nabla(\text{Hydro. Field}) = -\text{Response Flow}$$

where the response flow corresponds to the motion of a conserved quantity density.

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Examples

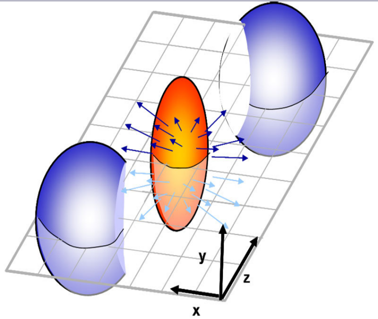
η shear viscosity; ζ bulk viscosity
κ thermal conductivity; σ electrical conductivity

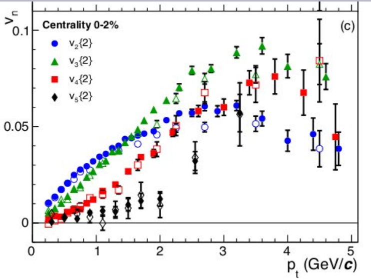
Note: Viscosities are usually normalized by the entropy density

Anisotropies in the expanding fireball in rel. heavy ion collisions (see talk by S. Voloshin)

Elliptic (v_2) and higher order flow harmonics ($v_3, v_4, ...$)

$v_n = \langle \cos n(\phi_i - \Psi_{RP}) \rangle$ $\phi_i \in OXY$ plane; $\Psi_{RP} = OXZ$ plane





ALICE Collaboration, 2011

Estimating the shear viscosity

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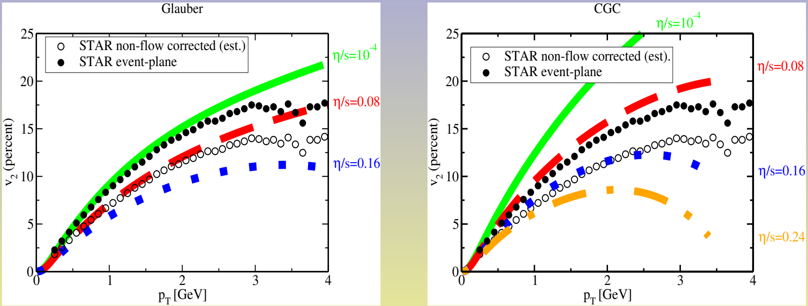
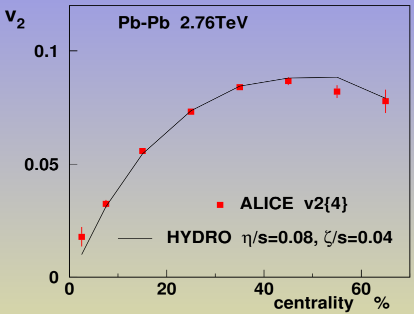


Glauber: $\eta/s \simeq 1/(4\pi)$ M. Luzum and P. Romatschke, 2008 CGC: $\eta/s \simeq 2 \times 1/(4\pi)$

Bulk viscosity is becoming quite popular too



P. Bozek, 2011

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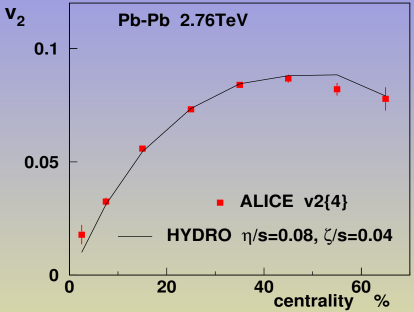

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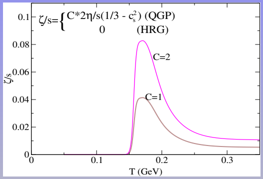
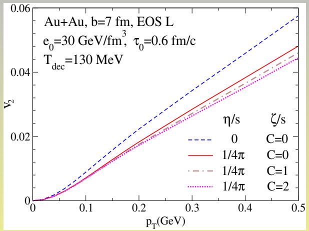


H. Song and U.W. Heinz, 2008



Theoretical Calculation

Pion Gas at low energies $\rightarrow$ Perturbative QCD is not valid

Effective theory for the pions:

$SU(2)$ Chiral Perturbation Theory with physical pion masses

(J. Gasser and H. Leutwyler, 1984)

Effective Lagrangian (see talk by R. Torres)

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \cdots \quad \text{with } \mathcal{L}_n \sim \mathcal{O}\left(\frac{p}{4\pi F_0}\right)^n, \mathcal{O}\left(\frac{m_\pi}{4\pi F_0}\right)^n$$

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At low temperatures ($T \lesssim m_\pi$) only $2 \rightarrow 2$ elastic scattering is relevant. Inelastic channels are suppressed at Lagrangian level and in the final phase-space. At moderate energies inelastic processes and the strange-carrying states ($SU(3)$ ChPT) must be included.

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Pion number is conserved $\rightarrow$ pseudo-chemical potential $\mu_\pi \leq m_\pi$.

Unitarized pion gas

Chiral Perturbation Theory

Satisfies unitarity order by order:

$$\text{Im } t^{(2)}_{IJ}(s) = 0; \quad \text{Im } t^{(4)}_{IJ}(s) = \sqrt{1 - 4m_\pi^2/s}\ |t^{(2)}_{IJ}(s)|^2$$

with $t^{(2n)}_{IJ} \sim \mathcal{O}(p^{2n}) \sim \mathcal{O}(s^n)$. This gives an increasing cross-section with energy.

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Inverse Amplitude Method (T.N. Truong, A. Dobado, J.R. Pelaez...)

Satisfies exact unitarity:

$$\tilde{t}_{IJ}(s) = \frac{t^{(2)}_{IJ}(s)}{1 - t^{(4)}_{IJ}(s)/t^{(2)}_{IJ}(s)} \rightarrow \text{Im } \tilde{t}_{IJ}(s) = \sqrt{1 - 4m_\pi^2/s}\ |\tilde{t}_{IJ}(s)|^2$$

The amplitude is a rational function of energy, the cross-section eventually saturates and we can dynamically generate the ρ and σ resonances.

Boltzmann-Uehling-Uhlenbeck Equation

Transport equation for pions

$$\frac{df_p}{dt} = C[f_p, f_p]$$

$$C[f_p, f_p] = \frac{g_\pi}{2} \int d\Gamma_{12,3p}[f_1 f_2 (1+f_3)(1+f_p) - f_3 f_p (1+f_1)(1+f_2)]$$

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Chapman-Enskog expansion (in powers of Knudsen number)

$$f_p = f_{Bose} + \epsilon f^{(1)} + \epsilon^2 f^{(2)} + \cdots$$

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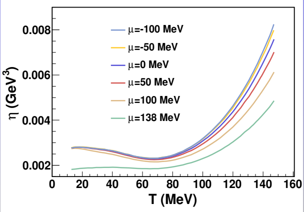
Chapman-Enskog expansion (in powers of Knudsen number)

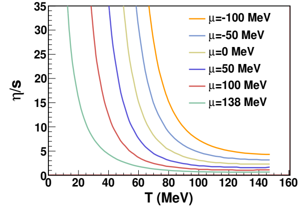
$$f_p = f_{Bose} + \epsilon f^{(1)} + \epsilon^2 f^{(2)} + \cdots$$

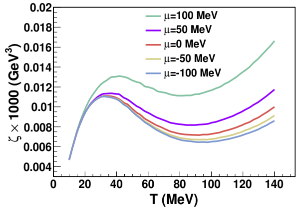
The TC can be expressed as a simple integral containing $f^{(1)}$:

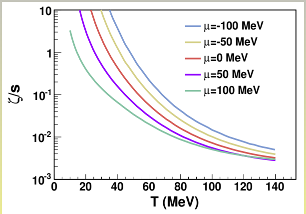
$$\text{TC} = \langle \ (\nabla[\text{Hydro. fields})]^{-1} | f^{(1)} \ \rangle$$

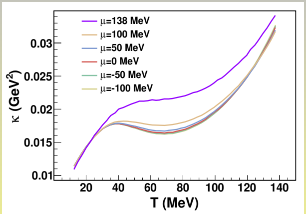
Results











In collaboration with F.J. Llanes-Estrada and A. Dobado

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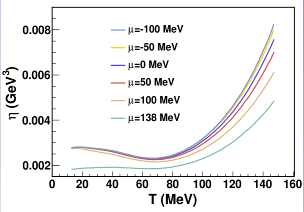

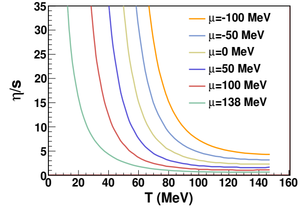

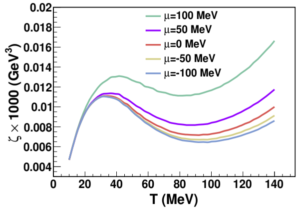

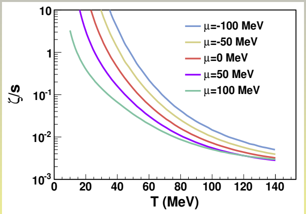

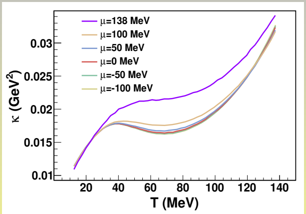

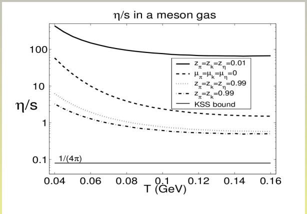

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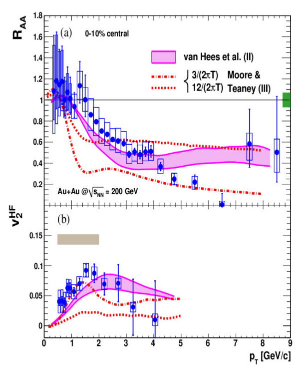
Charm Diffusion

Consider a c-quark (hadronized into a D or D^* meson) propagating in the medium and interacting with the thermalized pion gas.



(PHENIX Collaboration, 2010)

The charm diffusion coefficients tell us about the energy loss of the c-quark due to collisions with the pions. These diffusion coefficients are important to describe the nuclear modification factor and the elliptic flow of the electrons coming from semileptonic decays of heavy hadrons.

$$R_{AA} = \frac{dN^e_{AA}/dp_T}{<N_{coll}> dN^e_{pp}/dp_T}$$

Kinetic calculation

Boltzmann-Uehling-Uhlenbeck equation for f_c

$$\frac{df_c}{dt} = C[f_c, f_\pi]$$

Scale hyerarchy: $M_D, M_{D^*} \gg m_\pi \sim T \sim \text{transfered momentum.}$

Kinetic calculation

Fokker-Planck equation

$$\partial_t f_c(t,p) = \partial_{p_i} \left\{ F_i(p) f_c(t,p) + \partial_{p_j} \left[\Gamma_{ij}(p) f_c(t,p) \right] \right\}$$

Three different coefficients (but only two independent because of the fluctuation-dissipation theorem):

$F(p)$: Drag force $\Gamma_0(p)$, $\Gamma_1(p)$: Diffusion coefficients

In the static limit ($p \to 0$) only one is independent because of the Einstein relation $\Gamma = FTM_D$.

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Other relevant quantities

$$D_x = T^2/\Gamma; \quad -dp/dx = E \times F(p); \quad -dE/dx = p \times F(p)$$

$D-\pi$, $D^*-\pi$ interaction

Effective Field Theory

Eff. Lagrangian with Chiral symmetry + Heavy Quark symmetry + On-shell Unitarization (J.A.Oller and E. Oset, 1997)
(L. Roca, E. Oset and J. Singh, 2005)

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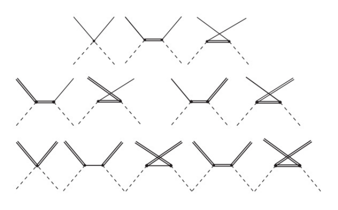
$$\mathcal{L}^{(1)} = \nabla^\mu D \nabla_\mu D^\dagger - m_D^2 D D^\dagger - \nabla^\mu D^{*\nu} \nabla_\mu D_\nu^{*\dagger} + m_D^2 D^{*\mu} D_\mu^{*\dagger}$$

$$+ig \left(D^{*\mu} U_\mu D^\dagger - D U^\mu D_\mu^{*\dagger} \right) + \frac{g}{2m_D} \left(D_\mu^* U_\alpha \nabla_\beta D_\nu^{*\dagger} - \nabla_\beta D_\mu^* U_\alpha D_\nu^{*\dagger} \right) \epsilon^{\mu\nu\alpha\beta}$$



(L.S. Geng, N. Kaiser, J. Martin-Camalich and W. Weise, 2010)

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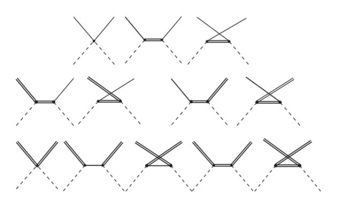
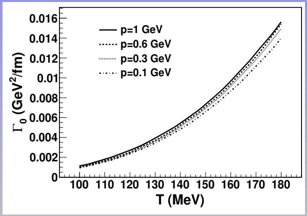
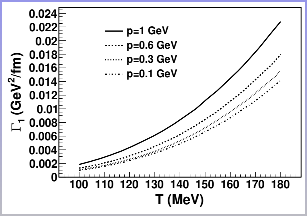


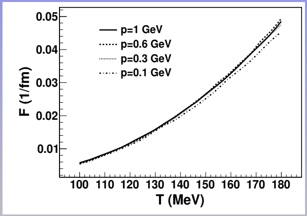
(L.S. Geng, N. Kaiser, J. Martin-Camalich and W. Weise, 2010)

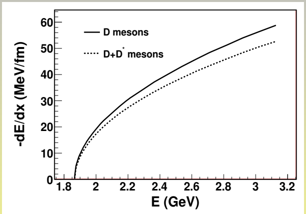
By unitarizing the scattering amplitudes we dynamically generate the $D_0(2400)$ and the $D_1(2430)$ resonances.

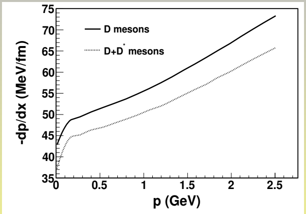
Results

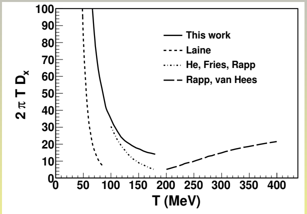












In collaboration with F.J. Llanes-Estrada, D. Cabrera and L. Abreu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(WORKING ON THIS PRESENTATION AT MADRID'S AIRPORT)