NUCLEAR CHIRAL THERMODYNAMICS
and PHASES of QCD

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Prelude: QCD Phase Diagram (Concepts, Models, Problems)

Main Theme: Nuclear Chiral Thermodynamics
- QCD interface with nuclear physics: Chiral Effective Field Theory
- Nuclear Equation of State and QCD phase diagram
- Density and temperature dependence of the Chiral (Quark) Condensate

Outlook: New constraints from Neutron Stars
Part I: Prelude

QCD PHASE DIAGRAM

Visions & Facts
QCD PHASE DIAGRAM

( theorists’ vision )

**Spontaneous Chiral Symmetry Breaking**

\[ \langle \bar{q}q \rangle \neq 0 \]

**High Density:**

- Color Superconductivity
- Cooper pairing

**Critical Point**

\[ T_c \]

**Quark – Gluon Phase**

\[ \langle \bar{q}q \rangle = 0 \]

**Hadron Phase**

\[ \langle \bar{q}q \rangle \neq 0 \]

**CSC Phases**

\[ \langle qq \rangle \neq 0 \]

**Baryon Chemical Potential**

\[ \mu_B \]

\[ T \text{ [GeV]} \]

\[ \bar{q}q \text{ condensation} \]

\[ q \quad [1] \quad \bar{q} \]

\[ q \quad [3] \quad q \]
QCD PHASE DIAGRAM

(reality ? facts ?)

Constraints from Nuclear Physics

New constraints from Neutron Stars

\[ \langle \bar{q}q \rangle \neq 0 \]

hadron phase

\[ \langle q \rangle \neq 0 \]

critical point

\[ \mu_B \]

T [GeV]

\[ T_c \]

\[ T \]

\[ \langle \bar{q}q \rangle \neq 0 \]

\( \bar{q}q \) condensation

Lattice QCD

quark – gluon phase

CSC phases

nuclear matter
MODELING the QCD PHASE DIAGRAM

Guiding principle:
QCD symmetries and symmetry breaking patterns

Spontaneously broken chiral symmetry
$\text{SU}(N_f)_R \times \text{SU}(N_f)_L$

non-local PNJL model

Centre $\mathbb{Z}(3)$ of $\text{SU}(3)_c$ gauge group

chiral and deconfinement crossover transitions (3 flavor PNJL model)

$\langle \bar{q}q \rangle_T / \langle \bar{q}q \rangle_0$

Polyakov loop

LQCD 2+1 flavors

pure gauge

non-local PNJL

$T$ [GeV]

$\langle \bar{q}q \rangle_T / \langle \bar{q}q \rangle_0$

Polyakov loop

LQCD 2+1 flavors

pure gauge

non-local PNJL

$T$ [GeV]

The orange band shows the confinement-deconfinement crossover transition as described by the Polyakov loop in the range \( k_L < \Phi < k_{nh} \). The dashed black line corresponds to the chiral crossover \( \bar{\psi} \psi/\bar{\psi} \psi < 0 < k_{rch} \). The solid black line indicates the chiral first-order transition. The temperature scale is set by \( T_c \times M_{GeV} \).

Including wavefunction renormalization effects requires a careful reassessment of chiral low-energy theorems. Pseudoscalar meson masses and corresponding decay constants at zero temperature have been derived. The results clearly show that the formalism incorporates fundamental chiral relations such as the Gell-Mann–Oakes–Renner and Goldberger-Treiman relations. In the three-flavor case, the inclusion of the 't Hooft-Kobayashi-Maskawa interaction leads to the correct mass splitting between the \( \eta \) and the \( \eta' \) meson.

The PNJL thermodynamics has now been developed with systematic inclusion of the quark quasiparticle renormalization factor \( Z_{bp} \). The temperature dependence of the chiral condensate and of the Polyakov loop has been calculated, indicating chiral and deconfinement crossover transitions. We have compared our results with recent lattice QCD computations. Finally, a quark chemical potential has been introduced that enables extensions to the finite-density region of the QCD phase diagram.

Does the first-order line really extend down to low temperatures?
baryon densities \( \rho_B = \frac{1}{3} \left( \frac{\partial P}{\partial \mu} \right)_T \)
in the range \( 0.1 - 0.2 \text{ fm}^{-3} \)

Quarks are not the relevant active quasiparticles at low temperatures and baryon chemical potentials
Part II:

NUCLEAR CHIRAL THERMODYNAMICS
NUCLEAR MATTER and QCD PHASES

Scales in nuclear matter:

- momentum scale: **Fermi momentum**
  \[ k_F \simeq 1.4 \text{ fm}^{-1} \sim 2m_\pi \]
- NN distance:
  \[ d_{NN} \simeq 1.8 \text{ fm} \simeq 1.3 \text{ m}_\pi^{-1} \]
- energy per nucleon:
  \[ E/A \simeq -16 \text{ MeV} \]
- compression modulus:
  \[ K = (260 \pm 30) \text{ MeV} \sim 2m_\pi \]
PIONS and NUCLEI in the context of LOW-ENERGY QCD

- **CONFINEMENT** of quarks and gluons in hadrons
  - Spontaneously broken **CHIRAL SYMMETRY**

LOW-ENERGY / LOW-TEMPERATURE QCD: 
Effective Field Theory of weakly interacting

Nambu-Goldstone Bosons (**PIONS**) representing QCD at (energy and momentum) scales

\[ Q << 4\pi f_\pi \sim 1 \text{ GeV} \]

\[ f_\pi = 92.4 \text{ MeV} \]

\[ m_\pi^2 f_\pi^2 = -m_q \langle \bar{\psi}\psi \rangle + O(m_q^2) \]

- **spontaneous symmetry breaking**
- **explicit symmetry breaking**
CHIRAL EFFECTIVE FIELD THEORY

- Systematic framework at interface of QCD and Nuclear Physics
- Interacting systems of **PIONS** (light / fast) and **NUCLEONS** (heavy / slow):

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_\pi(U, \partial U) + \mathcal{L}_N(\Psi_N, U, ...) \]

\[ U(x) = \exp[i\tau_a \pi_a(x)/f_\pi] \]

- Construction of Effective Lagrangian: **Symmetries**

  ![Diagrams showing interactions between pions and nucleons](image)

- Short distance dynamics: **contact terms**
Explicit $\Delta(1230)$ DEGREES of FREEDOM

- Large spin-isospin polarizability of the Nucleon

example: polarized Compton scattering

\[ \beta\Delta = \frac{g_A^2}{f_\pi^2(M\Delta - M_N)} \sim 5 \text{ fm}^3 \]

\[ M\Delta - M_N \simeq 2 m_\pi << 4\pi f_\pi \] (small scale)

- Pionic Van der Waals - type intermediate range central potential

N. Kaiser, S. Fritsch, W.W., NPA750 (2005) 259

\[ V_c(r) = -\frac{9 g_A^2}{32\pi^2 f_\pi^2} \beta A \frac{e^{-2m_\pi r}}{r^6} P(m_\pi r) \]

J. Fujita, H. Miyazawa (1957)
Pieper, Pandharipande, Wiringa, Carlson (2001)

strong 3-body interaction
Important pieces of the **CHIRAL NUCLEON-NUCLEON INTERACTION**

- **ISOVECTOR TENSOR FORCE**

  ![Isovector Tensor Force Diagram]

  Note: **no** $\rho$ meson

- **CENTRAL ATTRACTION** from **TWO-PION EXCHANGE**

  ![Central Attraction Diagram]

  Note: **no** $\sigma$ boson

**Van der WAALS - like force:**

$$V_c(r) \propto -\frac{\exp[-2m_\pi r]}{r^6}P(m_\pi r)$$

... at intermediate and long distance

PIONS (and DELTA isobars) as explicit degrees of freedom

\[ k_F \sim 2 m_\pi \sim M_\Delta - M_N < < 4\pi f_\pi \]

### IN-MEDIUM CHIRAL PERTURBATION THEORY

pion exchange processes in presence of filled Fermi sea

2nd order TENSOR force + nucleon’s SPIN-ISOSPIN polarizability

short-distance dynamics: contact interactions (incl. resummations)
**IN-MEDIUM CHIRAL PERTURBATION THEORY**

- **Loop expansion** of *(In-Medium) Chiral Perturbation Theory*
  
  Systematic expansion of **ENERGY DENSITY** $\mathcal{E}(k_F)$ in powers of **Fermi momentum** [modulo functions $f_n(k_F/m_\pi)$]
  
  (works for $k_F << 4\pi f_\pi \sim 1\text{ GeV}$)

- **Finite nuclei** ↔ **energy density functional**
  

  many quantitatively successful applications throughout the nuclear chart
  
  e.g. P. Finelli et al.: Nucl. Phys. A 770 (2007) 1

- **Nuclear thermodynamics**: compute **free energy density**
  
  (3-loop order)
  

**in-medium nucleon propagators incl. Pauli blocking**
- **In-medium ChPT**
  3-loop \((\pi, N, \Delta)\)

- **Input** parameters:
  two contact terms

- **Output**:
  - Binding & saturation
    \[ E_0 / A = -16 \text{ MeV}, \quad \rho_0 = 0.16 \text{ fm}^{-3}, \quad K = 290 \text{ MeV} \]
  - Realistic (complex, momentum dependent) single-particle potential
    ... satisfying Hugenholtz - van Hove and Luttinger theorems (!)
  - Asymmetry energy
    \[ A(k_F^0) = 34 \text{ MeV} \]

- **Landau parameters**
  J.W. Holt, N. Kaiser, W.W.
NUCLEAR THERMODYNAMICS

NUCLEAR CHIRAL (PION) DYNAMICS

BINDING & SATURATION:

Van der Waals + Pauli

\[ \pi \]

\[ N, \Delta \]

+ 

3-body forces

contact terms

\[ N \downarrow \quad \pi \quad \pi \quad \downarrow N \]

\[ N \downarrow \quad \pi \quad \pi \quad \downarrow N \]

Liquid - Gas Transition at Critical Temperature \( T_c = 15 \, \text{MeV} \)

(empirical: \( T_c = 16 - 18 \, \text{MeV} \))


NUCLEAR THERMODYNAMICS

Skyrme phenomenology

Multifragmentation and fission analysis

G. Sauer, H. Chandra, U. Mosel
Nucl. Phys. A 264 (1976) 221

V.A. Karnaukhov et al.:
In-medium 
**chiral effective field theory**
(3-loop calculation of free energy density)


- Pion-nucleon dynamics incl. delta isobars
- Short-distance NN contact terms
- Three-body forces

**PHASE DIAGRAM of NUCLEAR MATTER**

[Diagram showing the phase diagram of nuclear matter with critical point and phase boundaries labeled.]
Trajectory of CRITICAL POINT for asymmetric matter

as function of proton fraction \( Z/A \)

\[ T \] [MeV]

\[ \rho \] [fm\(^{-3}\)]

\[ Z/A = 0.5 \]

...determined almost entirely by **isospin** dependent (one- and two-) **pion** exchange dynamics

CHIRAL CONDENSATE at finite BARYON DENSITY

- Chiral (quark) condensate $\langle \bar{q}q \rangle$:
  \[
  m^2 \frac{f^2}{\pi} = -2 m_q \langle \bar{q}q \rangle
  \]
  Order parameter of spontaneously broken chiral symmetry in QCD

- Hellmann - Feynman theorem:
  \[
  \langle \Psi | \bar{q}q | \Psi \rangle = \langle \Psi | \frac{\partial H_{QCD}}{\partial m_q} | \Psi \rangle = \frac{\partial E(m_q; \rho)}{\partial m_q}
  \]

\[
\frac{\langle \bar{q}q \rangle_{\rho}}{\langle \bar{q}q \rangle_0} = 1 - \frac{\rho}{f^2} \left[ \frac{\sigma_N}{m^2_{\pi}} \left( 1 - \frac{3 p^2_F}{10 M^2_N} + \ldots \right) + \frac{\partial}{\partial m^2_{\pi}} \left( \frac{E_{\text{int}}(p_F)}{A} \right) \right]
\]

sigma term

in-medium chiral effective field theory

(free) Fermi gas of nucleons

nuclear interactions (dependence on pion mass)
Substantial change of symmetry breaking scenario between chiral limit $m_q = 0$ and physical quark mass $m_q \sim 5$ MeV

**Nuclear Physics** would be very different in the chiral limit!
CHIRAL CONDENSATE:
DENSITY and TEMPERATURE DEPENDENCE

- Free energy density
  \[ \mathcal{F}(m_q; \rho, T) \]

- In-medium Chiral Effective Field Theory
  (NLO 3-loop)
  constrained by realistic nuclear equation of state

- No indication of first order chiral phase transition for
  \[ \rho \lesssim 2 \rho_0, \quad T \lesssim 100 \text{ MeV} \]

\[ \langle \Psi | \bar{q} q | \Psi \rangle_{\rho, T} = \frac{\partial \mathcal{F}(m_q; \rho, T)}{\partial m_q} \]
CHIRAL CONDENSATE: Dependence on TEMPERATURE and BARYON CHEMICAL POTENTIAL

- **Liquid-gas** phase transition leaves its signature also in chiral condensate
- but: **no** tendency toward **chiral first order transition** in the range $\mu_B \lesssim 1$ GeV

**Summary: PHASE DIAGRAM with NUCLEAR PHYSICS CONSTRAINTS**

**Major challenge:** design QCD phase diagram in accordance with known realistic features from hadronic and nuclear physics.

from nuclear chiral thermodynamics:

- corridor of spontaneously broken chiral symmetry extends at least up to:
  \[
  \rho \lesssim 2 \rho_0 \\
  T \lesssim 100 \text{ MeV}
  \]

Outlook:

New Constraints from NEUTRON STARS
A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest\textsuperscript{1}, T. Pennucci\textsuperscript{2}, S. M. Ransom\textsuperscript{1}, M. S. E. Roberts\textsuperscript{3} & J. W. T. Hessels\textsuperscript{4,5}


direct measurement of neutron star mass from increase in travel time near companion

J1614-2230
most edge-on binary pulsar known (89.17\degree)
+ massive white dwarf companion (0.5 M\textsubscript{sun})

heaviest neutron star with 1.97±0.04 M\textsubscript{sun}
TWO-SOLAR-MASS NEUTRON STAR

... measured using Shapiro delay

P.B. Demorest et al., Nature 467 (2010) 1081

![Graph showing mass versus radius for neutron star equations of state (EOS) with different models and constraints.](image)
News from NEUTRON STARS

K. Hebeler, J. Lattimer, C. Pethick, A. Schwenk
PRL 105 (2010) 161102

A.W. Steiner, J. Lattimer, E.F. Brown

- Realistic “nuclear” EoS (Illinois)

- New constraints from EFT and neutron star observables

- “Exotic” equations of state ruled out?
NEUTRON STAR MATTER
Equation of State

Including new neutron star constraints plus Chiral Effective Field Theory at lower density

\[ P = \text{const} \cdot \rho^\Gamma \]

Low-density (crust) + ChEFT (FKW)
Constrained extrapolation (polytropes)
Akmal, Pandharipande, Ravenhall (1998)
SUMMARY

- Exploration of **QCD phase diagram**: progress concerning basic symmetry breaking patterns
  - Lattice QCD (restricted to small quark chemical potentials)
  - Models (PNJL, PQM) (but: nuclear physics constraints missing)
  - Dyson-Schwinger QCD ( -- same problem -- )

- **Nuclear thermodynamics** based on
  - In-medium Chiral Effective Field Theory
  - Fermi liquid ↔ interacting Fermi gas (1st order transition)
    - **No** indication of first order *chiral* transition in the range
      \[ \rho \lesssim 2 \rho_0, \ T \lesssim 100 \text{ MeV} \]
  - Major challenge: design **QCD phase diagram** that is consistent with established hadronic and nuclear physics

- New **dense & cold matter** constraints from **neutron stars**:
  - Mass - radius relation; observation of two-solar-mass n-star
  - “Non-exotic” equation of state works best!
The End

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