

Applications of Holography to Strongly Coupled Plasmas



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INTERNATIONAL
MAX PLANCK
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FOR PRECISION TESTS
OF FUNDAMENTAL
SYMMETRIES



in collaboration with Carlo Ewerz

34th International School of Nuclear Physics
Erice/Sicily

Holography, Gauge/Gravity duals, AdS/ CFT correspondence, ...

Heavy ion collisions and AdS/CFT

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Abstract.

We review some recent applications of the AdS/CFT correspondence to heavy ion collisions including a calculation of the jet quenching parameter in $\mathcal{N} = 4$ super-Yang-Mills theory and quarkonium suppression from velocity scaling of the screening length for a heavy quark-antiquark pair. We also briefly discuss differences and similarities between QCD and $\mathcal{N} = 4$ Super-Yang-Mills theory.

...many realisations, but one concept.

Holography, Gauge/Gravity duals, AdS/ CFT correspondence, ...

Heavy ion collisions and **AdS/CFT**

PHYSICAL REVIEW D **84**, 065009 (2011)

Model of a Fermi liquid using gauge-gravity duality

Subir Sachdev

Department of Physics, Harvard University, Cambridge Massachusetts 02138, USA
(Received 4 August 2011; published 21 September 2011)

We use gauge-gravity duality to model the crossover from a conformal critical point to a confining Fermi liquid, driven by a change in fermion density. The short-distance conformal physics is represented by an anti-de Sitter geometry, which terminates into a confining state along the emergent spatial direction. The Luttinger relation, relating the area enclosed by the Fermi surfaces to the fermion density, is shown to follow from Gauss's law for the bulk electric field. We argue that all low energy modes are consistent with Landau's Fermi liquid theory. An explicit solution is obtained for the Fermi liquid for the case of hard-wall boundary conditions in the infrared.

DOI: [10.1103/PhysRevD.84.066009](https://doi.org/10.1103/PhysRevD.84.066009)

PACS numbers: 11.25.Tq, 71.10.Hf

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Heavy ion collisions and **AdS/CFT**

PHYSICAL REVIEW D **84**, 065009 (2011)

Model of a Fermi liquid using gauge-gravity duality

Subir Sachdev

Departme

We use gauge-gravity duality to study a Fermi liquid, driven by an anti-de Sitter space. The Luttinger relations follow from Gauss's law. Landau's Fermi liquid theory and wall boundary conditions are also discussed.

DOI: [10.1103/PhysRe](https://doi.org/10.1103/PhysRevD.83.066004)

PHYSICAL REVIEW D **83**, 066004 (2011)

Vortex flow for a holographic superconductor

Kengo

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(Received 7 December 2010; published 2 March 2011)

We investigate energy dissipation associated with the motion of the scalar condensate in a holographic superconductor model constructed from the charged scalar field coupled to the Maxwell field. Upon application of constant magnetic and electric fields, we analytically construct the vortex-flow solution and find the vortex-flow resistance near the second-order phase transition where the scalar condensate begins. The characteristic feature of the nonequilibrium state agrees with the one predicted by the time-dependent Ginzburg-Landau (TDGL) theory. We evaluate the kinetic coefficient in the TDGL equation along the line of the second-order phase transition. At zero magnetic field, the other coefficients in the TDGL equation are also evaluated just below the critical temperature.

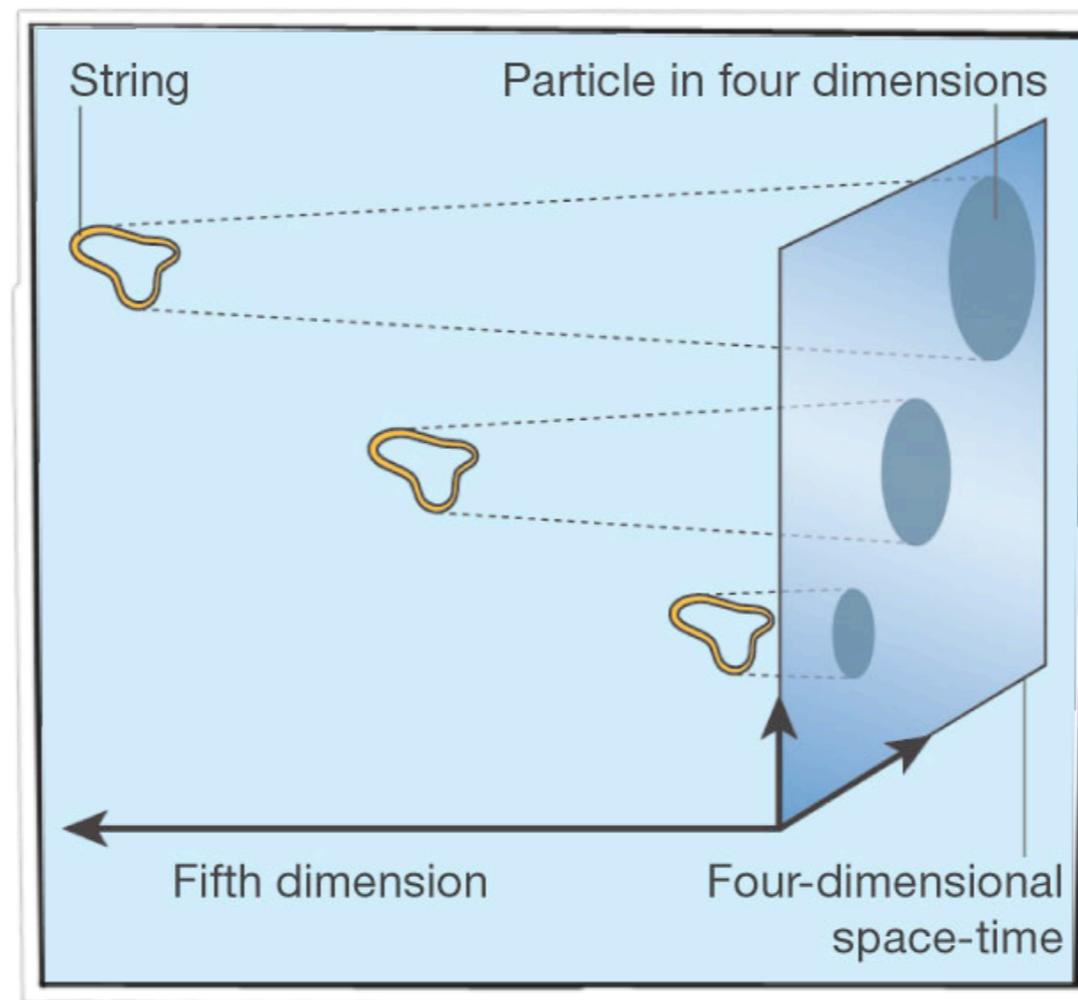
DOI: [10.1103/PhysRevD.83.066004](https://doi.org/10.1103/PhysRevD.83.066004)

PACS numbers: 11.25.Tq, 74.20.-z, 74.25.-q

...many realisations, but one concept.

Holographic Principle

The physics in a $(d+1)$ -dimensional volume can be described by a theory living on the d -dimensional boundary.



Maldacena, 2003

- e.g.: duality between gauge theories in d -dimension and gravity theories (string theories) in higher dimensions.

Practical Realisation of Holography

Condensed Matter Physics

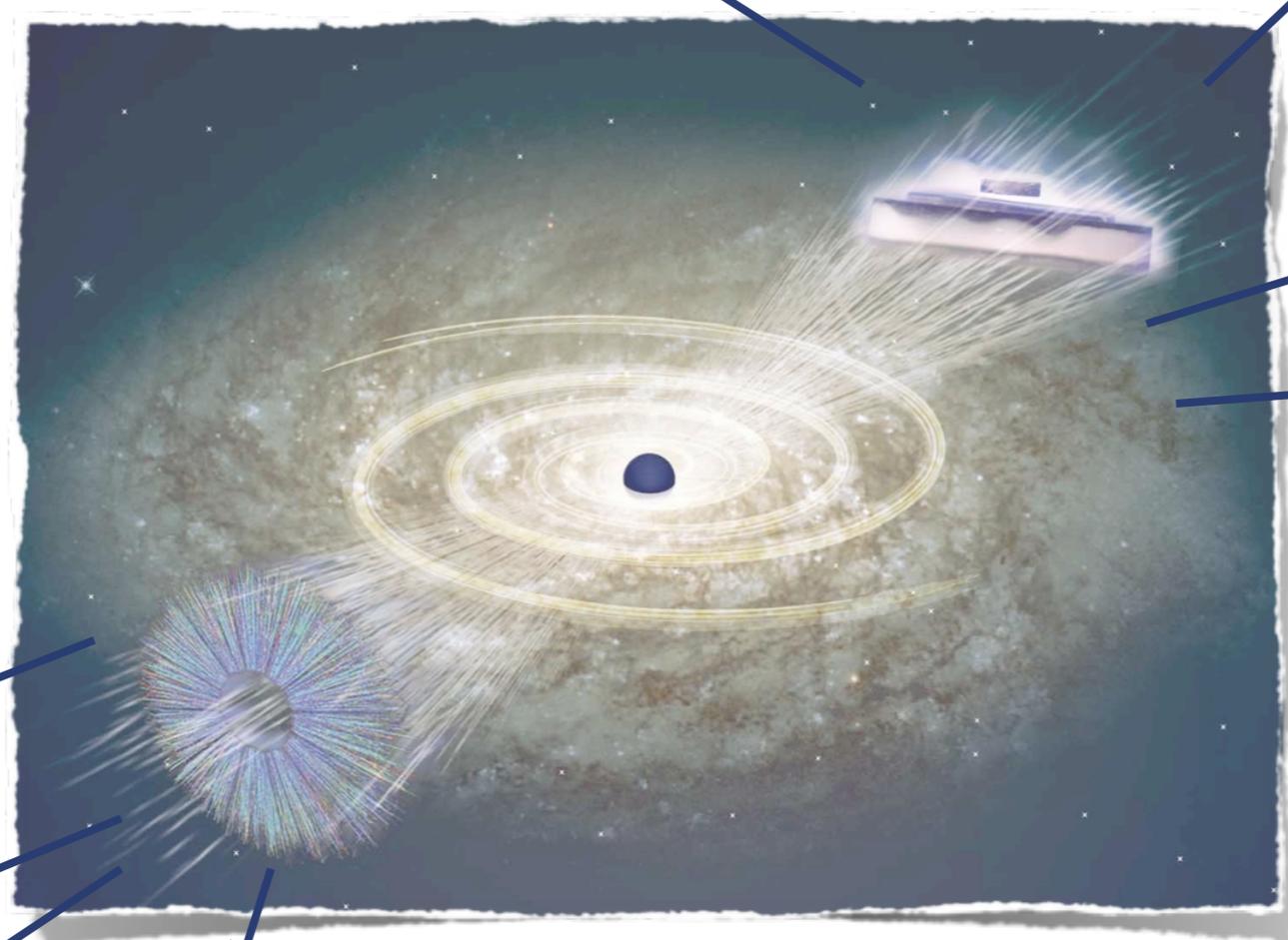
Holographic Superconductors

Non-Relativistic AdS/CFT

Non-Fermi liquids
using gauge/gravity duality

Holographic
Neutron Stars

Zaanen, 2007



AdS/CFT

AdS/QCD

Fluid/Gravity Correspondence

Gauge/Gravity Dualities

High Energy Physics

Gauge/Gravity Duality

$SU(N_c) \mathcal{N} = 4$ SYM

d-dim. **gauge theory**
(without gravity)

entropy of gauge theory
 \propto volume

\iff

$AdS_5 \times S^5$

d+1-dim. **gravitational
theory**

entropy of gravitational theory
 \propto area

$=$

Gauge/Gravity Duality

$SU(N_c) \mathcal{N} = 4$ SYM

d-dim. **gauge theory**
(without gravity)

entropy of gauge theory
 \propto volume

Duality

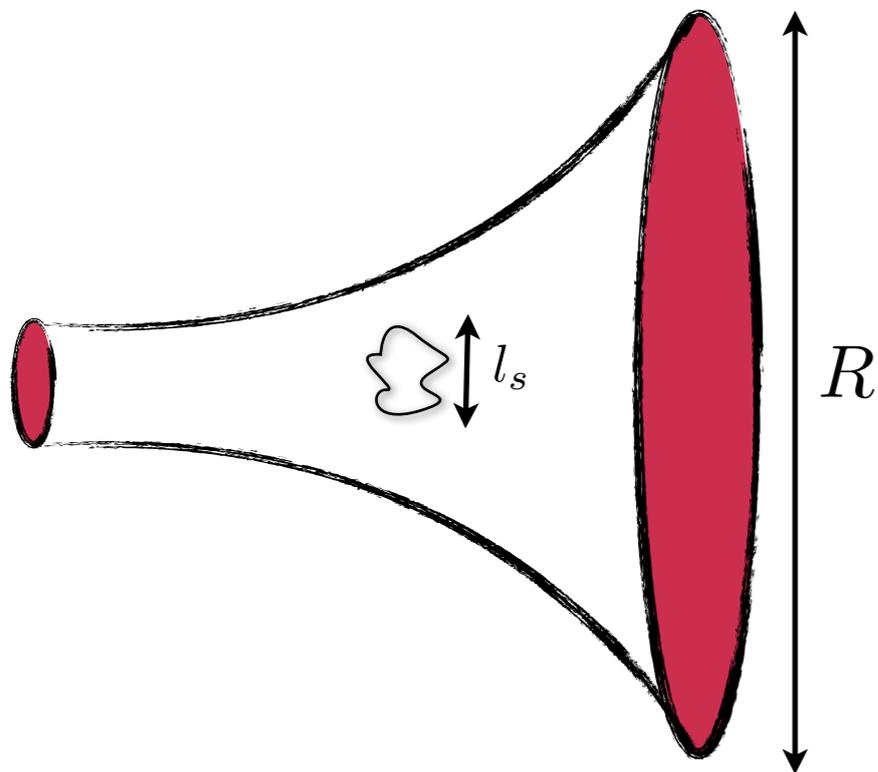
\iff

$=$

$AdS_5 \times S^5$

d+1-dim. **gravitational theory**

entropy of gravitational theory
 \propto area



Why is that duality useful?

$$g_{\text{YM}}^2 = 2\pi g_s, \quad R^4 = 4\pi g_s N_c l_s^4, \quad \lambda = g_{\text{YM}}^2 N_c$$

$$\lambda \text{ fixed, } N_c \longrightarrow \infty : \quad g_s \sim \lambda / N_c$$

$$\lambda \longrightarrow \infty : \quad R^4 \sim \lambda l_s^4$$

strongly coupled QFT \longleftrightarrow weakly coupled gravity

QCD \longleftrightarrow $\mathcal{N} = 4$ super Yang-Mills

- $\mathcal{N} = 4$ SYM very different from QCD:
 - Maximally supersymmetric
 - Conformal theory, coupling is constant
 - No confinement, no chiral symmetry breaking
 - $N_c \rightarrow \infty$ for duality

QCD \longleftrightarrow $\mathcal{N} = 4$ super Yang-Mills

- $\mathcal{N} = 4$ SYM very different from QCD
 - Maximally supersymmetric
 - Conformal theory, coupling is constant
 - No confinement, no chiral symmetry breaking
 - $N_c \rightarrow \infty$ for duality
- At finite T , differences are smaller:
 - Above $2T_c$ QCD almost conformal
 - No confinement in QCD above T_c
 - Finite T breaks supersymmetry

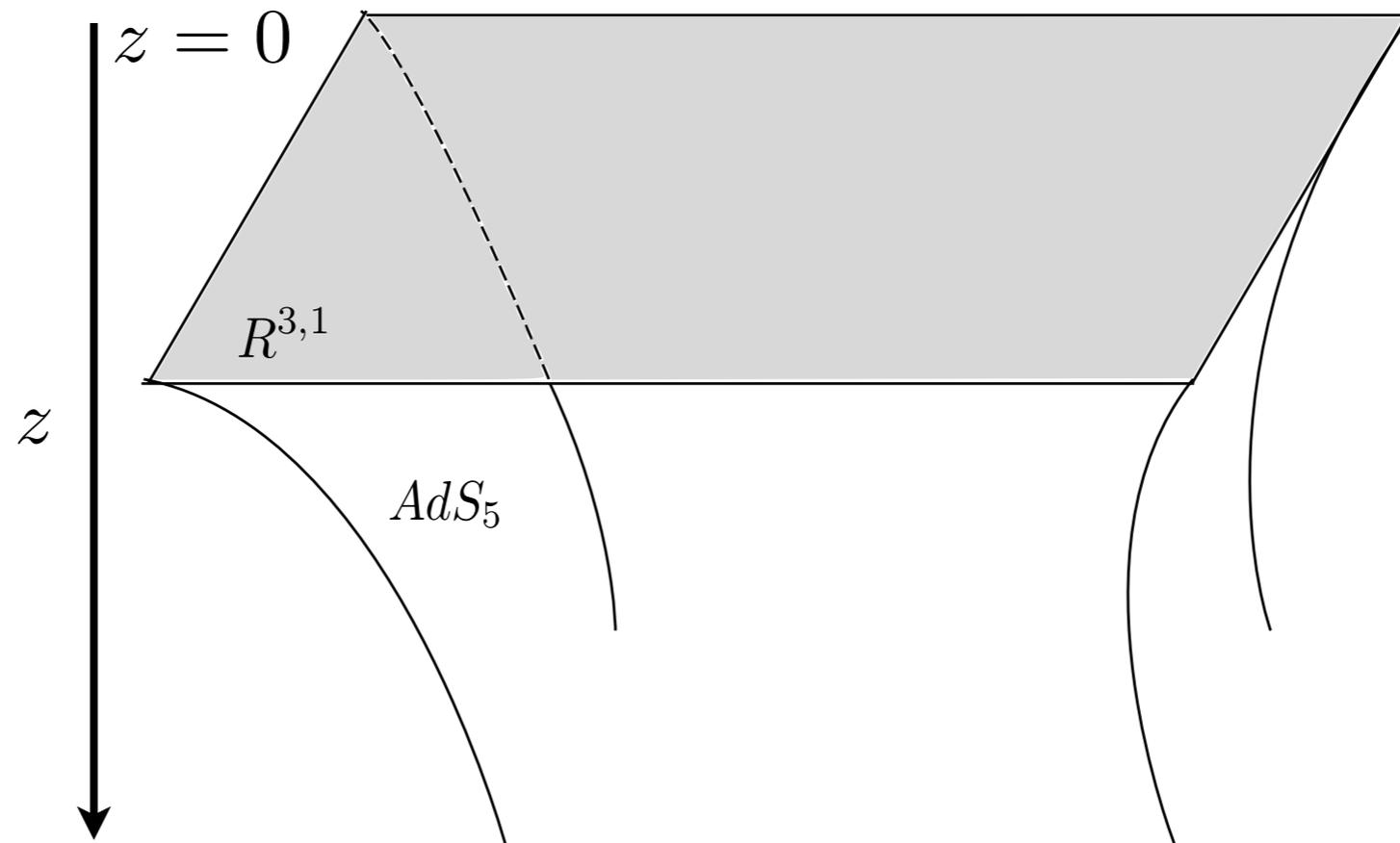
Basic Properties of AdS

- AdS_5 metric:

$$ds^2 = \frac{R^2}{z^2} \left(-dt^2 + d\vec{x}^2 + dz^2 \right) \text{ with } R \text{ being the AdS curvature}$$

- Solution to 5D Einstein-Hilbert action:

$$S = \frac{1}{16\pi G} \int d^5x \sqrt{-g} (\mathcal{R} - 2\Lambda)$$

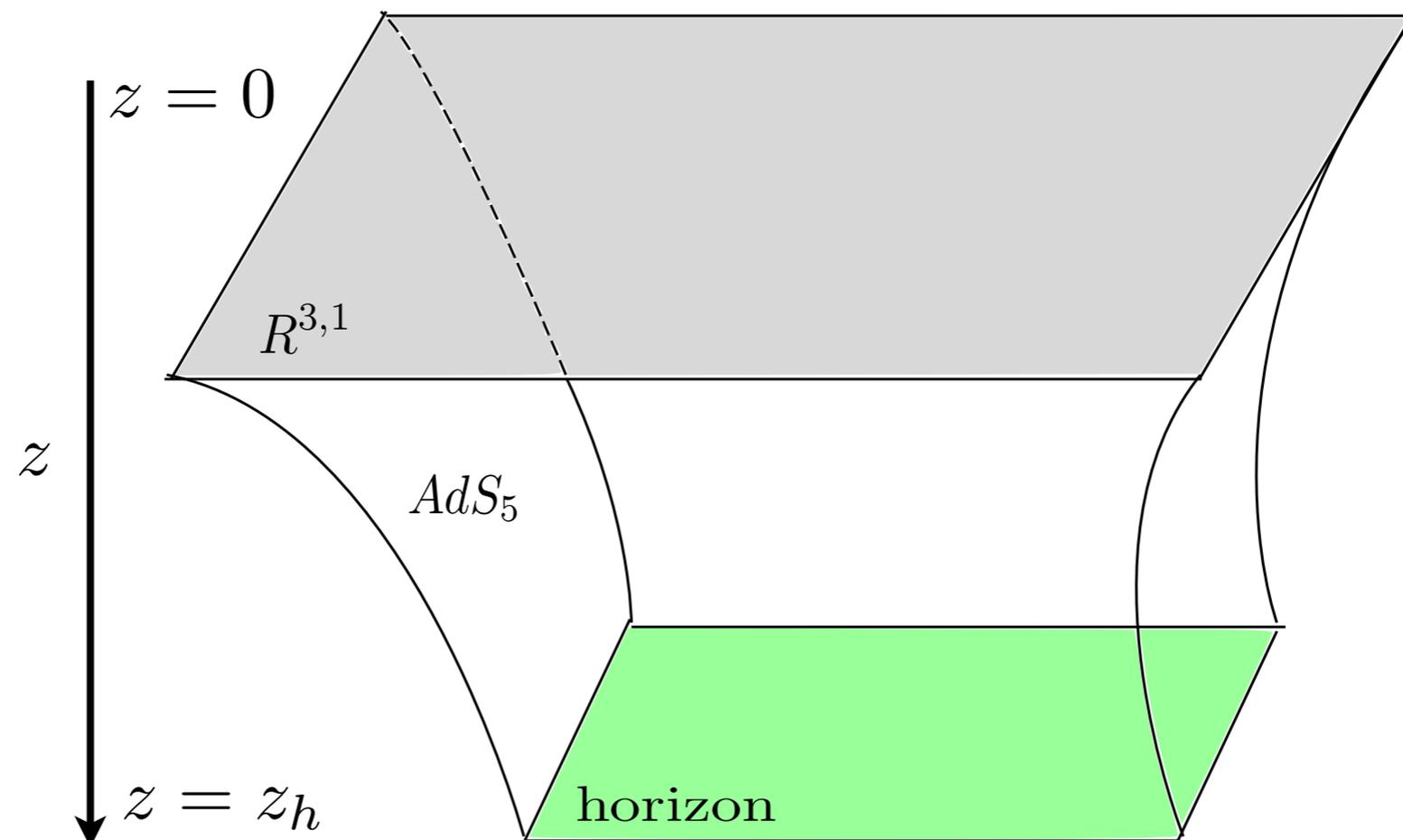


Basic Properties of AdS

- AdS_5 black hole metric:

$$ds^2 = \frac{R^2}{z^2} \left(-h dt^2 + d\vec{x}^2 + \frac{dz^2}{h} \right) \text{ with } h = 1 - \frac{z^4}{z_h^4} \text{ and } T = \frac{1}{\pi z_h}$$

- Solves the same e. o. m.:



Metric models at finite temperature

- AdS_5 BH metric at finite temperature:

$$ds^2 = \frac{R^2}{z^2} \left(-h dt^2 + d\vec{x}^2 + \frac{dz^2}{h} \right) \text{ with } h = 1 - \frac{z^4}{z_h^4} \text{ and } T = \frac{1}{\pi z_h}$$

- SW_T model:

Kajantie, Tahkokallio, Yee

$$ds^2 = \frac{R^2}{z^2} e^{cz^2} \left(-h dt^2 + d\vec{x}^2 + \frac{dz^2}{h} \right)$$

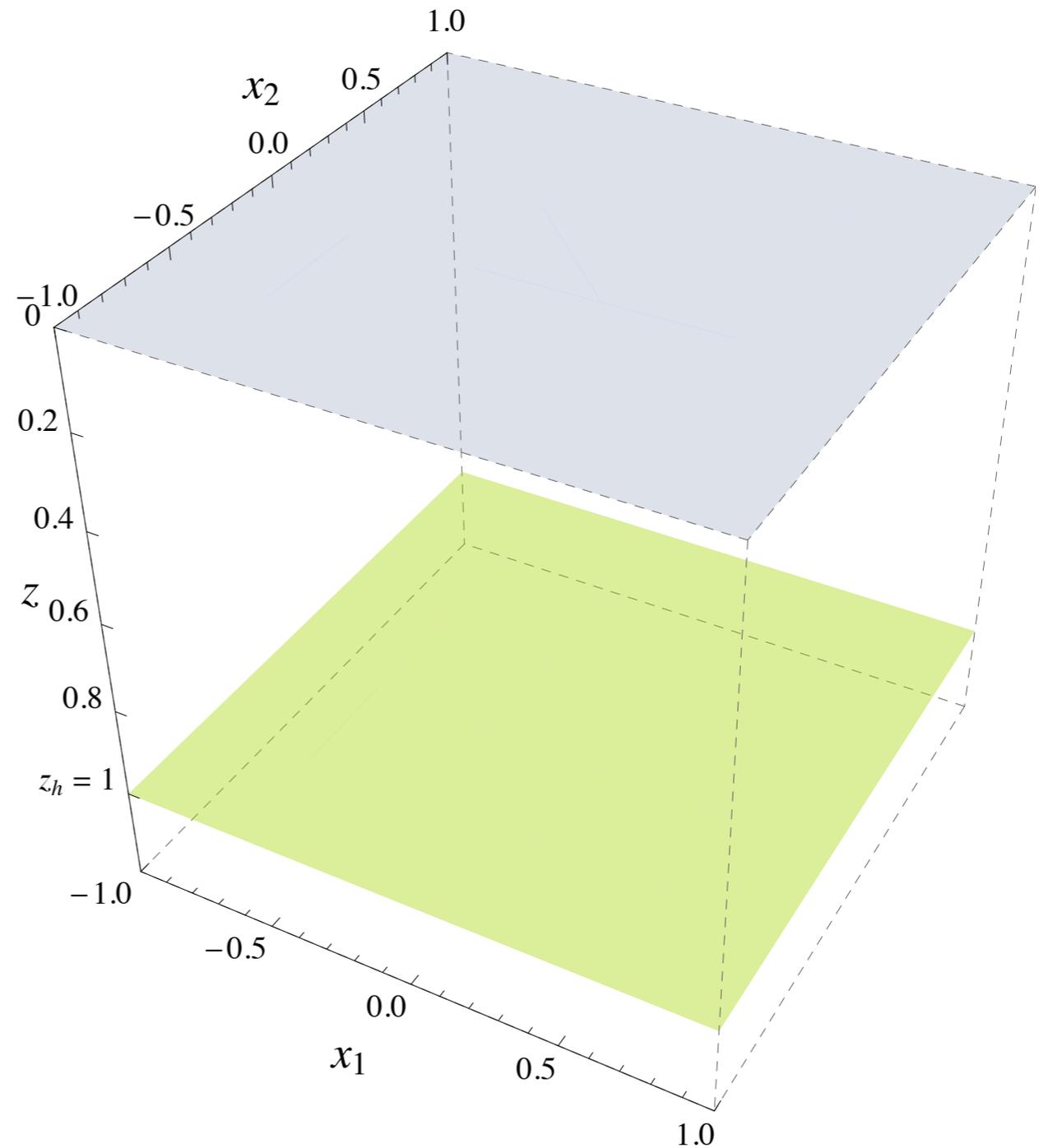
- 2-parameter model:

DeWolfe, Rosen; Gubser

$$ds^2 = e^{2A(\Phi)} \left(-h(\Phi) dt^2 + d\vec{x}^2 \right) + \frac{e^{2B(\Phi)}}{h(\Phi)} d\Phi^2$$

is a solution to equations of motion.

Screening distance in hot moving plasmas



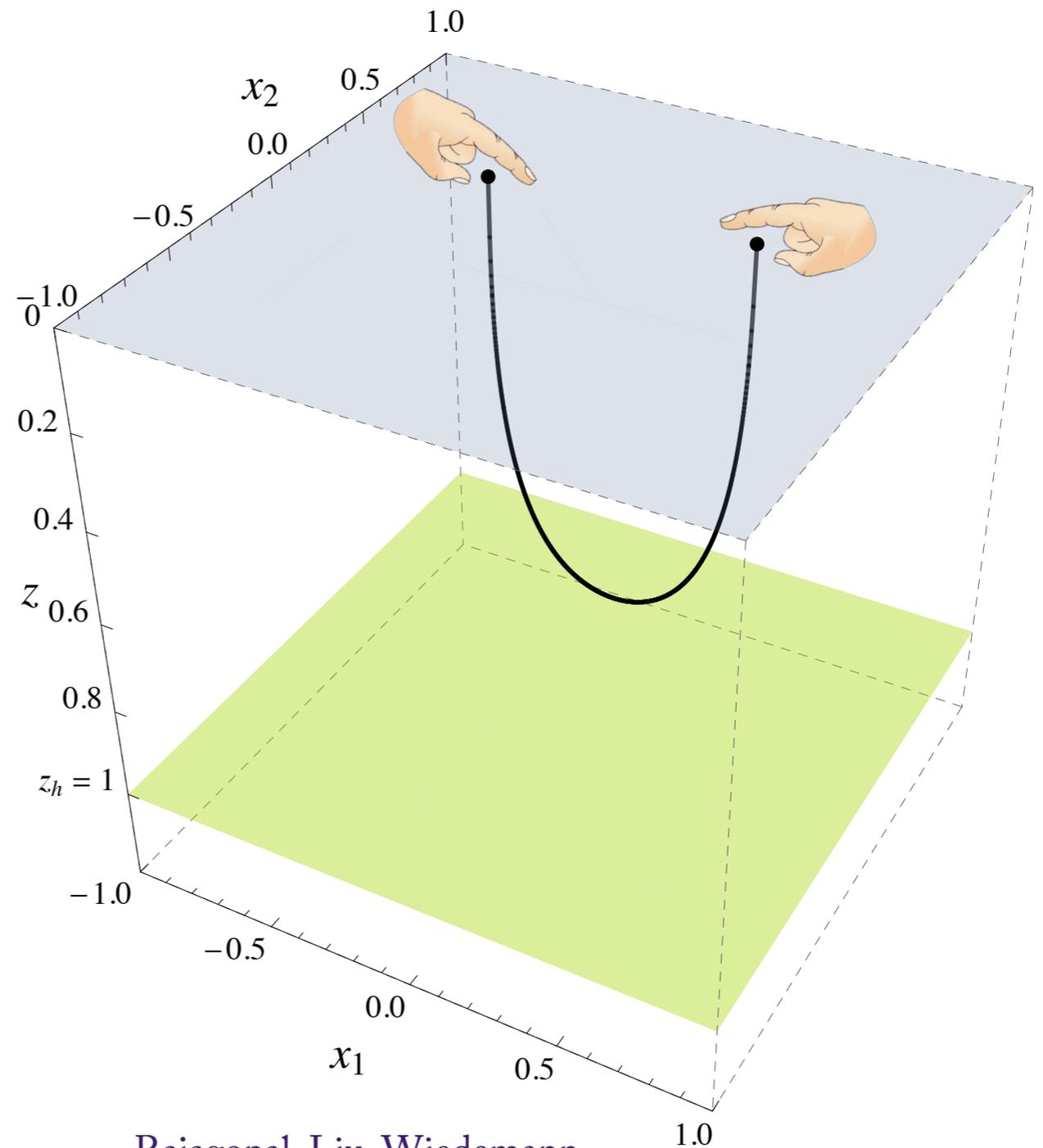
Rajagopal, Liu, Wiedemann

Screening distance in hot moving plasmas

Nambu-Goto action:

$$S = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-\det g_{\alpha\beta}}$$

with $g_{\alpha\beta} = G_{\mu\nu} \partial_\alpha x^\mu \partial_\beta x^\nu$



Rajagopal, Liu, Wiedemann

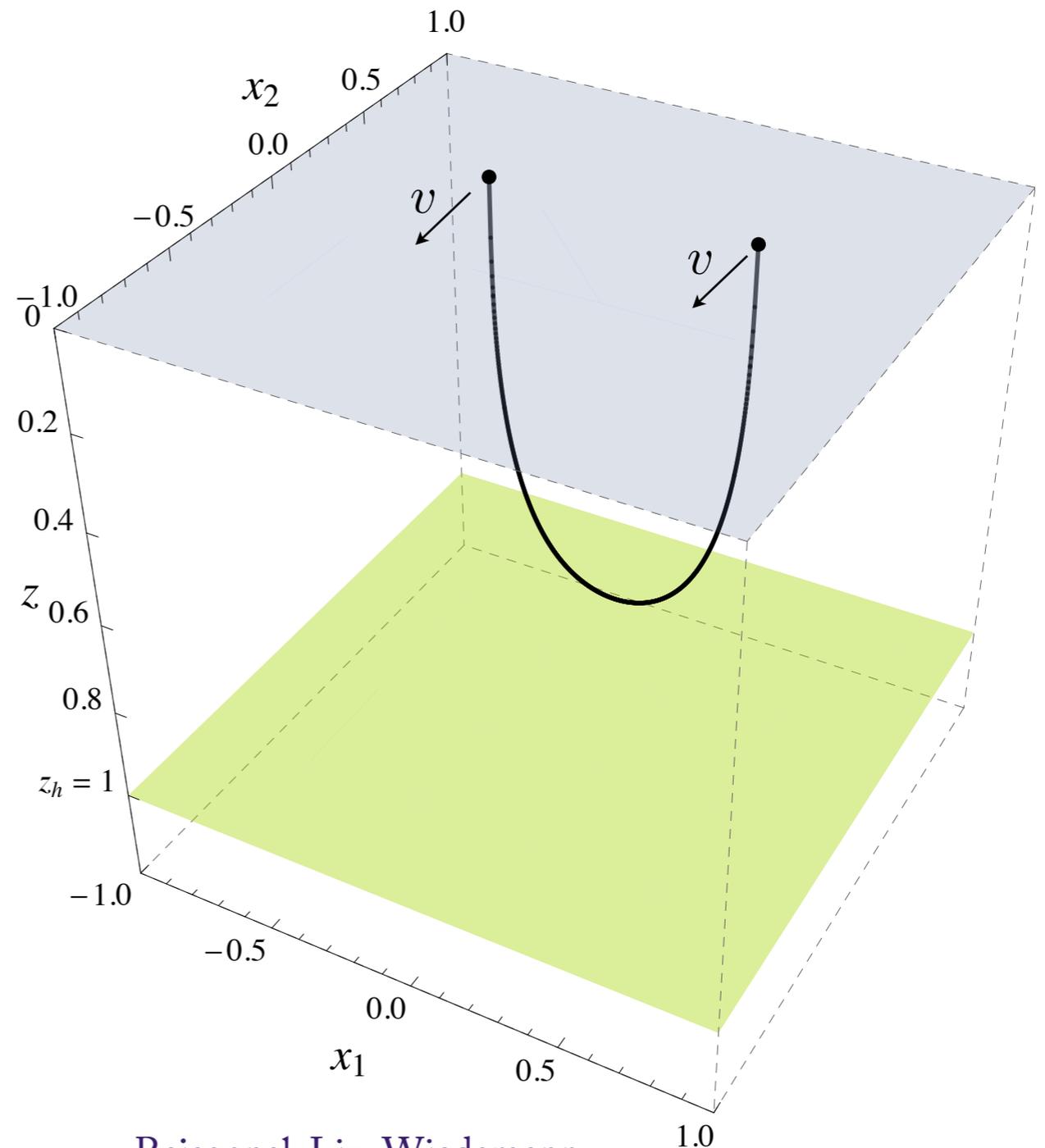
Screening distance in hot moving plasmas

- Static $q\bar{q}$ - pair in a hot moving plasma “wind” blowing in x_2 -direction
- velocity $v = \tanh \eta$
- orientation angle θ

Nambu-Goto action:

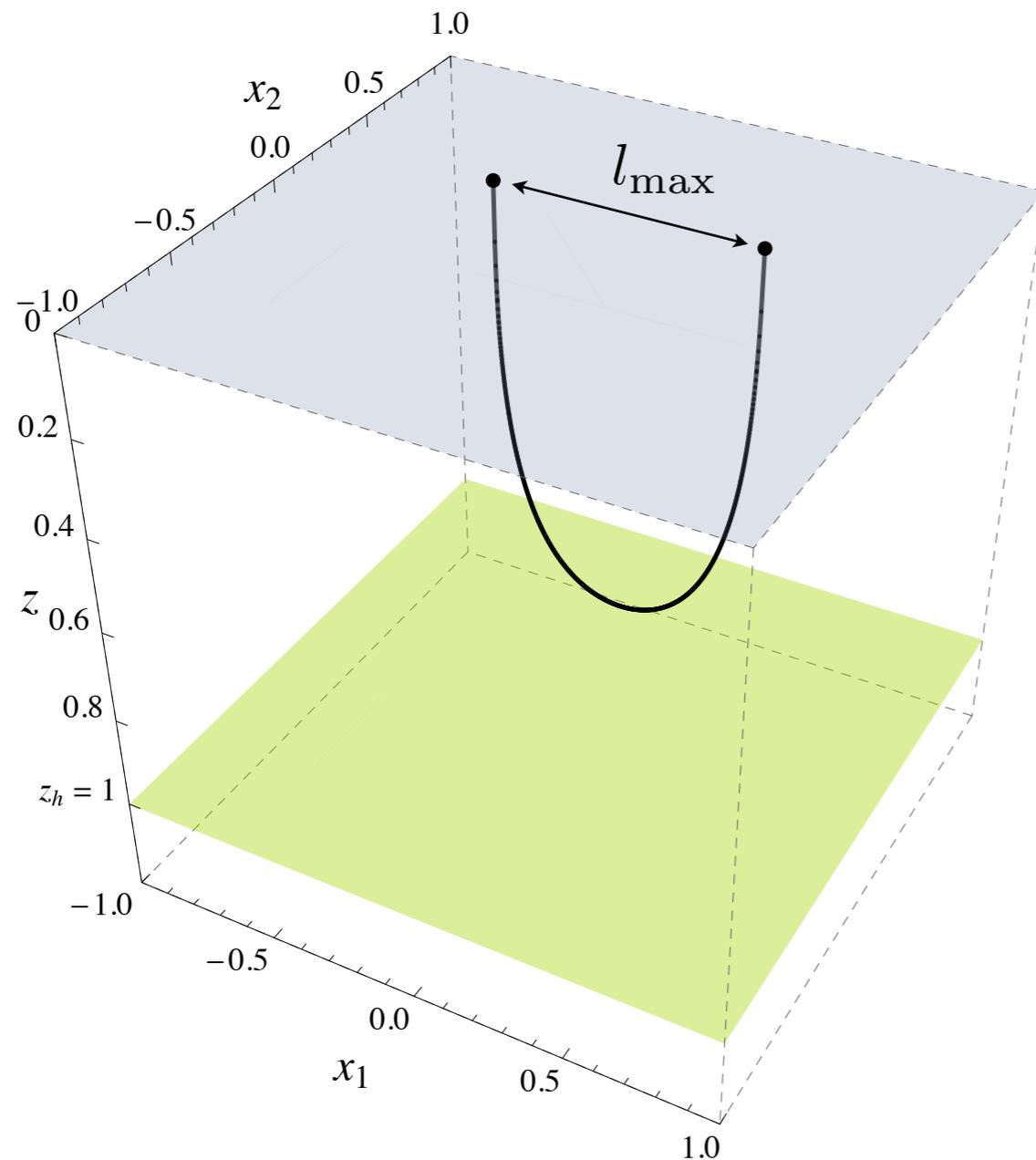
$$S = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-\det g_{\alpha\beta}}$$

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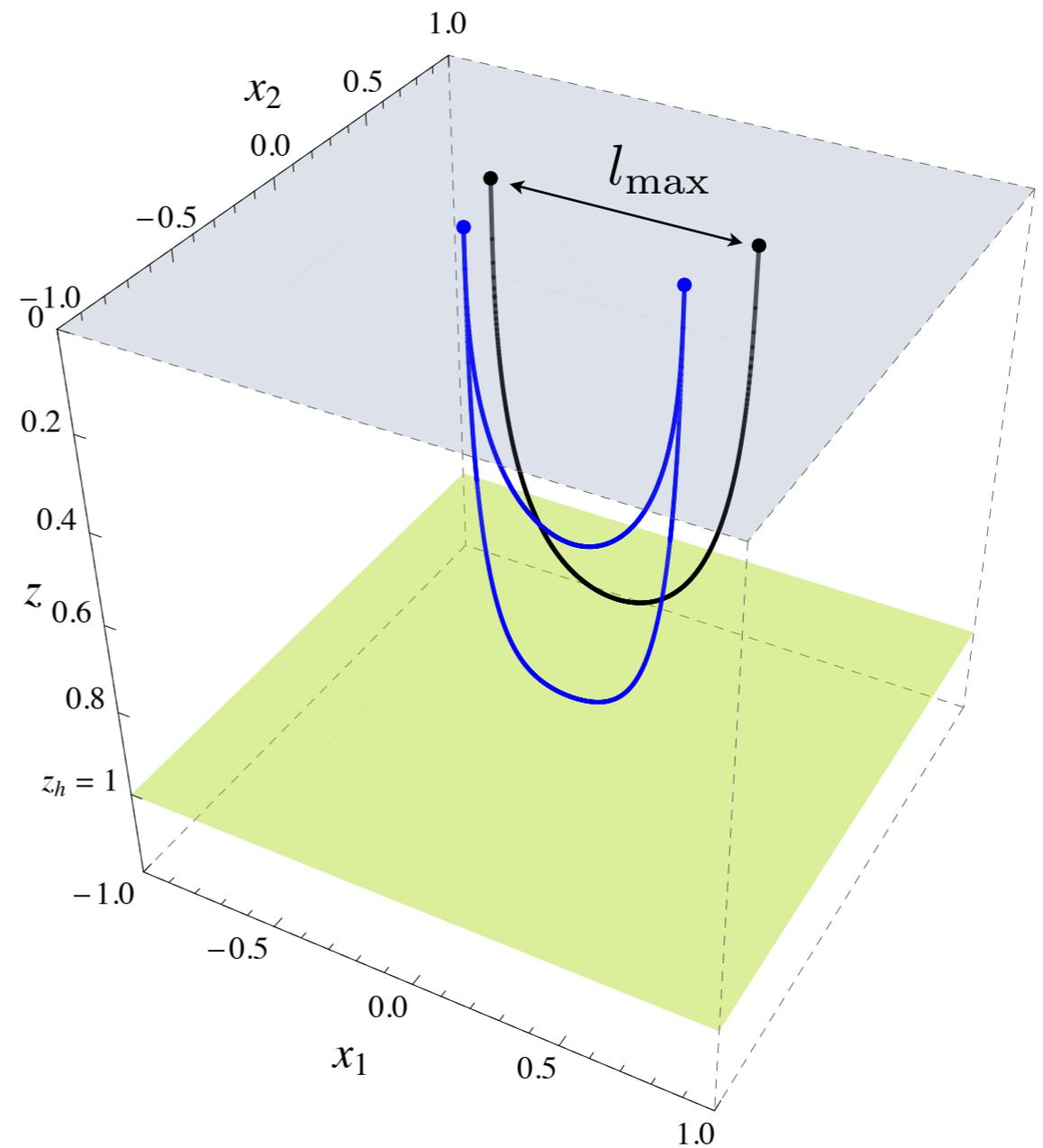
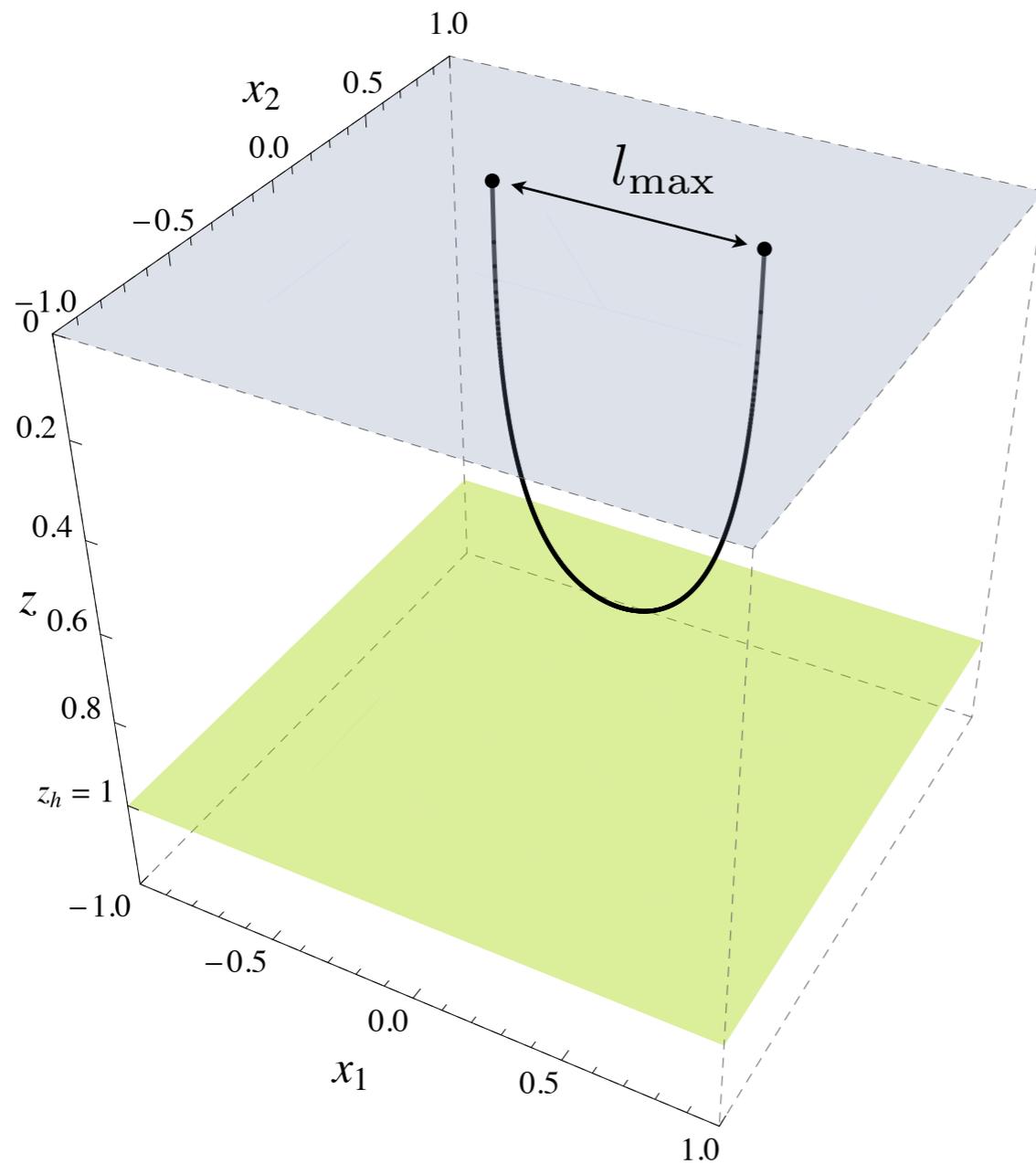
Rajagopal, Liu, Wiedemann

Configuration of the strings



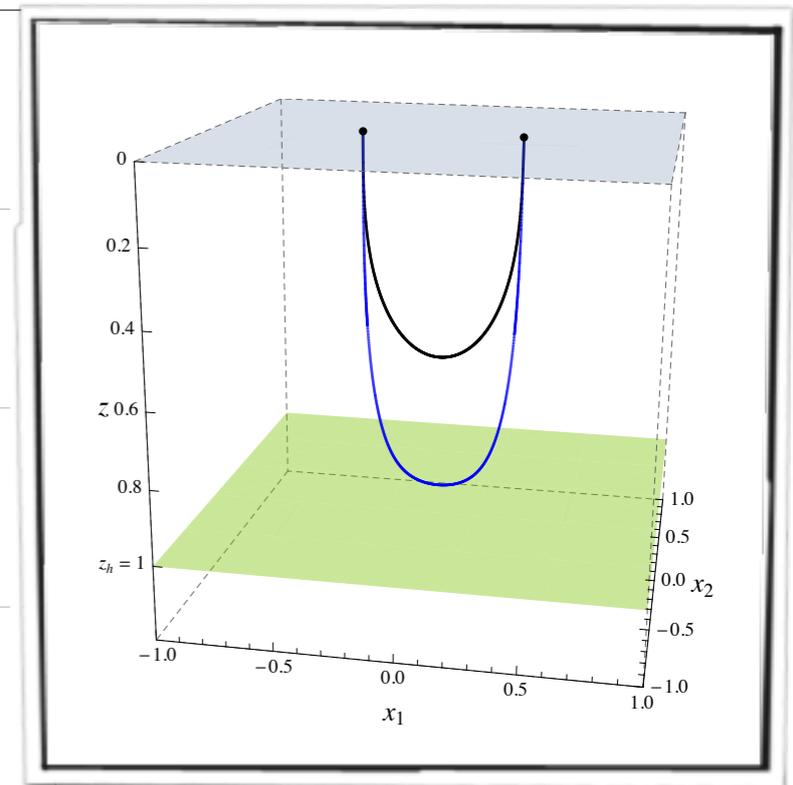
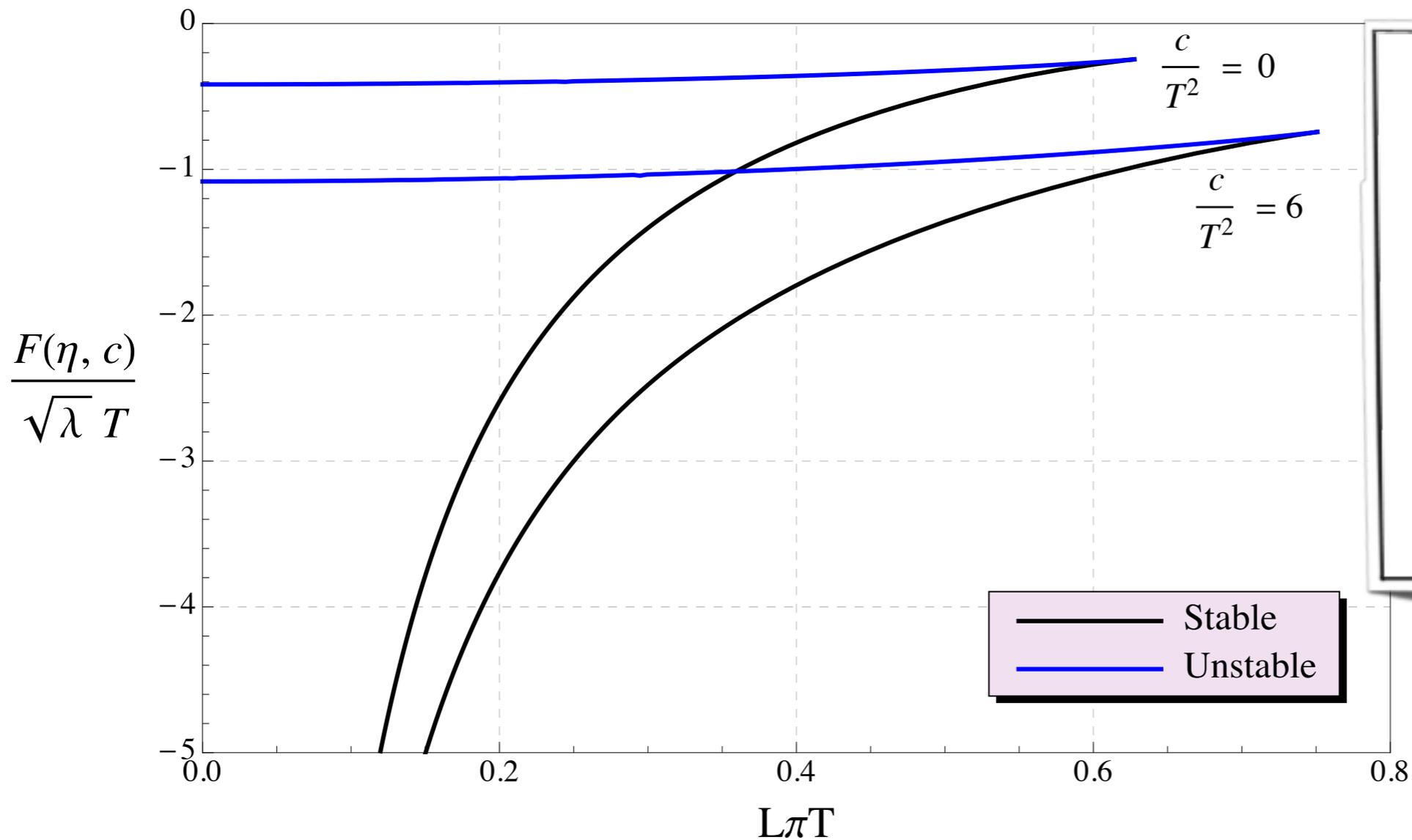
- l_{\max} defines the *screening distance*.
- The screening distance is a lower bound in $\mathcal{N} = 4$ SYM for all theories under investigation.

Configuration of the strings



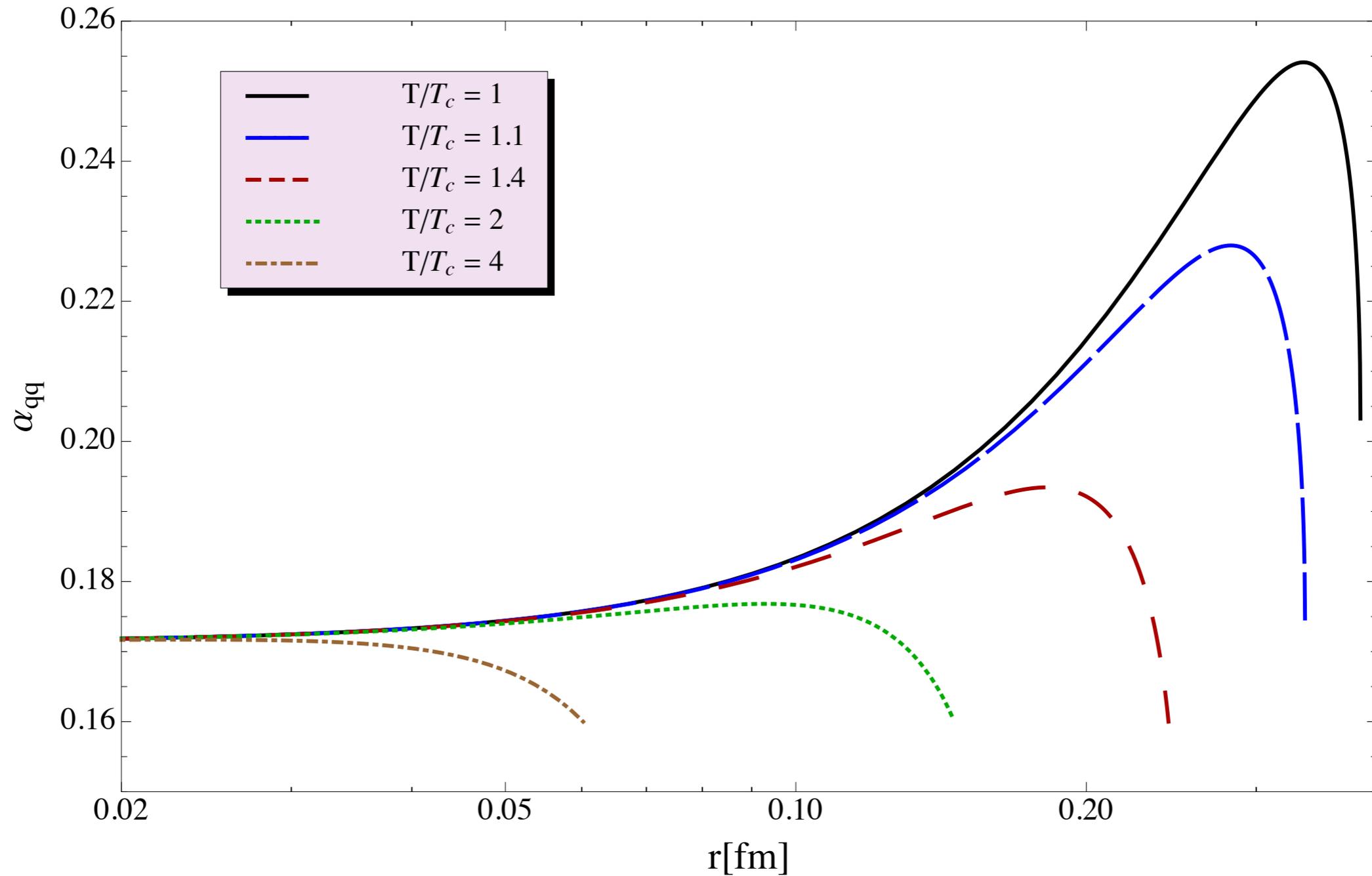
- The string configuration coming closer to the horizon is unstable.

$Q\bar{Q}$ -free energy: results



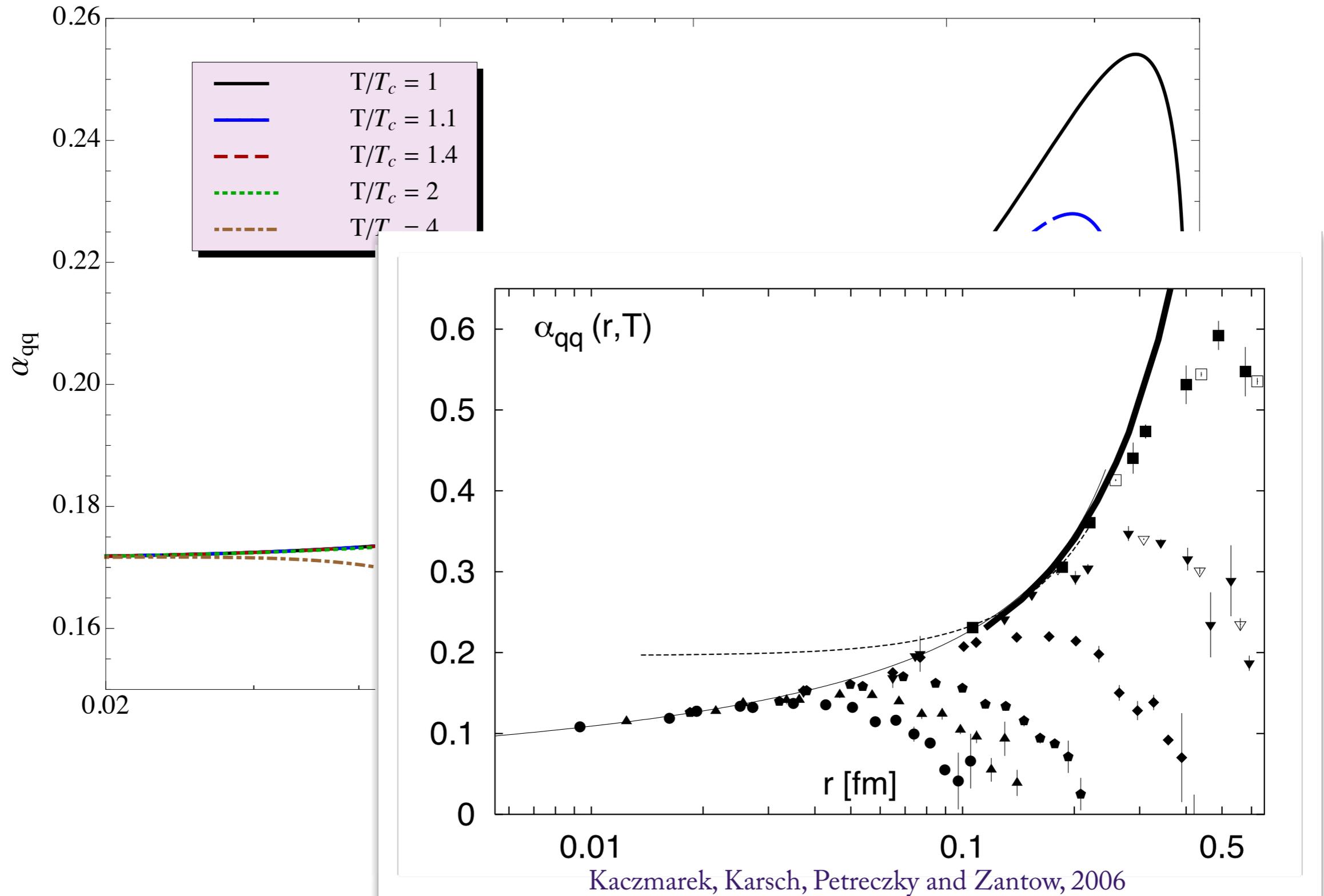
- Free energy of $q\bar{q}$ -pair at finite rapidity $\eta = 1$.
- Unstable configurations are weaker bounded.

Running Coupling from Free Energy



- Coupling α_{qq} is defined as $\alpha_{qq} = \frac{3r^2}{4} \frac{dF(r, T)}{dr}$ in QCD.
- Many possibilities by rescaling the parameters.

Running Coupling from Free Energy

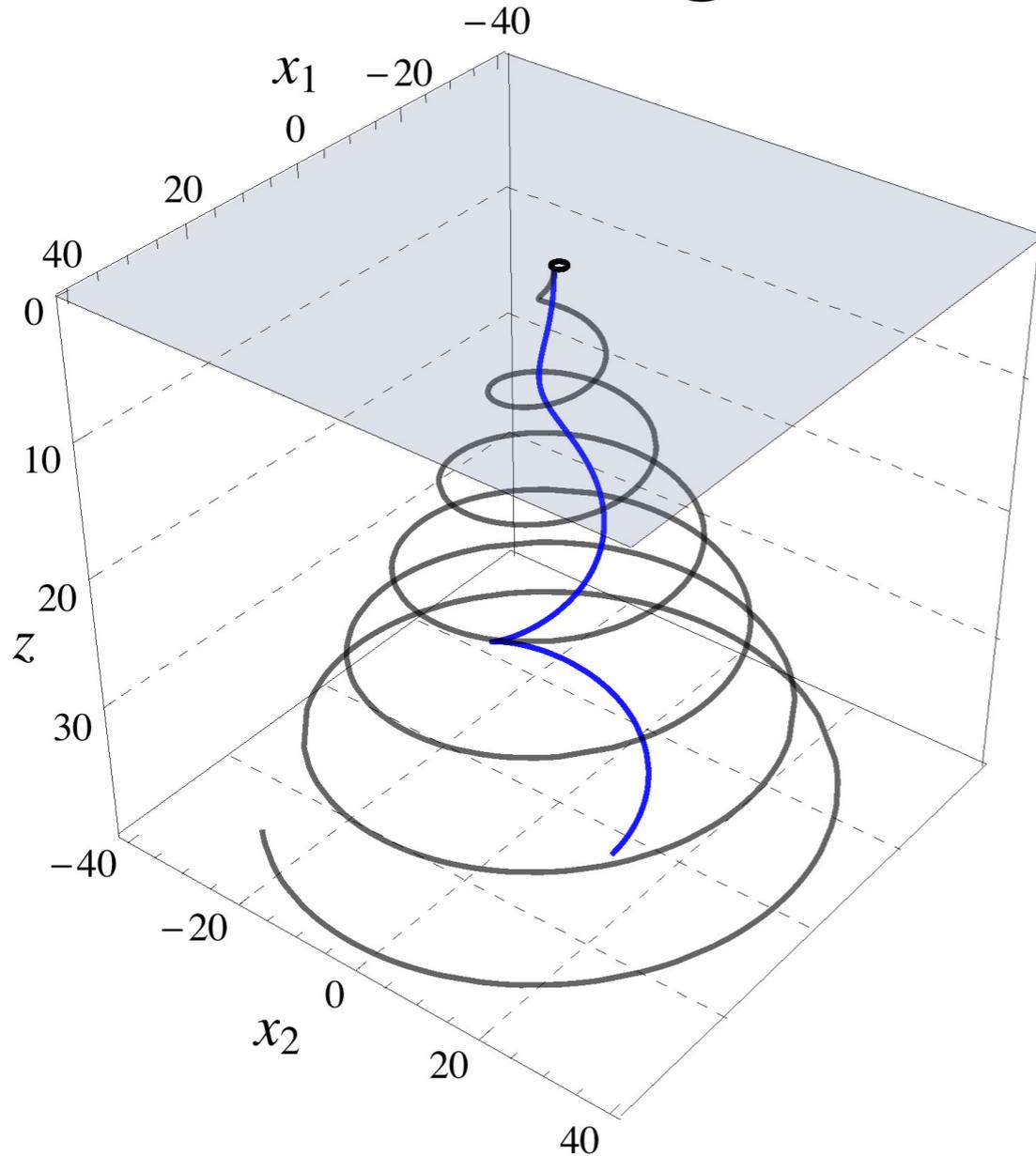


Conclusions

- Although being a conjecture the AdS/CFT correspondence as a realisation of the *Holographic principle* is a very powerful tool for qualitative and quantitative analysis, e.g.:
 - *Robustness and Universality* of the screening distance.
 - Running coupling of $q\bar{q}$ - pairs in good agreement with lattice QCD data.
- Many other more sophisticated models (e.g. including D3/D7 branes) available that nicely reproduce many QCD features and includes finite chemical potential.

Thank you for your
attention!

Rotating Quark at Zero Temperature

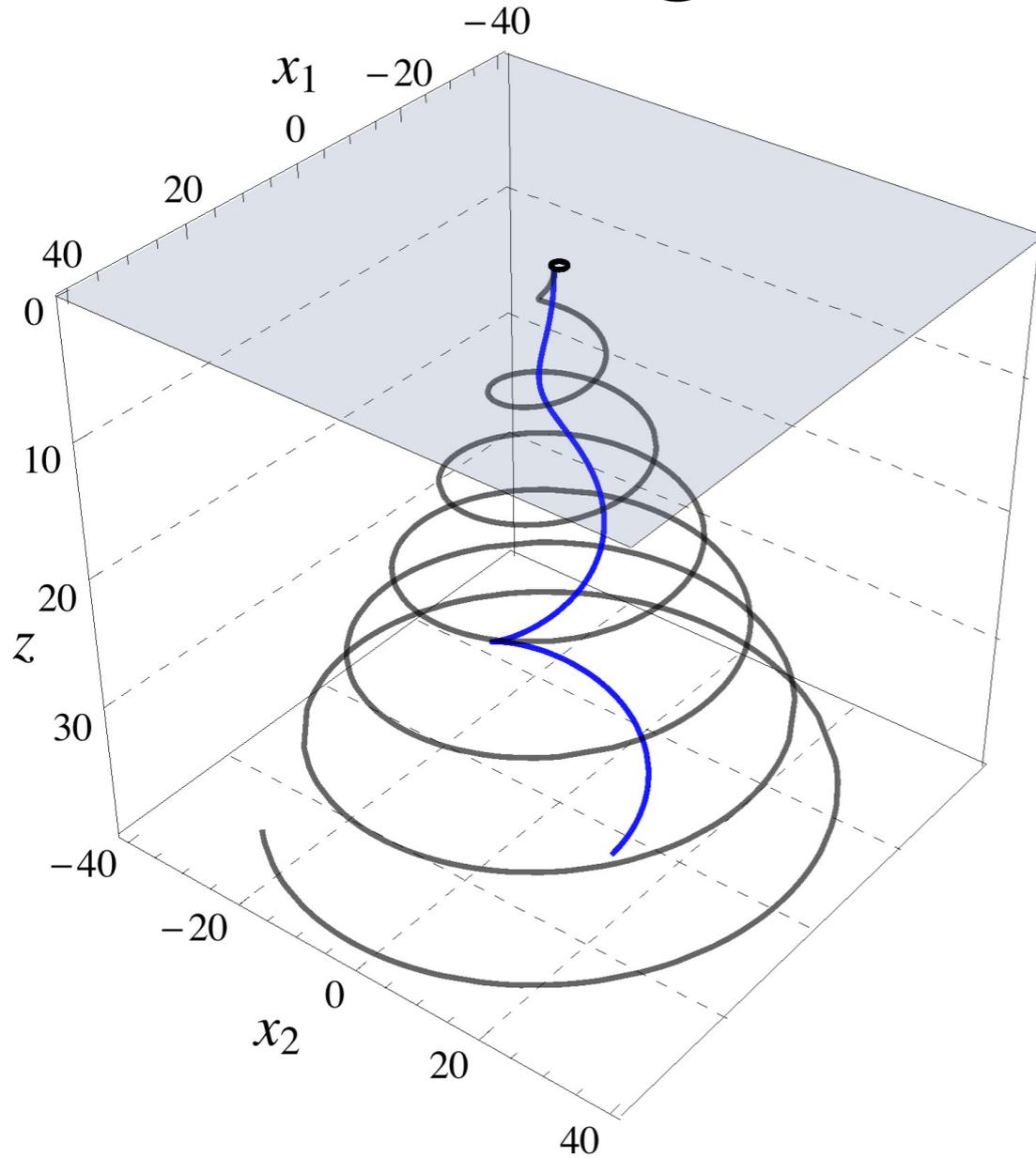


$$T = 0, \omega = 0.3, 0.7, R_0 = 1$$

Athanasίου, Rajagopal, Chesler, Liu, Nickel, 2010

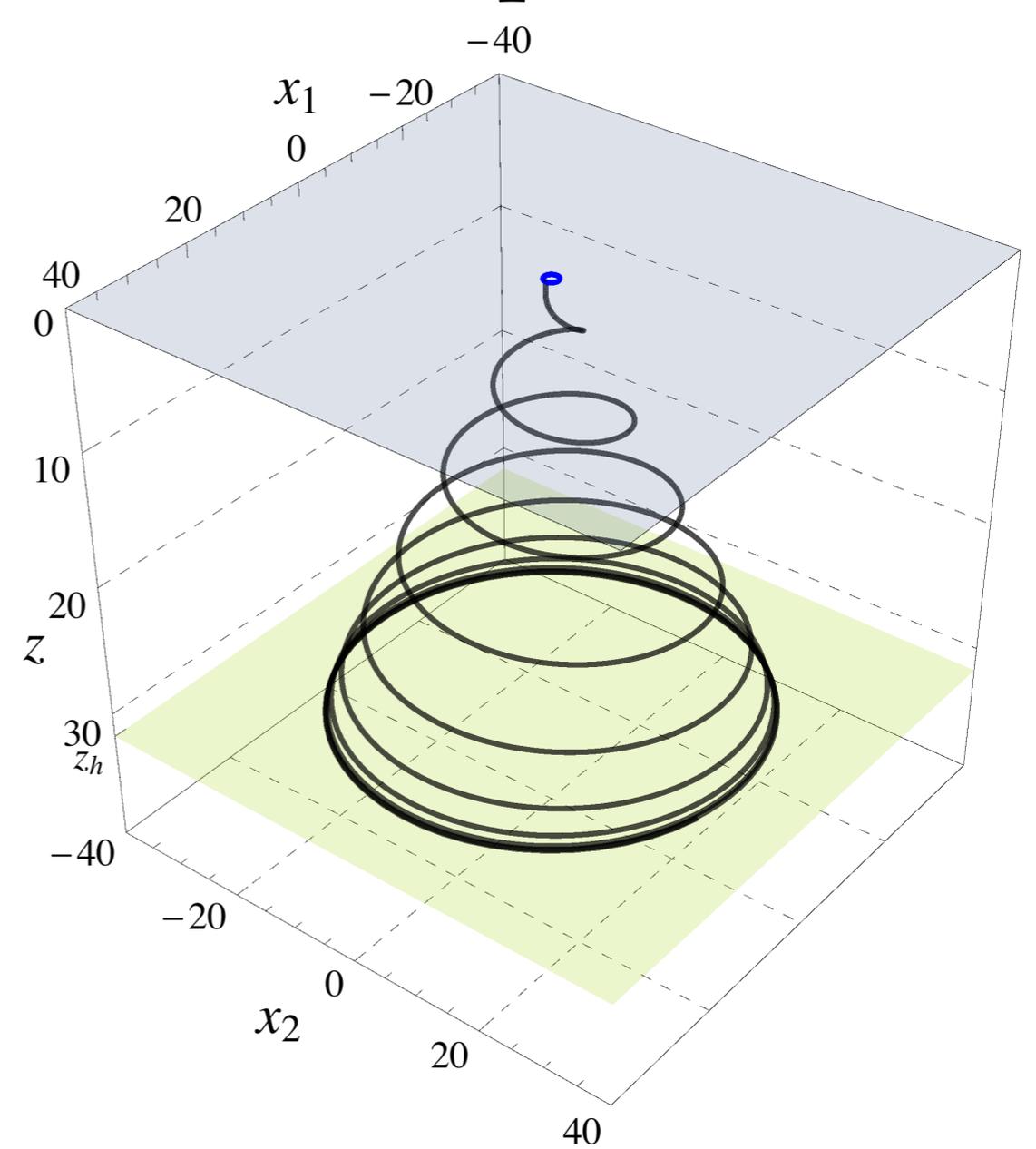
- What can we learn from a rotating quark in a hot moving plasma?

Rotating Quark at Finite Temperature



$$T = 0, \omega = 0.3, 0.7, R_0 = 1$$

Athanasίου, Rajagopal, Chesler, Liu, Nickel, 2010



$$T = 0.01, \omega = 0.7, R_0 = 1$$

Fadafan, Rajagopal, Wiedemann, 2009

- What can we learn from a rotating quark in a hot moving plasma?

Rotating Quark in Deformed Metric Models

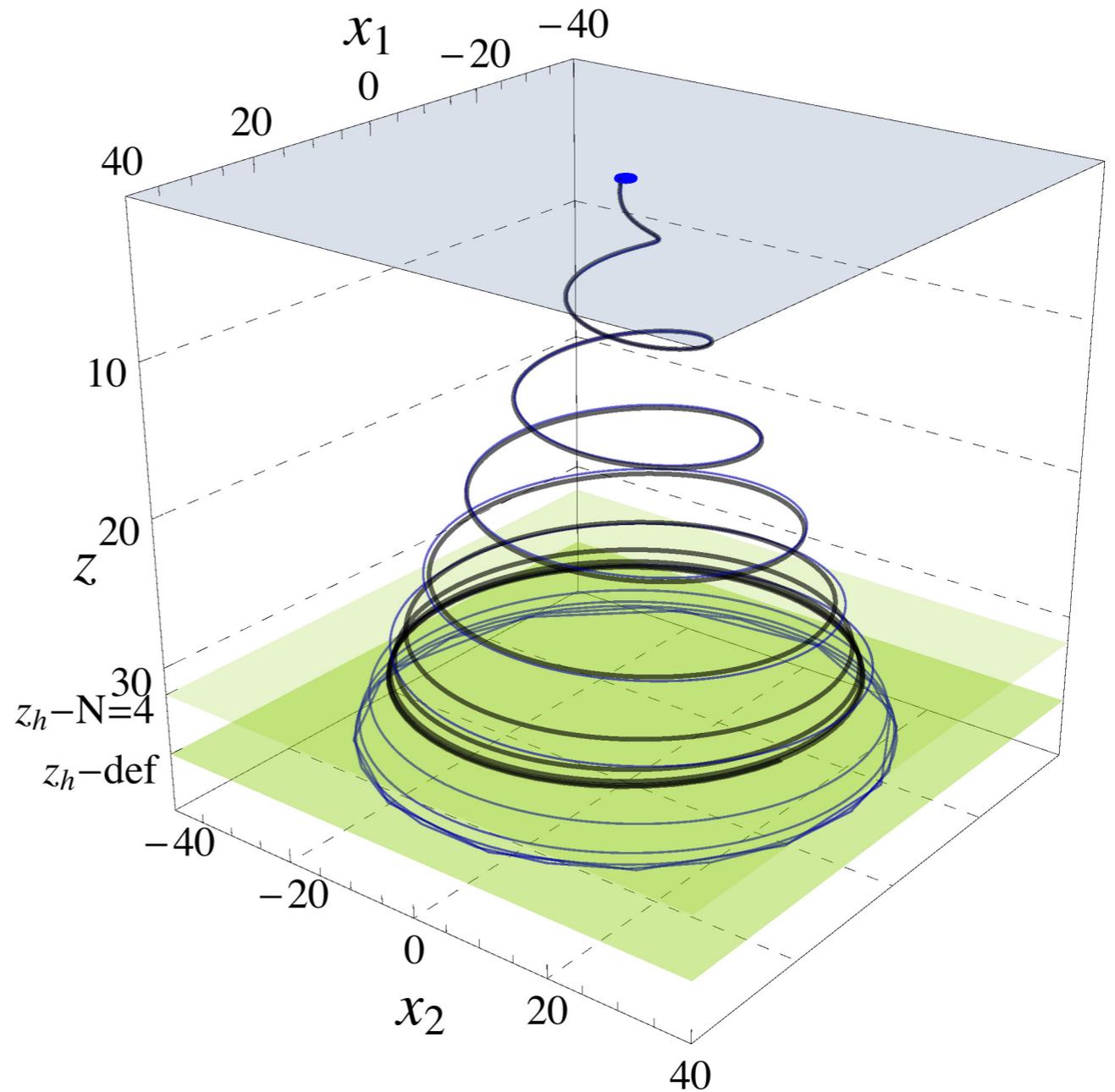
- Is $\omega \ll \pi T$, $R_0 \ll 1/\omega$

then $\left. \frac{dE}{dt} \right|_{\text{RotQ}} \approx \left. \frac{dE}{dt} \right|_{\text{Drag}}$

- Is $\omega \gg \pi T$, $R_0 \omega = v \approx 1$

then $\left. \frac{dE}{dt} \right|_{\text{RotQ}} \approx \left. \frac{dE}{dt} \right|_{\text{VacRad}}$

Vacuum radiation of conformal $\mathcal{N} = 4$



$$T = 0.01, \omega = 0.7, R_0 = 1, \phi = \phi_{\max}$$

- Vacuum radiation is independent of the deformation ϕ .
- Universal scaling in the crossover regime.

Conclusions

- Although being a conjecture the AdS/CFT correspondence as a realisation of the *Holographic principle* is a very powerful tool for qualitative and quantitative analysis, e.g.:
 - *Robustness* and *Universality* of the screening distance.
 - Running coupling of $q\bar{q}$ - pairs in good agreement with lattice QCD data.
 - *Robustness* of the energy loss of rotating quarks in deformed models.
- Many other more sophisticated models (e.g. including D3/D7 branes) available that nicely reproduce many QCD features and includes finite chemical potential.

Thank you for your
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Rotating Quark in Deformed Metric Models

■ Is

the

$$\frac{\left. \frac{dE}{dt} \right|_{\text{RotQ}}}{\left. \frac{dE}{dt} \right|_{\text{Drag}}} \rightarrow \frac{\Pi \omega z_*^2}{v^2}$$

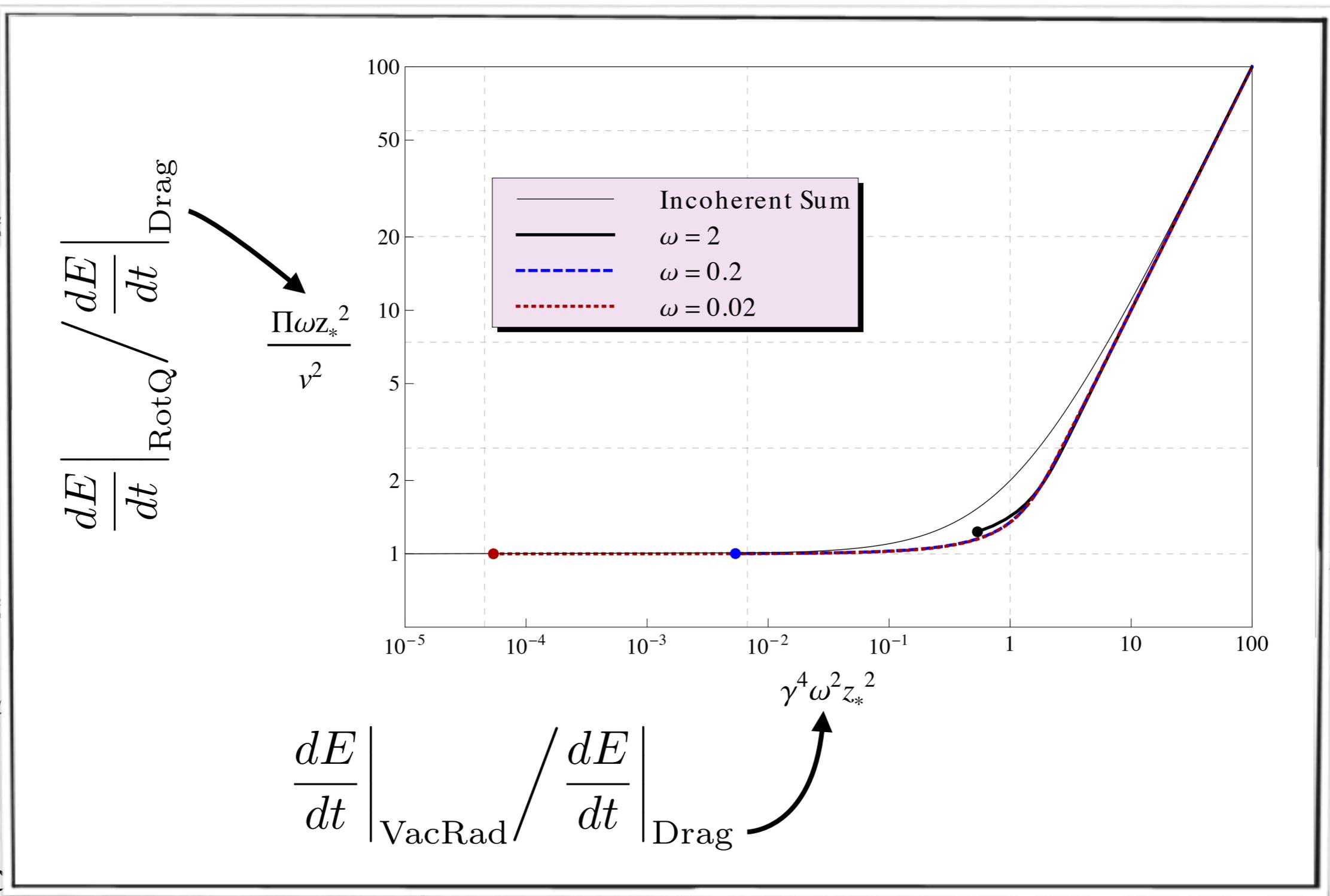
■ Is

the

Vac

$$\frac{\left. \frac{dE}{dt} \right|_{\text{VacRad}}}{\left. \frac{dE}{dt} \right|_{\text{Drag}}} \rightarrow \gamma^4 \omega^2 z_*^2$$

■ Vac



■ Universal scaling in the crossover regime.

Metric models at finite temperature

■ Deformed 2-parameter metric:

DeWolfe, Rosen; Gubser

● 5D-Einstein-Hilbert-Dilaton action:

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left(R - \frac{1}{2} (\partial_\mu \Phi)^2 - V(\Phi) \right)$$

● General ansatz for metric

$$ds^2 = e^{2A(\Phi)} (-h(\Phi) dt^2 + d\vec{x}^2) + \frac{e^{2B(\Phi)}}{h(\Phi)} d\Phi^2$$

● with parameters: $\frac{c}{T^2}, \alpha \equiv \frac{c}{\phi}$

● Temperature

$$T = \frac{e^{A(\Phi_h) - B(\Phi_h)} |h'(\Phi_h)|}{4\pi}$$

Metric models at finite temperature

- Deformed 2-parameter metric:

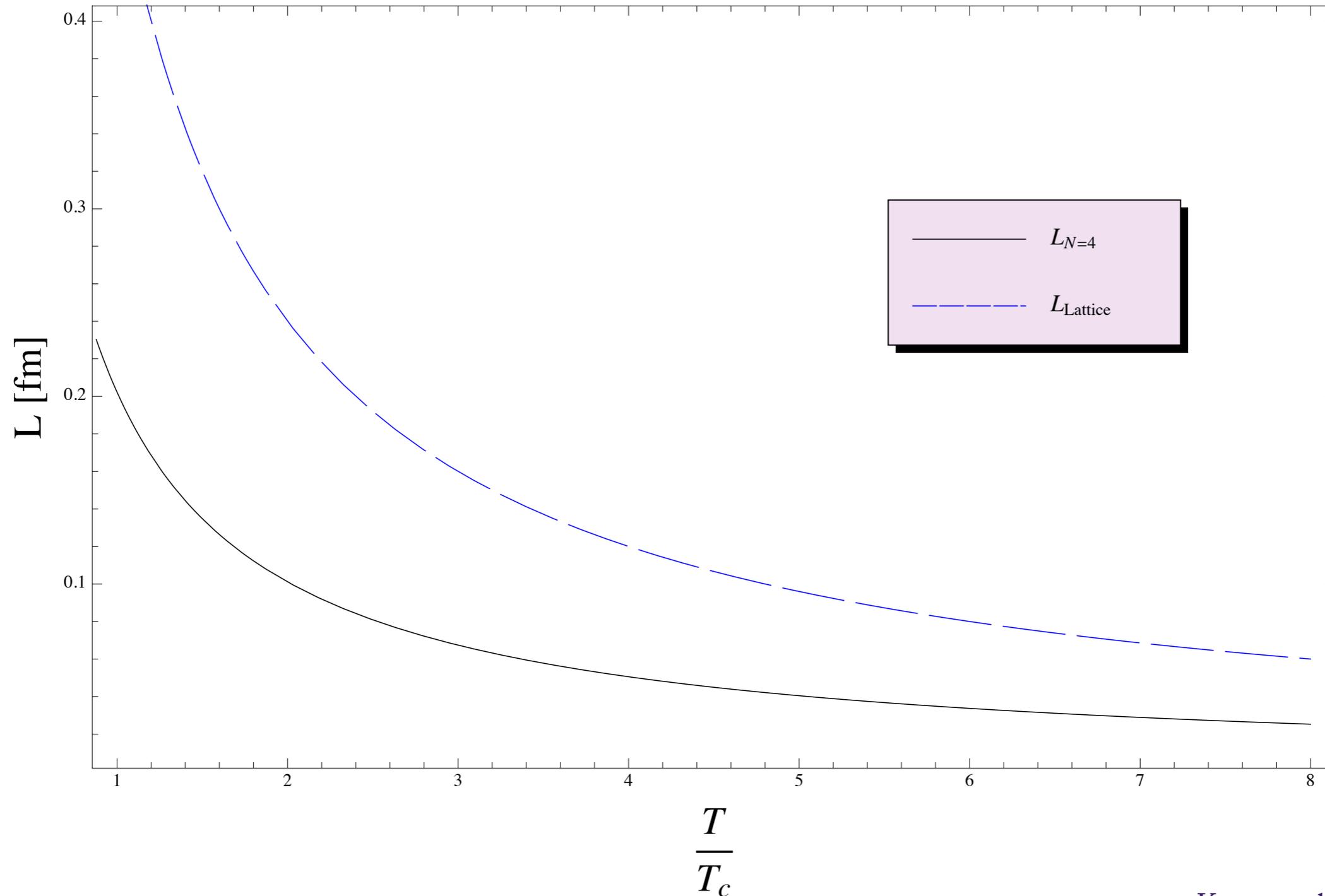
DeWolfe, Rosen; Gubser

$$A(\Phi) = \frac{1}{2} \ln \left(\sqrt{\frac{3}{2}} c \frac{R^2}{\alpha} \right) - \frac{1}{2} \ln \Phi - \frac{\alpha}{\sqrt{6}} \Phi$$

$$B(\Phi) = \ln \left(\frac{R}{2} \right) + \frac{1 + 2\alpha^2}{2\alpha^2} \ln \left(1 + \alpha \sqrt{\frac{2}{3}} \Phi \right) - \ln \Phi - \frac{1}{\alpha\sqrt{6}} \Phi$$

- 2-parameter model solves supergravity equations of motion for suitable dilaton potential V

Screening distance *in physical units*



- Singlet free energy from lattice (quenched)

Kaczmarek, Karsch, Zantow
and Petreczky, 2004

$Q\bar{Q}$ -free energy: computation

- General formulation of the $q\bar{q}$ free energy:

$$E(L, \eta, \theta)\mathcal{T} = S(L, \eta, \theta) - S_0$$

- The string action reads:

$$S = \sqrt{\lambda}TT \int_{y_c}^{\infty} \frac{d\sigma}{y'} \sqrt{(y^4 - \cosh^2 \eta) \left(1 + \frac{y'^2}{y^4 - 1}\right)} \quad \Rightarrow \quad \frac{S}{\sqrt{\lambda}TT} = \int_{y_c}^{\infty} dy \frac{y^4 - \cosh^2 \eta}{\sqrt{(y^4 - y_c)(y^4 - 1)}}$$

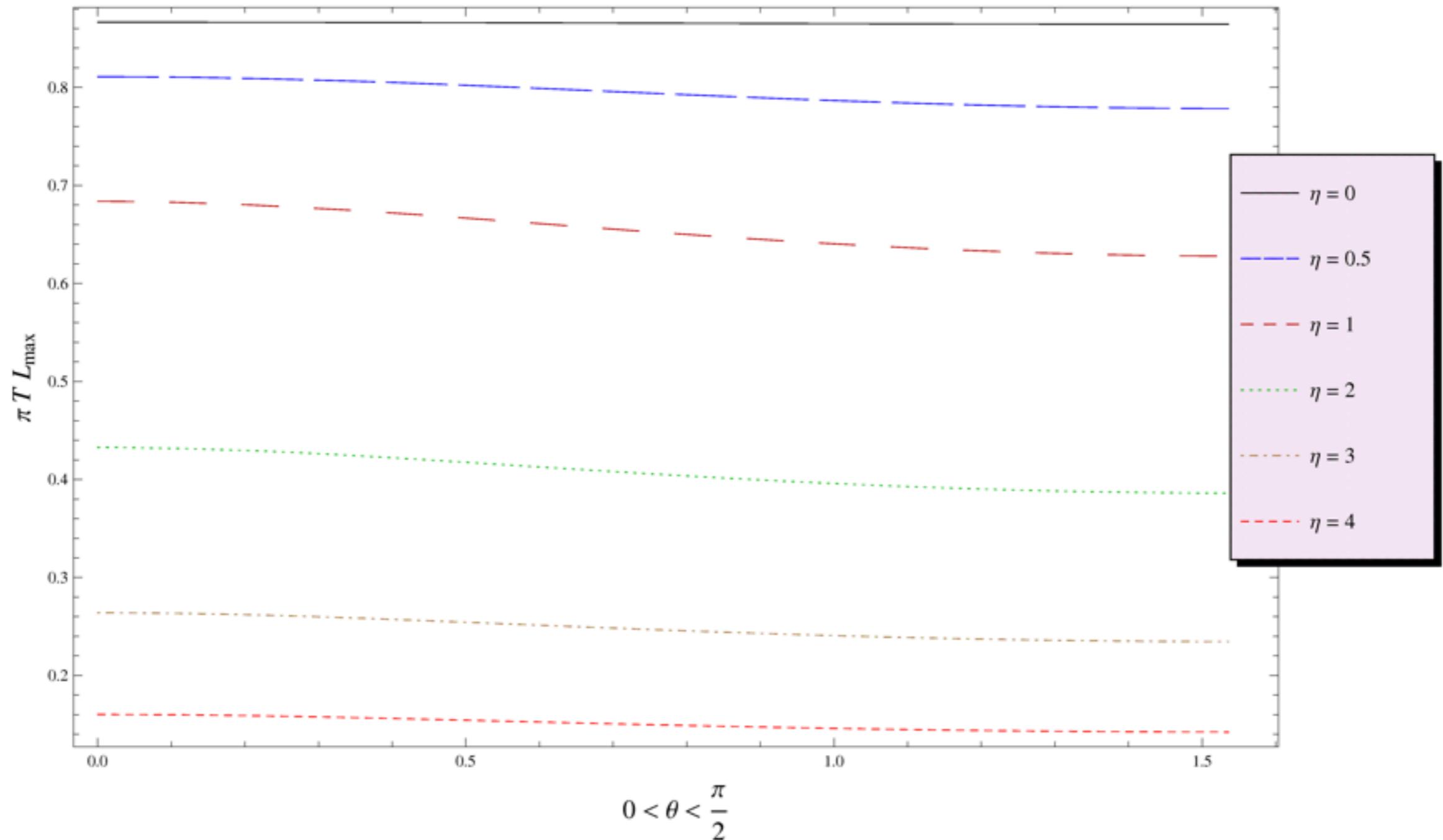
- Compute drag solution for a moving heavy quark:

$$S_0 = \sqrt{\lambda}TT \int_1^{\infty} dy$$

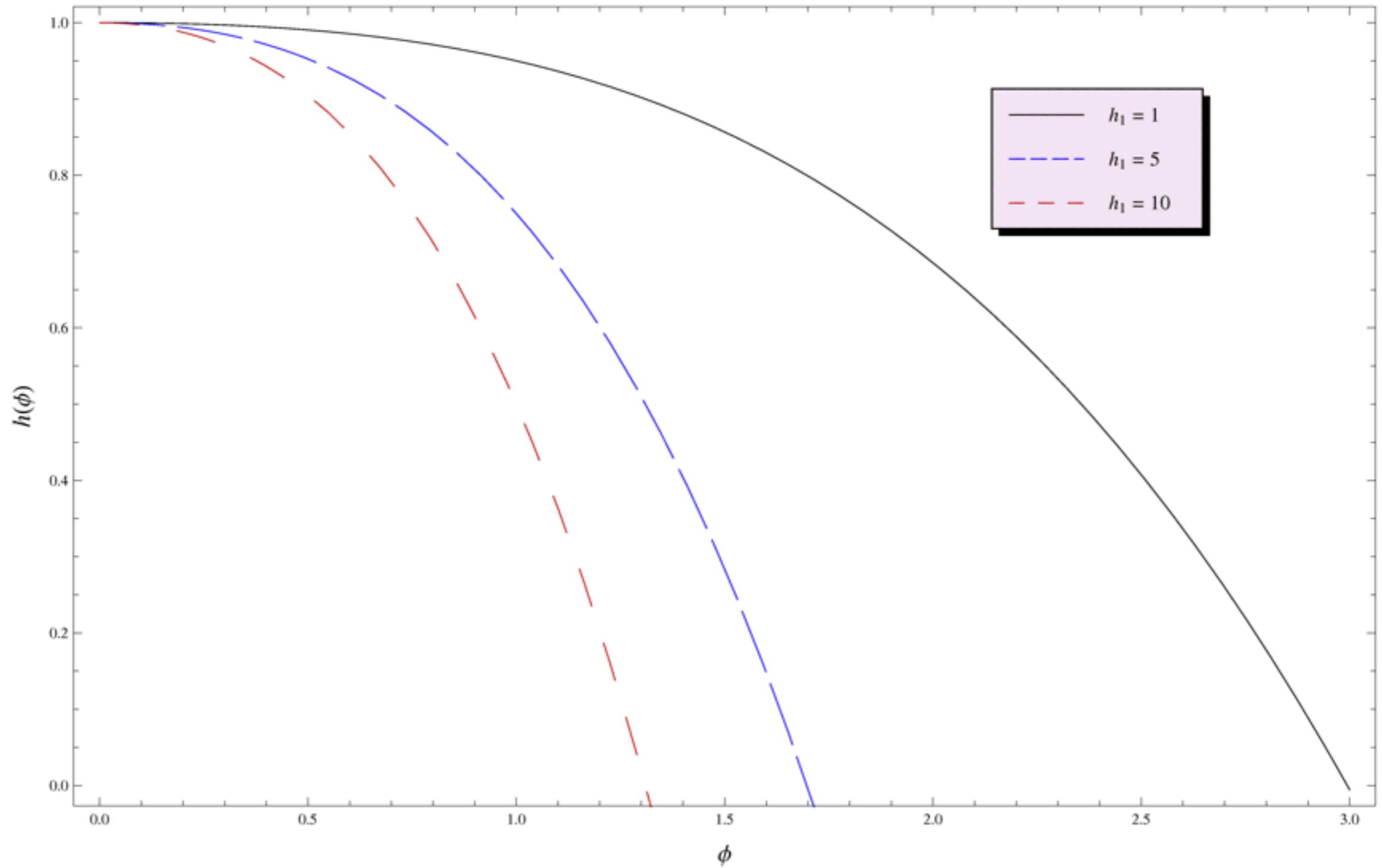
- Final equation for the free energy

$$E\left(q(L), \eta, \frac{\pi}{2}\right)\mathcal{T} = \sqrt{\lambda}TT \int_{y_c}^{\infty} \left[\frac{y^4 - \cosh^2 \eta}{\sqrt{(y^4 - y_c)(y^4 - 1)}} - 1 \right] dy - \sqrt{\lambda}TT(y_c - 1)$$

Different orientation angles

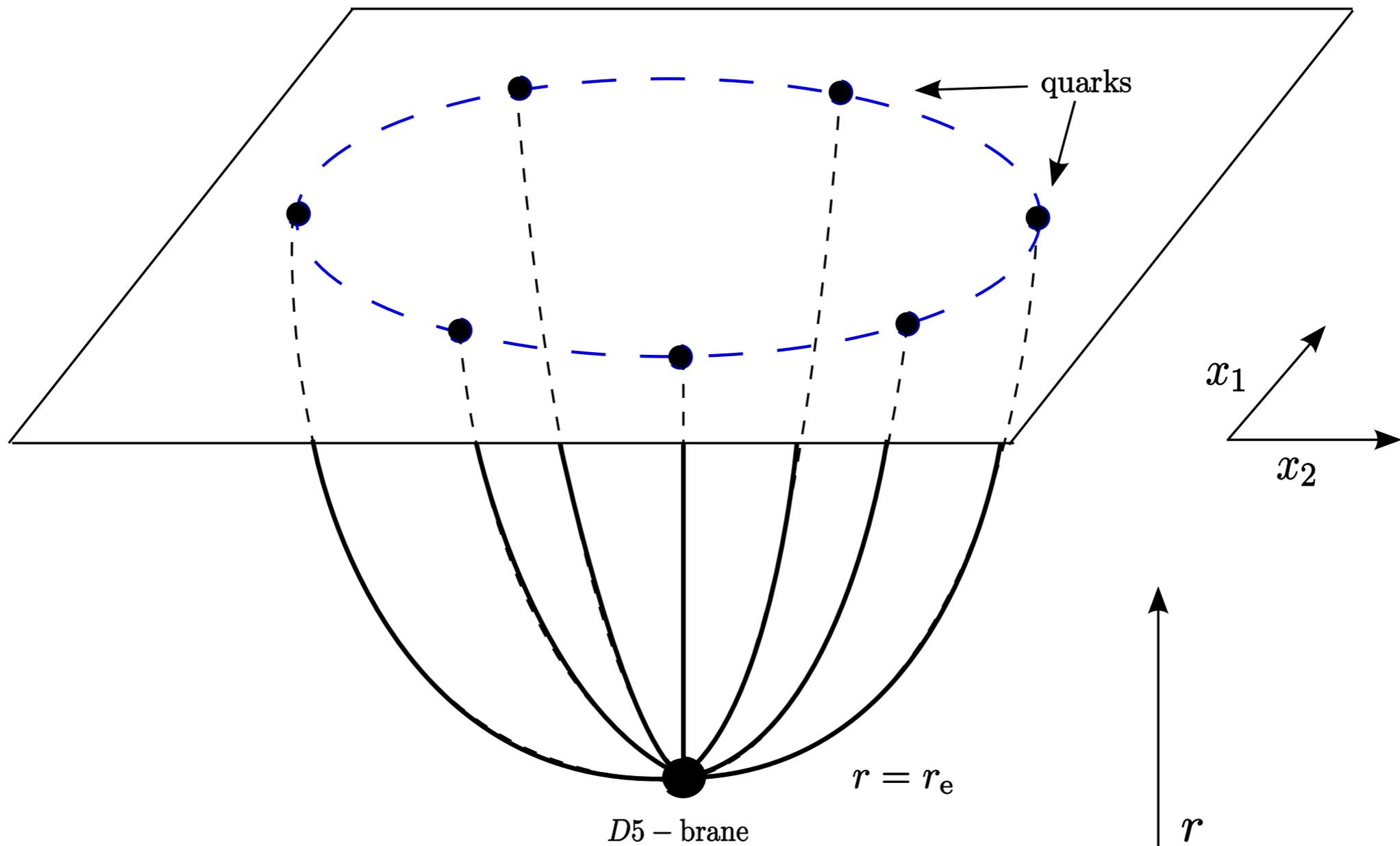


Horizon function



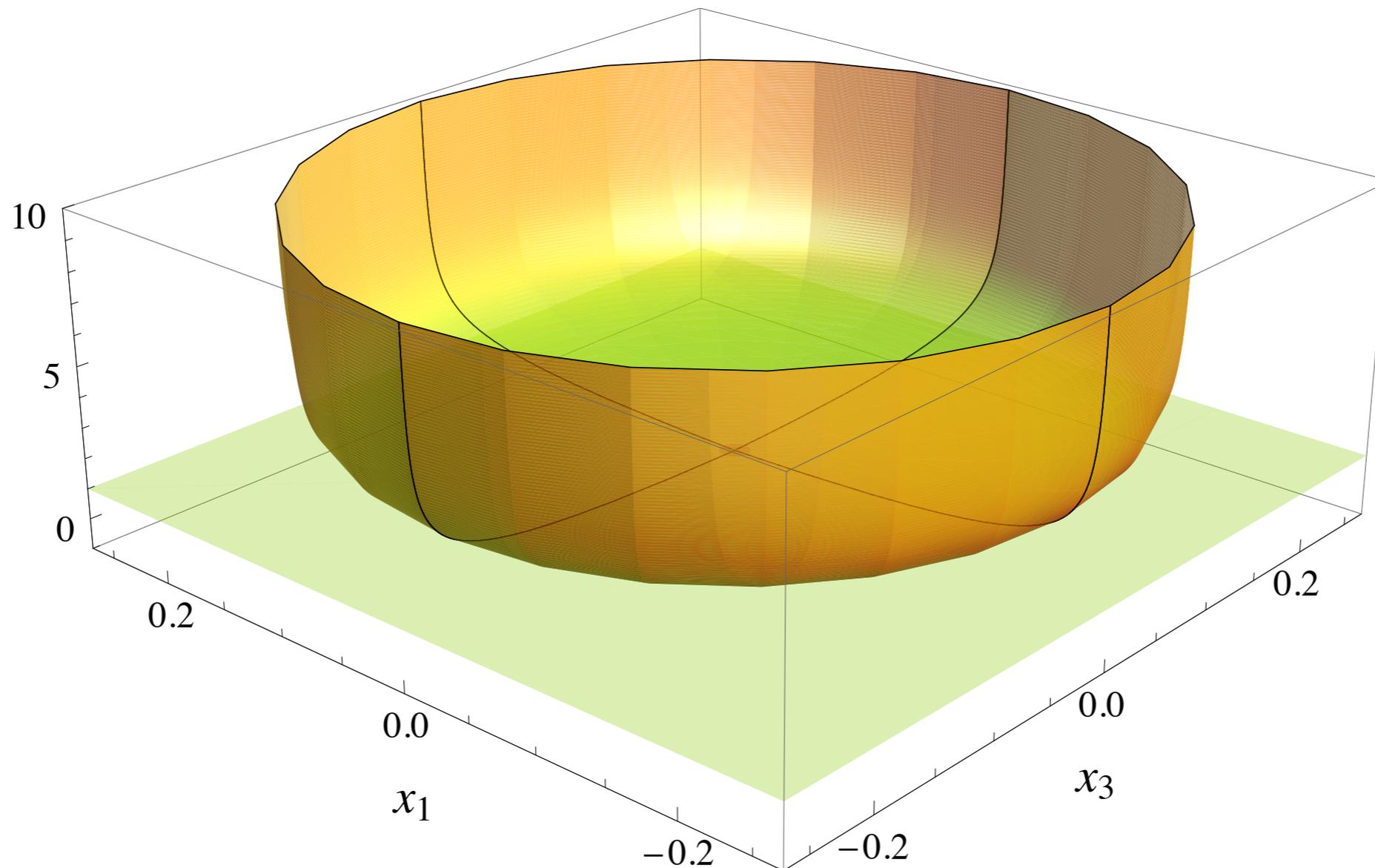
Heavy baryon screening

- Consider baryon configuration with N_c quarks arranged on a circle



- Introduce density of quarks along the circle

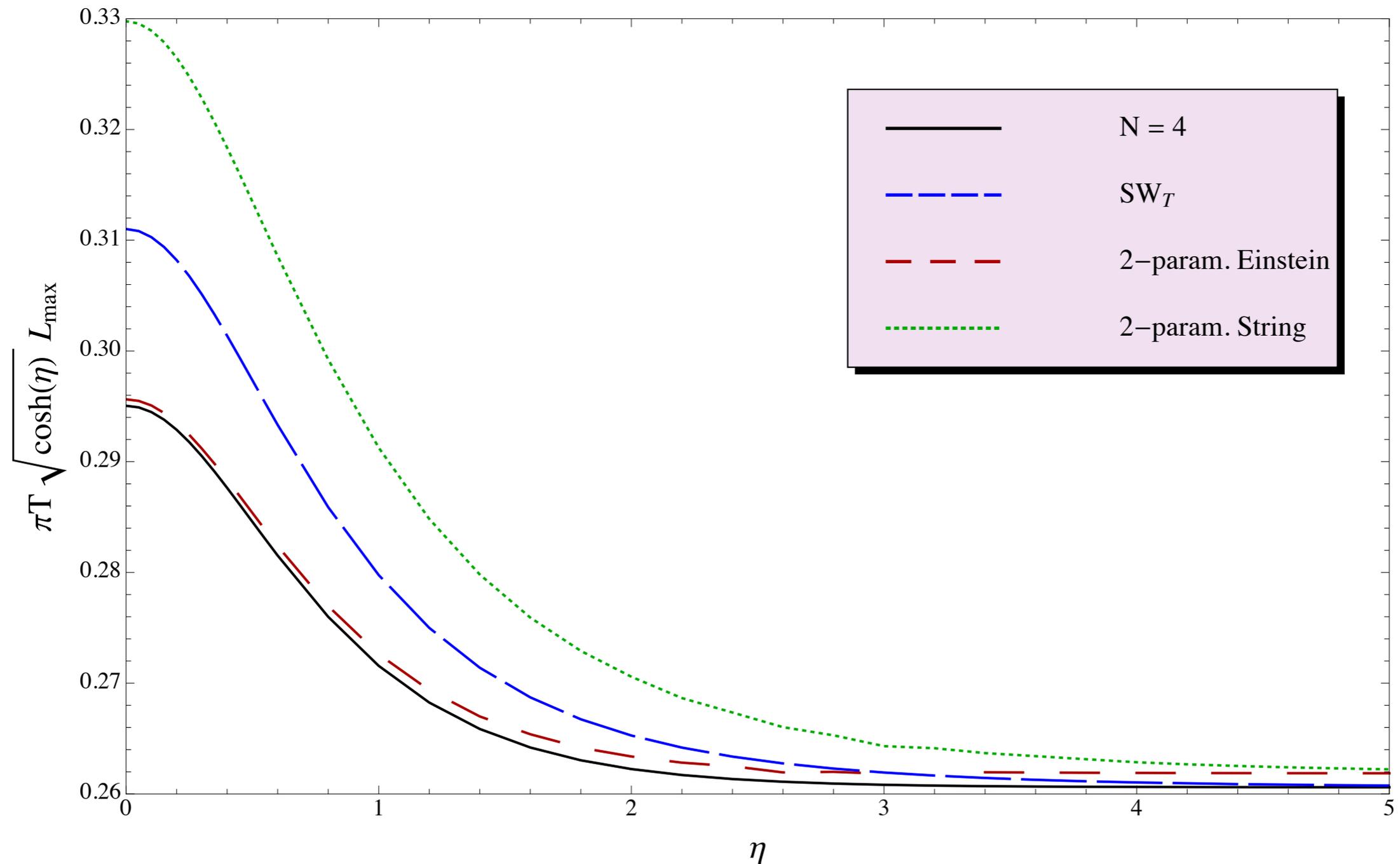
Baryon string configuration



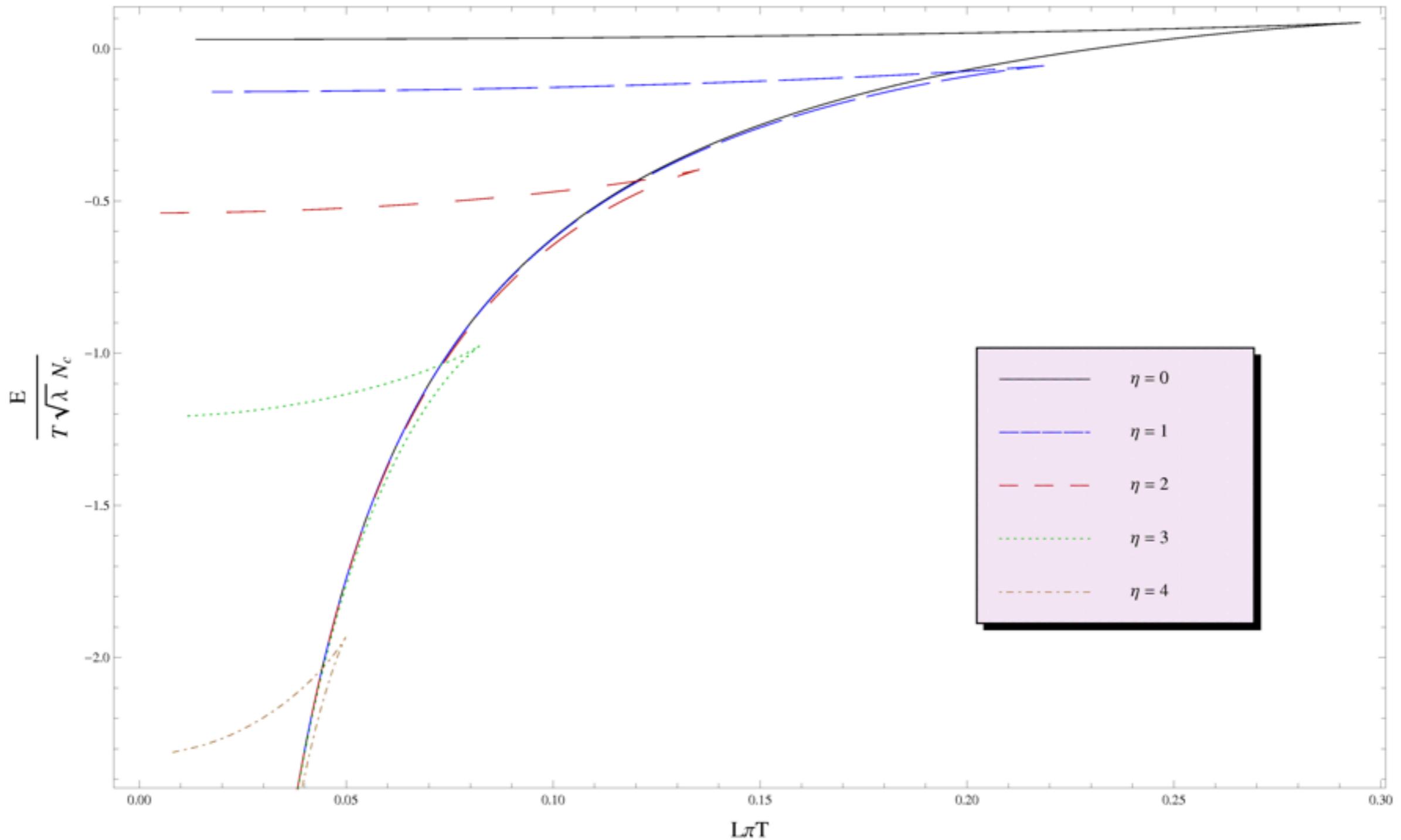
- We can find a stable and unstable solution. The stable one is plotted.

Screening distance bound for baryons

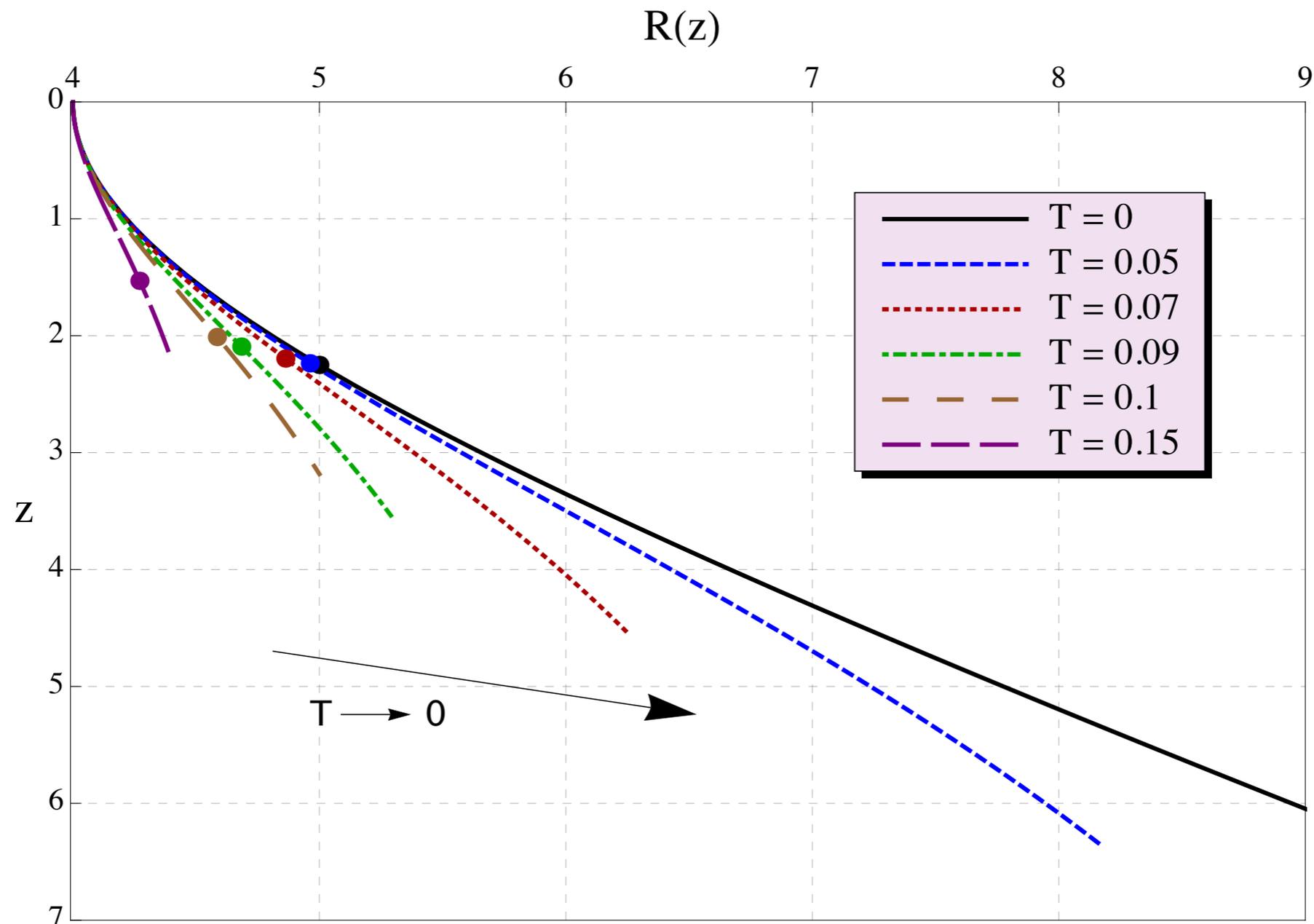
- Again L_{\max} is minimal for $\mathcal{N} = 4$



Free energy for baryons



Rotating Quark at Finite Temperature



- Radial function of the rotating quark. The dots denote the point where the string reaches the local speed of light in the bulk.