Search for the QCD Critical Point

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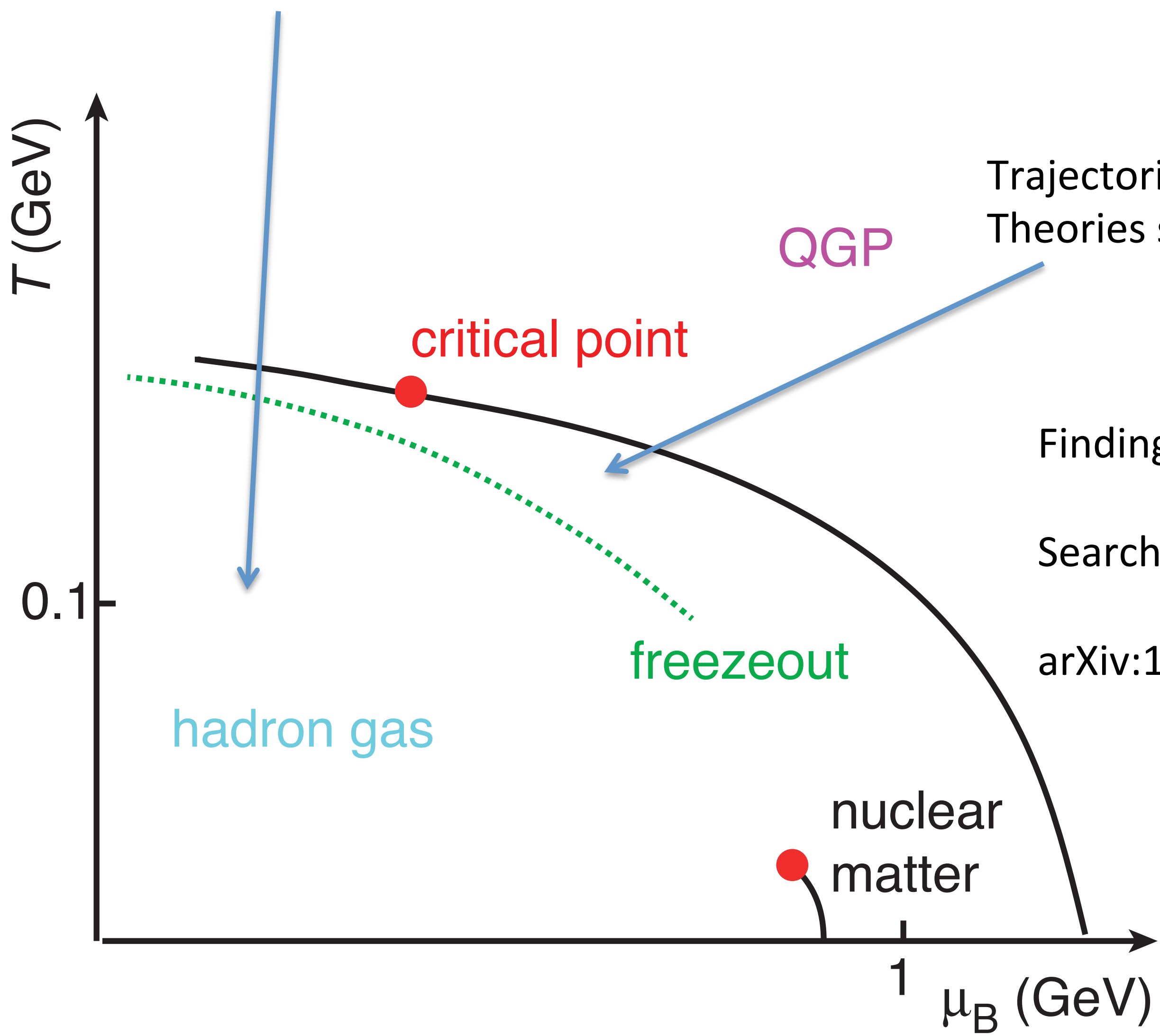
Search for Possible QCD Critical Point

Trajectories at low μ_B
Theories suggest a cross-over transition

Trajectories at high μ_B
Theories suggest a first-order transition

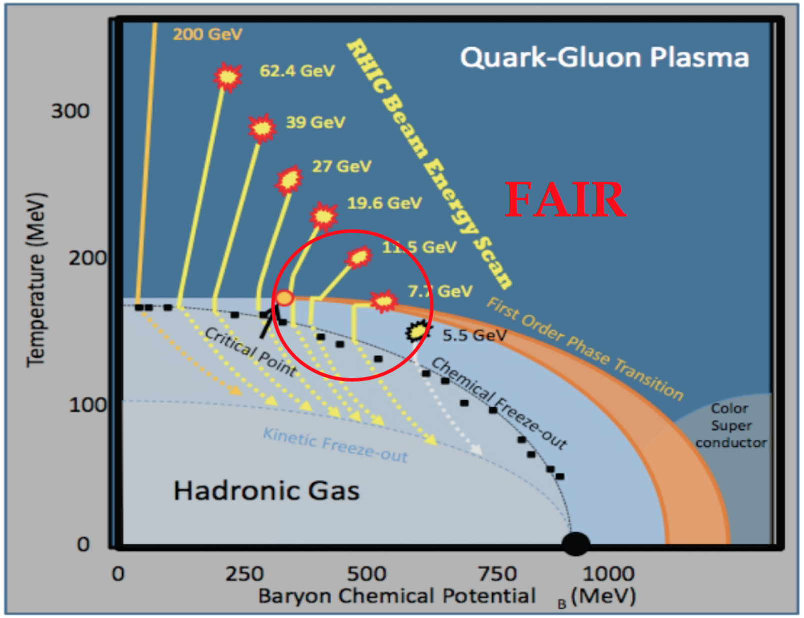


Finding the CP theoretically is difficult

Search for the CP experimentally

arXiv:1007.2613

The RHIC Beam Energy Scan



$\sqrt{s_{NN}}$ (GeV)	Min. Bias Events (10^6)
7.7	4.3
11.5	11.7
19.6	35.8
27	70.4
39	130.4
62.4	67.3
200	500

- Goals:
  - Search for signatures of a possible phase transition from hadronic matter to quark gluon matter
  - Search for signatures of a possible QCD critical point
- Strategy
  - Vary incident energy to span range in T-μ_B
  - Look for non-monotonic behavior of observables

Observables

- Particle ratio fluctuations
  - K/π
    - Strangeness fluctuations
  - p/π
    - Baryon fluctuations
  - K/p
    - Strangeness/baryon correlations
- Net particle higher moments
  - Net charge
    - Net charge fluctuations
  - Net kaons
    - Net strangeness fluctuations
  - Net protons
    - Net baryon fluctuations

Observable - Fluctuations

- Our observable is $\nu_{\mathrm{dyn},K\pi}$, which measures how correlated the event-by-event distributions are

$$\nu_{\mathrm{dyn},K\pi} = \frac{\langle N_K(N_K-1)\rangle}{\langle N_K\rangle^2} + \frac{\langle N_\pi(N_\pi-1)\rangle}{\langle N_\pi\rangle^2} - 2\frac{\langle N_K N_\pi\rangle}{\langle N_K\rangle\langle N_\pi\rangle}$$

  - $\nu_{\mathrm{dyn},K\pi}$ is zero for uncorrelated emission (Poisson)
- NA49 uses σ_{dyn}

$$\sigma_{\mathrm{dyn}} = \mathrm{sgn}(\sigma^2_{\mathrm{data}} - \sigma^2_{\mathrm{mixed}})\sqrt{|\sigma^2_{\mathrm{data}} - \sigma^2_{\mathrm{mixed}}|}$$

- With enough statistics and large denominator

$$\nu_{\mathrm{dyn}} \approx \mathrm{sgn}(\sigma_{\mathrm{dyn}})\sigma^2_{\mathrm{dyn}}$$

Observable – Net Particle Moments

- Fluctuations of conserved quantities such as net charge, net strangeness, and net baryon number may be related to the chiral phase transition or critical phenomena
  - Net charge
    $$N_{q,\mathrm{net}} = N_{+} - N_{-}$$
  - Net strangeness
    $$N_{\mathrm{s,net}} = N_{K^+} - N_{K^-}$$
  - Net baryon number
    $$N_{\mathrm{b,net}} = N_{p} - N_{\bar{p}}$$
- Higher moments of the net particle distributions are predicted to be sensitive to high powers of the susceptibility

Cumulants

- To describe the higher moments of net particle distributions, we employ cumulants
- Cumulants can be expressed in terms of the central moments
- The central moments are defined by

$$\delta x = x - \langle x \rangle$$

x is the number in an event

$\langle x \rangle$ is the average over an ensemple of events

second moment: $\left\langle \left\langle x^2 \right\rangle \right\rangle = \left\langle (\delta x)^2 \right\rangle$

third moment: $\left\langle \left\langle x^3 \right\rangle \right\rangle = \left\langle (\delta x)^3 \right\rangle$

fourth moment: $\left\langle \left\langle x^4 \right\rangle \right\rangle = \left\langle (\delta x)^4 \right\rangle$

Cumulants

- The first four cumulants are

$$C_1 = \langle (\delta x) \rangle \qquad C_2 = \langle (\delta x)^2 \rangle$$

$$C_3 = \langle (\delta x)^3 \rangle \quad C_4 = \langle (\delta x)^4 \rangle - 3\langle (\delta x)^2 \rangle^2$$

- We can then define

$$\text{Variance: } \sigma^2 = C_2 \qquad \text{Standard deviation: } \sigma = \sqrt{C_2}$$

$$\text{Skewness: } S = C_3 / C_2^{3/2} \qquad \text{Kurtosis: } \kappa = C_4 / C_2^2$$

$$S\sigma = C_3 / C_2 \quad \kappa\sigma^2 = C_4 / C_2$$

- Near the critical point, the cumulants will diverge with large powers of the correlation length (ξ)
- Higher moments scale with higher powers of the correlation length

Cumulants

- The susceptibilities χ and correlation lengths ξ scale as

$$\frac{C_3}{C_2} = S\sigma \sim \frac{\chi_3}{\chi_2} \sim \xi^{5/2}$$

$$\frac{C_4}{C_2} = \kappa\sigma^2 \sim \frac{\chi_4}{\chi_2} \sim \xi^{5}$$

- Various theories predict large enhancements in C_4/C_2 for net charge, net strangeness, and net baryon number
  - Karsch et al., PLB 695, 136 (2011). arXiv: 1203.0784
  - Cheng et al., PRD 79, 074505 (2009)
  - Hatta et al., PRL 91, 102003 (2003), Hatta et al. PRD 67, 014028 (2003)
  - Gavai, Gupta. PLB, 696, 459 (2011)
  - Stephanov PRL 102, 032301 (2009)

Expectations for Net Particle Moments

- We can define two expectations for net particle moments
  - Poisson distribution
    - We assume that the positive particle multiplicity and the negative particle multiplicity distributions are described by Poisson distributions characterized by μ^+ and μ^-
  - Negative binomial/binomial distribution
    - We assume that the positive particle multiplicity and the negative particle multiplicity distributions are described by negative binomial/binomial distributions characterized by (μ^+,σ^{2+}) and (μ^-,σ^{2-})

Difference of Two Negative Binomials/Binomials

- The positive particle and negative particle multiplicity distributions are negative binomials or binomial distributions, depending on μ/σ^2
- The positive and negative multiplicity distributions can be characterized by their cimmulants:

$$C_{1,+}, C_{2,+}, C_{3,+}, C_{4,+} \quad C_{1,-}, C_{2,-}, C_{3,-}, C_{4,-}$$

- The cumulants of the differences of the two cumulants are

$$C_1 = C_{1,+} - C_{1,-} \quad C_3 = C_{3,+} - C_{3,-}$$

$$C_2 = C_{2,+} + C_{2,-} \quad C_4 = C_{4,+} + C_{4,-}$$

Ole E. Barndorff-Nielsen, David G. Pollard and Neil Shephard
DISCRETE-VALUED LEVY PROCESSES AND LOW LATENCY FINANCIAL ECONOMETRICS
Oxford, ISSN 1471-0498

- The products of the moments are then

$$C_3 / C_2 = S\sigma \quad C_4 / C_2 = \kappa\sigma^2$$

Difference of Two Poisson Distributions

- μ_+ = mean of the positive particle distribution
- μ_- = mean of the negative particle distribution
- The cumulants of a Poisson distribution are given by

$$C_{1,+} = C_{2,+} = C_{3,+} = C_{4,+} = \mu_+$$

$$C_{1,-} = C_{2,-} = C_{3,-} = C_{4,-} = \mu_-$$

- So the cumulants of the difference are

$$C_1 = C_{1,+} - C_{1,-} = \mu_+ - \mu_- \quad C_3 = C_{3,+} - C_{3,-} = \mu_+ - \mu_-$$

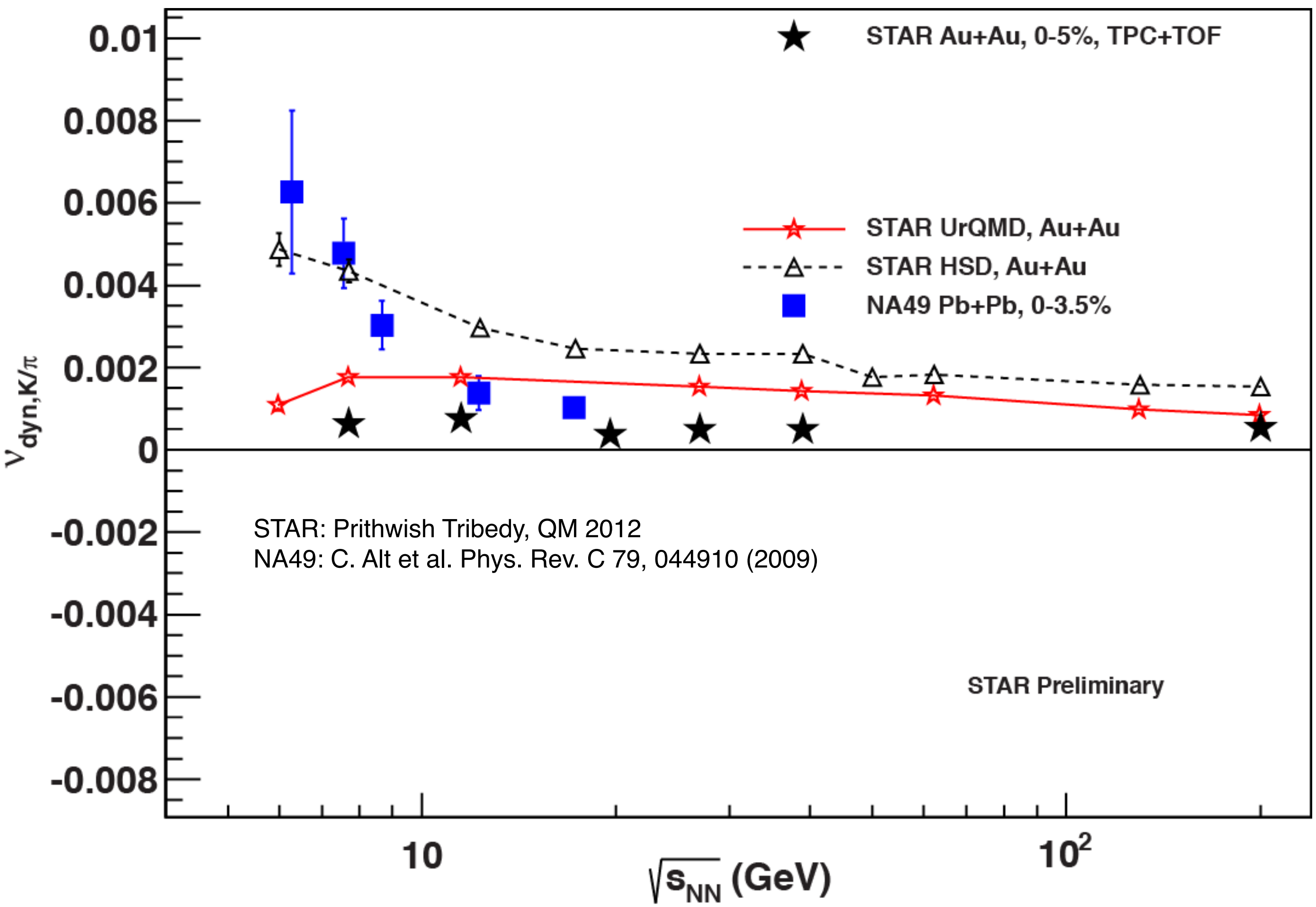
$$C_2 = C_{2,+} + C_{2,-} = \mu_+ + \mu_- \quad C_4 = C_{4,+} + C_{4,-} = \mu_+ + \mu_-$$

- The moment ratios are

$$C_3 / C_2 = \left(\mu_+ - \mu_-\right) / \left(\mu_+ + \mu_-\right)$$

$$C_4 / C_2 = \left(\mu_+ + \mu_-\right) / \left(\mu_+ + \mu_-\right) = 1$$

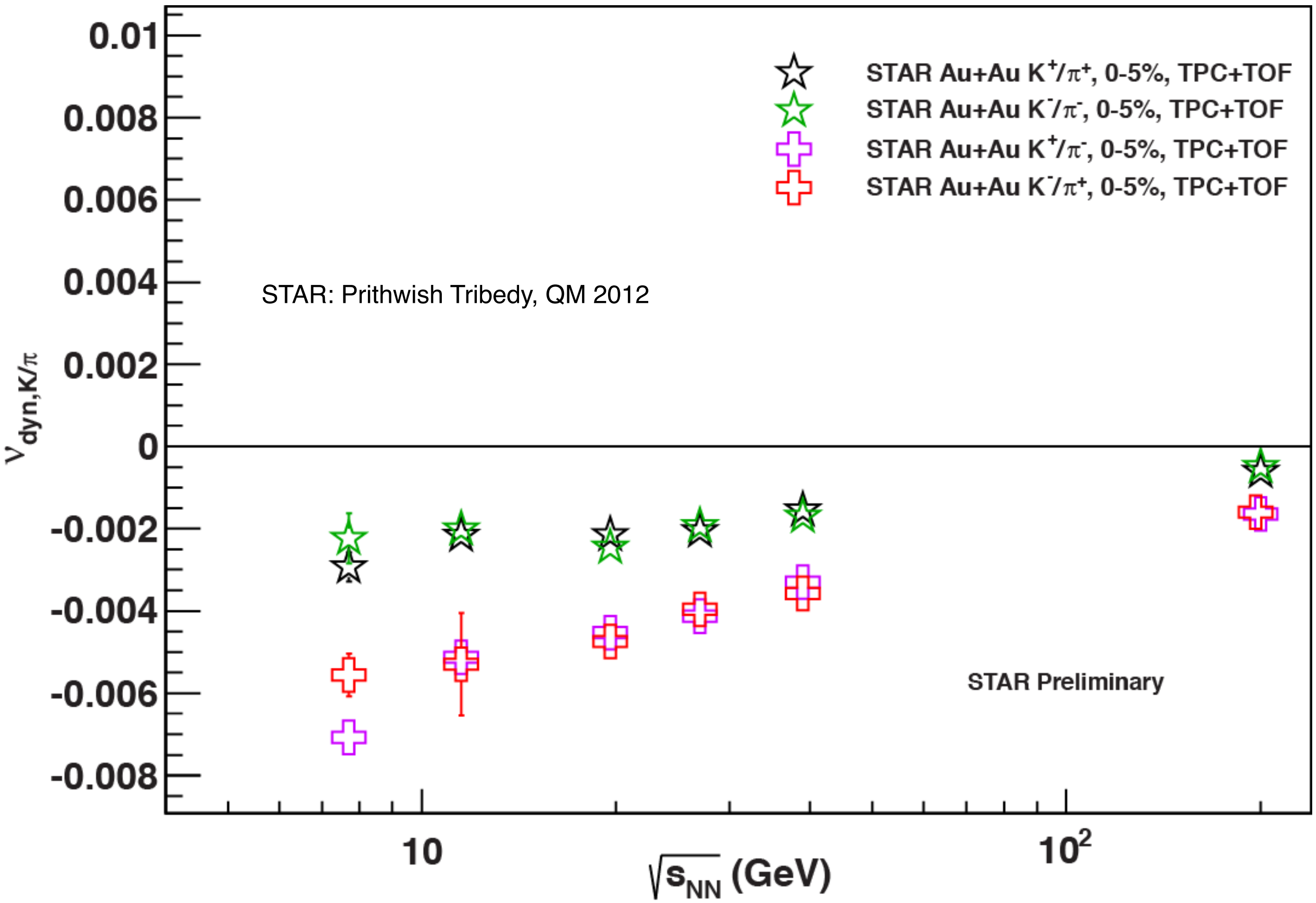
K/π Fluctuations – Summed Signs



$$\nu_{\mathrm{dyn},K\pi} = \frac{\langle N_K(N_K-1)\rangle}{\langle N_K\rangle^2} + \frac{\langle N_\pi(N_\pi-1)\rangle}{\langle N_\pi\rangle^2} - 2\frac{\langle N_K N_\pi\rangle}{\langle N_K\rangle\langle N_\pi\rangle}$$

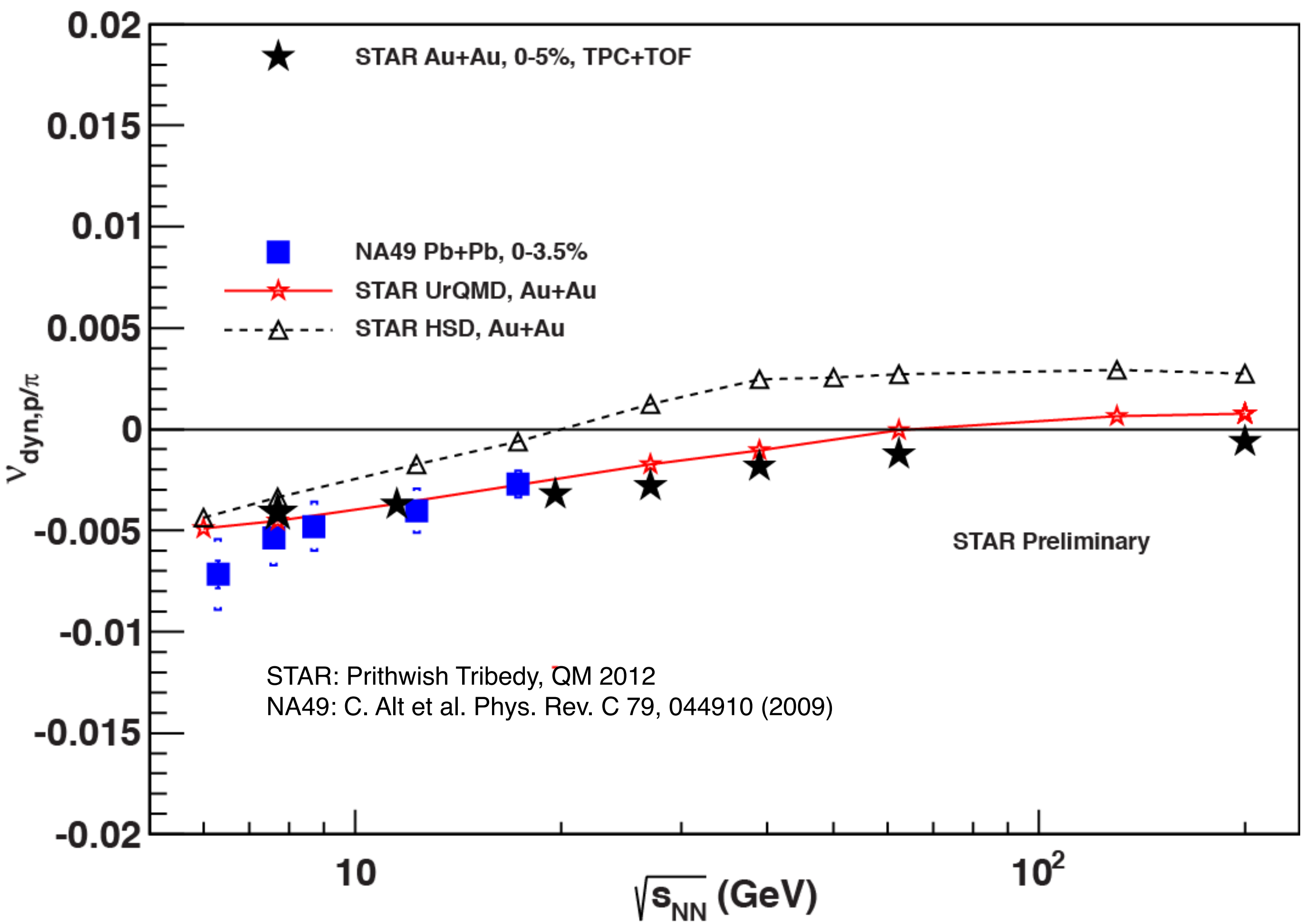
- $(K^+ + K^-)/(\pi^+ + \pi^-)$
- Central events (0 - 5%)
- STAR data show no significant energy dependence
- There appears to be a disagreement between STAR and NA49 results below 19.6 GeV
- The UrQMD model shows little energy dependence and over-predicts the fluctuations

K/π Fluctuations – Separated Signs



- K^+/π^- is strongly negative due to decays, possible candidate is $K^*(892) \rightarrow K^+ + \pi^-$
- K^+/π^+ is also negative, needs further study to investigate the origin

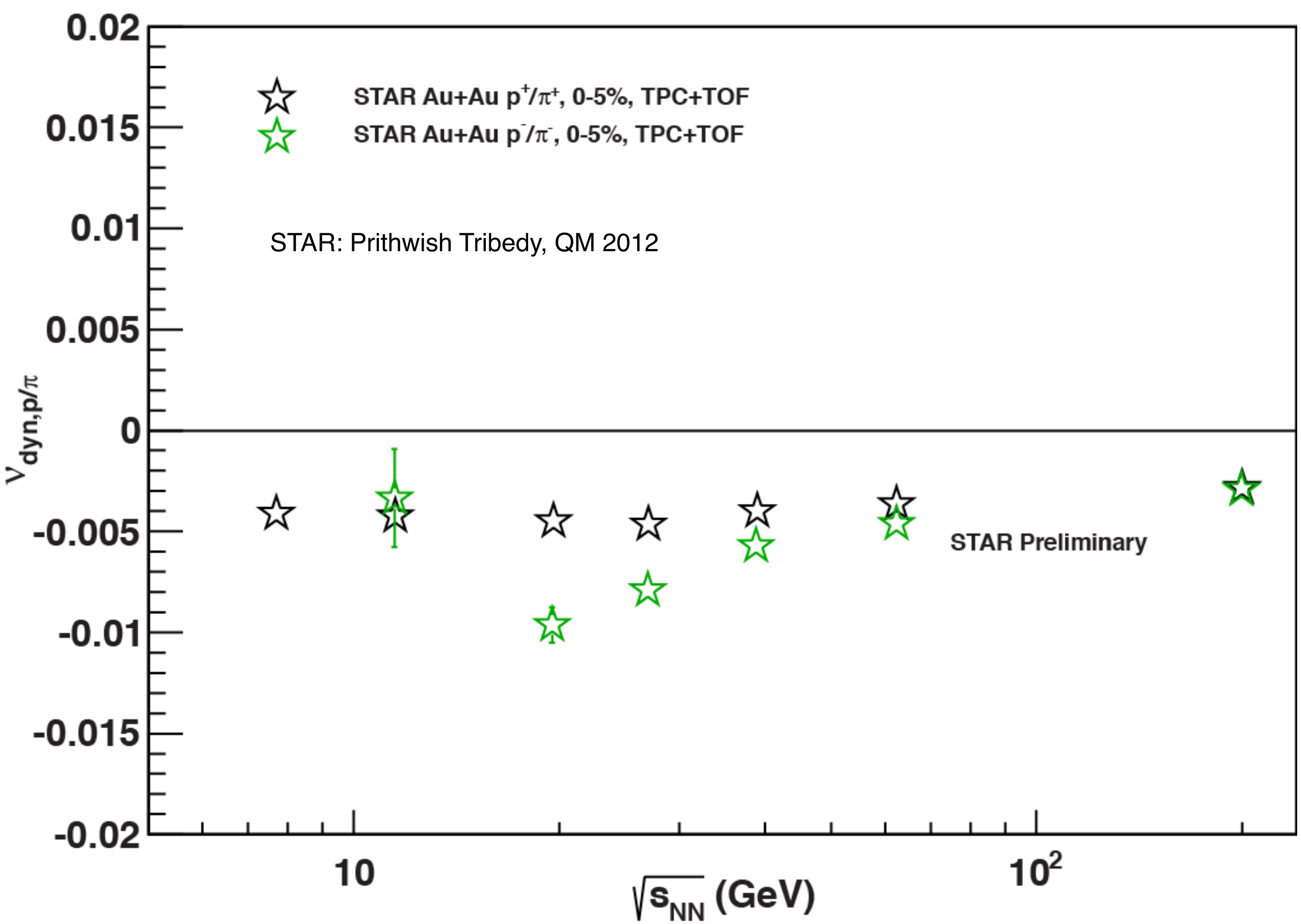
p/π Fluctuations – Summed Signs



$$\nu_{\mathrm{dyn},p\pi} = \frac{\langle N_p(N_p-1)\rangle}{\langle N_p\rangle^2} + \frac{\langle N_\pi(N_\pi-1)\rangle}{\langle N_\pi\rangle^2} - 2\frac{\langle N_p N_\pi\rangle}{\langle N_p\rangle\langle N_\pi\rangle}$$

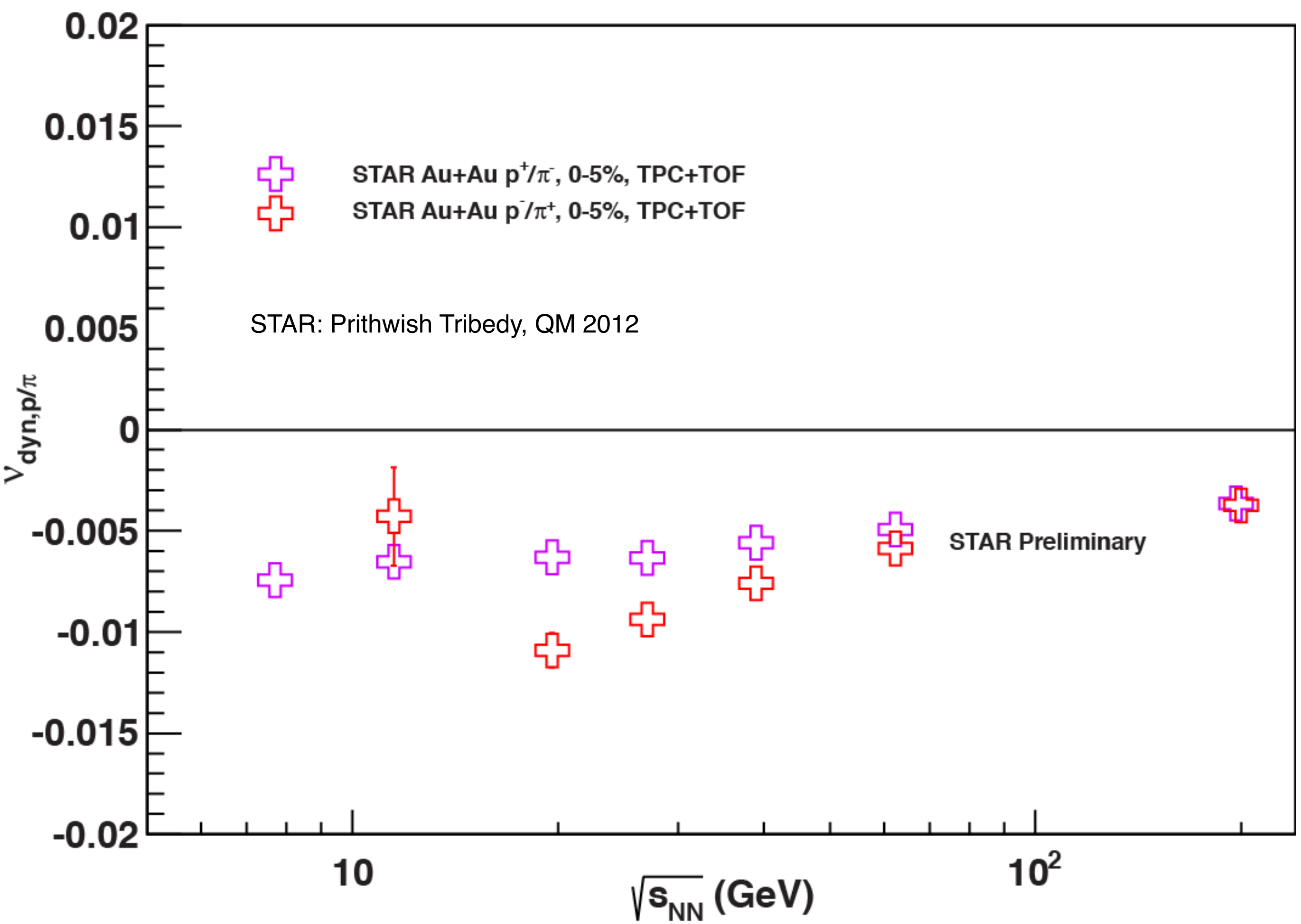
- $(p+\overline{p})/(\pi^+ + \pi^-)$
- Central events (0 – 5%)
- STAR results show a smooth decrease with decreasing incident energy
- There is good agreement between STAR and NA49 results
- The UrQMD model agrees with the data at low energy, but changes sign at high energy

p/π Fluctuations – Same Signs



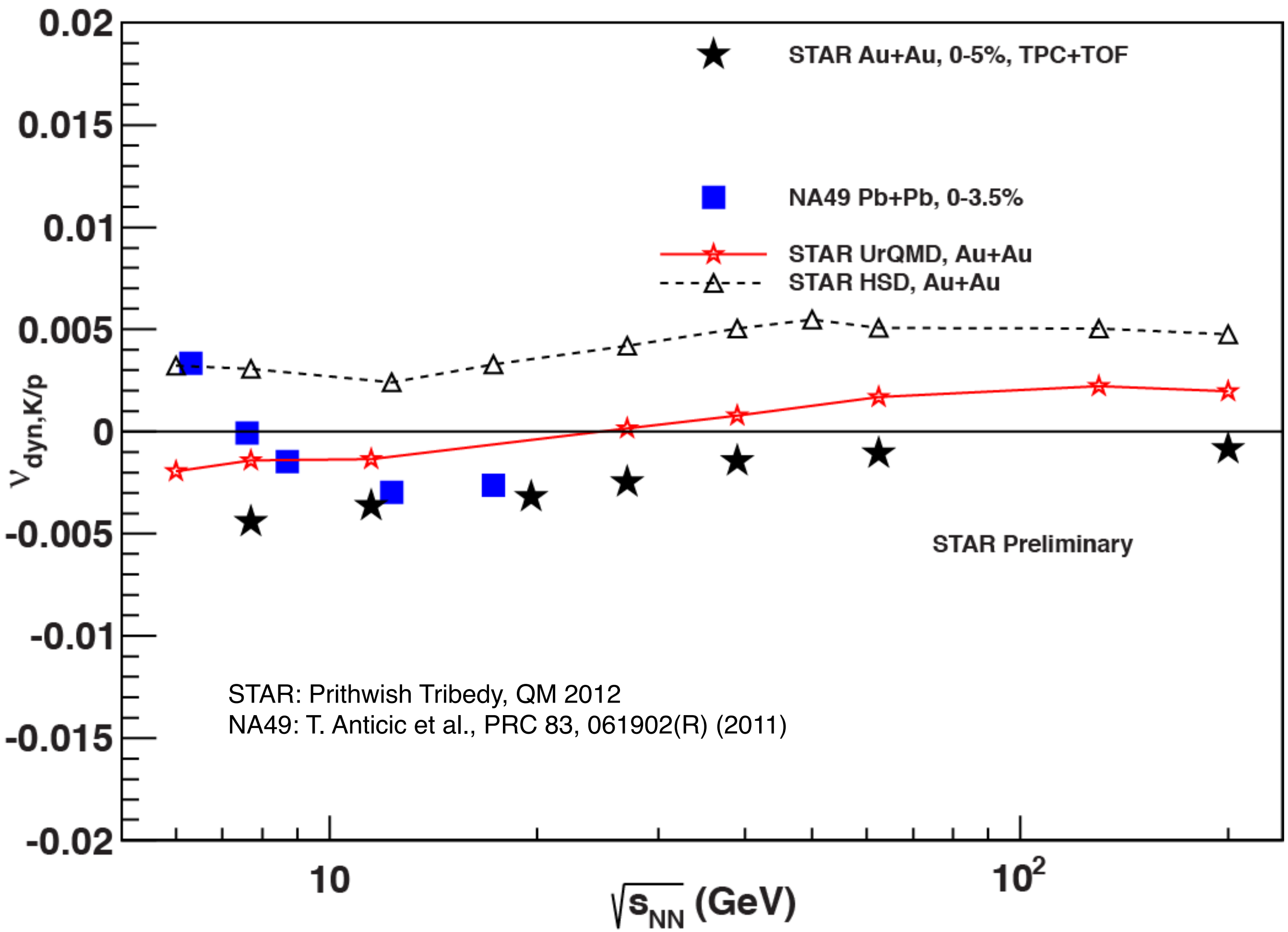
- p^+/π^+
- p^-/π^-
- Central events (0 – 5%)
- Little energy dependence for positive signs
- Very few antiprotons at the lowest energy

p/π Fluctuations – Opposite Signs



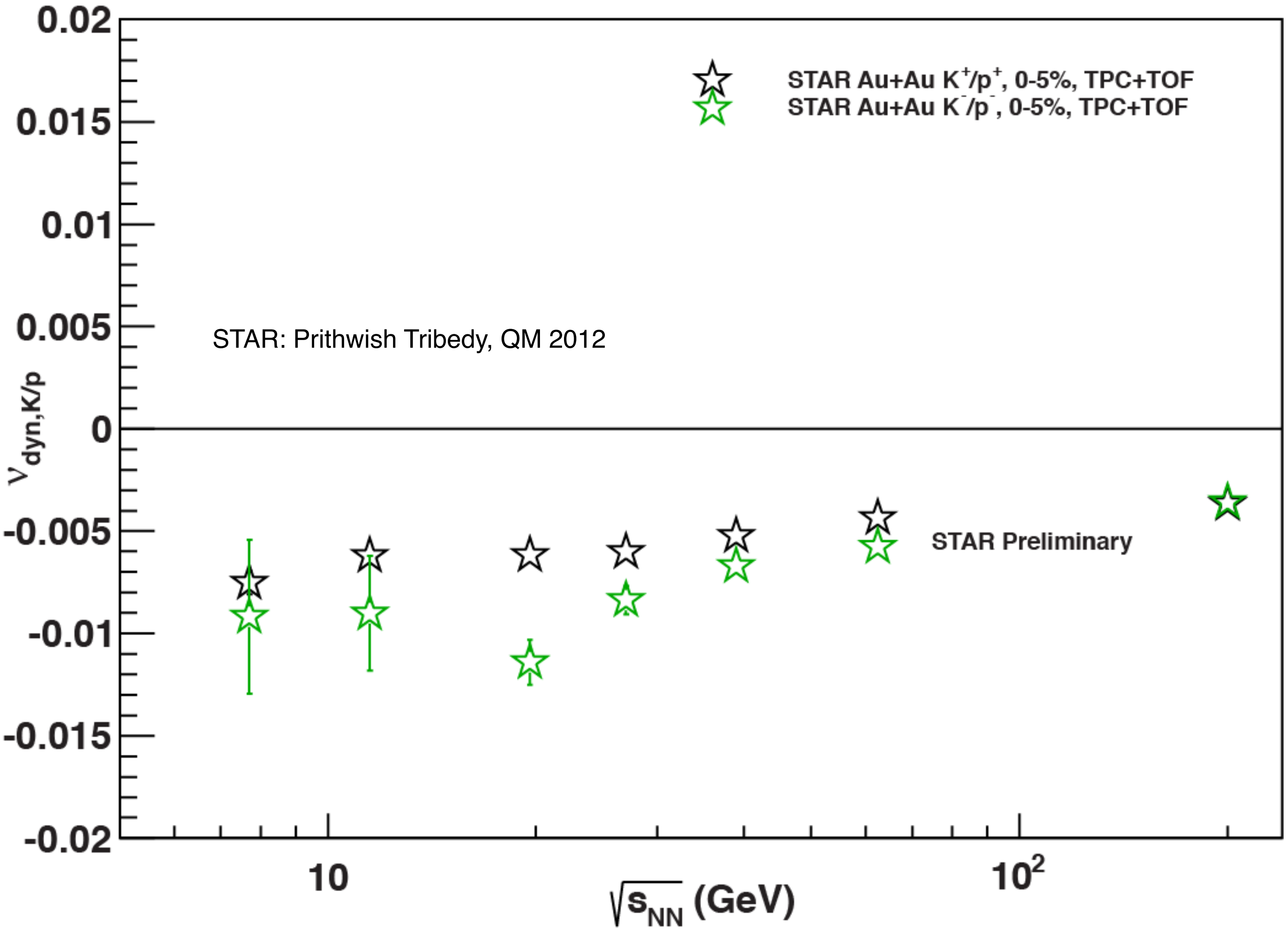
- p^{+}/π^{-}
- p^{-}/π^{+}
- Central events (0 – 5%)
- Little energy dependence for p^{+}/π^{-}
- Very few antiprotons at the lowest energy

K/p Fluctuations – Summed Signs



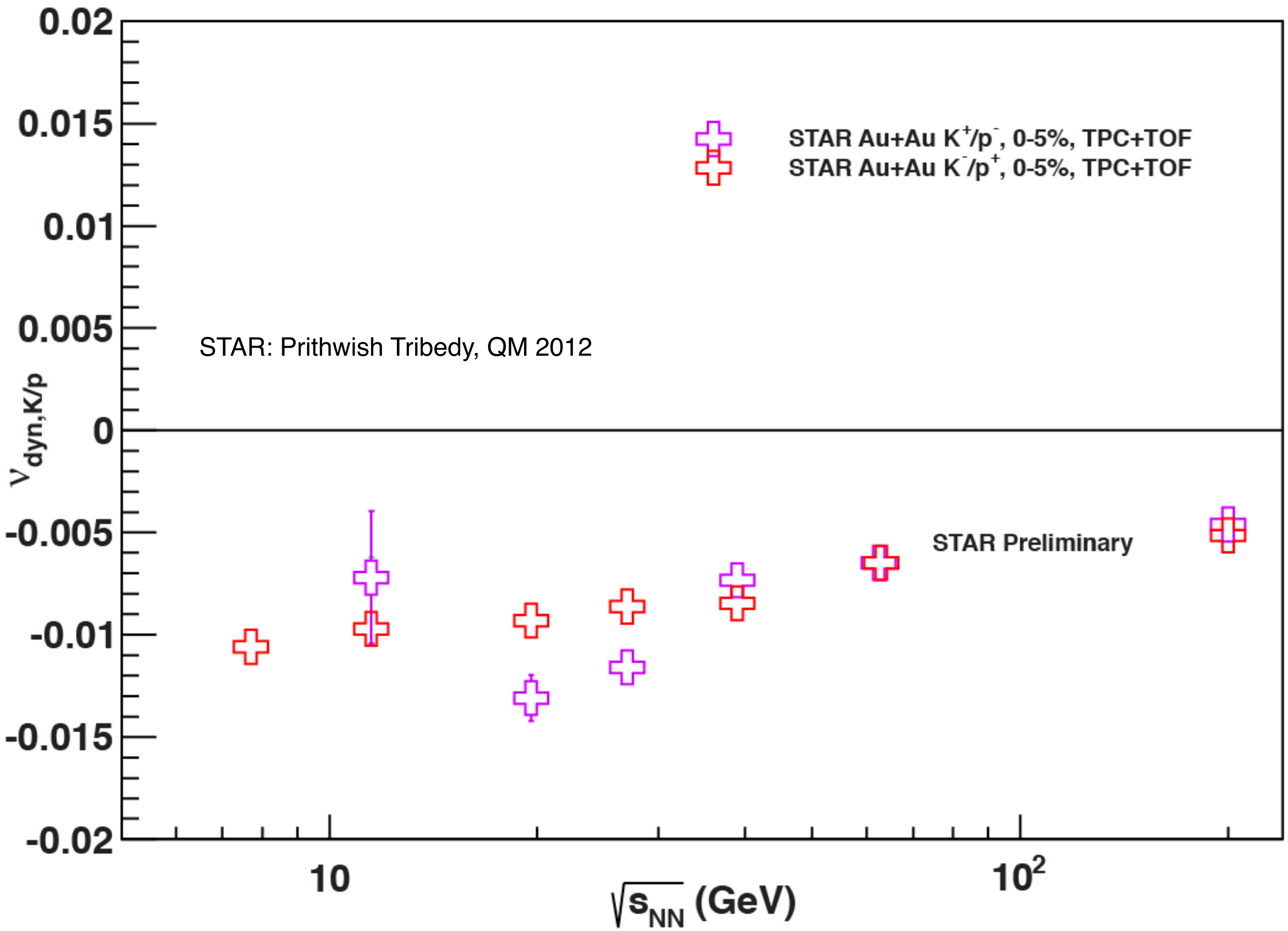
- $(p+\bar{p})/(K^{+}+K^{-})$
- Central events (0 - 5%)
- STAR results decrease smoothly with decreasing incident energy
- The seems to be disagreement between STAR and NA49 results at 7.7 GeV
- The UrQMD model over-predicts the fluctuations and changes sign at high energy

K/p Fluctuations – Same Sign



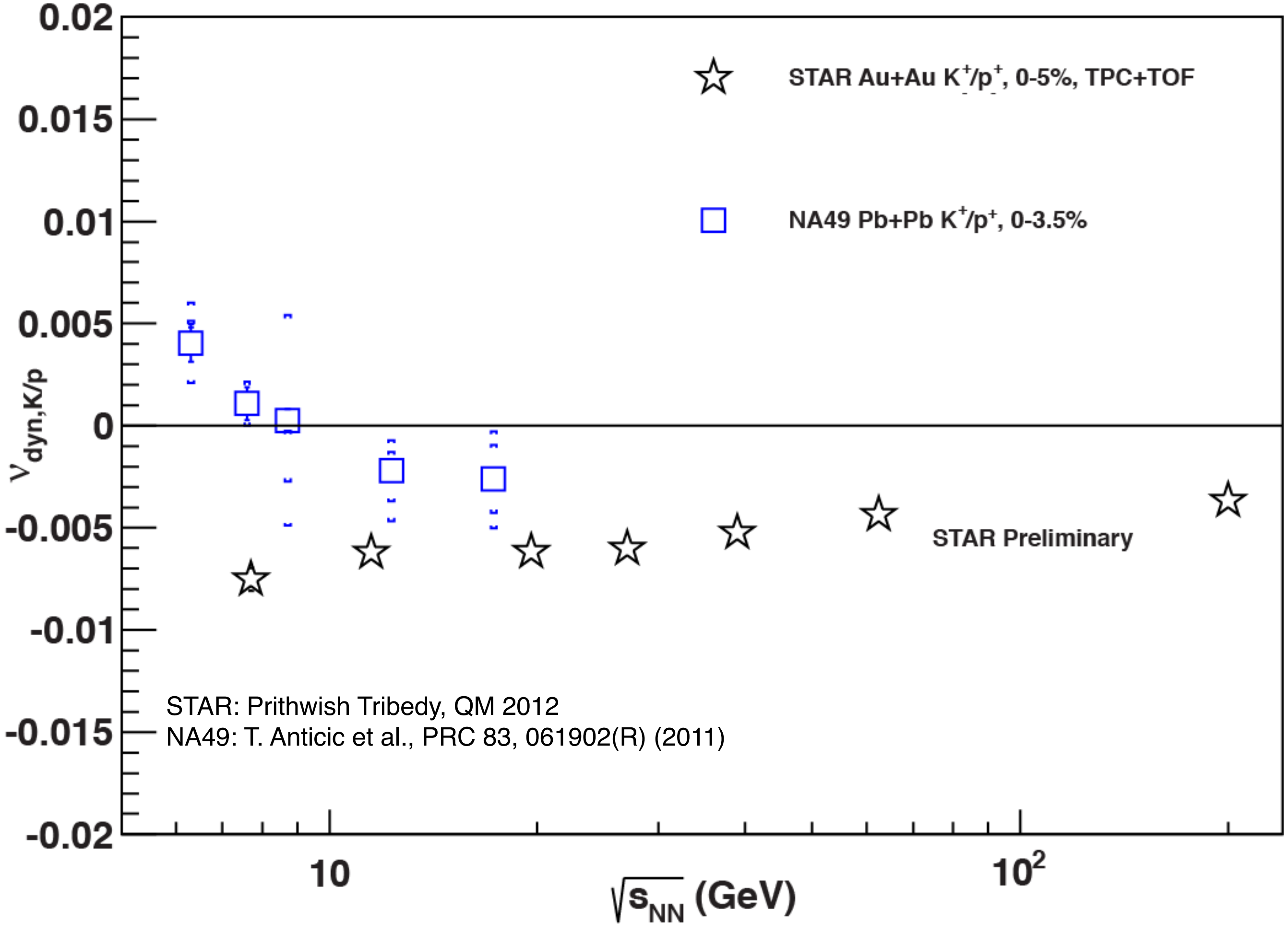
- K^{+}/p^{+}
- K^{-}/p^{-}
- Central events (0 – 5%)
- Little energy dependence

K/p Fluctuations – Opposite Sign



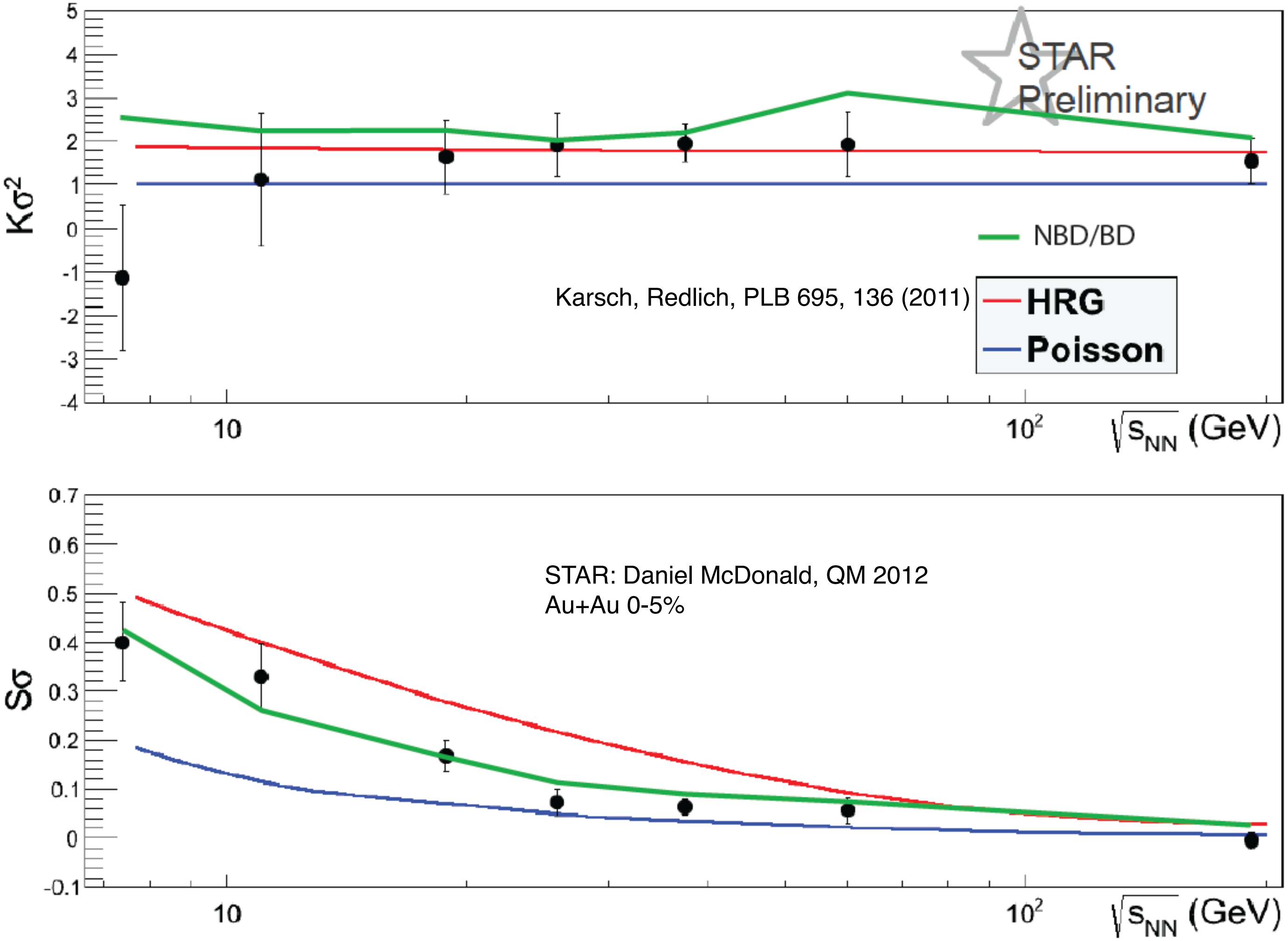
- K^+/p^-
- K^-/p^+
- Central events (0 – 5%)
- Little energy dependence
- Very few antiprotons at the lowest energy

K^+/p^+ Fluctuations – Compare NA49

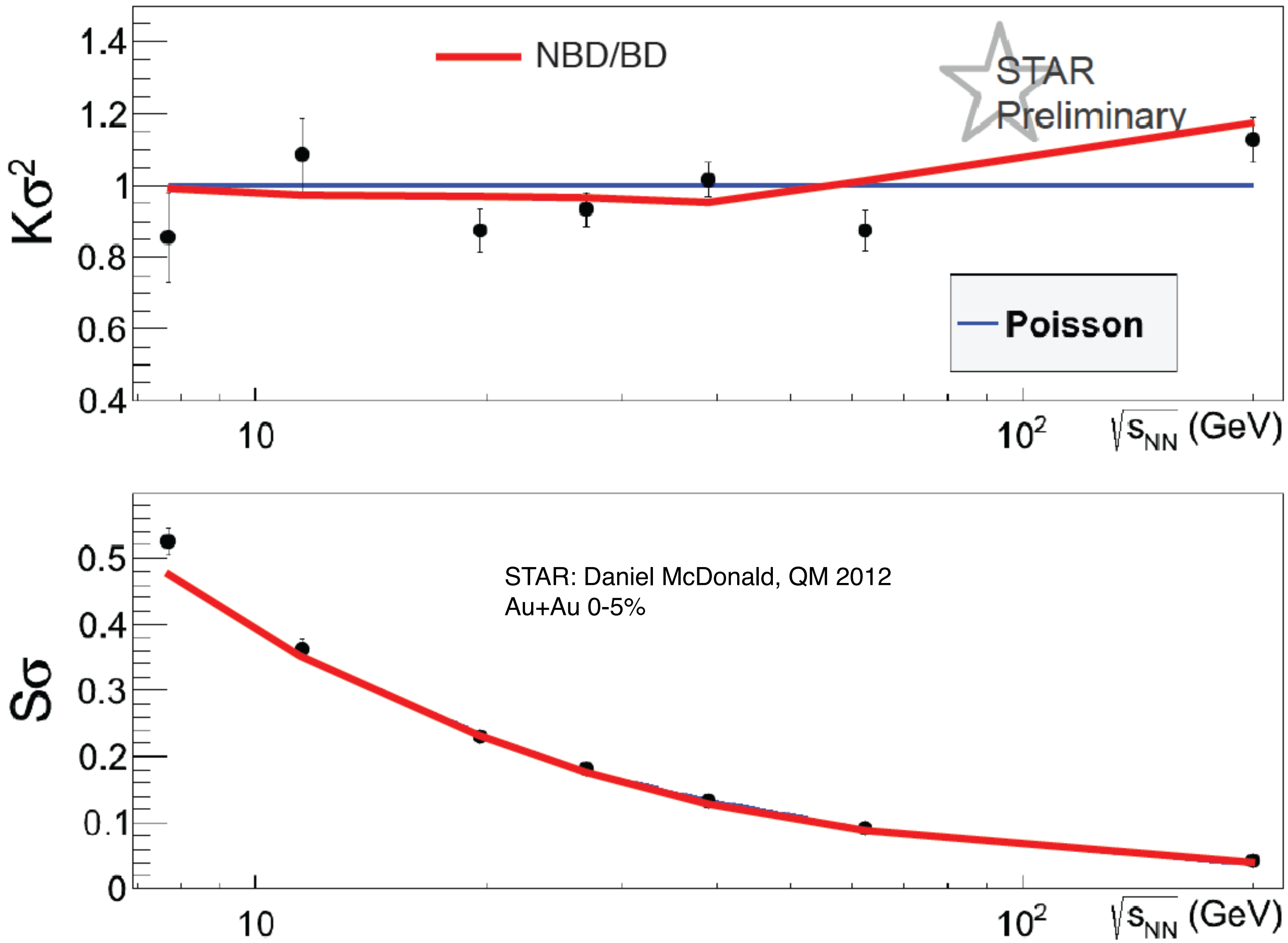


- K^+/p^+
- Central events
- Disagreement between STAR and NA49 at lowest energies

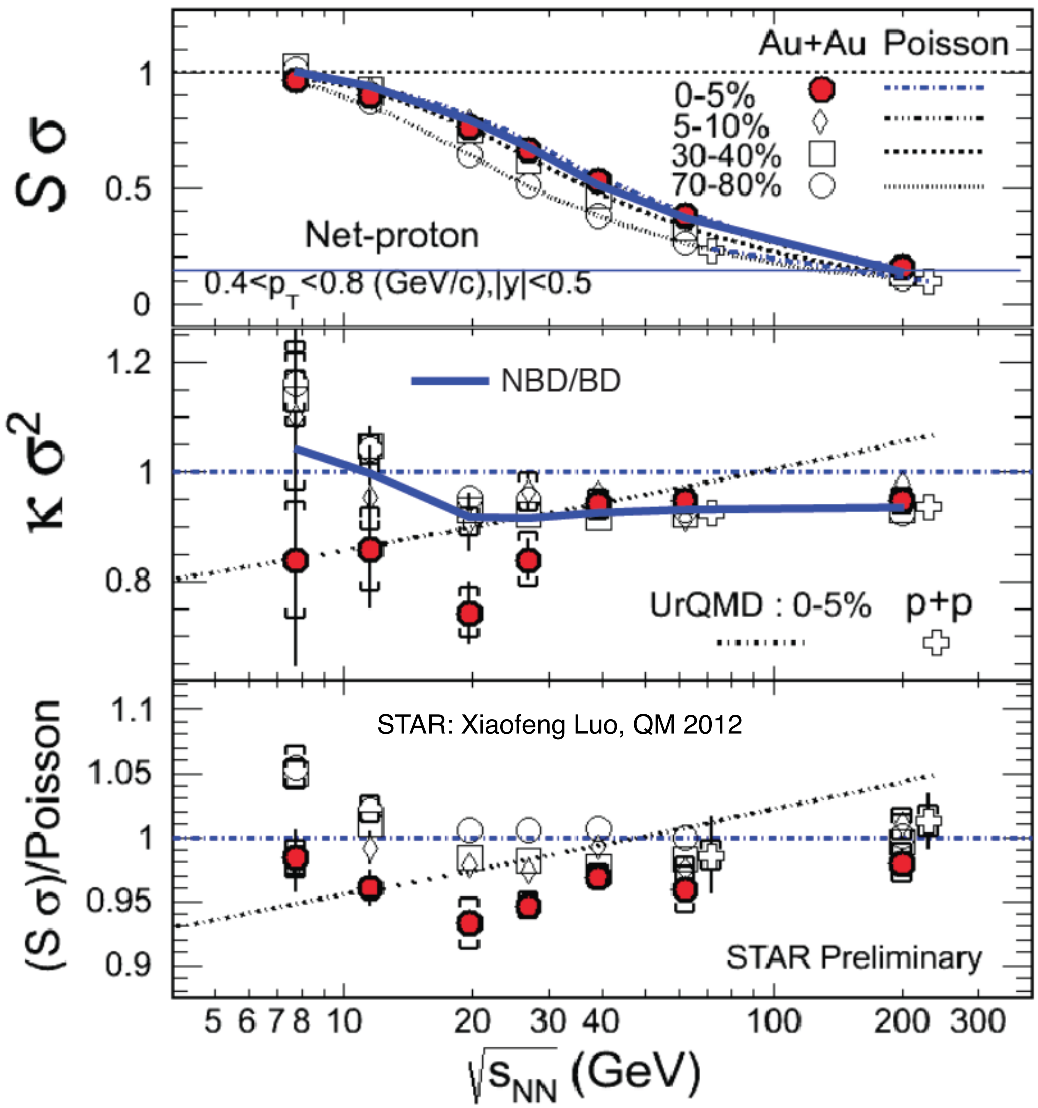
Net Charge Moments Energy Dependence



Net Kaon Moments
Energy Dependence



Net Proton Moments Energy Dependence



Conclusions – Particle Ratio Fluctuations

- Particle ratio fluctuations in central collisions
  - K/π
    - Summed sign fluctuations are positive
    - Separate sign fluctuations are negative
    - Disagreement with NA49 at lowest energy
  - p/π
    - Summed sign fluctuations are negative
    - Separate sign fluctuations are negative
    - Agreement with NA49
  - K/p
    - Summed sign fluctuations are negative
    - Separate sign fluctuations are negative
    - Disagreement with NA49 at lowest energy
  - No non-monotonic behavior observed with incident energy

Conclusions – Net Particle Higher Moments

- Net particle higher moments in central collisions
  - Net charge
    - $S\sigma = C_3/C_2$ consistent with negative binomial/binomial expectation, above Poisson expectation, and below HRG
    - $\kappa\sigma^2 = C_4/C_2$ between Poisson and negative binomial/binomial expectations and consistent with HRG
  - Net kaons
    - $S\sigma = C_3/C_2$ consistent with both the Poisson and negative binomial/binomial expectations
    - $\kappa\sigma^2 = C_4/C_2$ consistent with both Poisson and negative binomial/binomial expectations
  - Net protons
    - $S\sigma = C_3/C_2$ consistent with both the Poisson and negative binomial/binomial expectations
    - $\kappa\sigma^2 = C_4/C_2$ below Poisson expectation but more consistent with the negative binomial/binomial expectation
  - No non-monotonic behavior observed with incident energy