The phase diagram of QCD from Dyson-Schwinger equations

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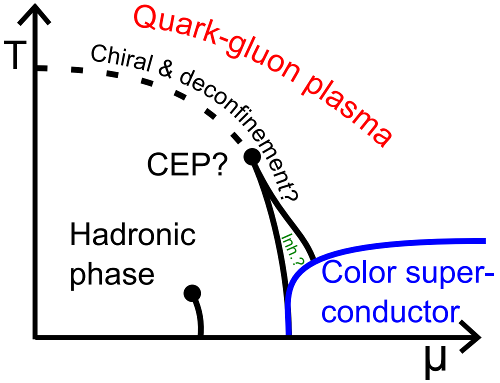
C. S. Fischer, JL, arXiv:1206.5191

C. S. Fischer, JL, J. A. Mueller, Phys.Lett. B702 (2011) 438-441

The phase diagram of QCD



- Critical end-point (CEP)?
- Chiral symmetry restoration and deconfinement coincide?

Why Dyson-Schwinger?

Lattice QCD

- ☺ Ab-initio
- ☹ Only for small μ (sign problem)

Effective field theories

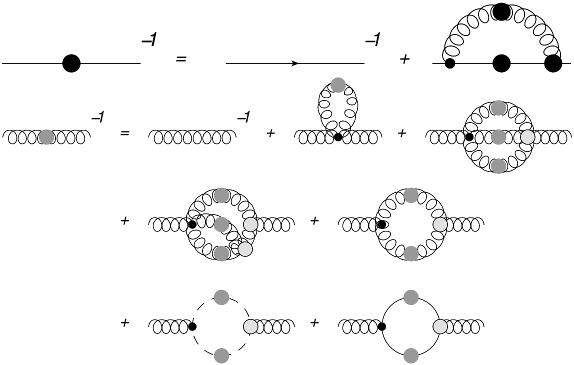
- ☺ No sign problem
- ☹ Effective degrees of freedom

Dyson-Schwinger

- ☺ No sign problem
- ☺ QCD degrees of freedom
- ☹ Truncation scheme needed

Dyson-Schwinger equations

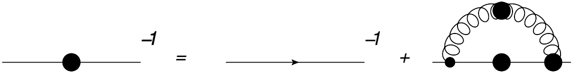
Derive from generating functional Z_{QCD}:



+ DSEs for all higher *n*-point functions

Order parameters: chiral symmetry breaking

Quark DSE



$$S^{-1}(p) = Z_2 S_0^{-1}(p) + \Sigma(p)$$

$$S^{-1}(\vec{p}, \omega_n) = \mathrm{i}C(\vec{p}^2, \omega_n)(\omega_n + \mathrm{i}\mu)\gamma_4 + \mathrm{i}A(\vec{p}^2, \omega_n)\vec{p}\vec{\gamma} + B(\vec{p}^2, \omega_n)$$

Quark condensate

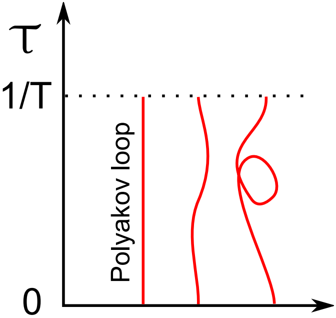
$$\langle \bar{\psi}\psi \rangle = \mathrm{Tr}[S] \propto T \sum \int_l \frac{B(l)}{C^2(l)\tilde{\omega}_n^2 + A^2(l)\vec{l}^2 + B^2(l)}$$

$\Rightarrow$ order parameter for chiral symmetry breaking

Confinement: dual condensate

$$\Sigma_{\pm 1} = \int \frac{d\varphi}{2\pi} e^{\mp i\varphi} \langle \bar{\psi}\psi \rangle_\varphi$$

- $\langle \bar{\psi}\psi \rangle_\varphi$ for quarks with $\psi(\vec{x}, 1/T) = e^{i\varphi}\psi(\vec{x}, 0)$
- **Dressed Polyakov loop**
- Sensitive to center symmetry

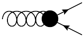


$\Rightarrow$ confinement/deconfinement

C. Gattringer, Phys. Rev. Lett. **97** (2006)
F. Synatschke, A. Wipf, C. Wozar, Phys. Rev. **D75** (2007)
E. Bilgici, F. Bruckmann, C. Gattringer, C. Hagen, Phys. Rev. **D77** (2008)

Remember:

Needed:

- Gluon propagator 
- Quark-gluon vertex 

Problems:

- T, μ dependence
- Reaction on phase transitions

Quark-gluon vertex

$$\Gamma_\mu = \gamma_\mu \cdot \Gamma(p^2, k^2, q^2) \cdot \left(\delta_{\mu,4} \frac{C(p) + C(q)}{2} + \delta_{\mu,i} \frac{A(p) + A(q)}{2} \right)$$

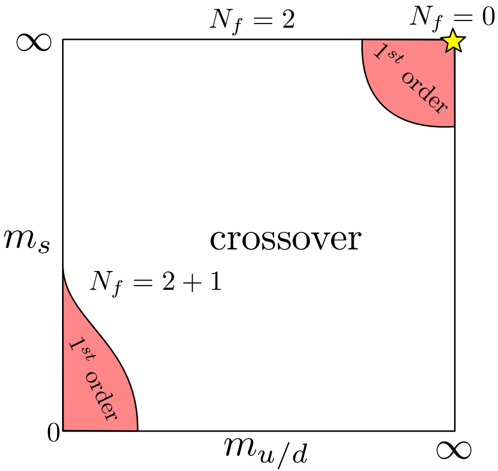
- Dependence on dressings from Slavnov-Taylor
- Ansatz function Γ

Ansatz function

$$\Gamma(p^2, k^2, q^2) = \frac{d_1}{d_2 + q^2} + \frac{q^2}{\Lambda^2 + q^2} \left(\frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2 + 1]}{4\pi} \right)^{2\delta}$$

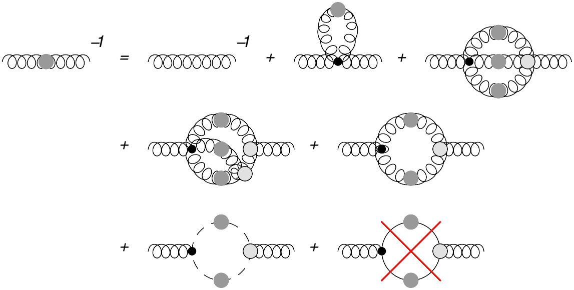
We fix the IR part by f_π at $T = 0$

The Columbia plot



- We will start with $N_f = 0$

Gluon propagator

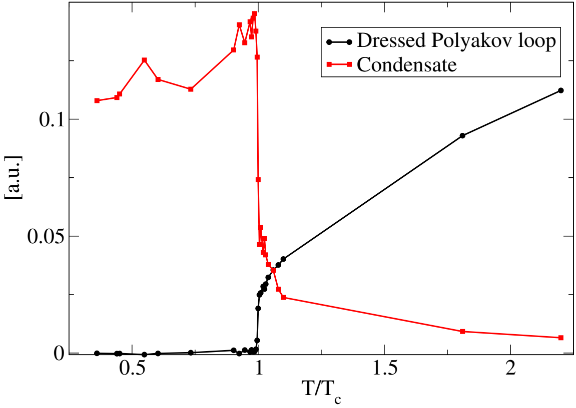


⇒ from temperature-dependent lattice QCD

Fischer, Maas, Mueller EPJ C68

Maas, Pawlowski, von Smekal, Spielmann, arXiv:1110.6340 (hep-lat)

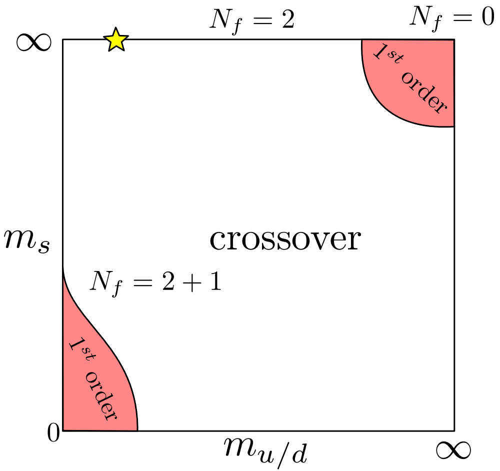
Results from quark DSE



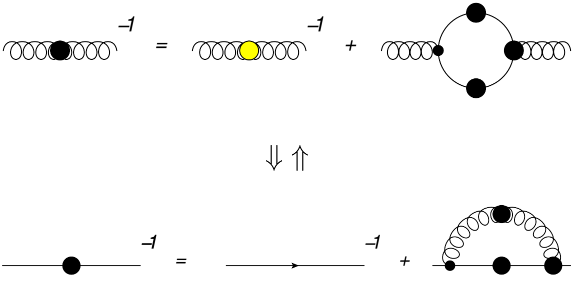
JL and C. S. Fischer, arXiv:1111.0180 (hep-ph).

⇒ Pattern of chiral and center symmetry breaking reproduced

The Columbia plot



- $N_f = 2$: introduce a chemical potential

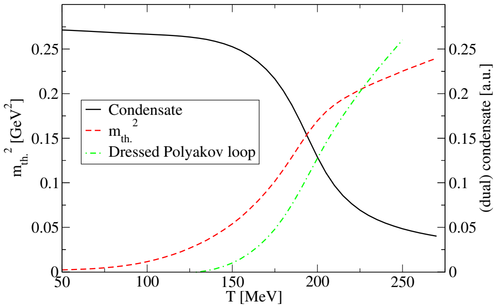


- Use quenched gluon as input, unquench via DSE

$\Rightarrow$ coupled equations

$N_f = 2$ QCD

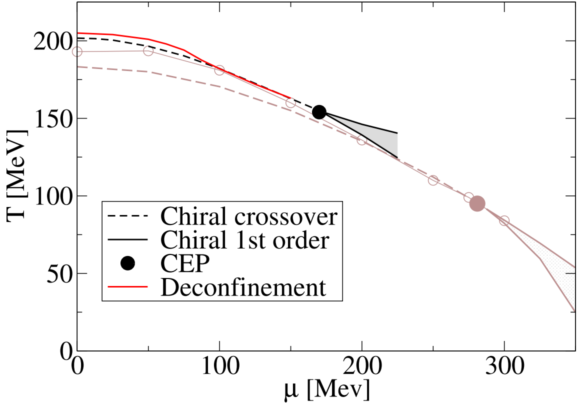
$$m_{th}^2 = \Pi_L^{ql}(0)/2$$



- Gluon sensitive to chiral symmetry breaking

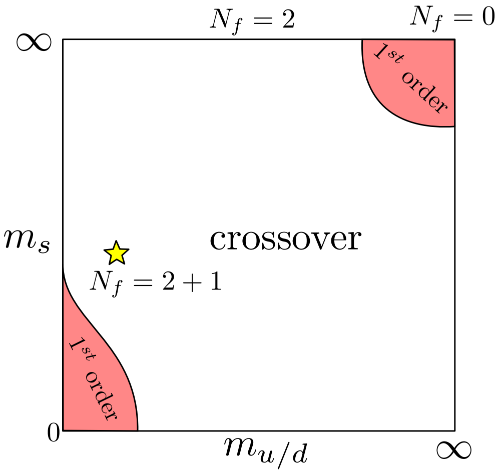
$\Rightarrow$ Back coupling leads to steeper crossover

Phase diagram for two flavours



- CEP at $\mu/T \approx 1.1$
- Chiral and deconfinement phase transitions coincide
- Back coupling brings CEP to smaller μ

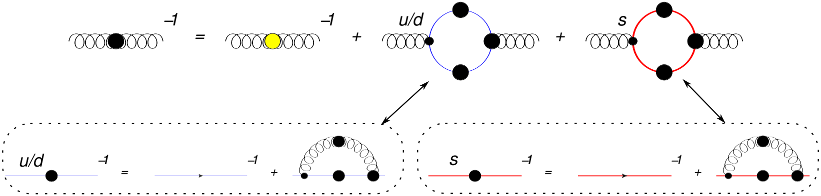
The Columbia plot



- Include strange quarks: $N_f = 2 + 1$

Strange quarks

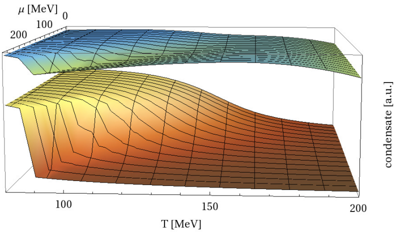
Coupled light-quark, gluon, strange-quark DSEs



⇒ quark loop couples light and strange quarks

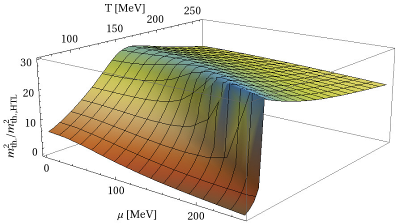
Light/strange condensates and Debye masses

Condensates



- Coupling of strange and light quarks visible

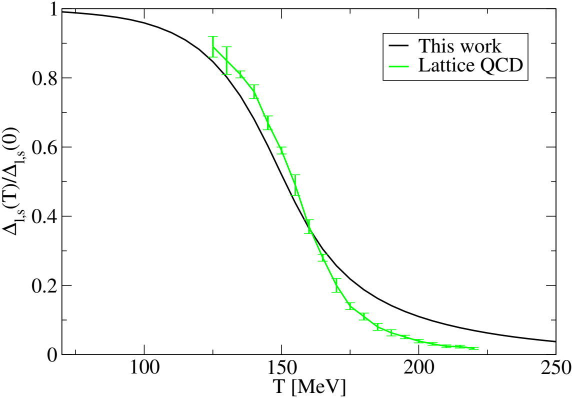
Thermal masses



- $m^2_{th.HTL} \propto T^2 + \frac{3}{\pi^2}\mu^2$

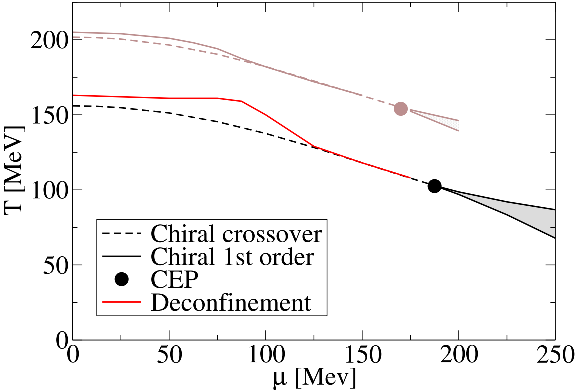
Strange quarks

$$\Delta_{l,s} = \langle\bar{\psi}\psi\rangle_l - \frac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_s$$



Lattice data from S. Borsanyi *et al.* (Wuppertal-Budapest Collaboration), JHEP **1009** (2010) 073

Phase diagram for two+one flavours



- CEP at $\mu/T \approx 1.9$
- Strange quarks reduce T_c, CEP moves to larger μ

Summary

- Quenched QCD: lattice data reproduced
- Unquenching from quark loop
  - Chiral transition accelerated
  - Strange quarks can be taken into account
- A CEP is found
- Deconfinement coincides with chiral symmetry restoration