Equation of state of a meson gas from the PNJL model for interacting quarks

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in collaboration with
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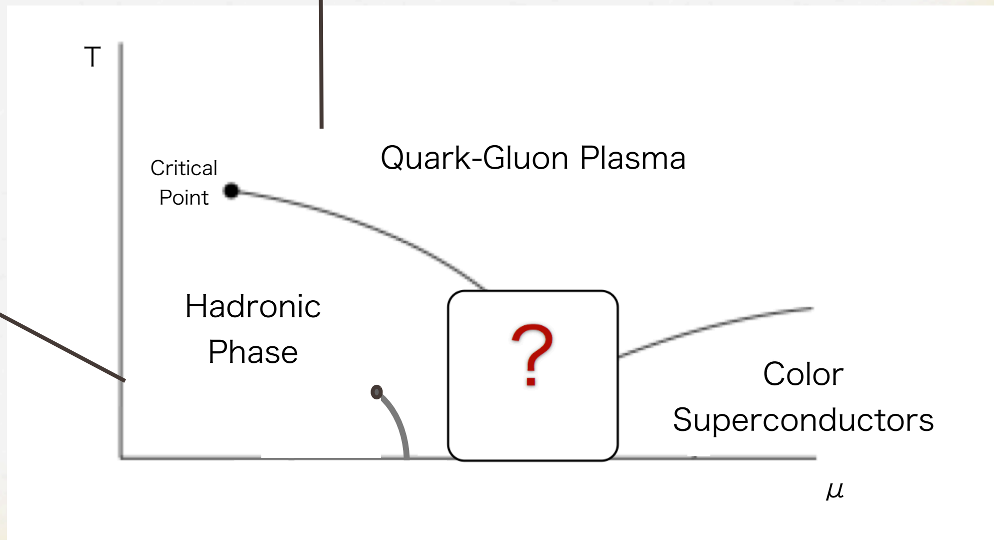
Introduction

Quark-Gluon Plasma

- **Chiral Phase Transition**
- **Deconfined Phase Transition**

Hadronic Phase

- Chiral symmetry is broken
- Color is confined



Motivation

Study phase transitions by effective model

We use the model which has the following features :

- Model which can deal with **chiral** and **decondined phase transition**
- There are **hadrons** at **low temperatures**

Nambu-Jona-Lasinio model with Polyakov loop (PNJL model)
Mean field approximation + Mesonic correlation

In this work, we calculate a equation of state at zero chemical potential

PNJL model

previous works

- K. Fukushima, 2004
- C. Ratti, S. Rößner, W.Weise, 2007
- H. Hansen, W. M. Alberico, A. Beraudo, A. Molinari, M. Nardi, C. Ratti, 2007
- D. Blaschke, M. Buballa, A. E. Radzhabov, M. K. Volkov, 2008
- . . .

New points of our work

- **In the chiral limit**, we can separate the collective meson modes from non-collective modes
- And show that collective meson modes dominate to equation of state at low T
- **With finite bare quark mass**, we can still find isolated meson pole which generates equation of state at low T

The rest of this talk

1. Formalism : pass integral, bosonization

2. Mean Field Approximation

   - Pressure of quarks quasi particle

   - Effect of confinement on the quark distribution function via Polyakov loop

3. Mesonic correlation

   - Pressure of mesonic correlation : collective vs non-collective

   - Chiral limit

   - Finite bare quark mass

4. Summary

Formalism

Partition function

$$Z(T,A_4)=\int[dq][d\bar{q}]\exp\left[\int_0^\beta d\tau\int d^3x\mathcal{L}_{\rm NJL}(q,\bar{q},A_4)\right]$$

$$\mathcal{L}_{\rm NJL}(q,\bar{q},A_4)=\bar{q}(i\gamma^\mu D_\mu-m_0)q+G\left[(\bar{q}q)^2+(i\bar{q}\gamma_5\tau q)^2\right],\quad D_\mu=\partial_\mu+gA_4\delta_{\mu,0}$$

- **Bosonization** (Introduce **four auxiliary boson fields**) $\phi_i=(\sigma,\boldsymbol{\pi}),\quad i=0,1,2,3$
- Fermion integral

$$Z(T,A_4)=\int[d\phi]e^{-I(\phi,A_4)}$$

Expand to the **second order** of the bosonic field fluctuation and integrate the fluctuation

$$\Omega(T,A_4)=T\left(I_0+\frac{1}{2}\mathrm{Tr_M}\ln\frac{\delta I}{\delta\phi_i\delta\phi_j}\right)$$

from **mean filed** from **mesonic excitation**

Mean field approximation

$$\Omega_{MF}(T, A_4) = T I_0 = -p_{MF} V$$

- Take statistical average over the external color gauge field A_4
- Replace A_4 with the thermal average of the Polyakov loop Φ

$$p_{MF}(T, \Phi) = p^0_{MF}(M_0) - \Delta p_{\text{vac}} + 4 \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E_p} f_\Phi(E_p) - \mathcal{U}(T, \Phi)$$

zero point motion **vacuum** pressure

Polyakov loop

$$L(\boldsymbol{r}) = \mathcal{P}\exp\left[ig \int_0^\beta d\tau A_4(\tau, \boldsymbol{r})\right] \longrightarrow e^{\beta A_4}$$

$$\Phi = \frac{1}{3} \langle \text{tr}_c L \rangle$$

Quark distribution function

$$f_\Phi(E_p) = \frac{1}{3} \text{tr}_c \frac{1}{L e^{\beta E_p} + 1}$$

$$E_p = \sqrt{p^2 + M_0^2}$$

M_0 is determined by gap equation

Mean field approximation

$$\Omega_{MF}(T, A_4) = TI_0 = -p_{MF}V$$

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zero point motion **vacuum** pressure

Quark distribution function

- Take trace over color index

$$f_\Phi(E_p) = \frac{1}{e^{3\beta E_p}+1} \qquad f_\Phi(E_p) = \frac{1}{e^{\beta E_p}+1}$$

$$\Phi = 0 \qquad \Phi = 1$$

$$E_p = \sqrt{p^2 + M_0^2}$$

M_0 is determined by gap equation

Mean field approximation

$$\Omega_{MF}(T, A_4) = TI_0 = -p_{MF}V$$

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zero point motion　　**vacuum** pressure

Effective potential of Φ

- Represent dynamics of gluon phenomenologically
- Parameters are chosen so that :
    - Φ=0 at T=0
    - Φ gets close to 1 at high T
    - U becomes gluon's pressure at high T

Mesonic correlation energy

$$\Omega_M(T,A_4) = \frac{T}{2}\mathrm{Tr}_M \ln \frac{\delta^2 I}{\delta\phi_i\delta\phi_j}\bigg|_{\phi=\phi_0}, \qquad p_M = -\frac{\Omega_M}{V}$$

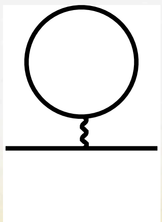
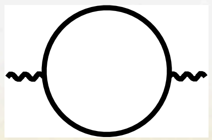
- contribution from **2nd order term of auxiliary field**
- trace is to be performed over the space-time coordinates of auxiliary field

$$p_{\mathrm{M}}(T,A_4) = -\frac{T}{2}\sum_n \int \frac{d^3q}{(2\pi)^3}\left\{\ln\left[\beta^2\left(\frac{1}{2G} - \Pi_\sigma(\omega_n,q,A_4)\right)\right]\right.$$

$$\left. +3\ln\left[\beta^2\left(\frac{1}{2G} - \Pi_\pi(\omega_n,q,A_4)\right)\right]\right\}, \qquad \omega_n = 2\pi n T$$

Sigma meson

Pion

$\Pi =$  $+$ $(\omega_n^2 + q^2)F(\omega_n,q,A_4)$ 

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Mesonic correlations : collective v.s. non-collective

$$\frac{1}{2G} - \Pi_\sigma(\omega_n, q, A_4) = (\omega_n^2 + q^2 + 4M_0^2)F(\omega_n, q, A_4) + \frac{m_0}{2GM_0}$$

$$\frac{1}{2G} - \Pi_\pi(\omega_n, q, A_4) = (\omega_n^2 + q^2)F(\omega_n, q, A_4) + \frac{m_0}{2GM_0}$$

Collective mode

Non-collective mode

- In the chiral limit, the second term becomes zero.

We can separate the contributions of the collective bare meson modes from non-collective individual excitations.

Pressure of mesons (Chiral limit)

Pressure of collective modes

$$p_M^{\text{free}}(T) = p_M^0 + \int \frac{d^3q}{(2\pi)^3}\left[\frac{q^2}{3\omega_q} f_B(\omega_q) + 3 \times \frac{q}{3} f_B(q)\right]$$

Vacuum pressure

Bosonic single particle distribution function : $f_B(\omega) = \dfrac{1}{e^{\beta\omega} - 1}$

Pressure of non-collective modes

$$p_M^{\text{non-coll.}}(T, A_4) = -2T \sum_n \int \frac{d^3q}{(2\pi)^3} \ln F(\omega_n, q, A_4)$$

$\omega_n = 2\pi n T$: bosonic Matsubara frequensies

$m_0 \neq 0$

cannot separate the non-collective modes from the collective meson modes easily

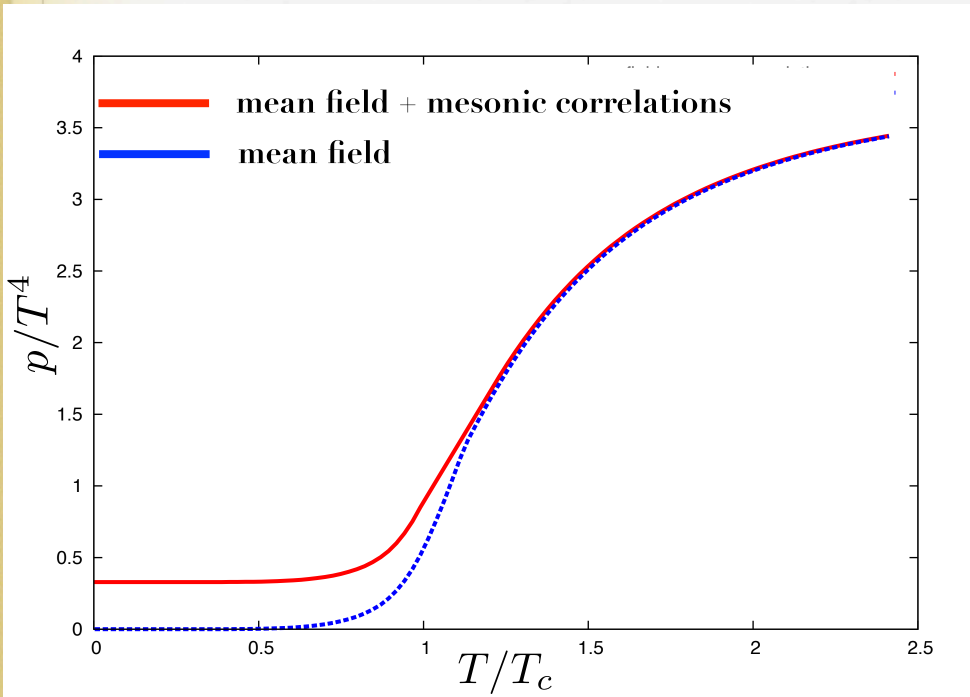
$$\mathcal{M}_\pi = \frac{1}{2G} - \Pi_\sigma(\omega_n, q, A_4) = (\omega_n^2 + q^2 + 4M_0^2)F(\omega_n, q, A_4) + \frac{m_0}{2GM_0}$$

$$\mathcal{M}_\sigma = \frac{1}{2G} - \Pi_\pi(\omega_n, q, A_4) = (\omega_n^2 + q^2)F(\omega_n, q, A_4) + \frac{m_0}{2GM_0}$$

→ cannot factorize the collective meson modes

$$p_M(T, A_4) = -2\int \frac{d^3q}{(2\pi)^3}\frac{1}{2\pi i}\int_0^\omega \Big[1 + \frac{2}{e^{\beta\omega} - 1}\Big] \times\Big\{\ln\Big[\frac{\mathcal{M}_\sigma(\omega + i\epsilon, q)}{\mathcal{M}_\sigma(\omega - i\epsilon, q)}\Big] + 3\ln\Big[\frac{\mathcal{M}_\pi(\omega + i\epsilon, q)}{\mathcal{M}_\pi(\omega - i\epsilon, q)}\Big]\Big\}$$

Pressure (Numerical result)

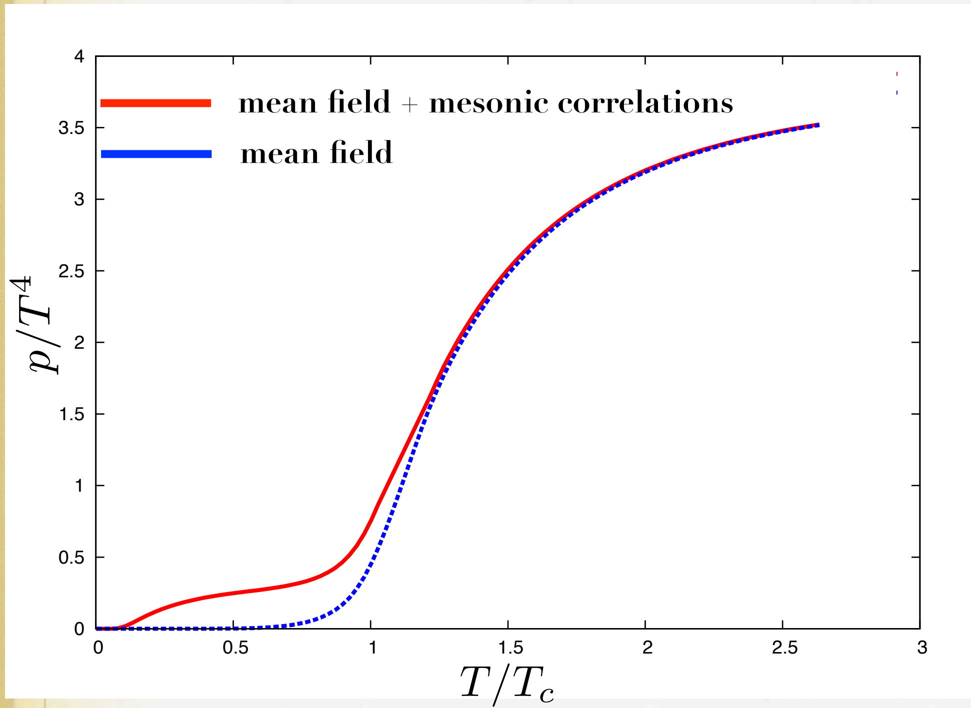


- Result computed in the **chiral limit**

- At low temperatures, pressure from **quark excitations are suppressed** by the Polyakov loop

- At high temperatures, **massless quark excitations dominate** the pressure

Pressure (Numerical result)



- Result of finite bare quark mass

- m_0 is chosen to reproduce physical pion mass

- Pion mass : 135MeV

Interpretation of result (finite bare quark mass)

- Contribution of mesonic correlation becomes small at high T . Why ?

→ Study $\mathcal{M}_\pi$, $\mathcal{M}_\sigma$ in detail

Position of meson poles

- With finite bare quark mass, collective modes and non-collective modes are mixed in $\mathcal{M}$
- Collective modes cannot be factorized
- We can find position of meson poles :

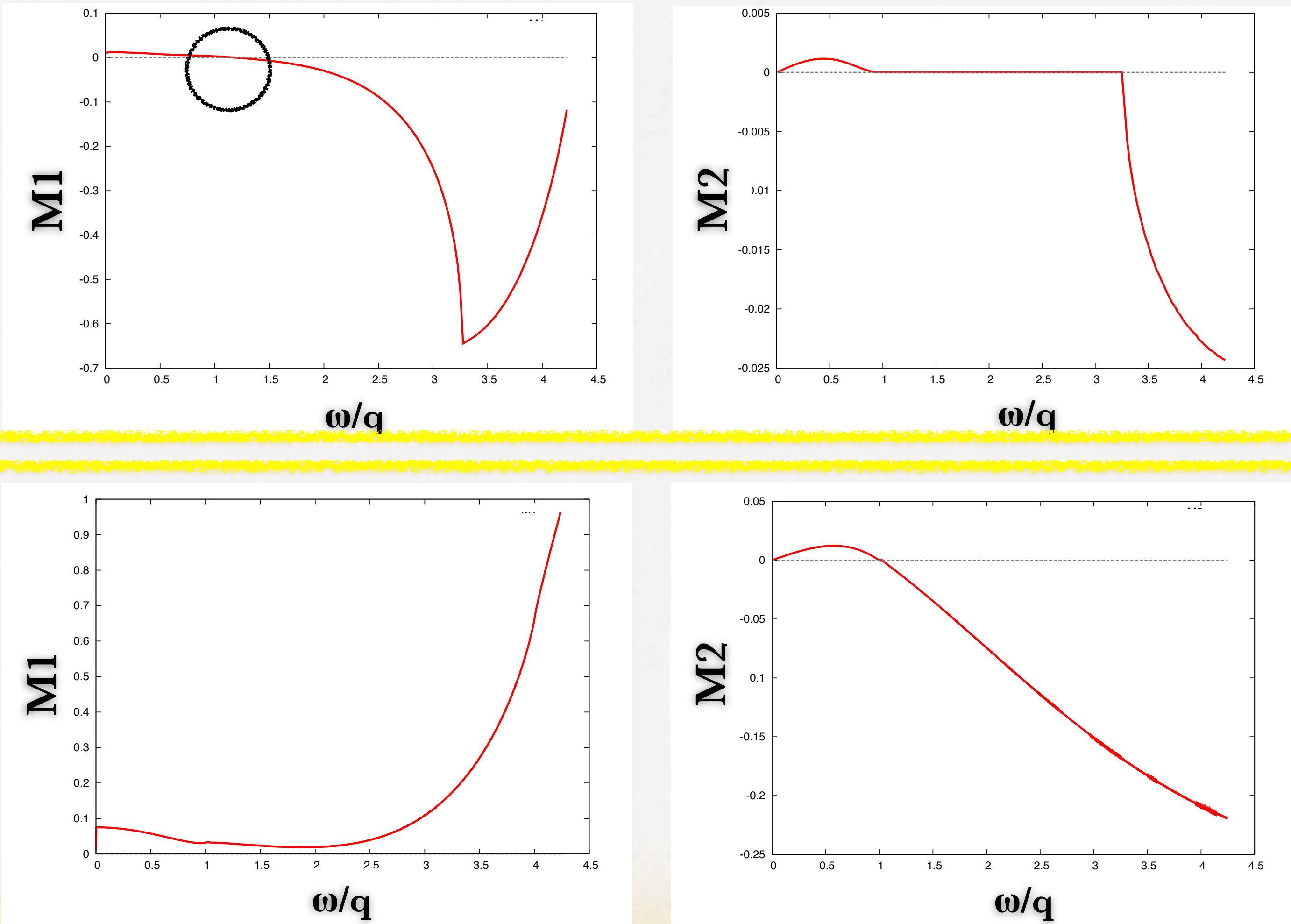
$$\mathcal{M}_1 = \mathrm{Re}[\frac{1}{2G} - \Pi] = 0 \ , \quad \mathcal{M}_2 = \mathrm{Im}[\frac{1}{2G} - \Pi] = 0$$

- Plot M1, M2 at low T and at high T

Position of pion pole

$T < T_0$

$T > T_0$



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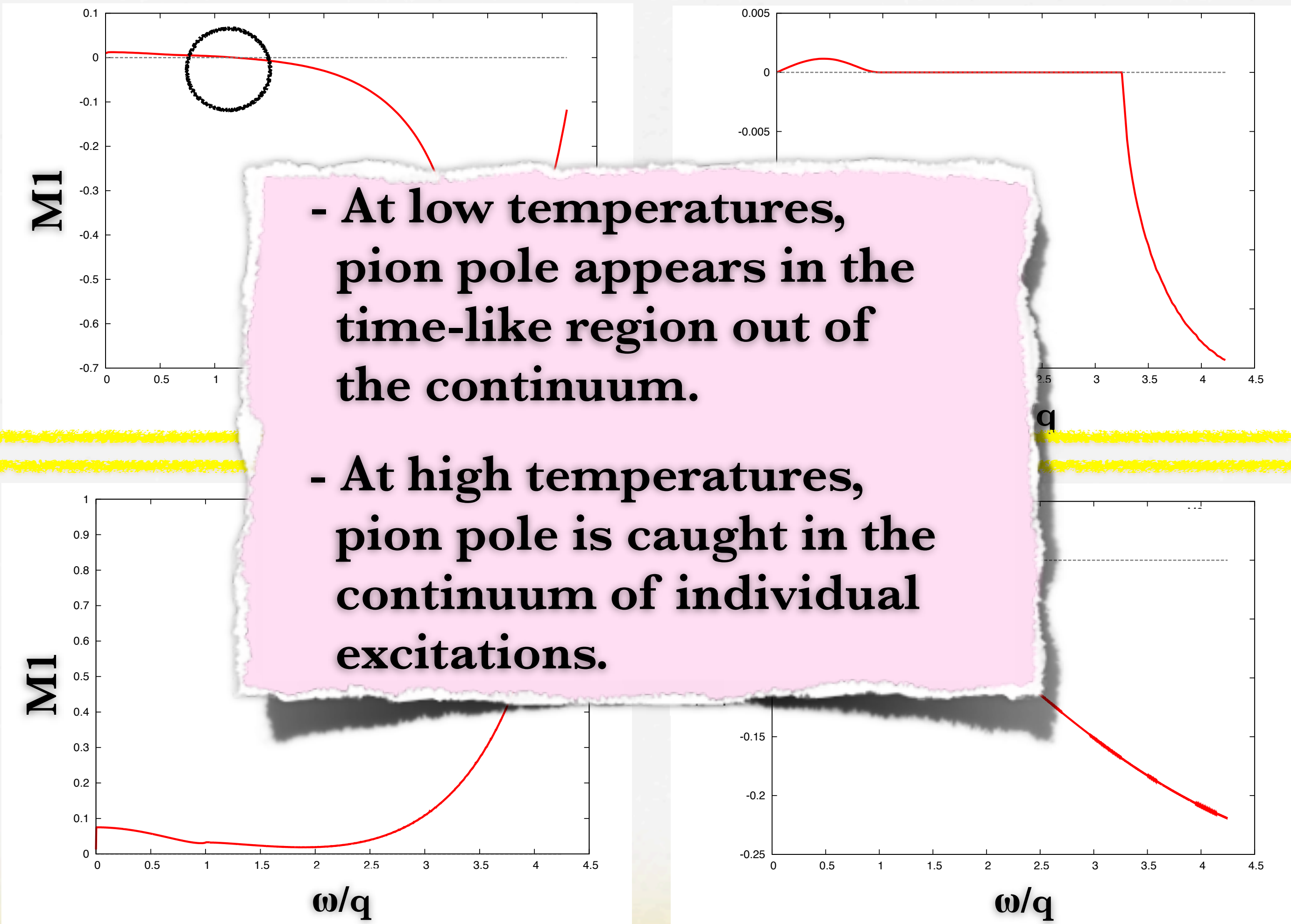
Position of pion pole

$T < T_0$

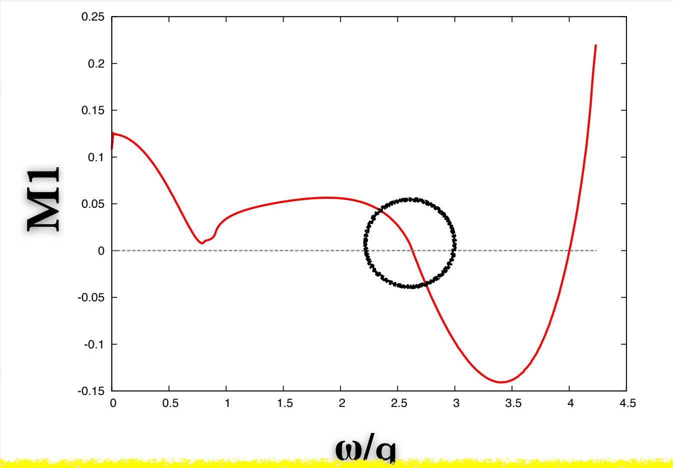
$T > T_0$

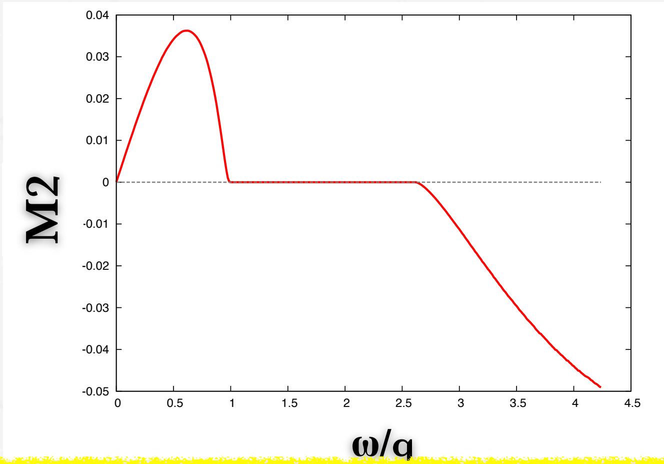


- At low temperatures, pion pole appears in the time-like region out of the continuum.
- At high temperatures, pion pole is caught in the continuum of individual excitations.

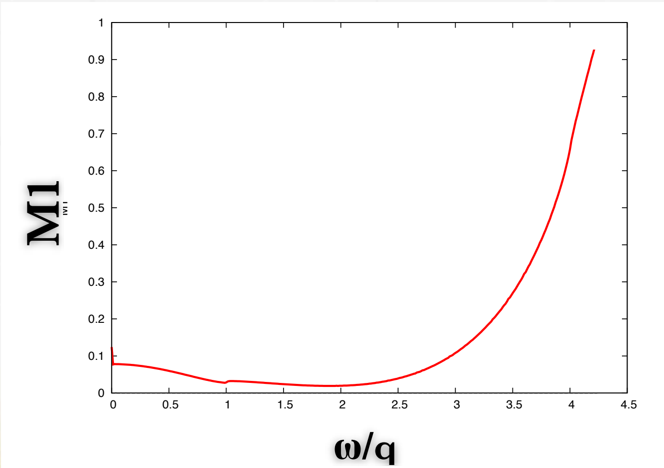
Position of sigma meson pole

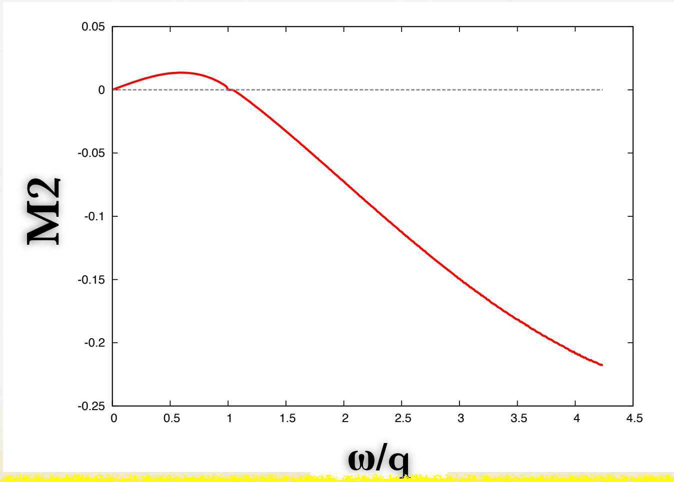
$T < T_0$





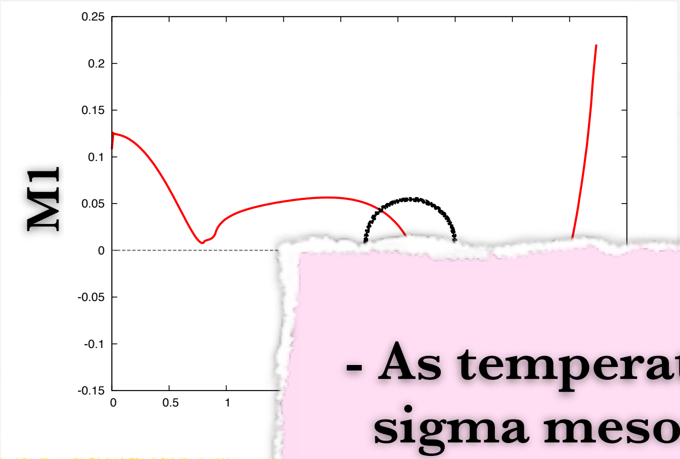
$T > T_0$

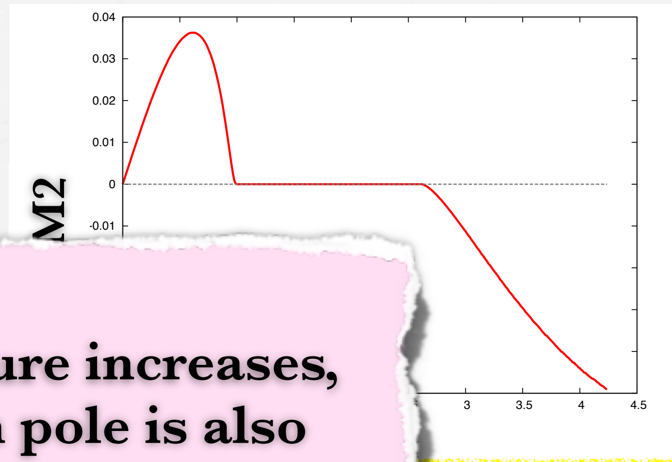




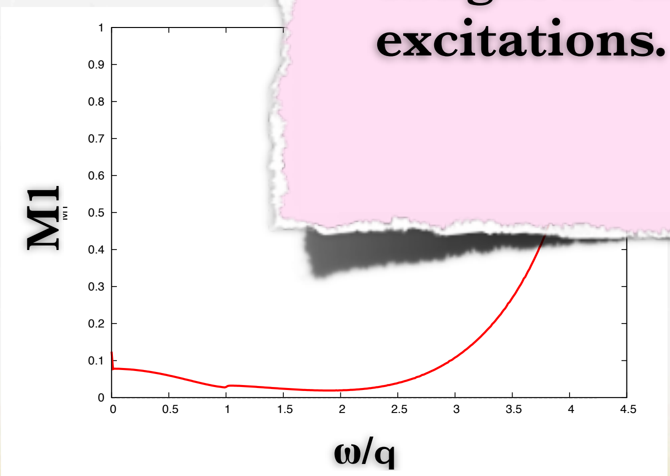
Position of sigma meson pole

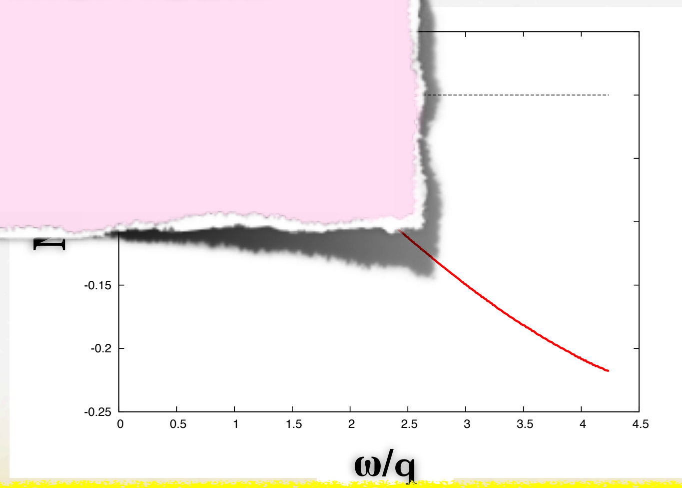
$T < T_0$





$T > T_0$





- **As temperature increases, sigma meson pole is also caught in the individual excitations.**

Position of meson pole

- At low temperatures, meson poles appear in the time-like region out of the continuum.

- At high temperatures, meson poles disappear.

- As a result, contribution of mesonic excitations are almost zero at high temperatures.

Summary

- We have shown that an equation of state of meson gas can be derived from the PNJL model for interacting quarks.

- In the chiral limit, we can separate the collective meson modes from the non-collective individual excitations.

- We show that the the mesonic excitations dominate in the low temperature phase, while quarks and gluons dominate in the high temperature phase.

Open Problems

- How to put baryonic correlation into quark triad

- Equation of state with baryon at finite chemical potential

- Reexamine the QCD phase diagram with quark confinement

Back Up

Formalism

Bosonization (Hubbard-Stratonovich transformation)

- **Introduce four auxiliary boson fields** $\phi_i = (\sigma, \boldsymbol{\pi})$, $i = 0, 1, 2, 3$
- **Perform fermion integral**

$$Z(T, A_4) = \int [d\phi] e^{-I(\phi, A_4)}$$

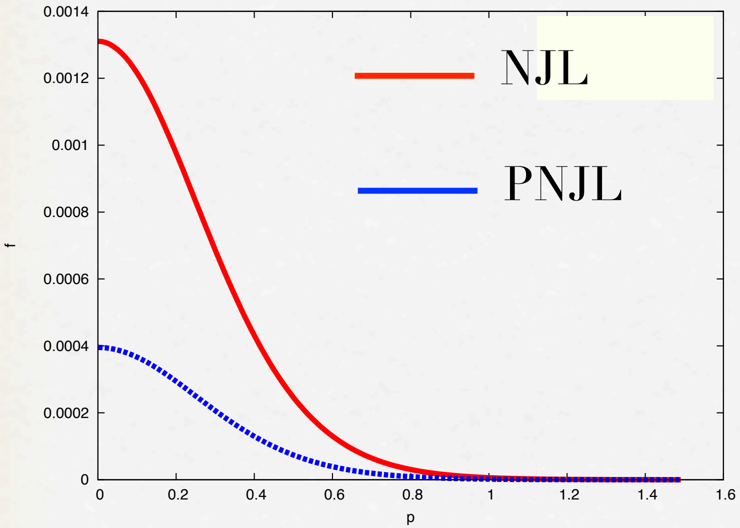
$$I(\phi, A_4) = \frac{1}{4G} \int_0^\beta d\tau \int d^3x \left((\sigma' - m_0)^2 - \boldsymbol{\pi}^2 \right) - \mathrm{Tr} \ln \left[\beta (i\gamma^\mu D_\mu + \sigma' + i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi}) \right]$$

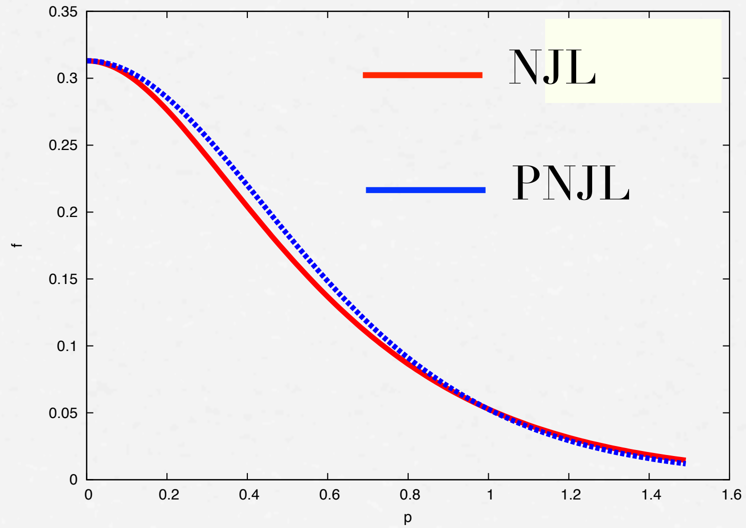
Expand to the **second order** of the bosonic field fluctuation and integrate the fluctuation

$$\Omega(T, A_4) = T \left(I_0 + \frac{1}{2} \mathrm{Tr}_{\mathrm{M}} \ln \frac{\delta I}{\delta \phi_i \delta \phi_j} \right)$$

from **mean filed** from **mesonic excitation**

Fermi distribution function





$T < T_0$ $T > T_0$

Interpretation of result (Chiral Limit)

- Contribution of non-collective modes is very small. Why ?
- Study non-collective modes in detail

Pressure of non-collective modes

- Sum over the Matsubara frequencies can be performed by contour integration
- Contour encircles counterclockwise around the imaginary axis
- Change the contour to the pass which encircles the real axis

$$p_M^{\text{non-coll.}}(T, A_4) = -2\int \frac{d^3q}{(2\pi)^3}\frac{1}{2\pi i}\int_0^\infty d\omega\left[1+\frac{2}{e^{\beta\omega}-1}\right]\ln\left[\frac{F(\omega+i\epsilon, q, A_4)}{F(\omega-i\epsilon, q, A_4)}\right]$$

Function F

$$F(\omega_n, q, A_4) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p 2E_{p+q}}$$

$$\times \Big[\Big(\frac{1}{\omega + E_p - E_{p+q}} - \frac{1}{\omega - E_p + E_{p+q}}\Big)\mathrm{tr}_c(f(E_p - igA_4) - f(E_{p+q} - igA_4))$$

$$+\Big(\frac{1}{\omega + E_p + E_{p+q}} - \frac{1}{\omega - E_p - E_{p+q}}\Big)\mathrm{tr}_c(1 - f(E_p - igA_4) - f(E_{p+q} - igA_4))\Big]$$

Non-collective mode

$$F(\omega \pm i\epsilon, q) = F_1(\omega, q) \pm iF_2(\omega, q) = \sqrt{F_1^2 + F_2^2} e^{\pm i\phi(\omega, q)}$$

$$\phi = \tan^{-1} \frac{F_2(\omega, q)}{F_1(\omega, q)}$$

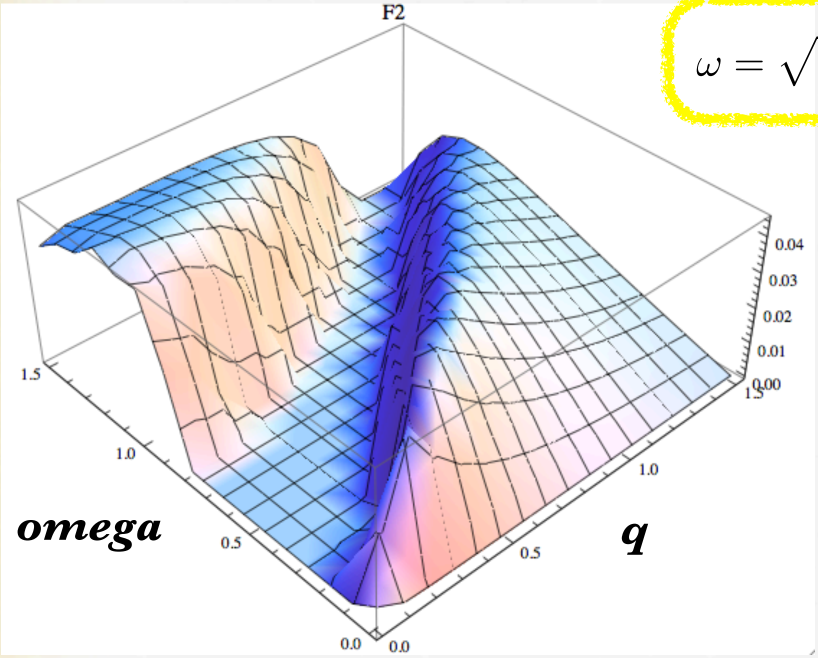
Using ϕ , pressure is given by

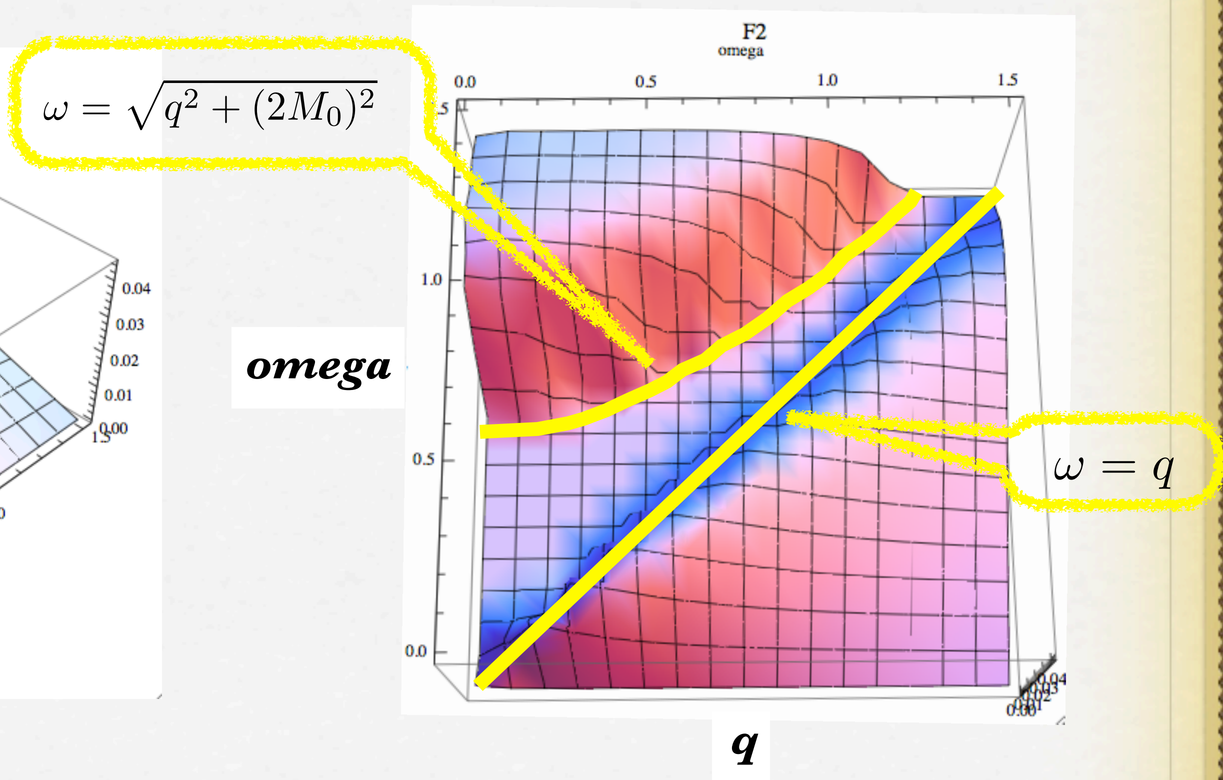
$$p_M(T) = -2 \int \frac{d^3q}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} \left[1 + \frac{2}{e^{\beta\omega} - 1}\right] 2\phi(\omega, q)$$

The region where F2 = 0 doesn't contribute to the pressure.

Number of Individual Excitations

$F_2(\omega, q)$

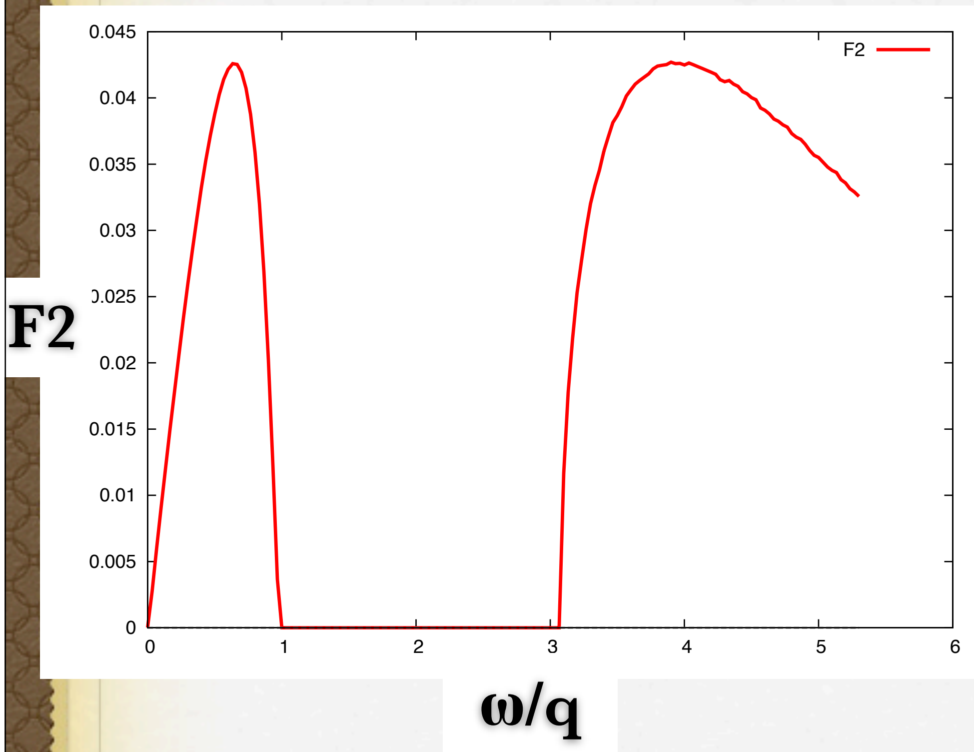


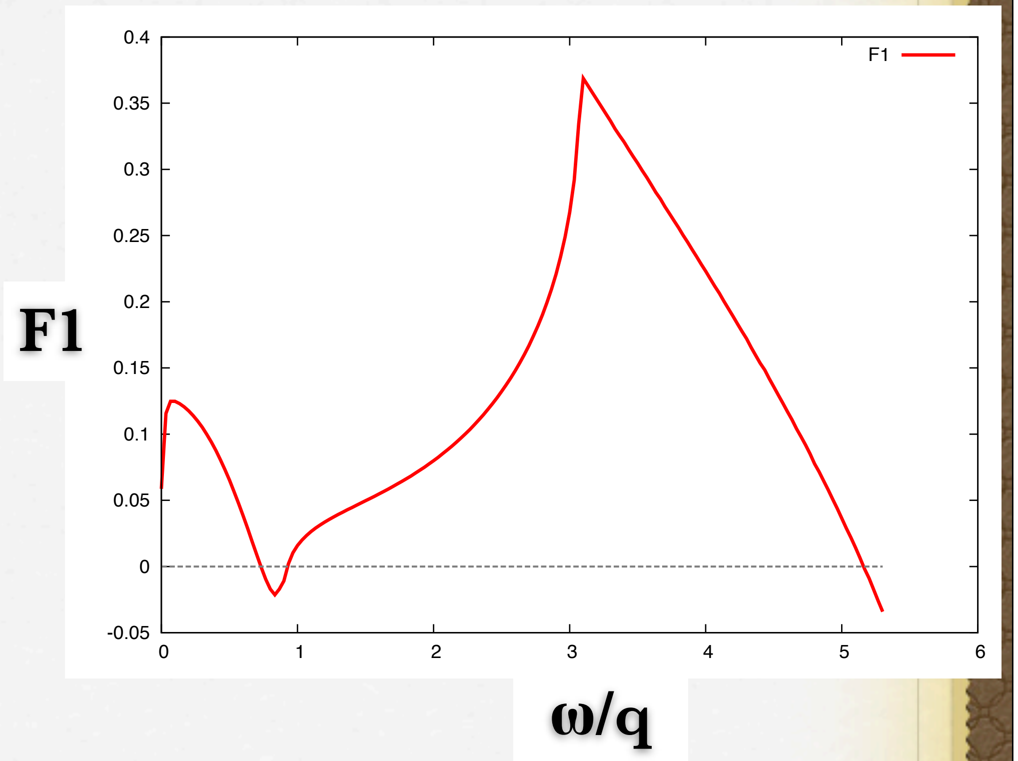


Individual Excitations

F1 and F2 as a function of ω scaled by q

$T < T_0$





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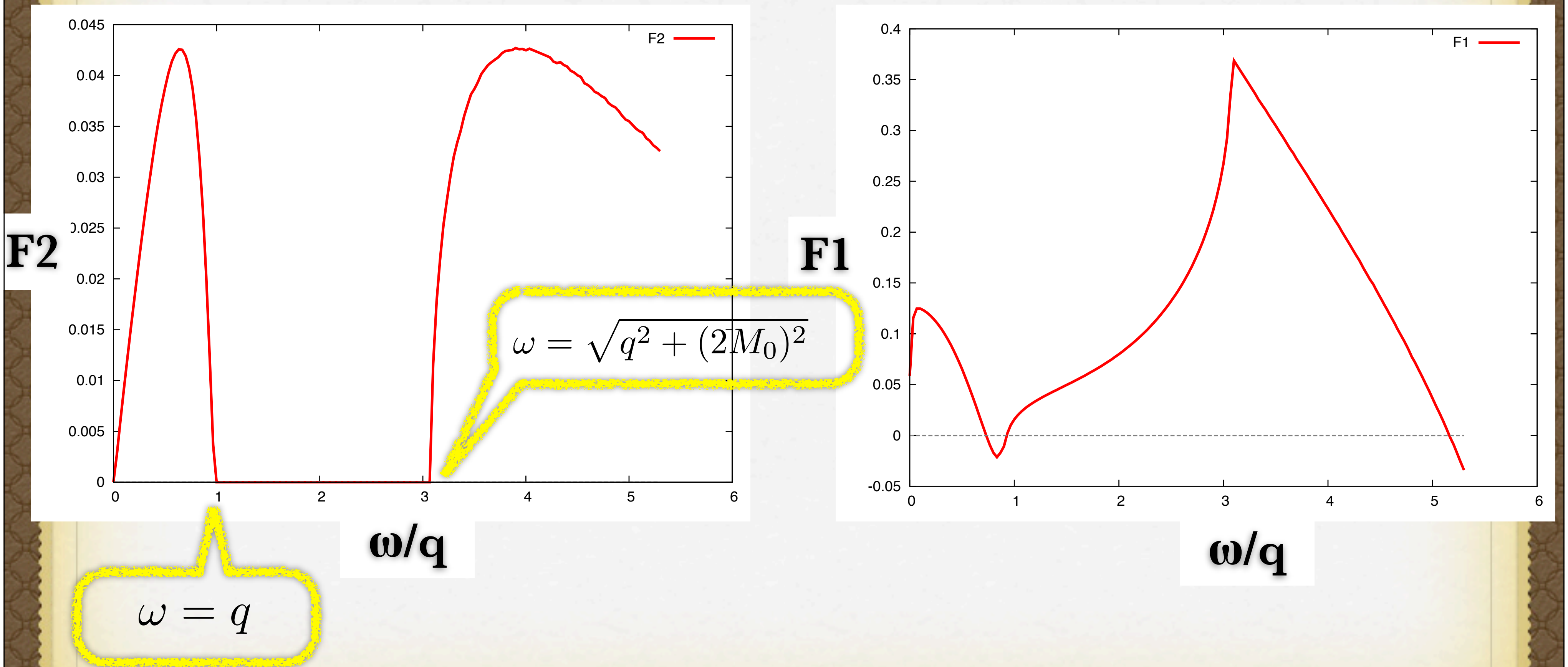

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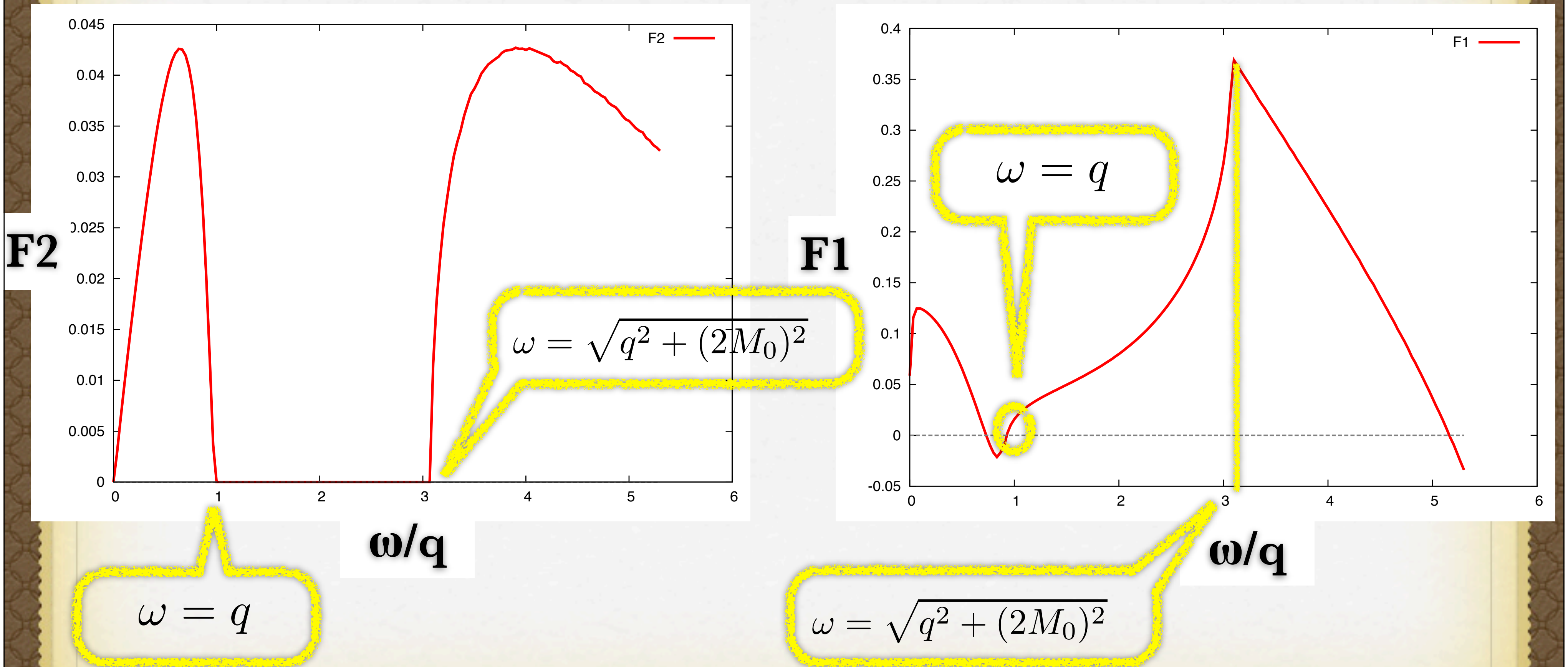

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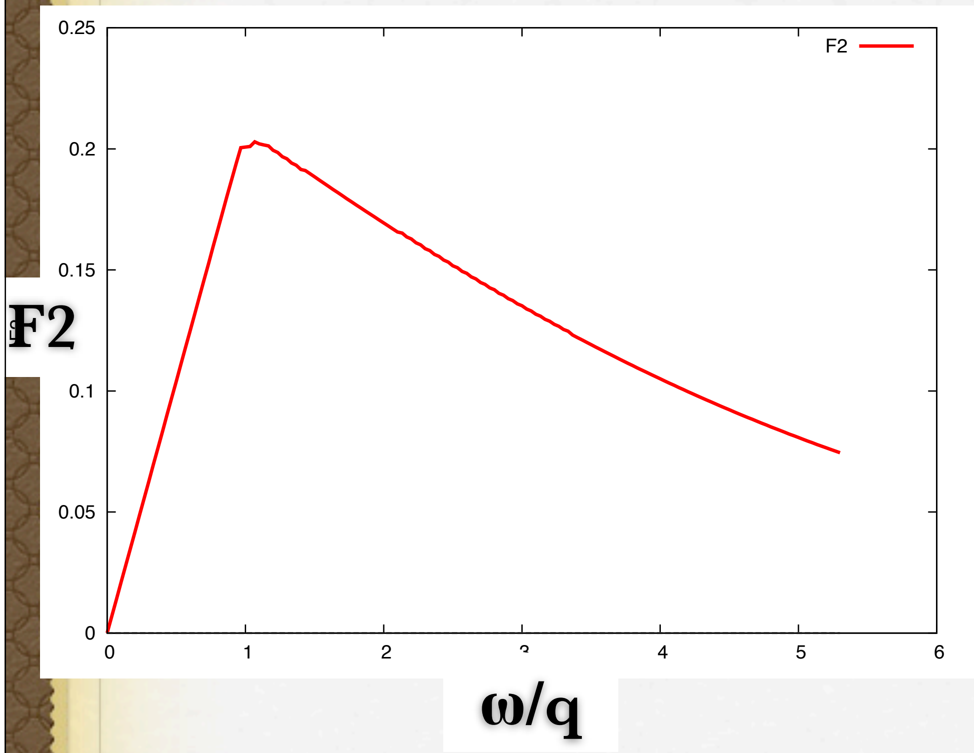
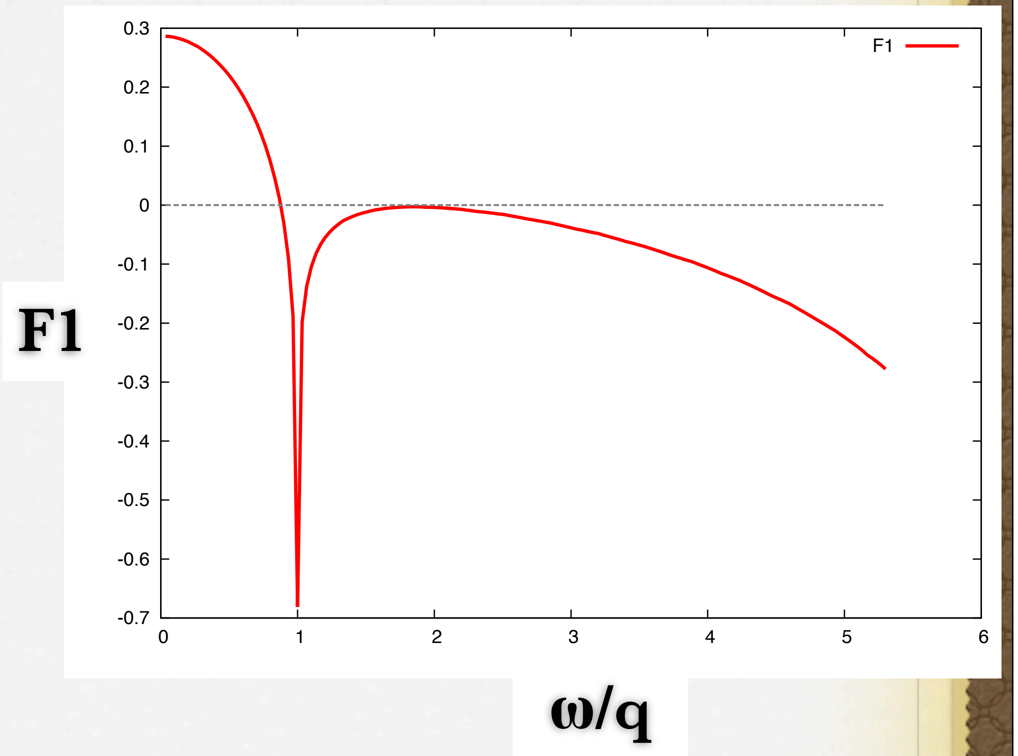
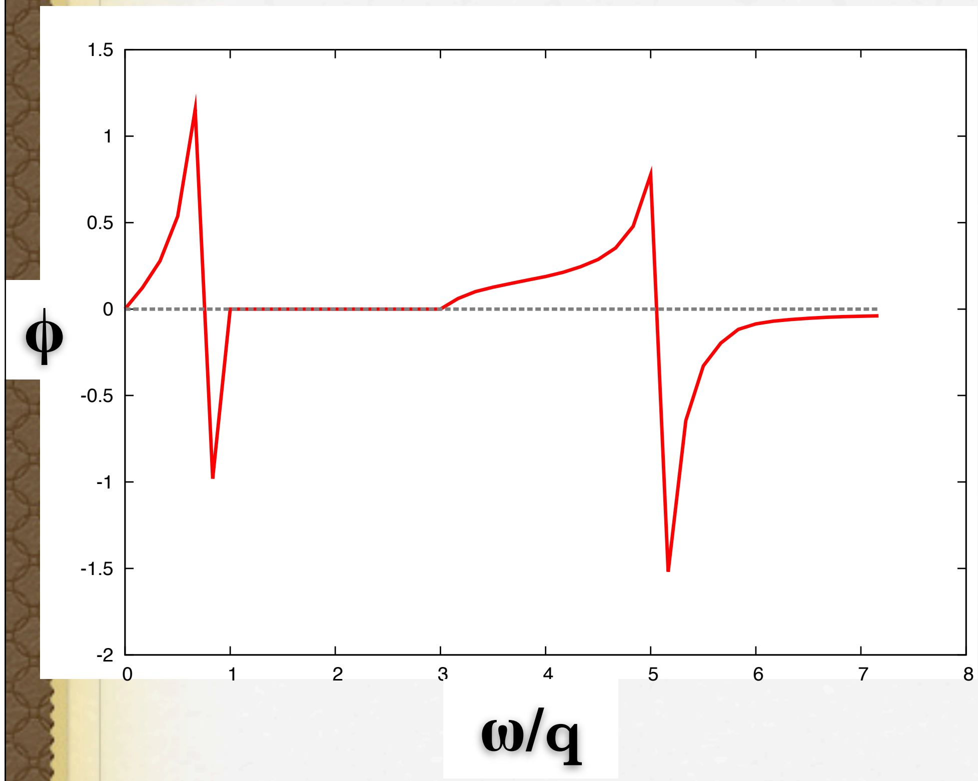




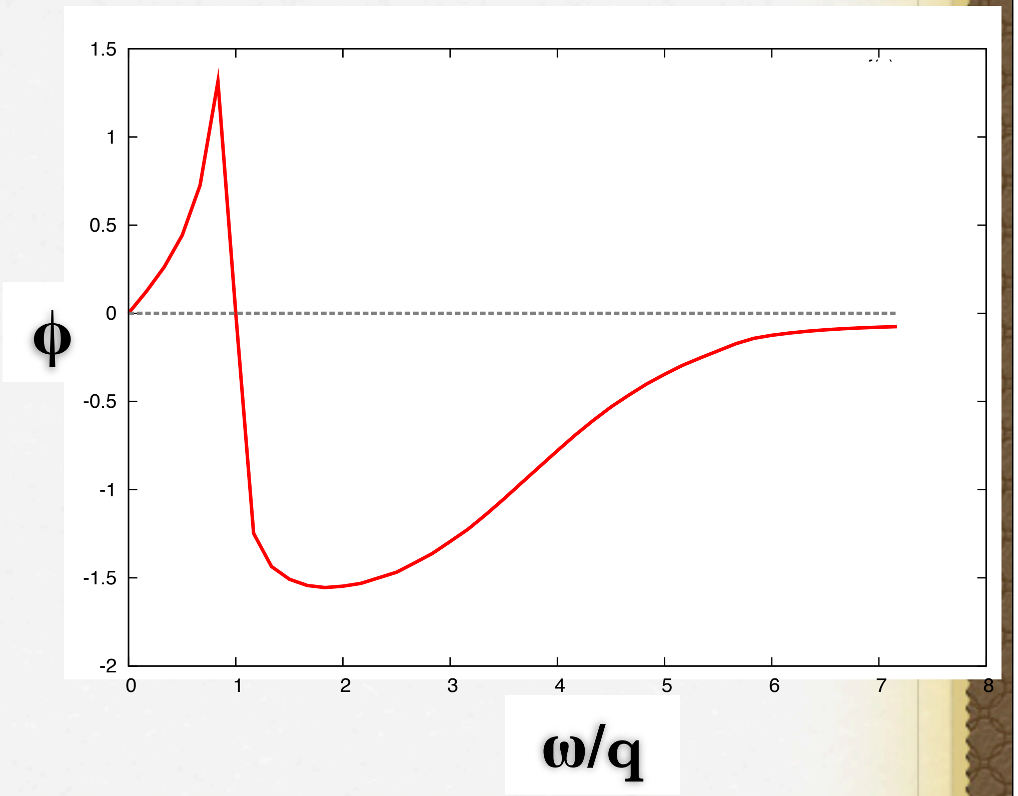
Individual Excitation

ϕ : Argument F

$T < T_0$



$T > T_0$



Position of pion pole

$T < T_0$

$T > T_0$