LIDER

Narodowe Centrum Badań i Rozwoju

Saturation: from production of entropy to coherent emission of gluons

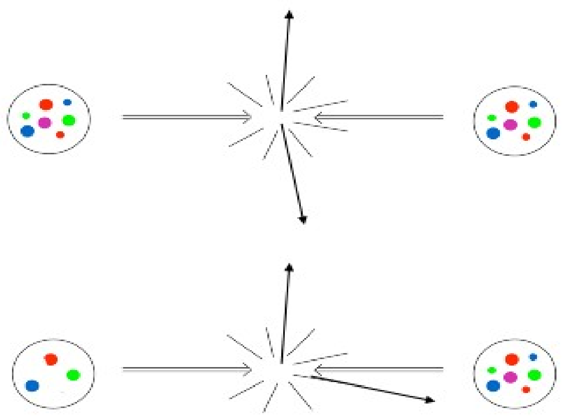
Krzysztof Kutak

Instytut Fizyki Jądrowej
im. Henryka Niewodniczańskiego
Polskiej Akademii Nauk

Research supported by: Polish Research Agency with grant LIDER/02/35/L-2/10/NCBiR/2011.

LHC as a scanner of gluon

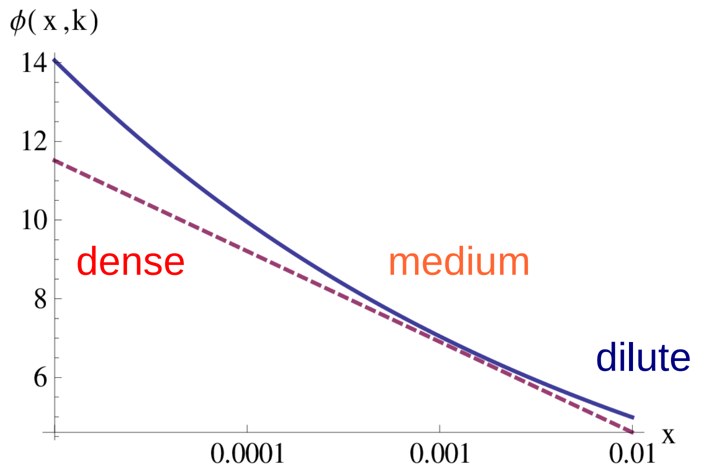
$S = 2P_1 \cdot P_2$



central-central i.e.
medium-medium

forward-central i.e.
dilute – dense

From C. Marquet



$$x_1 = \frac{1}{\sqrt{S}}\left(p_{t1}e^{y_1} + p_{t2}e^{y_2}\right) \xrightarrow{y_1 \sim 0,\, y_2 \gg 0} \sim 1$$

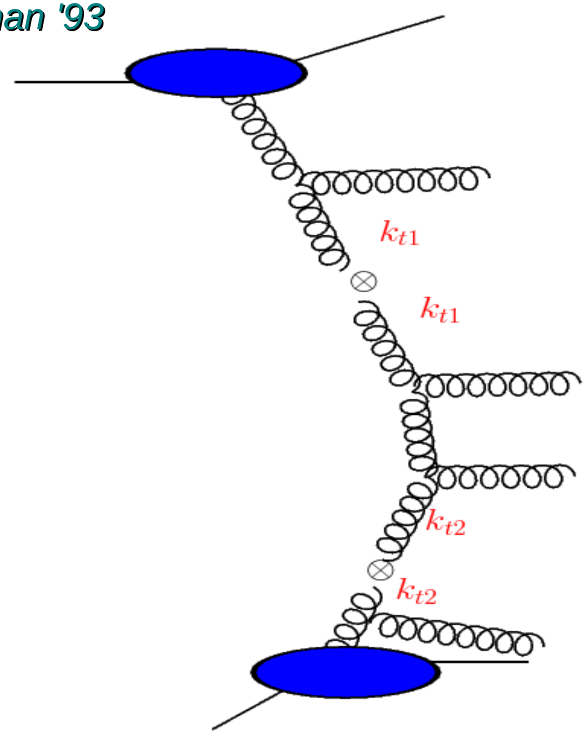
$$x_2 = \frac{1}{\sqrt{S}}\left(p_{t1}e^{-y_1} + p_{t2}e^{-y_2}\right) \qquad \ll 1$$

QCD at high energies

$$\frac{d\sigma}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} = \sum_{a,b,c,d} \int \frac{d^2k_{1t}}{\pi} \frac{d^2k_{2t}}{\pi} \frac{1}{16\pi^2 (x_1 x_2 S)^2} \overline{|\mathcal{M}_{ab\to cd}|}^2 \delta^2(\vec{k}_{1t} + \vec{k}_{2t} - \vec{p}_{1t} - \vec{p}_{2t})$$

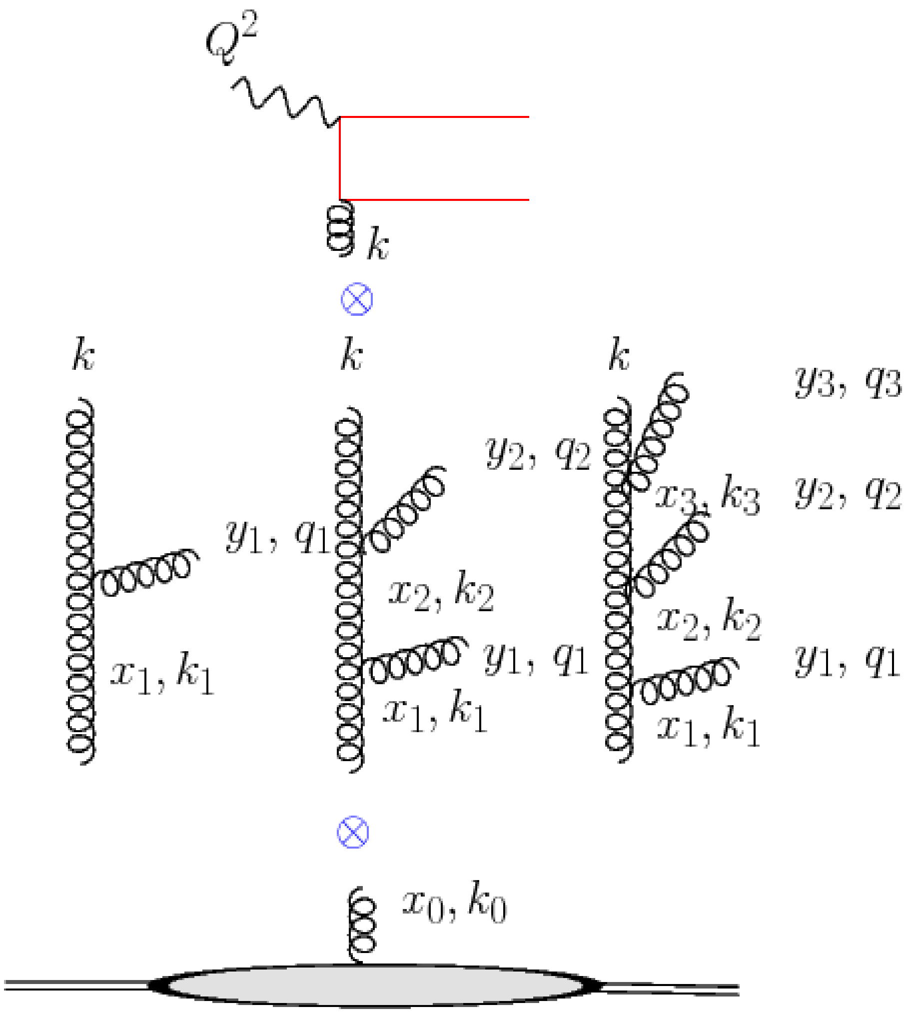
$$\times\ \phi_{a/A}(x_1, k_{1t}^2, \mu^2)\, \phi_{b/B}(x_2, k_{2t}^2, \mu^2)\, \frac{1}{1+\delta_{cd}}$$

Gribov, Levin, Ryskin '81
Ciafaloni, Catani, Hautman '93



- *Longitudinal and transversal parton degrees of freedom taken into account also hard scale*
  - *Capable of taking into account finite transversal size of the hadron*
    - *Ralistic kinematics at lowest order*
      - *Gluon density depends on k_t*
      - *Gauge invarint matrix elements with off-shell gluons* *Lipatov '95.*

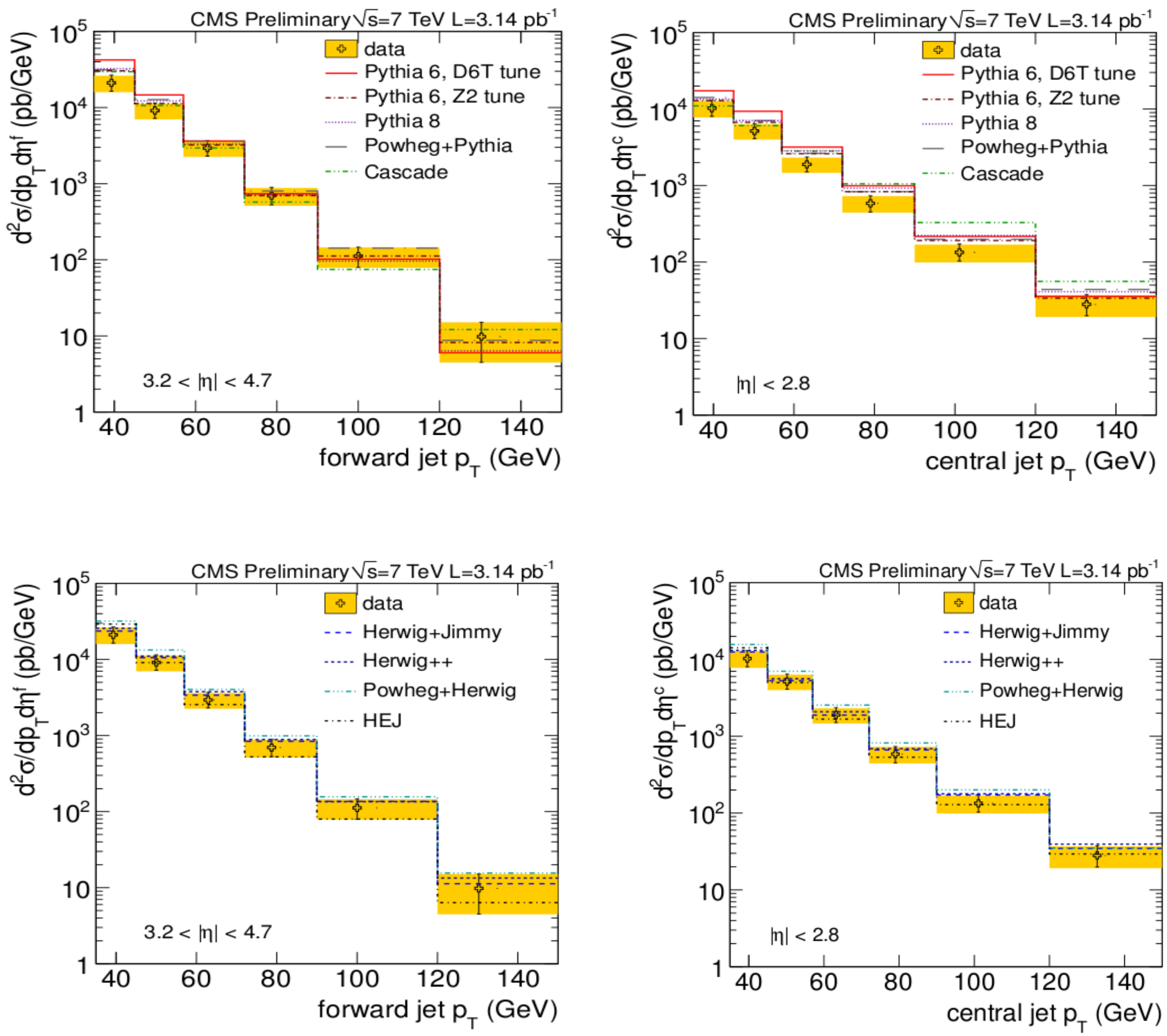
CCFM evolution equation - evolution with observer

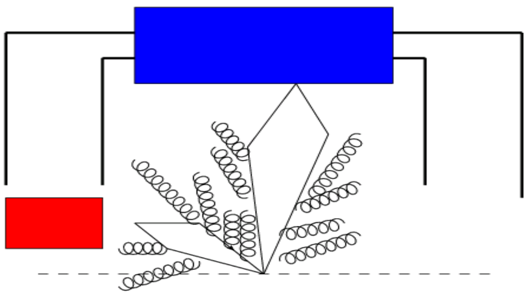


- p - incoming proton, $p = (1, 0, 0, 1)P$
- q_i - emitted gluons, $q_i = y_i p + \bar{y}_i \bar{p} + q_{i\perp}$
- axial gauge with the gauge vector $\bar{p} = (1, 0, 0, -1)P$
- gluon polarization vector purely transverse $\varepsilon_\mu^{(\lambda)}(q) = g_\mu^{(\lambda)} - \frac{q_\mu \bar{p}^{(\lambda)}}{q\bar{p}}$

Implemented in CASCADE Monte Carlo ***H. Jung 02***

Forward-central di-jet production





- *HEJ and Cascade based on unordered in k_t emissions but use different parton densities*
- *Herwig and PYTHIA use k_t odered shower but differ in approximations in ME and ordering conditions in shower*

CCFM based approach from

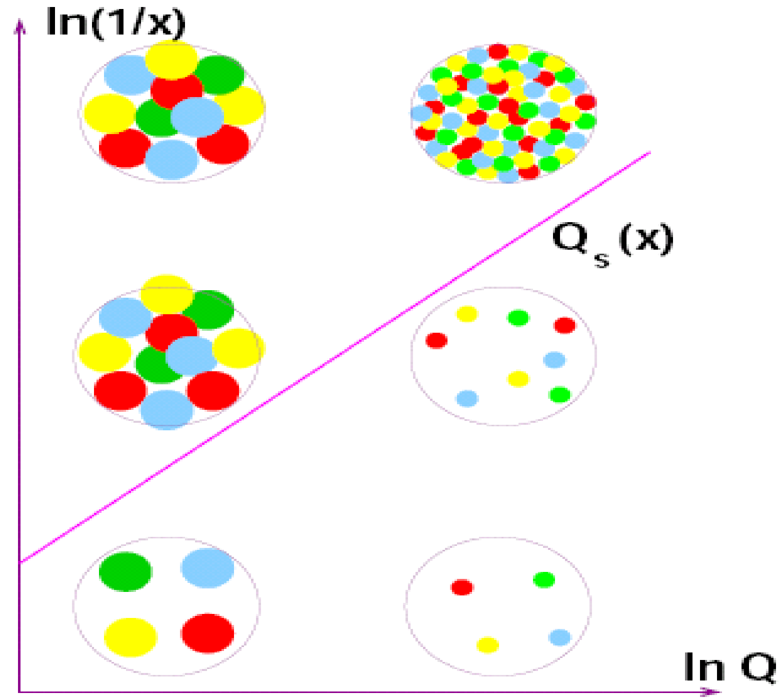
Deak, Jung, Hautmann, Kutak,' 10

Observable sensitive to saturation of gluon density

Kharzeev, Levin,McLerran '05, Marquet '07

High energy factorization and saturation

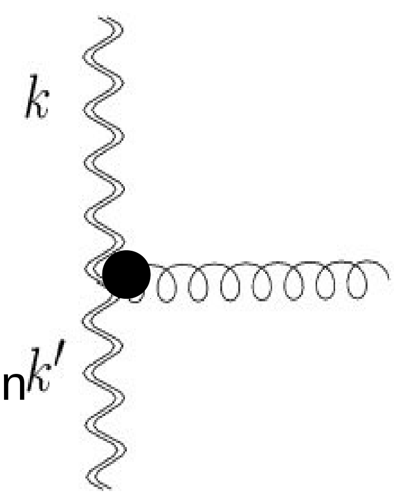
Saturation – state where number of gluons stops growing due to high occupation number.
More generally saturation is an example of percolation which has to happen since partons have size $1/k_t$ and hadron has finite size



Cross sections change their behavior from power like to logarithmic like.

On microscopic level it means that gluon apart splitting recombine

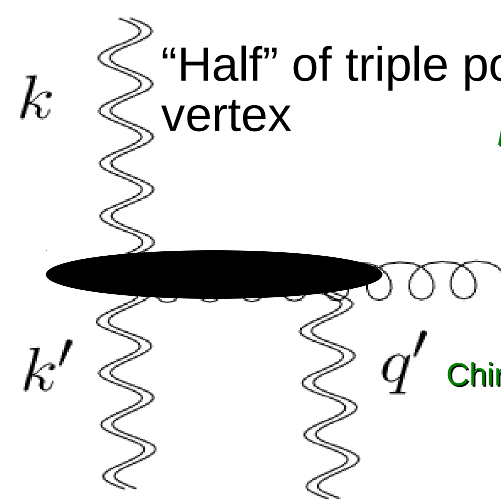
splitting



Linear evolution equation

recombination

Nonlinear evolution equations
BK, JIMWLK
CGC framework
DIPSY



"Half" of triple pomeron vertex

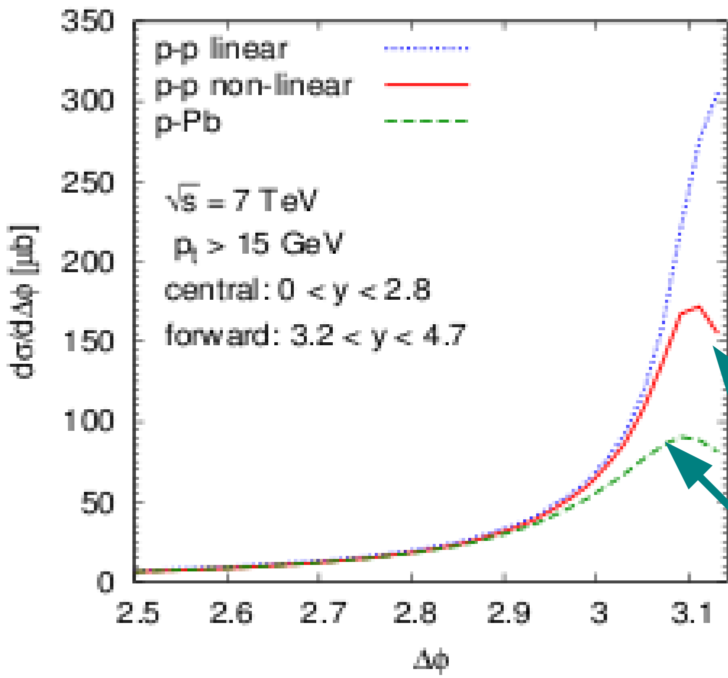
Bartels,Wusthoff

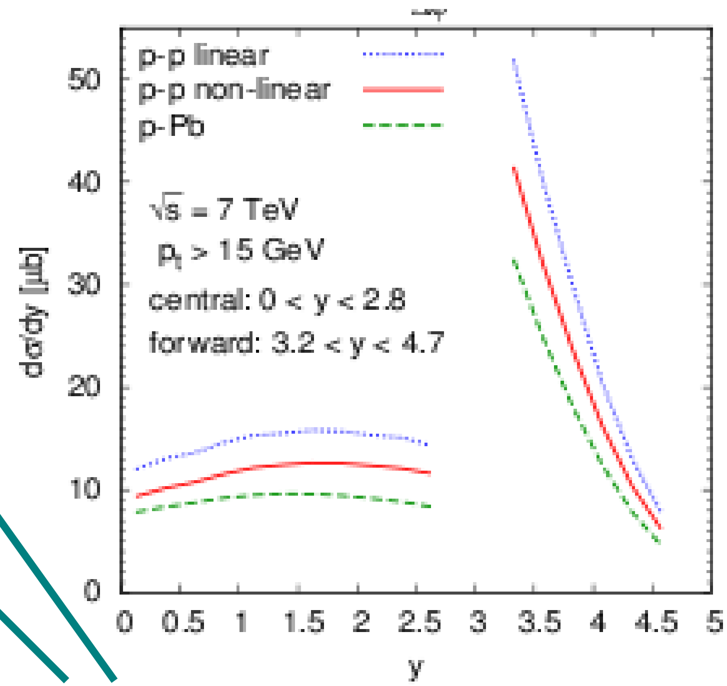
Z.Phys. C66 (1995) 157-180

Chirilli,Szymanowski,Wallon '10

Jest in p-p and p-Pb providing signatures of saturation

Kutak .Sapeta arxiv:1205.5035

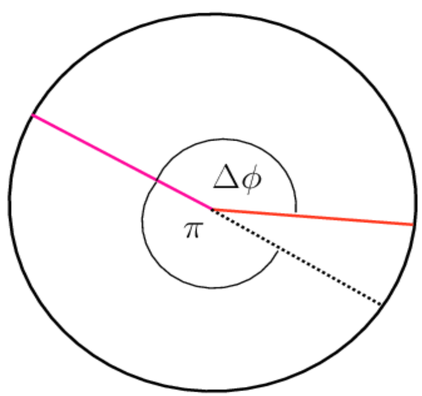


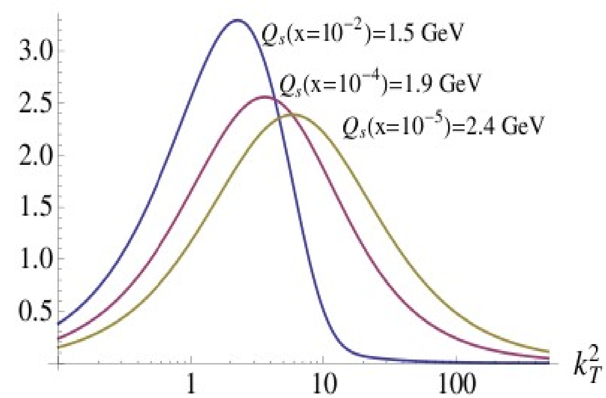


Reflects~ k^2
behavior of gluon density

Observable suggested to study BFKL and saturation effects

Kharzeev, Levin,05 Sabio-Vera, Schwensen '06, Marquet ' 07

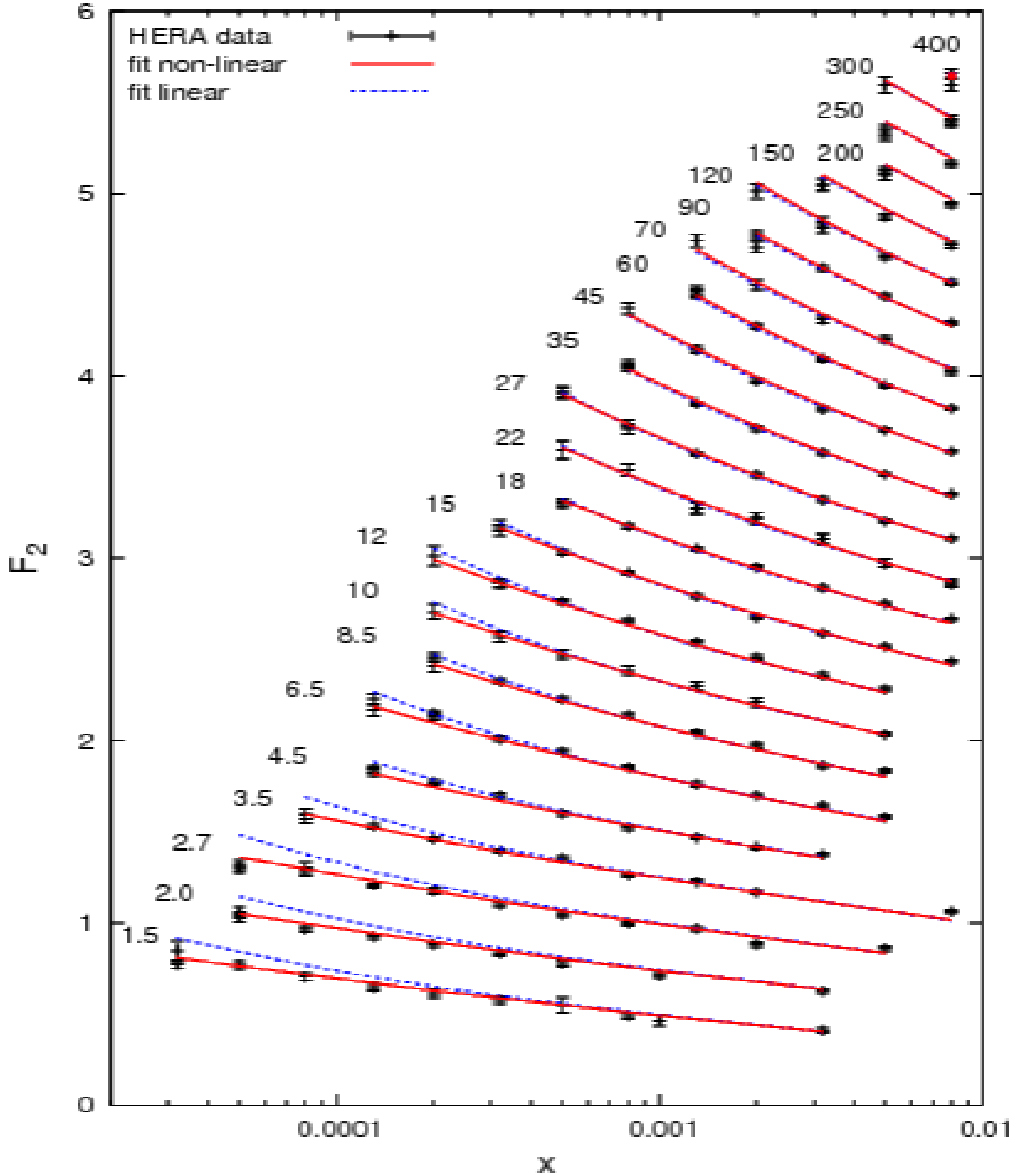




Calculation based on BK
with higher order corrections

From Dumitru et al. Phys.Lett.B697:21-25,2011

Further hints for saturation in F_2 data



S.Sapeta. KK
arxiv:1205.5035

Fit of BK-DGLAP and BFKL-DGLAP to combined H1-ZEUS data

Very good description with BK-DGLAP in range $Q^2 > 4.5\ GeV^2$

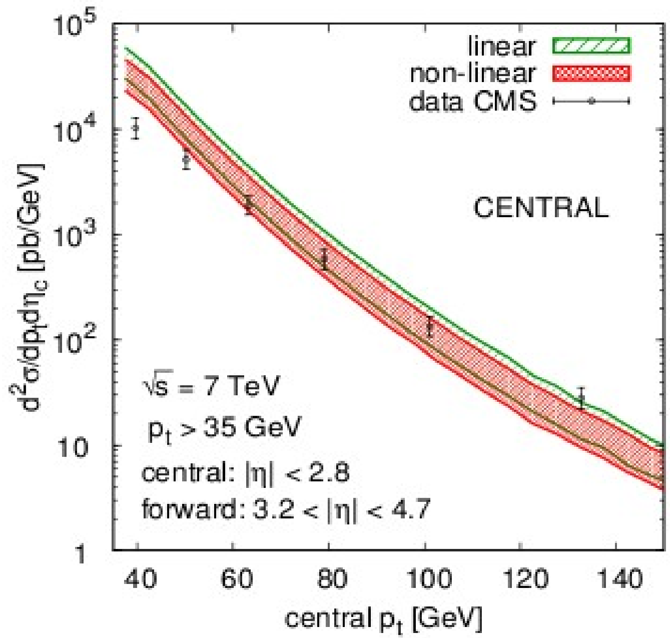
$$\chi^2 = 1.73$$

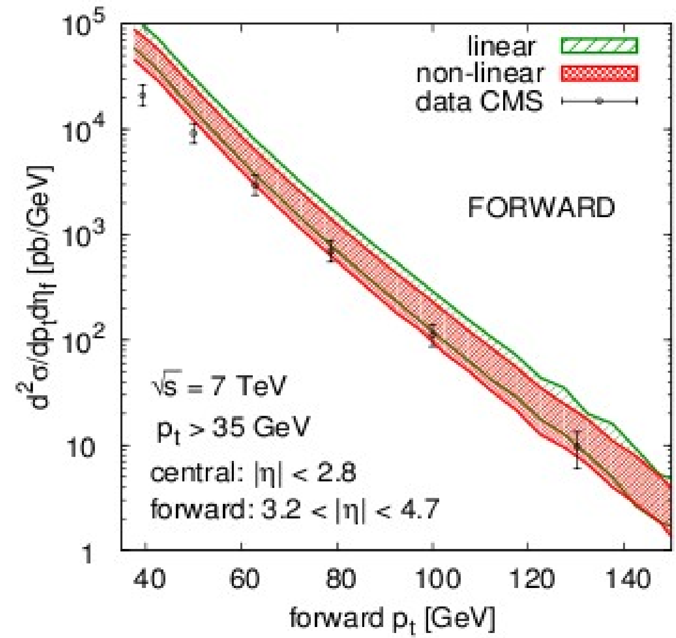
Very good description with BFKL-DGLAP in range $Q^2 > 4.5\ GeV^2$

$$\chi^2 = 1.5$$

Jets and saturation

S.Sapeta. KK
arxiv:1205.5035





$$\mathcal{F}_p(x,k^2) = \mathcal{F}_p^{(0)}(x,k^2) + \frac{\alpha_s(k^2)N_c}{\pi}\int_x^1 \frac{dz}{z}\int_{k_0^2}^\infty \frac{dl^2}{l^2}\left\{\frac{l^2\mathcal{F}_p(\frac{x}{z},l^2)\,\theta(\frac{k^2}{z}-l^2) - k^2\mathcal{F}_p(\frac{x}{z},k^2)}{|l^2-k^2|} + \frac{k^2\mathcal{F}_p(\frac{x}{z},k^2)}{|4l^4+k^4|^{\frac{1}{2}}}\right\}$$
$$+ \frac{\alpha_s(k^2)}{2\pi k^2}\int_x^1 dz\left(P_{gg}(z)-\frac{2N_c}{z}\right)\int_{k_0^2}^{k^2} dl^2\,\mathcal{F}_p(\frac{x}{z},l^2)$$
$$- \frac{2\alpha_s^2(k^2)}{R^2}\left[\left(\int_{k^2}^\infty \frac{dl^2}{l^2}\mathcal{F}_p(x,l^2)\right)^2 + \mathcal{F}_p(x,k^2)\int_{k^2}^\infty \frac{dl^2}{l^2}\ln\left(\frac{l^2}{k^2}\right)\mathcal{F}_p(x,l^2)\right]$$

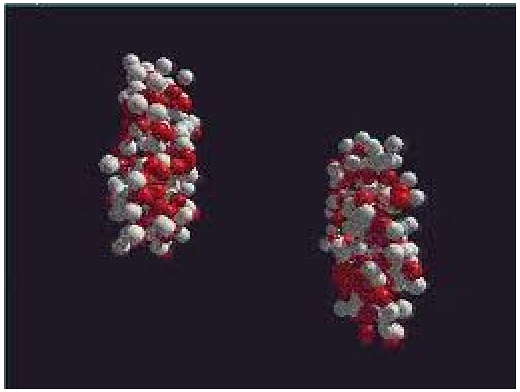
Kwiecinski,Kutak '03

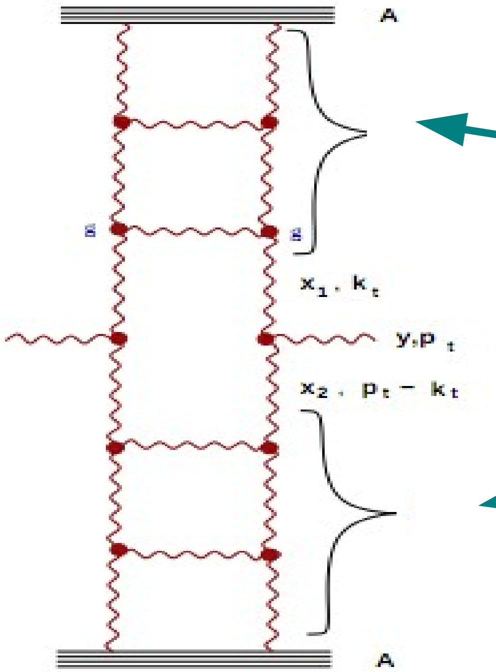
Corrections of higher orders Included. Kin. Constr DGLAP spf

Andersson, Gustafson,Sammuelsson '96
Kwiecinski, Martin, Sutton '96

Production of gluons in dilute vs. dense

Kovchegov; Levin,,; Praszłowicz,; Albacete, ...





$$\frac{d\sigma}{d^2p_t\,dy} = \frac{2\,\alpha_s}{C_F}\,\frac{1}{p_t{}^2}\int d^2k\,\phi(k,x_1)\,\phi(k-p_t,x_2)$$

$$x_1 \sim \frac{e^y}{\sqrt{s}} \qquad x_2 \sim \frac{e^{-y}}{\sqrt{s}}$$

$$Q_{s1} < p_t < Q_{s2}$$

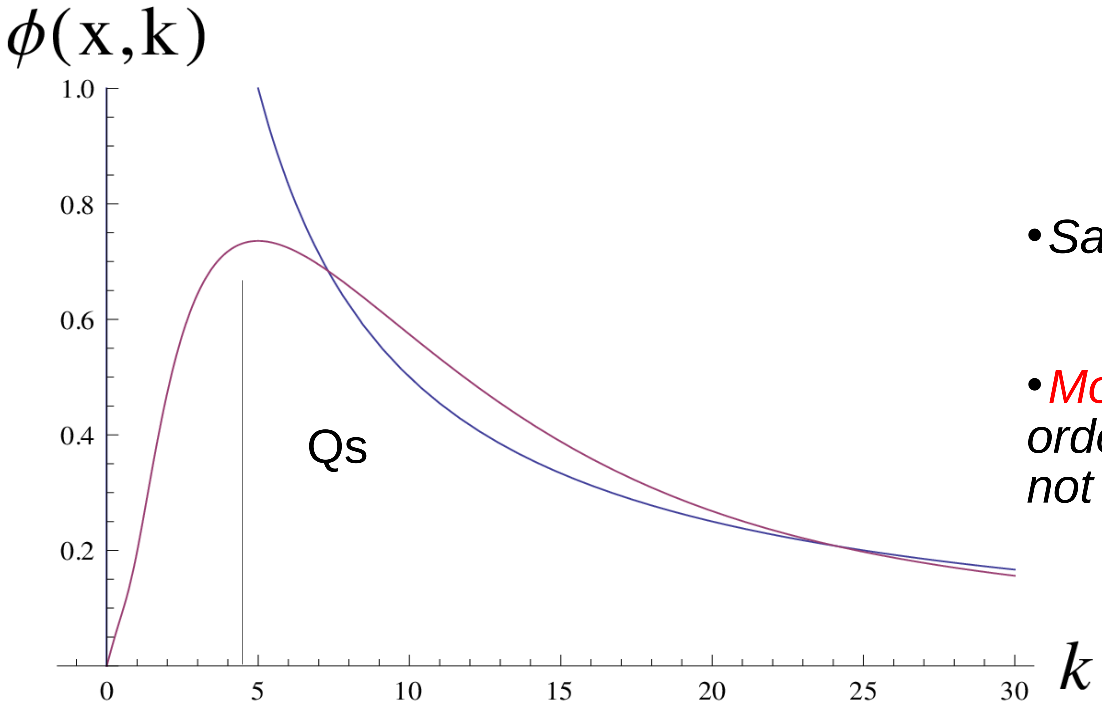
Dumitru, McLerran,Blaizot,...

$$\frac{d\sigma}{dy} = \frac{2A_\perp^2 C_F Q_{s1}^2 Q_{s2}^2 e^{-\frac{p_{t\,min}^2}{Q_{s1}^2+Q_{s2}^2}}\left(p_{t\,min}^2 Q_{s1}^2 Q_{s2}^2 + Q_{s1}^6 + Q_{s2}^6\right)}{\pi^2\alpha_s(Q_{s1}^2+Q_{s2}^2)^4} + \frac{4A_\perp^2 C_F Q_{s1}^4 Q_{s2}^4\,\Gamma\left(0,\frac{p_{t\,min}^2}{(Q_{s1}^2+Q_{s2}^2)}\right)}{\pi^2\alpha_s(Q_{s1}^2+Q_{s2}^2)^3}$$

K.Kutak, Physics Letters B 705 (2011),

$$\frac{d\sigma}{dy} = \frac{2A_\perp^2 C_F Q_{s1}^2}{\pi^2\alpha_s} - \frac{2A_\perp^2 C_F Q_{s1}^4}{\pi^2\alpha_s Q_{s2}^2}\left(2\log\left(\frac{Q_{s1}^2}{Q_{s2}^2}\right) + 5 + 2\gamma_E\right) + O\left((Q_{s1}^2/Q_{s2}^2)^3\right)$$

The gluon density - features



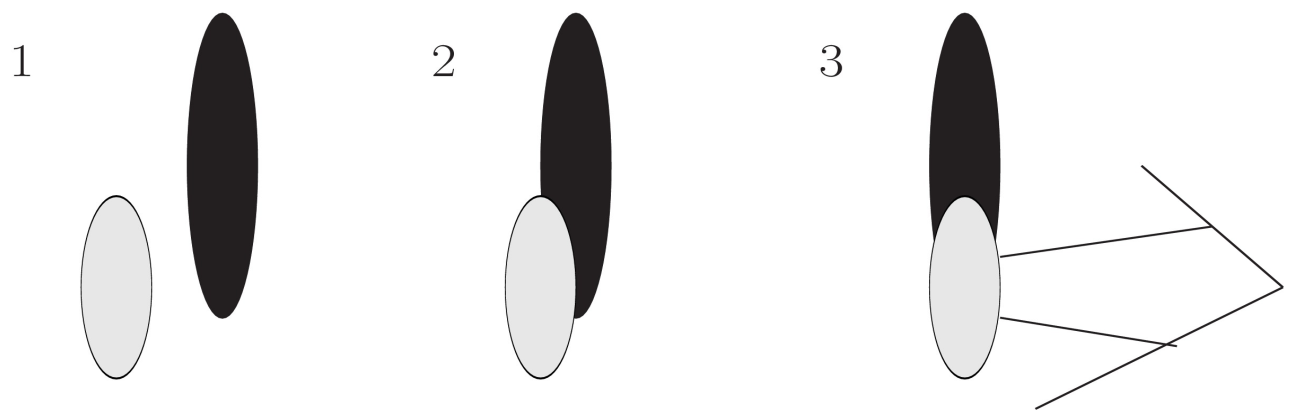
- *Saturation scale regulates the divergence*

- *Most probable momentum of the order of Qs. In BFKL or DGLAP not possible to define*

Colliding hadrons and Kharzeev Tuchin setup

Stages of collision

Kharzeev, Tuchin '05



$$P(M \leftarrow m) = 2\pi |\mathcal{T}(M \leftarrow m)|^2 \, \rho(M),$$

$$\int dM \, P(M \leftarrow m)$$

should be finite

Probability for transition to final state

density of states determined by typical momentum. Qs emerges

$$|\mathcal{T}(M \leftarrow m)|^2 \sim \exp(-2\pi M/a)$$

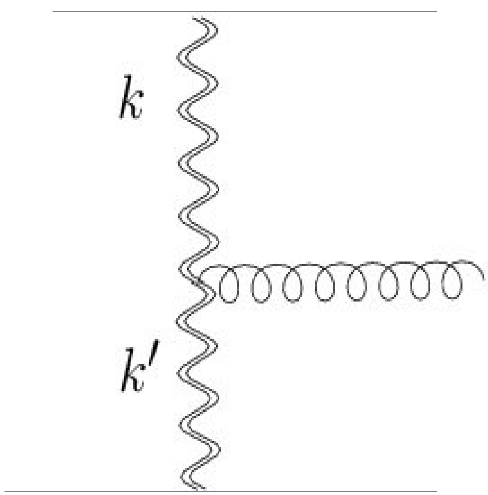
$$\frac{a}{2\pi} \equiv T \leq \frac{\sqrt{6}}{4\pi} \frac{1}{\sqrt{b}} \equiv T_{Hag}$$

$$T = \frac{Q_s(x)}{2\pi}$$

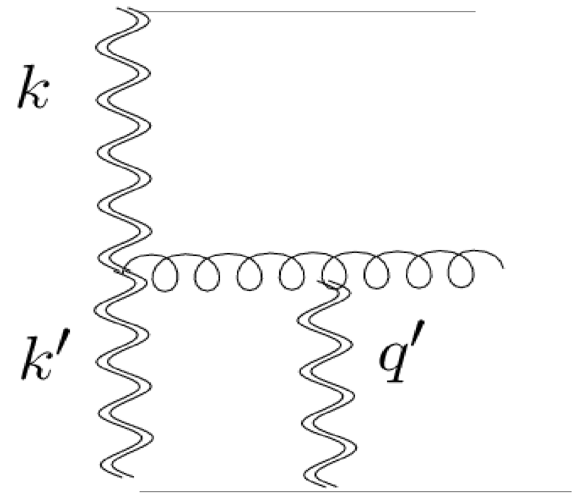
also motivated by Unruh effect

Saturation nonlinearities and fluctuations

BFKL



BK



The thermal fluctuations are vacum fluctuations experienced by accerelated apparatures. In the linear QCD equations as BFKL no fluctuation on a level or Reggeized gluon since

they collapse to linear equation. BK equation brings nonlinearities and loops...

"During deceleration the horizon forms so there are regions of the global spacetime which will never be observer by us, so there should be a certain entropy." L. Susskind

Towards entropy

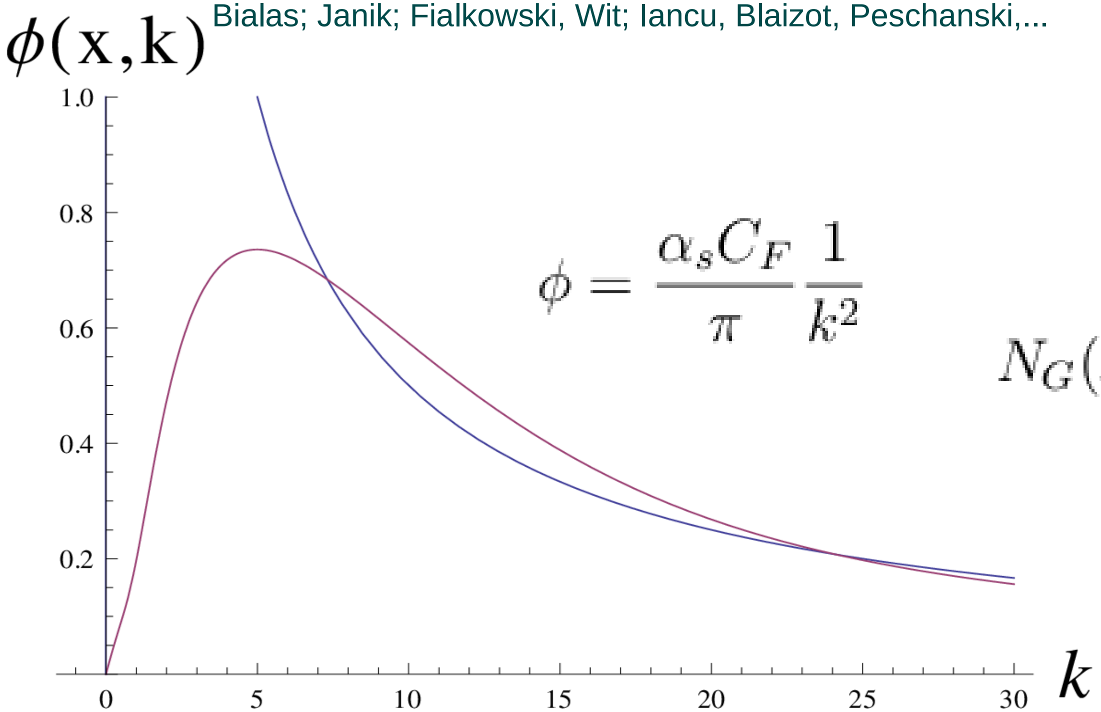
The relation $T = \frac{Q_s(x)}{2\pi}$ can be understood in a generalized sense i.e. that saturation scale defines some temperature.

Equilibrium thermodynamics relations → *Lower bound on produced entropy*

Gluon production and entropy

K.Kutak, Physics Letters B 705 (2011),

Bialas; Janik; Fialkowski, Wit; Iancu, Blaizot, Peschanski,...



$$M_G(x) = Q_s(x)$$ energy dependent gluon's mass

$$M(x) = N_G(x) M_G(x)$$ mass of system of gluons

$$N_G(x) \equiv \frac{dN}{dy} = \frac{1}{S_\perp} \frac{d\sigma}{dy}$$ number of gluons

$$dE = TdS$$

$$dM = TdS$$

$$d\left[N_G(x)\, M_G(x)\right] = \frac{Q_s(x)}{2\pi} dS$$

Gluon density build due to many-body interactions

Generated mass of gluon.

Framework of HTL.

Blaizot at.al Nucl.Phys. A873 (2012) 68-80

Entropy due to less dense hadron

$$S = \frac{6 C_F A_\perp}{\pi \alpha_s} Q_s^2(x) + S_0$$

$$S = 3\pi \left[N_G(x) + N_{G0}\right]$$

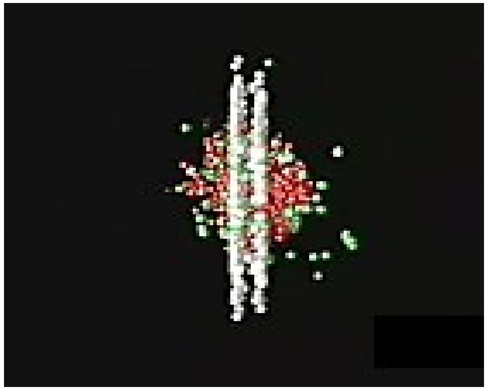
Similarly in QED. Cut on photon`s kt Is equivalent to introducing mass.

In presented approach mass is x dependent

Entropy and gluon distribution

Number of gluons in the updf:

$$n_G(x) \equiv \frac{1}{\pi}\int d^3r\, d^2k\, \Phi(x,k,r) = \frac{1}{\pi}\int d^2k\, \phi(x,k^2)$$



Using GBW gluon we get: $$n_G(x) = \frac{C_F A_\perp}{2\pi^2\alpha_s} Q_s^2(x)$$

$$d^3r \equiv d^2b\, dl$$

$$r \equiv (l, \mathbf{b})$$

And entropy expressed in terms of number of gluons.

$$S = 12\pi\, n_G(x) + 3\pi\, N_{G\,0}$$

Remark: possible definition of entropy of gluon density via :

$$s(x) = -\int d^2k \phi(x,k^2) \ln\phi(x,k^2) \sim Q_s^2(x)$$

Towards constraints on entropy
resummed form of the BK

The strategy:

- Use the equation for WW gluon density. Simple nonlinear term
- Split linear kernel into resolved and unresolved parts
- Resumm the virtual contribution and unresolved ones in the linear part
- Use analogy to postulate nonlinear CCFM

The starting point:

$$\Phi(x,k^2) = \Phi_0(x,k^2) + \overline{\alpha}_s \int_{x/x_0}^{1} \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[\frac{l^2\Phi(x/z,l^2) - k^2\Phi(x/z,k^2)}{|k^2 - l^2|} + \frac{k^2\Phi(x/z,k^2)}{\sqrt{(4l^4 + k^4)}} \right] - \frac{\overline{\alpha}_s}{\pi R^2} \int_{x/x_0}^{1} \frac{dz}{z} \Phi^2(x/z,k^2)$$

$$\pi R^2 = 1$$

Resummed form of the BK

K. Kutak, K. Golec-Biernat, S. Jadach, M. Skrzypek

JHEP 1202 (2012) 117

$$\Phi(x,k^2) = \Phi^0(x,k^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\mathbf{q}}{\pi q^2} \left[\Phi(x/z, |\mathbf{k}+\mathbf{q}|^2) - \theta(k^2-q^2)\Phi(x/z,k)\right] - \overline{\alpha}_s \int_x^1 \frac{dz}{z} \Phi^2(x/z,k)$$

Write in exclusive form

$$\Phi(x,k^2) = \Phi^0(x,k^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\mathbf{q}}{\pi q^2} \Phi(x/z, |\mathbf{k}+\mathbf{q}|^2)\theta(q^2-\mu^2) + \overline{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2\mathbf{q}}{\pi q^2} \left[\Phi(x/z, |\mathbf{k}+\mathbf{q}|^2)\theta(\mu^2-q^2) - \theta(k^2-q^2)\Phi(x/z,k)\right] - \overline{\alpha}_s \int_x^1 \frac{dz}{z} \Phi^2(x/z,k)\,.$$

Resolution scale introduced

Perform Mellin transform w.r.t x to get rid of "z" integral

$$\overline{\Phi}(\omega,k^2) = \int_0^1 dx x^{\omega-1}\Phi(x,k^2)$$

$$\Phi(x,k^2) = \int_{c-i\infty}^{c+i\infty} d\omega\, x^{-\omega}\overline{\Phi}(\omega,k^2)$$

Extension of CCFM to non linear equation

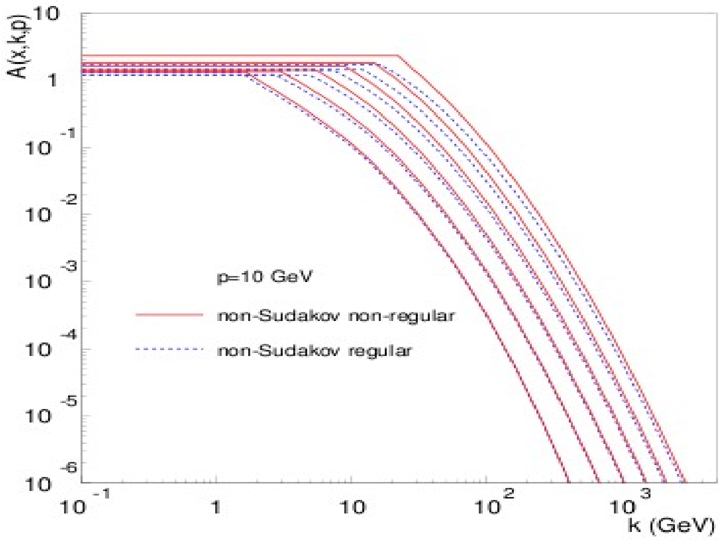
- The second argument should be kt motivated by analogy to BK
- The third argument should reflect locally the angular ordering

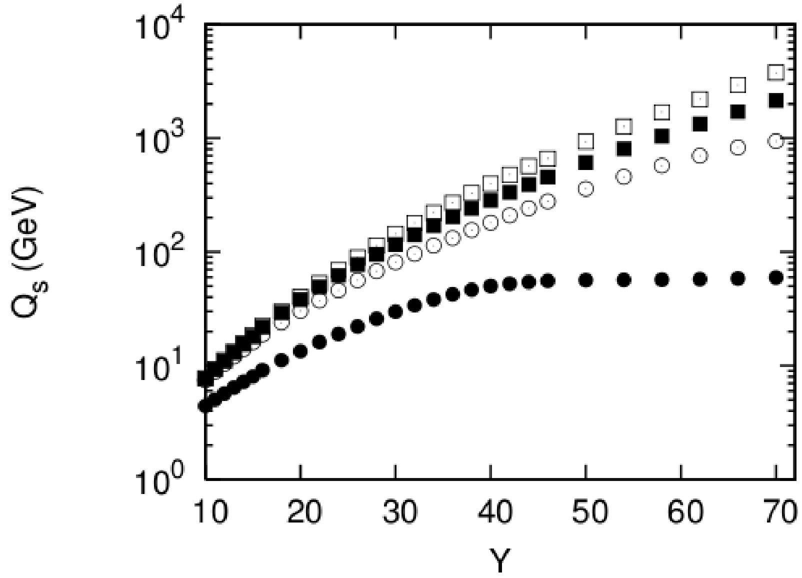
$$\Phi(x,k^2) = \tilde{\Phi}^0(x,k^2) + \overline{\alpha}_s \int_x^1 d\,z \int \frac{d^2\mathbf{q}}{\pi q^2}\,\theta(q^2-\mu^2)\frac{\Delta_R(z,k,\mu)}{z}\Big[\Phi(\frac{x}{z},|\mathbf{k}+\mathbf{q}|^2) - q^2\delta(q^2-k^2)\,\Phi^2(\frac{x}{z},q^2)\Big]$$

$$\mathcal{E}(x,k^2,p) = \mathcal{E}_0(x,k^2,p) + \bar{\alpha}_s \int_x^1 dz \int \frac{d^2\bar{\mathbf{q}}}{\pi\bar{q}^2}\,\theta(p-z\bar{q})\Delta_s(p,z\bar{q})\left(\frac{\Delta_{ns}(z,k,q)}{z} + \frac{1}{1-z}\right)\Big[\mathcal{E}\left(\frac{x}{z},k'^2,\bar{q}\right) - \bar{q}^2\delta(\bar{q}^2-k^2)\,\mathcal{E}^2(\frac{x}{z},\bar{q}^2,\bar{q})\Big].$$

Some estimates Unitarity corrections via boundary conditions in CCFM

Avsar, Stasto '10





Gluon density supressed in the saturated region. Arbitrary but...

saturation scale is bounded
because of limited phase space
due to existence of hard scale

Kutak, Jung '09
Avsar, Iancu '09
Avsar, Stasto 10

From *Avsar, Stasto*

Consequence: at given hard pt the saturation stops to depend on energy
Therefore there will be some maximal entropy from saturated region for a given pt

Conclusions and further comments

- With our framework we predict possibility to see saturation at high pt in p Pb via studies of jets

- Generation of saturation scale leads to generation of certain entropy which has intuitive meaning. It behaves like number of produced gluons and scales like the target's size

- Results justified by non-equlibrium thermodynamics. Provate communications with R. Peschanski

- Coherence combined with saturation gives bound on entropy

- Calculation using AdS/CFT duality via area of trapped surface with energy cut off on scale factor. Similar result for entropy. A. Kiritsis. A. Tsalios *PoS EPS-HEP2011 (2011) 121*

- I learned on Friday about similar result for entropy in AA by R. J. Fries, B, Muller, A. Schafer *10.1103/PhysRevC.79.034904.*