Quark number susceptibilities from resummed perturbative QCD

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In collaboration with Jens O. Andersen (Trondheim), Nan Su and Aleksi Vuorinen (Bielefeld)

Why finite T and μ QCD via analytical methods?

- Alternative to Lattice field theory methods at finite density
  - Heavy ion collisions
  - Neutron star physics
  - Early universe
- Different approach, deeper understanding of the nature of QGP
- Connection with the $T \to \infty$ case

What are the challenges (at least some of)?

- Lower the range of T for which predictions are reliable
- Improve the apparent convergence of the perturbative series
- Set up a self consistent and systematic framework
- Generalization to finite chemical potential, and far from $\frac{\mu}{T} \ll 1$

What do we intend to do here?

- Studying at which extent one can understand high precision lattice data, using weakly coupled quasiparticle picture

- Imaginary time formalism: $P_0 \equiv (2n+1)\,\pi T - i\mu$
- Dimensional regularization in the momentum space:

$$\sum\!\!\!\!\!\!\!\int_{\{K\}} \sim T \sum_{n=-\infty}^{n=+\infty} \int \frac{\mathrm{d}^{3-2\epsilon} \boldsymbol{p}}{(2\pi)^{3-2\epsilon}}$$

- Lagrangian density of QCD:

$$\begin{aligned} \mathcal{L}_{\rm QCD} &= -\frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + i\bar{\psi}\gamma^{\mu}D_{\mu}\psi \\ &\quad +\mathcal{L}_{\rm gf} + \mathcal{L}_{\rm ghost} + \Delta\mathcal{L}_{\rm QCD} \end{aligned}$$

- Relation between the thermodynamic potential Ω and the diagonal quark number susceptibility (QNS) χ:

$$\chi\left(T\right) \doteq -\left.\frac{\partial^2 \Omega\left(T,\mu\right)}{\partial \mu^2}\right|_{\mu=0}$$

- p the pressure: $p\left(T,\mu\right) \equiv -\Omega\left(T,\mu\right)$

- Reorganizing the perturbative series of thermal QCD:

$$\mathcal{L} = (\mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm HTL})\Big|_{g\to\sqrt{\delta}g} + \Delta\mathcal{L}_{\rm HTL}$$

- With the HTL improvement term:

$$\mathcal{L}_{\rm HTL} = -\frac{1}{2}(1-\delta)m_D^2 \,{\rm Tr}\left(F_{\mu\alpha}\left\langle\frac{y^\alpha y^\beta}{(y\cdot D)^2}\right\rangle_y F^\mu{}_\beta\right) +(1-\delta)\, i m_q^2 \bar{\psi}\gamma^\mu \left\langle\frac{y_\mu}{y\cdot D}\right\rangle_y \psi$$

- δ: formal expansion parameter, set to 1 in the end, # of HTL dressed loops
  $\Delta\mathcal{L}_{\rm HTL}$: HTL counterterm(s); y^μ: lightlike 4-vector
  m_D/m_q: Debye/quark thermal mass parameters
- Adding $\mathcal{L}_{\rm HTL}$ "SHIFTS" the expansion to an ideal gas of thermal quasiparticles...

- Leading order, 1-loop sum-integrals with dressed propagators:

$$\text{Typical integral:} \quad \sum\!\!\!\!\!\!\!\int_{\{P\}} \log\left[\frac{A_S^2 - A_0^2}{P^2}\right]$$

where A_0 and A_S are tricky functions of P_0, p, m_q and $\mathcal{T}_P$ with:

$$(\mathcal{T}_P)^m \doteq \left(\frac{\Gamma\left(\frac{3}{2}-\epsilon\right)}{\Gamma\left(\frac{3}{2}\right)\Gamma\left(1-\epsilon\right)}\right)^m \left(P_0^2\right)^m$$

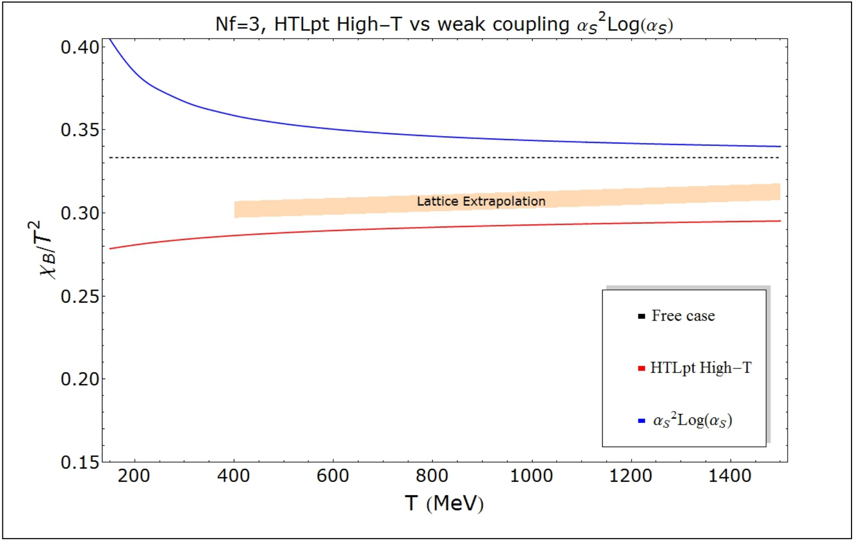
$$\int_0^1 \mathrm{d}c_1 \;...\; \int_0^1 \mathrm{d}c_m \left\{\frac{\left(1-c_1^2\right)^{-\epsilon}}{\left(P_0^2+\boldsymbol{p}^2 c_1^2\right)} ... \frac{\left(1-c_m^2\right)^{-\epsilon}}{\left(P_0^2+\boldsymbol{p}^2 c_m^2\right)}\right\}$$

- Here, result for the High-T truncation (no branch cuts): Expansion in power of $m_{q,D}/T$, truncation at order $O(m_{q,D}^4)$
- Prescription for the mass parameters: $m_D^2 \equiv \alpha_S \pi \left(\frac{4N_c}{3}T^2 + 2N_f\frac{\mu^2}{\pi^2}\right)$; $m_q^2 \equiv \alpha_S \pi \frac{N_c^2-1}{4N_c}\left(T^2+\frac{\mu^2}{\pi^2}\right)$

$$\Omega_{\rm HTL}^{\rm LO,\ high\text{-}T} \equiv -\frac{d_A\pi^2T^4}{45}\left\{1+\frac{d_F}{d_A}\left(\frac{7}{4}+30\hat{\mu}^2+60\hat{\mu}^4\right)\right.$$
$$-\frac{15}{2}\hat{m}_D^2-\left[30\frac{d_F}{d_A}\left(1+12\hat{\mu}^2\right)\right]\hat{m}_q^2$$
$$+30\hat{m}_D^3+\left[60\frac{d_F}{d_A}\left(6-\pi^2\right)\right]\hat{m}_q^4$$
$$+\left[\frac{45}{4}\left(\gamma_E-\frac{7}{2}+\frac{\pi^2}{3}+\log\left(\frac{\bar{\Lambda}}{4\pi T}\right)\right)\right]\hat{m}_D^4$$
$$\left.+O(m_D^6,m_q^6)\right\}$$

where $\hat{m} \equiv \frac{m}{2\pi T}$, $d_A \equiv N_c^2-1$, and $d_F \equiv N_c N_f$. As a preview,...

High-T truncation vs weak coupling up to $\alpha_S^2 \log \alpha_S$



- Separation of the pressure into different contributions:

$$p_{\mathrm{QCD}} \equiv p_{\mathrm{hard}} + p_{\mathrm{soft}}(m_{\mathrm{E}}, \lambda, g_{\mathrm{E}})$$

$\Rightarrow$ *Typical in-medium momentum scales*

- p_{hard}: From the hard modes ($\propto 2\pi T$), via strict perturbative expansion in the 4D theory
- p_{soft}: From the soft modes ($\propto gT$), via the effective 3D Yang-Mills plus adjoint Higgs theory i.e. EQCD
- Lagrangian density of EQCD:

$$\mathcal{L}_{\mathrm{EQCD}} = \frac{1}{2}\mathrm{Tr}\, F_{ij}^2 + \mathrm{Tr}\,[D_i, A_0]^2 + m_{\mathrm{E}}^2 \mathrm{Tr}\, A_0^2 + \frac{ig^3}{3\pi^2}\sum_f \mu_f\, \mathrm{Tr}\, A_0^3 + \delta\mathcal{L}_{\mathrm{E}}$$

See Aleksi Vuorinen's talk (on Tuesday)

Already known 4-loop result (at finite μ) [A. Vurinen; 2003]

BUT

- Effect of the strange quark mass within 5%
  Ratio of free gas result [M. Laine, Y. Schroder; 2006]:
  $$p(m_s) \approx \frac{p_{SB}(m_s)}{p_{SB}(m_s\!=\!0)} \times p(0)$$
- New resummation scheme applied to the finite μ case
  Keep the EQCD parameters unexpanded
  [M. Laine, Y. Schroder; 2006]
  $\Rightarrow$ Resums a certain class of diagrams to all order
  $\Rightarrow$ Substantially reduces the renormalization scale dependance!

As a preview,...

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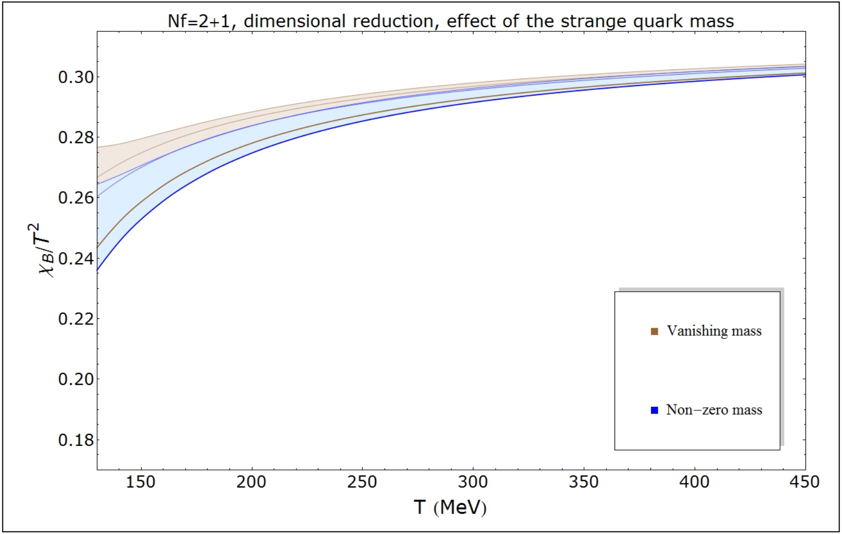
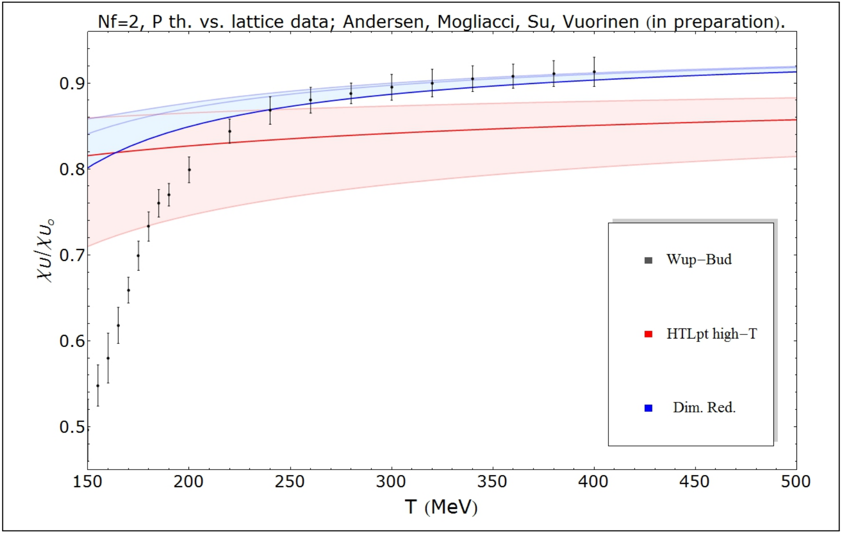
Effect of the strange quark mass in DR

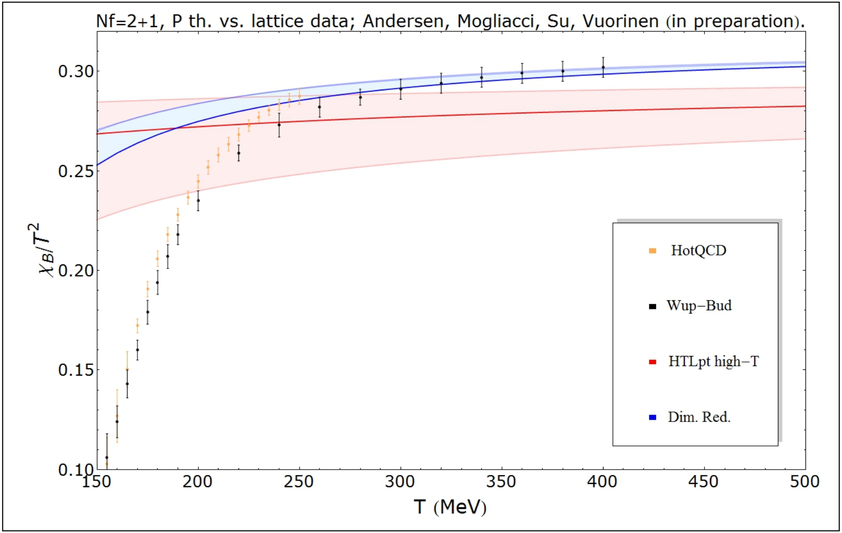


- Running of α_S: 1-loop for HTLpt, and 2-loop for DR
- Renormalization scale varied by a factor of 2 around:
  $\Lambda_{\text{Ren.}}^{N_f=2-3} \approx (1.29 - 1.44) \times 2\pi T$ for DR (optimal scale)
  [K. Kajantie et al.; 1997]
  $\Lambda_{\text{Ren.}}^{N_f=2-3} \equiv 2\pi T$ for HTLpt
- QCD scale:
  $\Lambda_{\overline{MS}}^{N_f=2-3} = 180 - 200$ MeV for DR (fitted to HRG pressure)
  $\Lambda_{\overline{MS}}^{N_f=2-3} \approx 140 - 160$ MeV for HTLpt (using lattice α_S value)
- Comparison to recent high precision lattice data:
  [S. Borsanyi et al., A. Bazavov et al.; 2012]

$N_f = 2$ perturbation theory vs Lattice data



$N_f = 2 + 1$ perturbation theory vs Lattice data



- Computation of χ in QCD via 2 different resummation schemes
- The results are consistent with recent lattice data down to $T \approx 250 - 300$ MeV!!
- HTLpt results are from the high T truncation of the free quasiparticle gas (1-loop)
  Seems to account for a next to leading order computation (full computation underway)

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BACKUP

Perturbative pressure and problem of apparent convergence

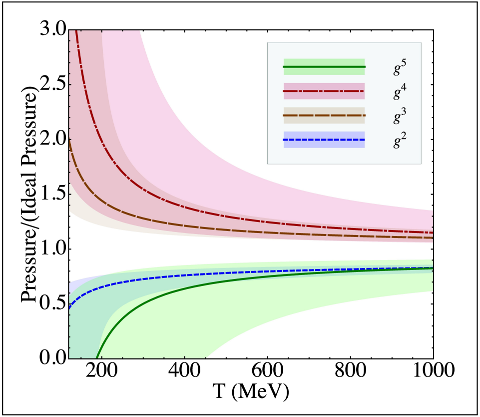

Figure : Weak coupling expansion of the pressure up to g^5.

HTLpt pressure up to 3-loops

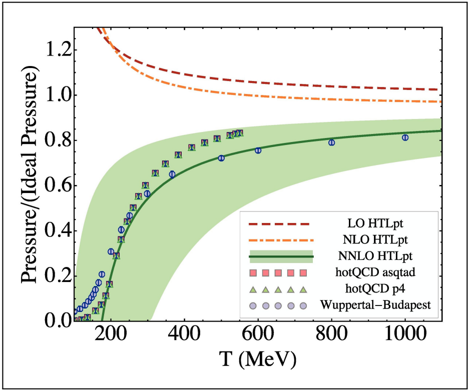

Figure : HTLpt pressure for QCD with $N_f = 3$ up to 3-loops at zero μ. Reference: [J.O. Andersen, L.E. Leganger, M. Strickland, N. Su; 2011].