Fluctuat Nec Mergitur:
Event-by-Event Fluctuations in Relativistic Heavy Ion Collisions

BM & A. Schäfer,
PRD 85 (2012) 114030

J. Kapusta, BM & M.Stephanov,
PRC 85 (2012) 054906

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Int. School on Nuclear Physics
34th Course
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Sources of fluctuations

- Initial-state fluctuations
  - Quantum fluctuations of the nucleon distribution
  - Quantum fluctuations of fast moving color charges
  - Quantum fluctuations of the gauge field
- Hydrodynamic fluctuations
- Fluctuations during hadronization
- Jet-medium interactions
  - Mach cones?

- Initial-state fluctuations give rise to final correlations at the largest rapidity difference !

Initial state fluctuations

Rapidity correlations

Initial separation: $\Delta x_0 = \Delta\left(\tau_0 \sinh \eta\right) \approx t_0 \Delta\eta \quad \rightarrow \quad \Delta\eta \approx \frac{\Delta x_0}{t_0}$

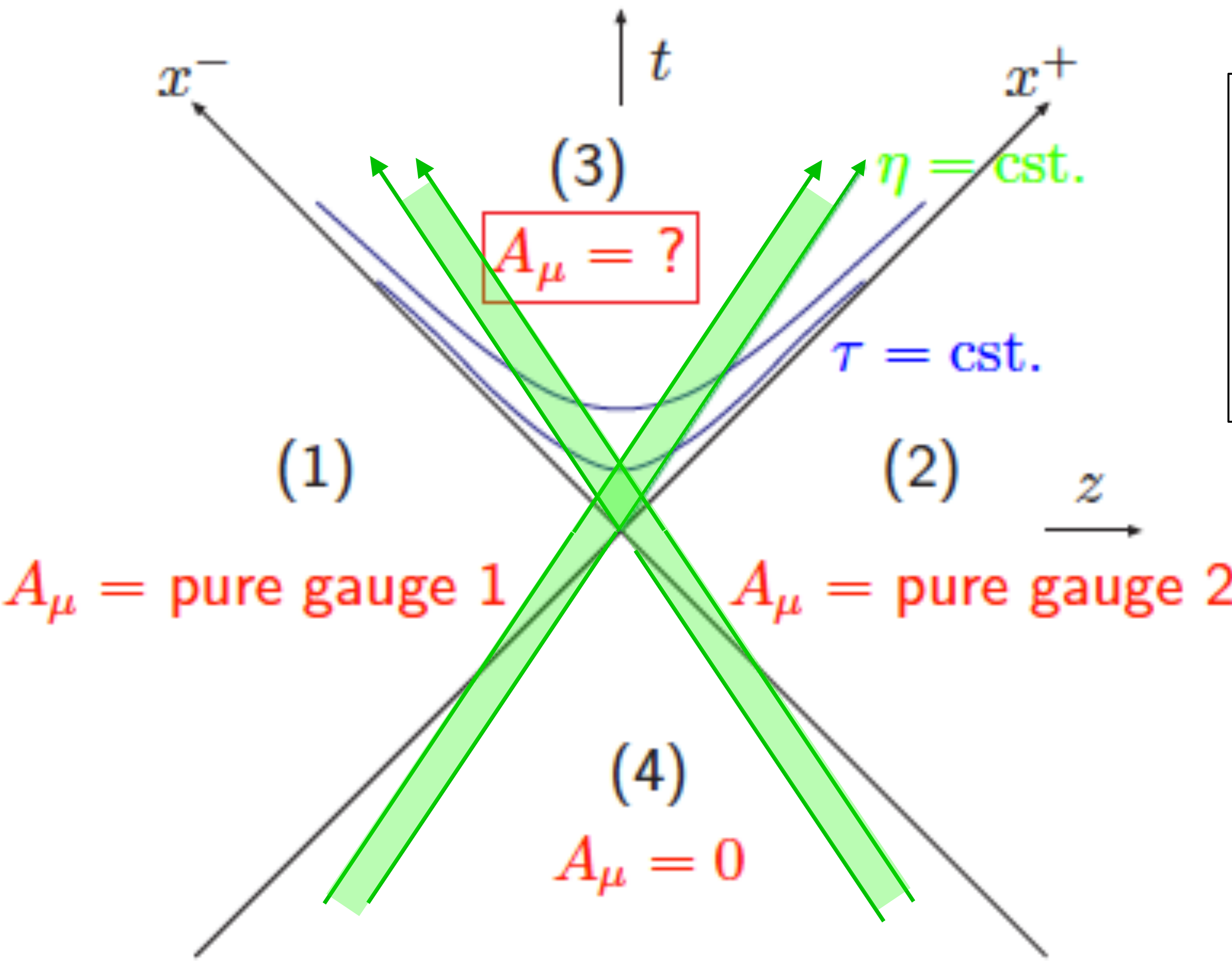
Relevant length scales for correlated fluctuations:

$\Delta x_0 \sim R/\gamma$	Contracted nuclei
$\Delta x_0 \sim 1/\alpha_s Q_s$	CGC coherence length
$\Delta x_0 \sim 1/T$	Thermal noise correlation length

Hydro propagation in longitudinally expanding medium increases separation by:

$$\Delta x_s = 2\mathrm{v}_s \ln\left(\tau_f / \tau_0\right)$$

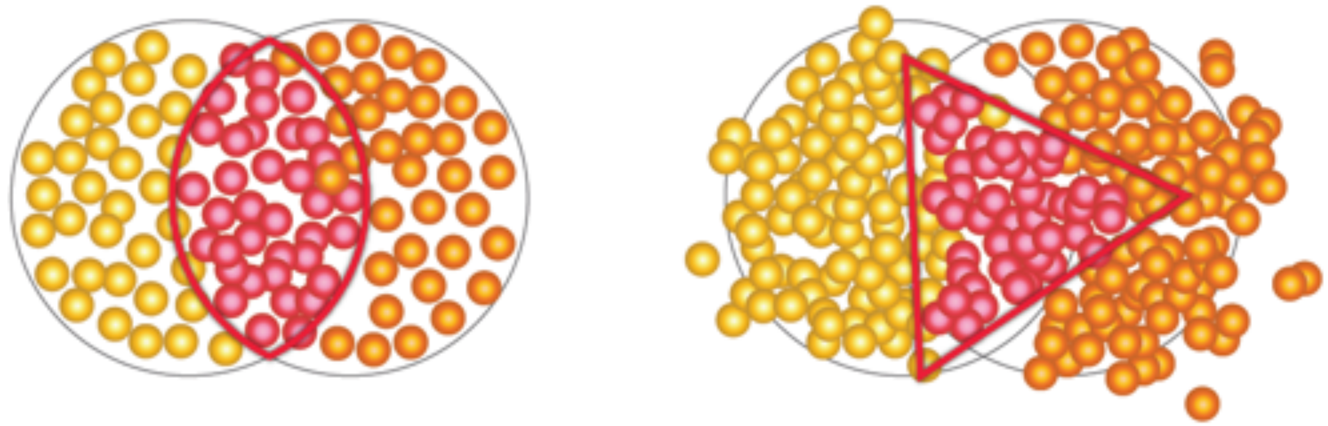
Space-time picture



Lappi, PLB 643 (2006) 11

$$
\begin{aligned}
A^i &= A^i_{(1)} + A^i_{(1)} \\
A^\eta &= \frac{ig}{2}\left[A^i_{(1)}, A^i_{(2)}\right] \\
\partial_\tau A_i &= 0 \\
\partial_\tau A^\eta &= 0.
\end{aligned}
$$

Color charge fluctuations



Quantum fluctuations in the positions of the colliding nucleons give rise to a position dependent density of valence partons and other hard partons: $\mu^2(x)$.

For given μ, color charges of the partons combine in a random walk in SU(3). This generates an approximately Gaussian distribution of color charges $\rho^a(x)$.

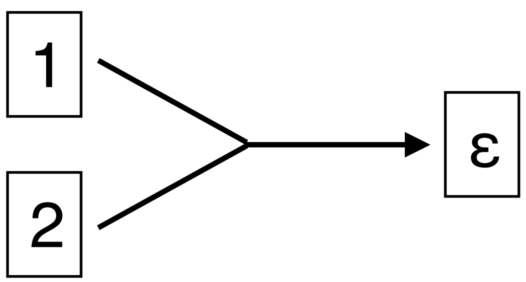
$$P[\rho] \propto \exp\left(-\frac{1}{2g^2\mu^2} \int d^2x \, \rho^a(x)\rho^a(x) \right)$$

Neglected: transverse correlations among color charges, x-dependence of μ, confinement related effects, etc.

Energy density fluctuations

Quantity to calculate: $\langle \varepsilon(\mathbf{x})\varepsilon(\mathbf{y})\rangle - \langle \varepsilon(\mathbf{x})\rangle \langle \varepsilon(\mathbf{y})\rangle$

Energy density deposited by two colliding sheets of CGC:



$$\varepsilon(\mathbf{x}) = \frac{1}{4}F^c_{ij}(\mathbf{x})F^c_{ij}(\mathbf{x}) + 2A^{\eta c}(\mathbf{x})A^{\eta c}(\mathbf{x})$$

$$F^c_{ij}(\mathbf{x}) \; = \; g f_{abc}\Big(A^a_i(1;\mathbf{x})A^b_j(2;\mathbf{x}) + A^a_i(2;\mathbf{x})A^b_j(1;\mathbf{x})\Big)$$

$$A^{\eta c}(\mathbf{x})A^{\eta c}(\mathbf{x}) \; = \; \frac{g^2}{4} f_{abc}f_{a'b'c} \;\; A^a_i(1;\mathbf{x})A^b_i(2;\mathbf{x})A^{a'}_j(1;\mathbf{x})A^{b'}_j(2;\mathbf{x})$$

CGC field correlator (light-cone gauge):

$$\Big\langle A^a_i(n;\mathbf{x})A^b_j(m;\mathbf{y})\Big\rangle \; = \; \delta_{mn}\delta_{ab} \int \frac{d^2p}{(2\pi)^2} \; \cos[\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})] \; \frac{p_i p_j}{\mathbf{p}^2} \; G(|\mathbf{p}|)$$

$$G(|\mathbf{x}|) = \frac{4}{g^2 N |\mathbf{x}|^2}\left[1 - \exp\left(\frac{g^2 N}{8\pi} g^2 \mu^2 |\mathbf{x}|^2 \ln(\Lambda|\mathbf{x}|)\right)\right] \; \Theta(1 - \Lambda|\mathbf{x}|)$$

$\langle \varepsilon \rangle$

Decompose p-quadrupole tensor: $p_i p_j = \frac{p_1^2 + p_2^2}{2}\,\delta_{ij} + \frac{p_1^2 - p_2^2}{2}\,\sigma_{ij}^3 + p_1 p_2\,\sigma_{ij}^1$

$$\left\langle A_i^a(n;\mathbf{x}) A_j^b(m;\mathbf{y}) \right\rangle = \frac{1}{2}\,\delta_{mn}\delta_{ab}\left(\delta_{ij} D(\mathbf{x}-\mathbf{y}) + \sigma_{ij}^3 E(\mathbf{x}-\mathbf{y}) - \sigma_{ij}^1 F(\mathbf{x}-\mathbf{y})\right)$$

$$D(0) = \int \frac{d^2p}{(2\pi)^2}\, G(|\mathbf{p}|) = \lim_{|\mathbf{x}|\to 0} G(|\mathbf{x}|); \qquad E(0) = F(0) = 0.$$

$G(0)$ diverges logarithmically; divergence can be regulated by “triumvirate” coupling:

$g^4 \longrightarrow g^2(\mu^2) g^2(1/|\mathbf{x}|^2)$ where $g^2(1/x^2) = \frac{16\pi^2}{9\ln(1/(\Lambda^2 x^2))}$

Average energy density: $\langle \varepsilon(\mathbf{x}) \rangle = \frac{g^2}{2} N(N^2-1) D^2(0)$

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Note: Logarithmic divergence exists only for τ = 0. UV components of glasma field get out of phase for τ > 0 and render ε finite [Fries, Kapusta & Li; Lappi].

$\langle\varepsilon(x)\varepsilon(y)\rangle$

$$
\begin{aligned}
\langle\varepsilon(\mathbf{x})\varepsilon(\mathbf{y})\rangle \;=\; & \frac{g^4}{4} f_{abc} f_{a'b'c} f_{efd} f_{e'f'd} \Big(\Big\langle A_i^a(\mathbf{x}) A_i^{a'}(\mathbf{x}) A_m^e(\mathbf{y}) A_m^{e'}(\mathbf{y})\Big\rangle\Big\langle A_j^b(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_n^f(\mathbf{y}) A_n^{f'}(\mathbf{y})\Big\rangle \\
& + \Big\langle A_i^a(\mathbf{x}) A_i^{a'}(\mathbf{x}) A_m^e(\mathbf{y}) A_n^{f'}(\mathbf{y})\Big\rangle\Big\langle A_j^b(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_n^f(\mathbf{y}) A_m^{e'}(\mathbf{y})\Big\rangle \\
& + \Big\langle A_i^a(\mathbf{x}) A_i^{a'}(\mathbf{x}) A_m^e(\mathbf{y}) A_n^{e'}(\mathbf{y})\Big\rangle\Big\langle A_j^b(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_m^f(\mathbf{y}) A_n^{f'}(\mathbf{y})\Big\rangle \\
& + \Big\langle A_i^a(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_m^e(\mathbf{y}) A_m^{e'}(\mathbf{y})\Big\rangle\Big\langle A_j^b(\mathbf{x}) A_i^{a'}(\mathbf{x}) A_n^f(\mathbf{y}) A_n^{f'}(\mathbf{y})\Big\rangle \\
& + \Big\langle A_i^a(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_m^e(\mathbf{y}) A_n^{f'}(\mathbf{y})\Big\rangle\Big\langle A_j^b(\mathbf{x}) A_i^{a'}(\mathbf{x}) A_n^f(\mathbf{y}) A_m^{e'}(\mathbf{y})\Big\rangle \\
& + \Big\langle A_i^a(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_m^e(\mathbf{y}) A_n^{e'}(\mathbf{y})\Big\rangle\Big\langle A_j^b(\mathbf{x}) A_i^{a'}(\mathbf{x}) A_m^f(\mathbf{y}) A_n^{f'}(\mathbf{y})\Big\rangle \\
& + \Big\langle A_i^a(\mathbf{x}) A_j^{a'}(\mathbf{x}) A_m^e(\mathbf{y}) A_m^{e'}(\mathbf{y})\Big\rangle\Big\langle A_i^b(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_n^f(\mathbf{y}) A_n^{f'}(\mathbf{y})\Big\rangle \\
& + \Big\langle A_i^a(\mathbf{x}) A_j^{a'}(\mathbf{x}) A_m^e(\mathbf{y}) A_n^{f'}(\mathbf{y})\Big\rangle\Big\langle A_i^b(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_n^f(\mathbf{y}) A_m^{e'}(\mathbf{y})\Big\rangle \\
& + \Big\langle A_i^a(\mathbf{x}) A_j^{a'}(\mathbf{x}) A_m^e(\mathbf{y}) A_n^{e'}(\mathbf{y})\Big\rangle\Big\langle A_i^b(\mathbf{x}) A_j^{b'}(\mathbf{x}) A_m^f(\mathbf{y}) A_n^{f'}(\mathbf{y})\Big\rangle\Big)
\end{aligned}
$$

Products of <AAAA> correlators of the CGC fields in the individual nuclei.

Gaussian approximation

Exact analytical evaluation of <AAAA> correlators in the CGC model is extremely complicated. In LC gauge, it requires evaluation of gauge links between **x** and **y** through (–∞) along the light-cone. Alternatively, one can use link representation:

$$A^i_{(1,2)}(\mathbf{x}_T) = \frac{i}{g} U_{(1,2)}(\mathbf{x}_T) \partial_i U^\dagger_{(1,2)}(\mathbf{x}_T)$$

with $$U_{(1,2)}(\mathbf{x}_T) = P \exp\left\{ -ig \int \mathrm{d}x^\pm \frac{1}{\boldsymbol{\nabla}_T^2} \rho_{(1,2)}(\mathbf{x}_T, x^\pm) \right\}$$

Generates infinite series of terms $\langle \rho^n \rangle$.

For techniques for an effective evaluation of $\langle U^4 \rangle$ correlators in the Gaussian approximation using color dipoles, see: Marquet & Weigert [NPA 843 (2010) 68].

General discussion for the validity of the Gaussian approximation:
Iancu & Triantafyllapoulos, arXiv:1112.1104 [hep-ph].

Abelian Gaussian limit

To obtain a simple result, we make the Gaussian approximation for the <AAAA> correlator, which assumes Gaussian charge correlations and abelian dominance. Rationale: ε is dominated by high-field regions, where fields are approximately abelian.

$$\left\langle A_j^b(\mathbf{x}) A_i^{a'}(\mathbf{x}) A_n^f(\mathbf{y}) A_m^{e'}(\mathbf{y}) \right\rangle = \left\langle A_j^b(\mathbf{x}) A_i^{a'}(\mathbf{x}) \right\rangle \left\langle A_n^f(\mathbf{y}) A_m^{e'}(\mathbf{y}) \right\rangle + \left\langle A_j^b(\mathbf{x}) A_n^f(\mathbf{y}) \right\rangle \left\langle A_i^{a'}(\mathbf{x}) A_m^{e'}(\mathbf{y}) \right\rangle + \left\langle A_j^b(\mathbf{x}) A_m^{e'}(\mathbf{y}) \right\rangle \left\langle A_n^f(\mathbf{y}) A_i^{a'}(\mathbf{x}) \right\rangle$$

Many lines of algebra later ..... one finds a simple result:

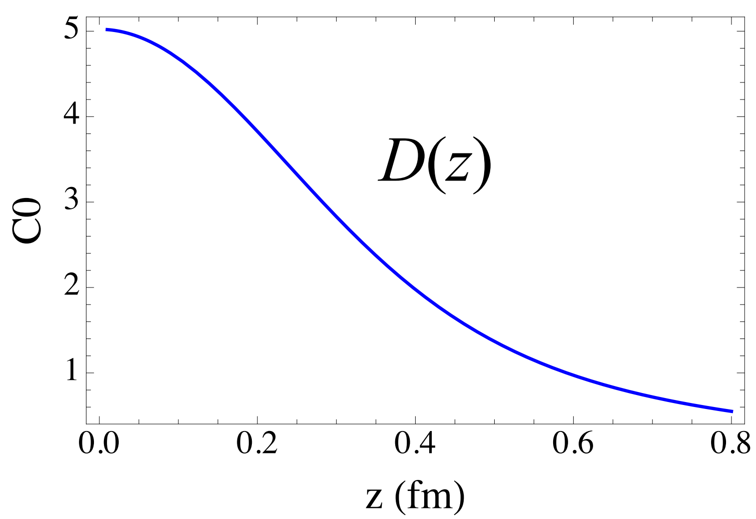
$$\langle \varepsilon(x)\varepsilon(y)\rangle - \langle \varepsilon(x)\rangle\langle \varepsilon(y)\rangle = \frac{g^4}{2} N^2(N^2-1) D(0)^2 K(x-y) + \frac{7g^4}{16} N^2(N^2-1) K(x-y)^2$$

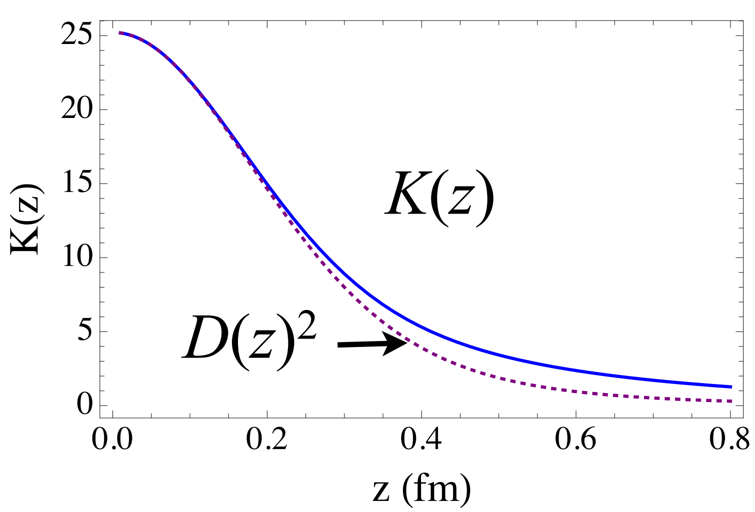
$$K(z) = D(z)^2 + E(z)^2 + F(z)^2$$

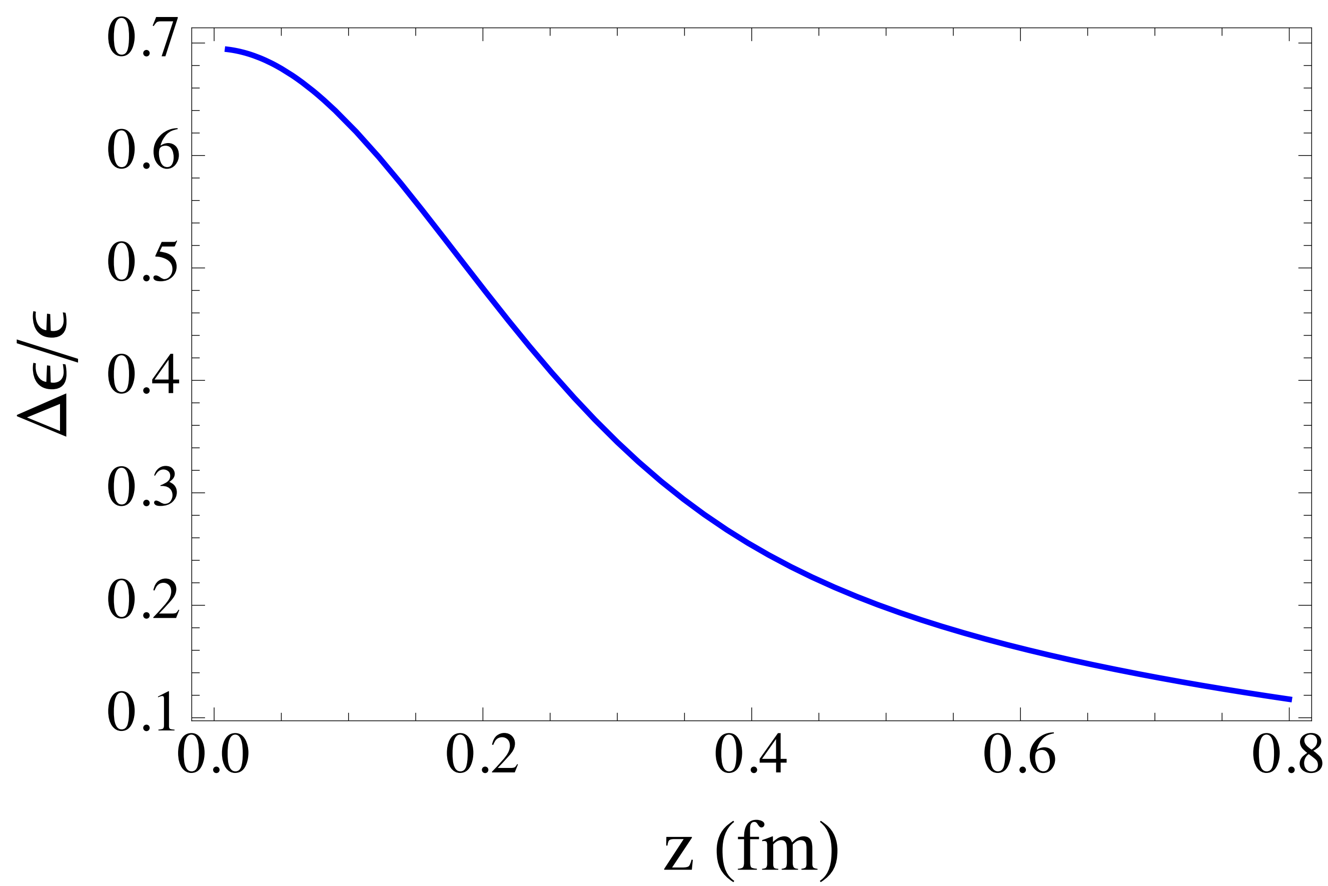
Δε/ε is large

Parameter choices:

$$Q_s^2 = (g^2\mu)^2 = 2\ \mathrm{GeV}^2;$$
$$g^2(\mu^2) = 3.785;$$
$$g^2(1/x^2) = \frac{16\pi^2}{9\ln(1/(\Lambda^2 x^2))}.$$







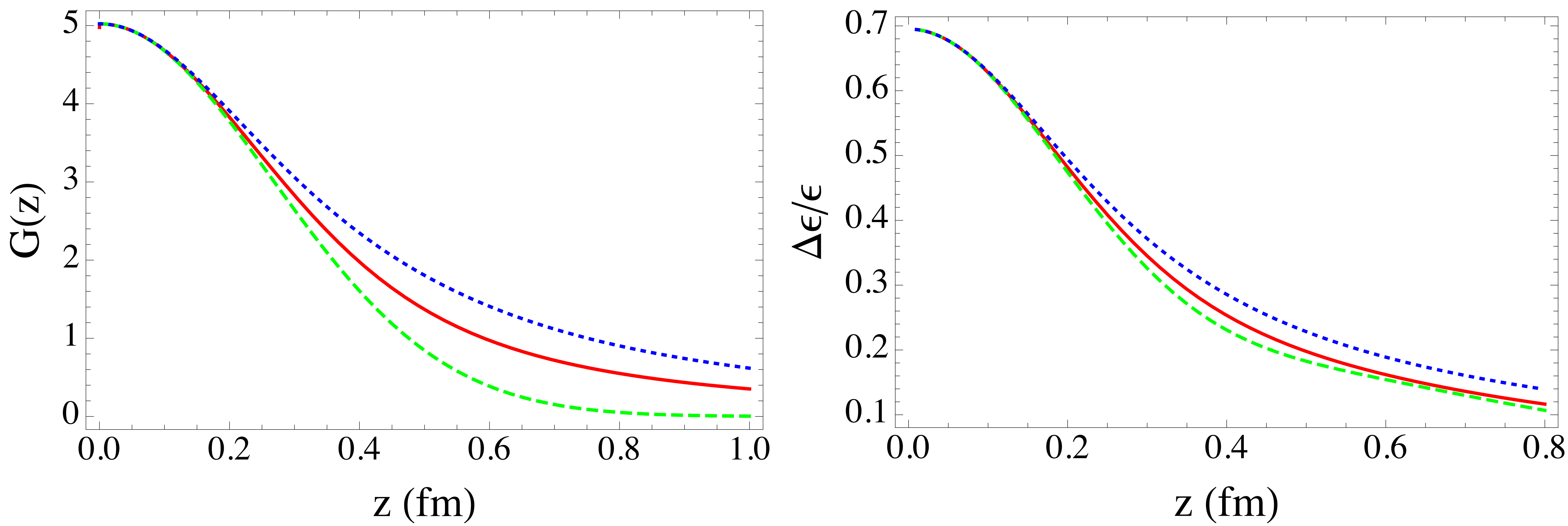
Alternative models

$$G(z) = G_0\,\phi(z^2/\xi^2) \quad \text{with} \quad G_0 = \tfrac{4}{9}\pi\mu^2, \quad 1/\xi^2 = \tfrac{1}{9}N\pi(g\mu)^2$$

$$\phi_{\mathrm{MV}}(u) = (1 - e^{-u})/u$$

$$\phi_1(u) = e^{-u/2}$$

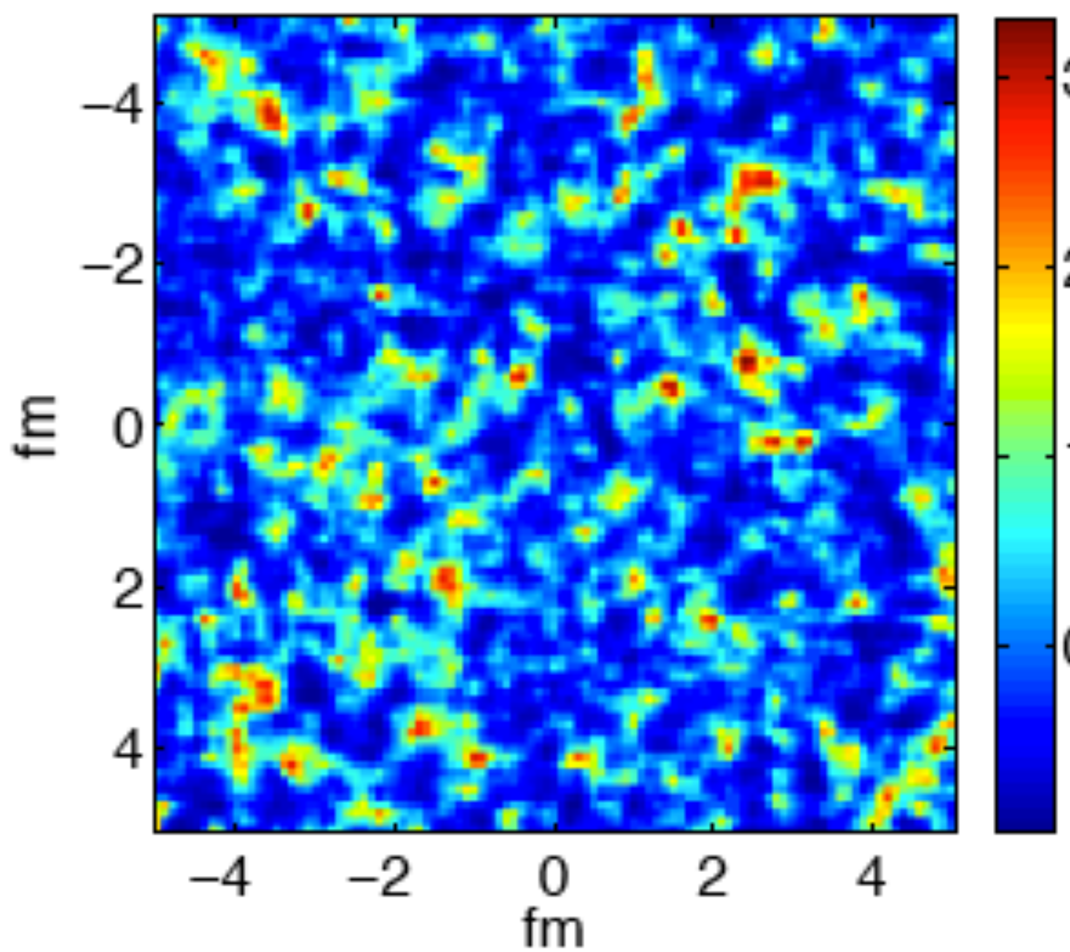
$$\phi_2(u) = \left(1 + \tfrac{u}{2}\right)^{-1}$$

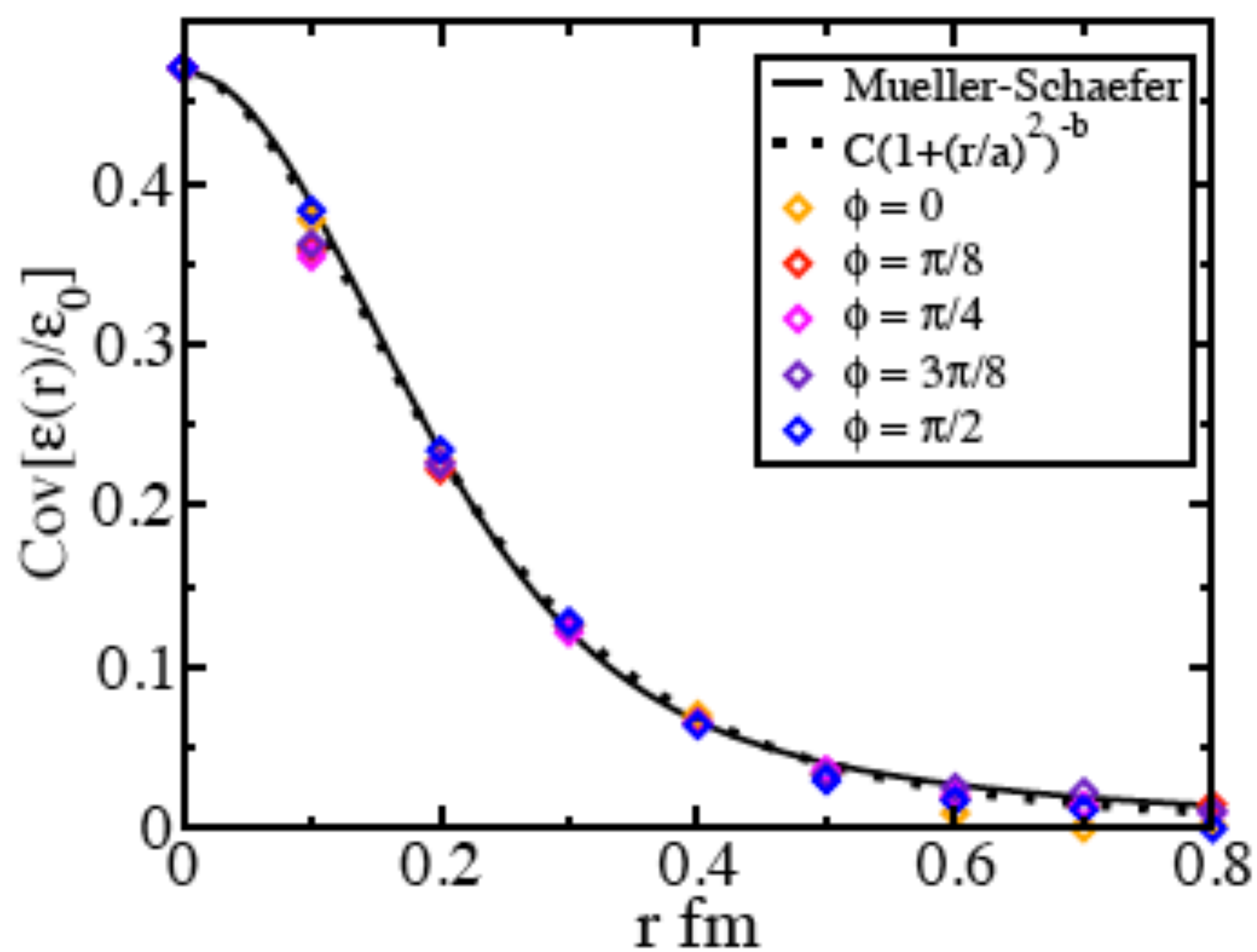


Implementation 1

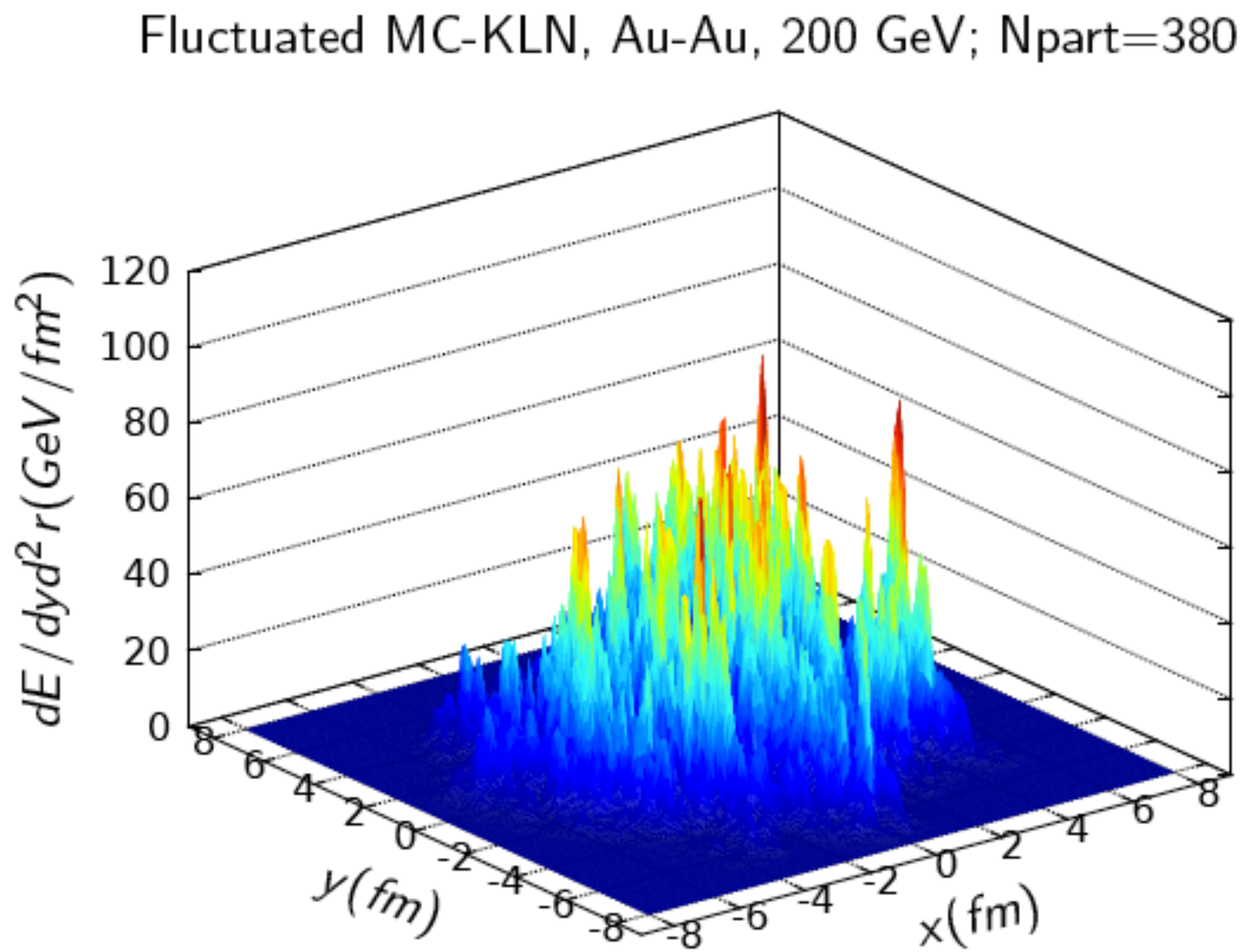
S. Moreland (OSU/Duke), Z. Qiu((OSU), Talk at QM2012

Map Gaussian Δε, <ε> on positive definite negative binomial distribution with same mean and standard deviation, generate random sample:



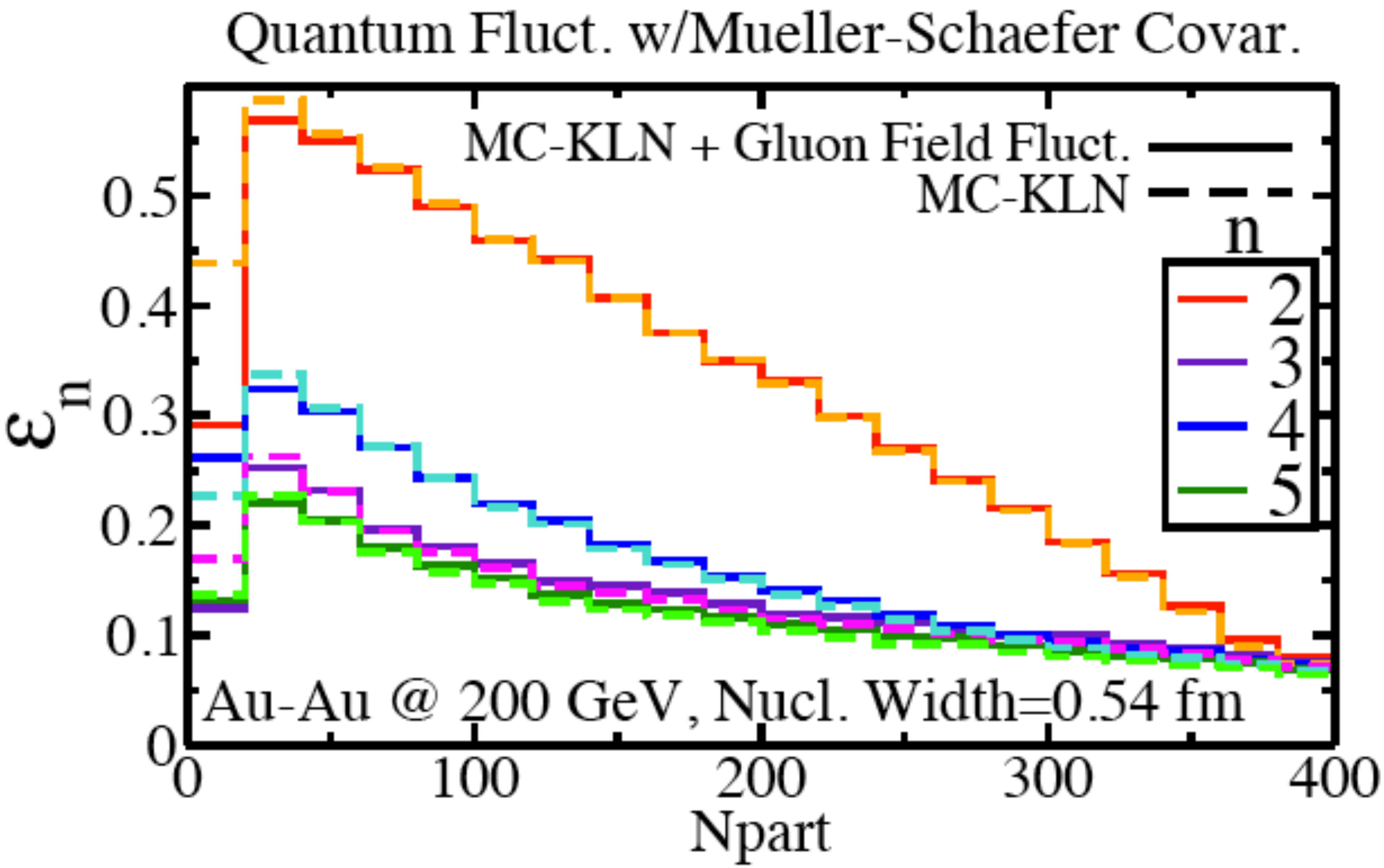


ε(x,y) in central event



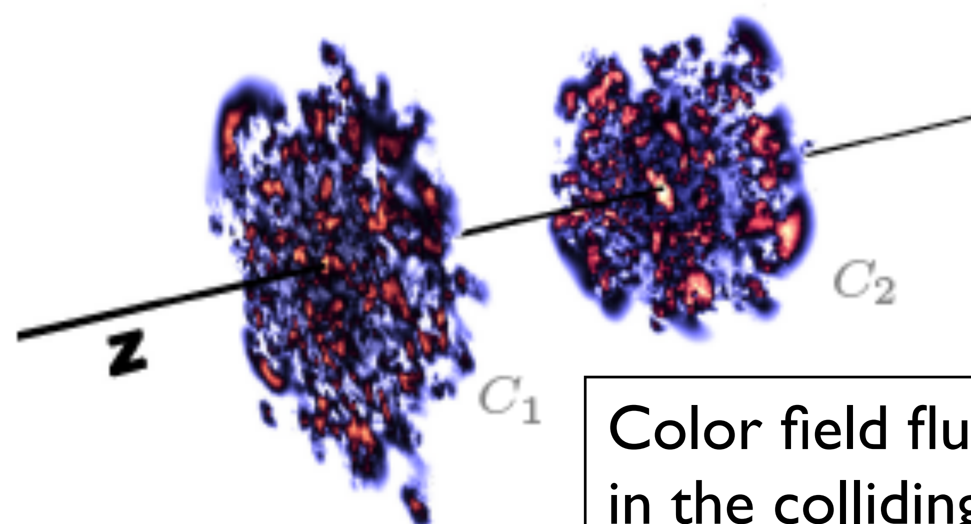
No influence on ε_n ?

Using definition: $\epsilon_n e^{in\Psi_n^{PP}} = -\frac{\int dxdy r^2 e^{in\phi}\rho(x,y)}{\int dxdy r^2 \rho(x,y)}$

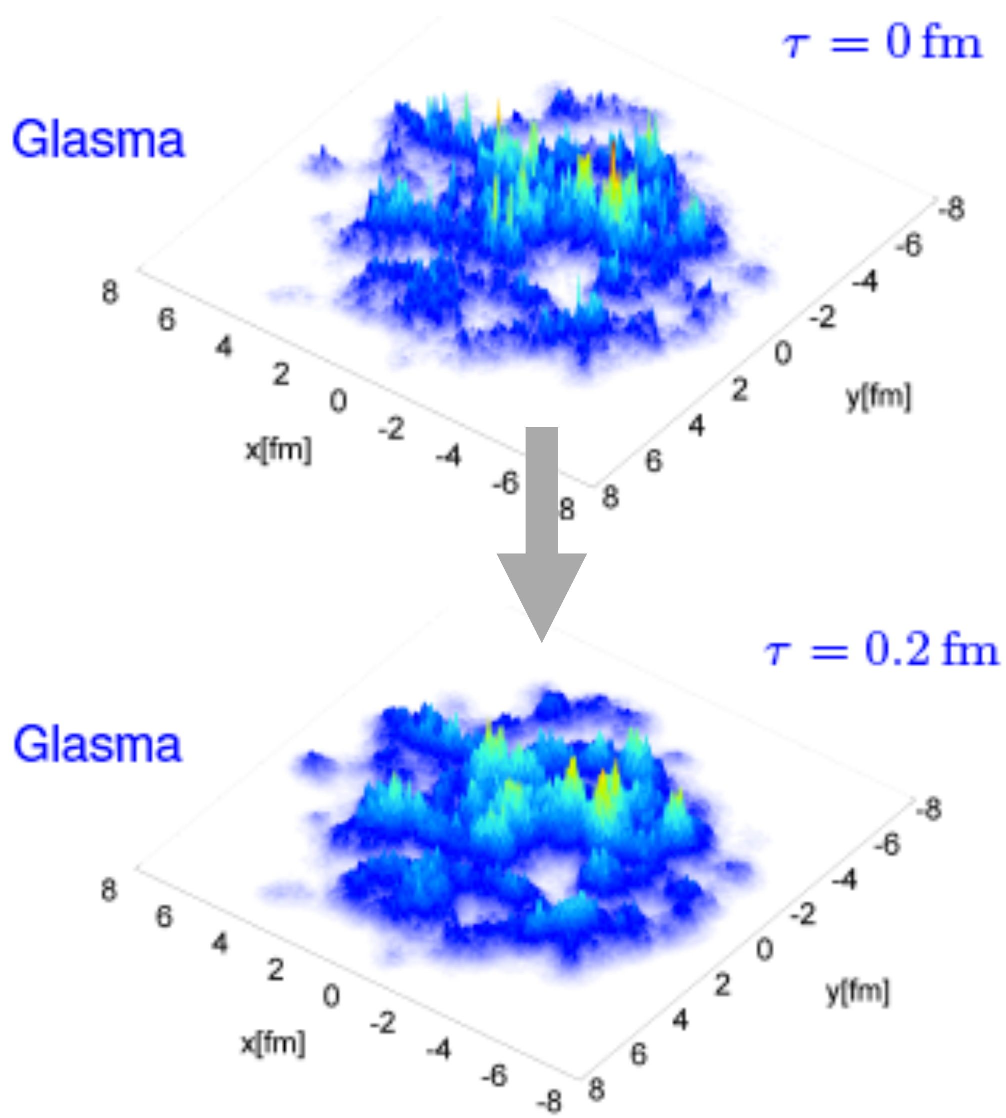


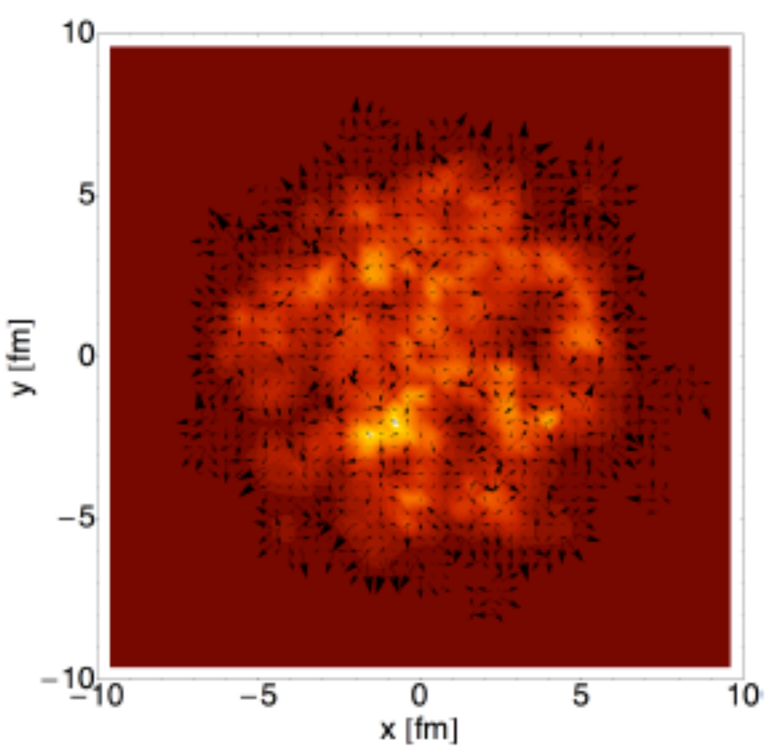
Implementation 2

Numerical implementation of color field fluctuations: Schenke, Tribedy & Venugopalan, arXiv:1202.6646, 1206.6805



Color field fluctuations in the colliding nuclei



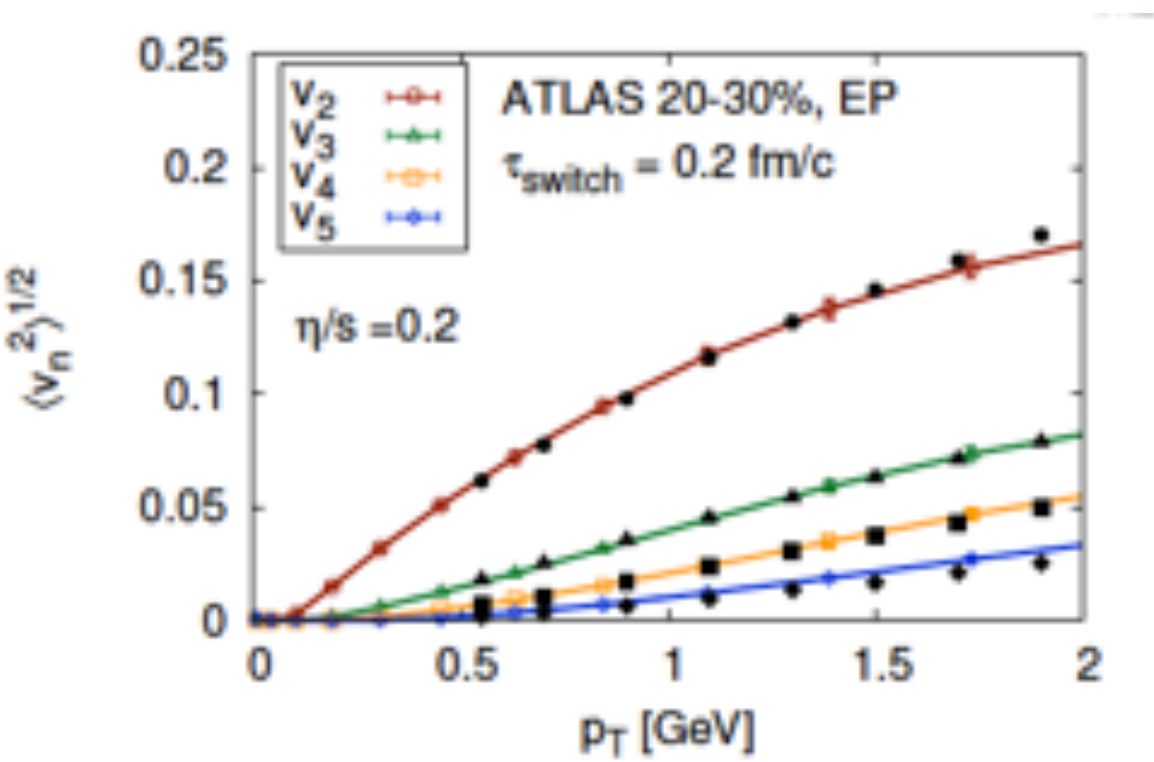


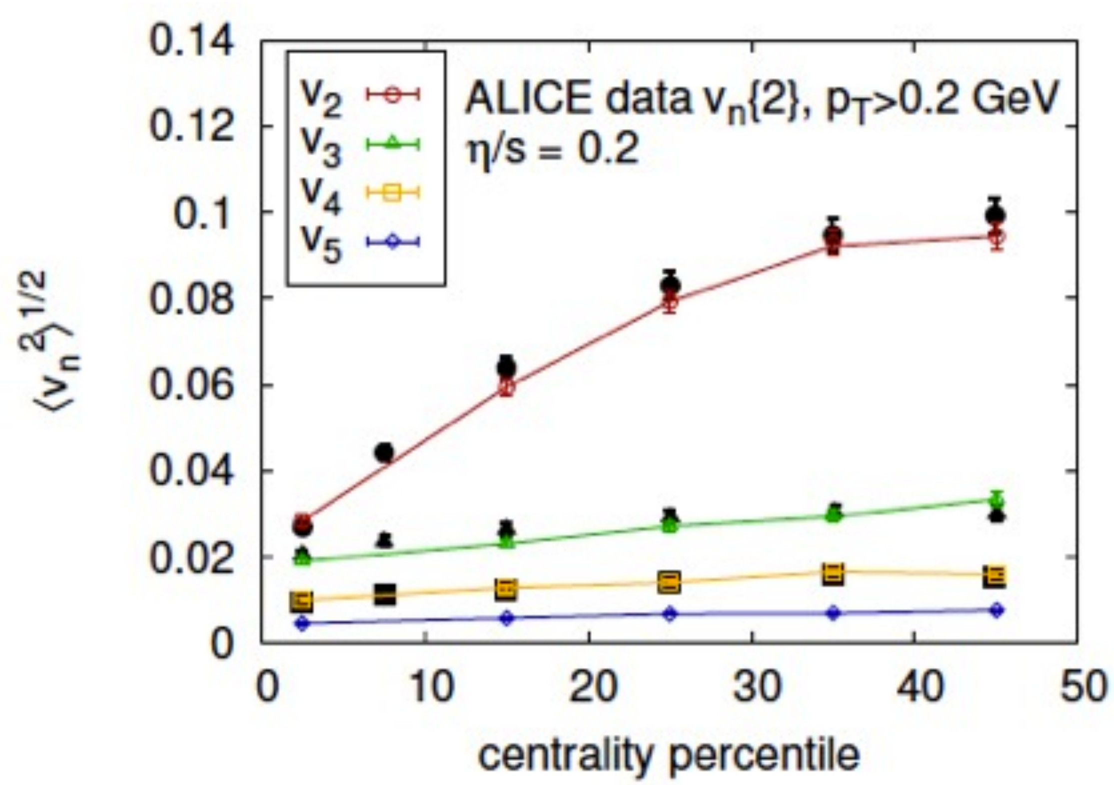
Energy density and (u_x, u_y) at $\tau = 0.4\,\mathrm{fm}/c$

Flow analysis and LHC data

Schenke, Tribedy, Venugopalan (QM 2012):

Nice agreement with Pb+Pb data for $\eta/s = 0.2$ in $v_n(p_T)$ and $v_n(b)$





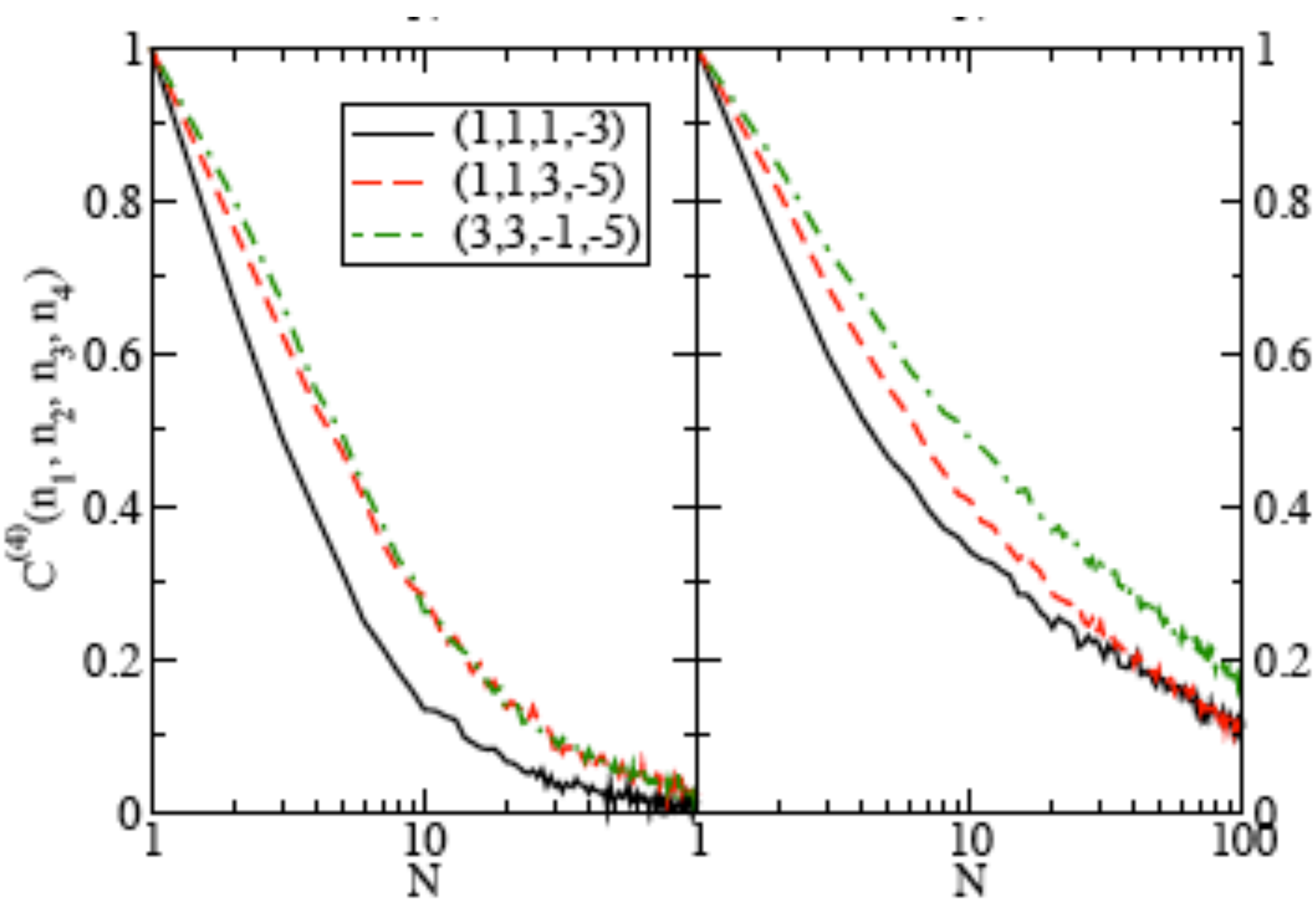
Inconsistency with Moreland et al: ε_n are influenced by finer gluonic fluctuations!

Counting hot/cold spots

$$C^{(k)}(n_1,...,n_k) = (-1)^k \left\langle \cos\left(n_1\Psi_1 + \cdots n_k\Psi_k\right)\right\rangle$$

ψ_m = event plane angle

G. Qin & BM, PRC 85 (2012) 061901



Hydrodynamic fluctuations

PRL 98, 064504 (2007) PHYSICAL REVIEW LETTERS week ending 9 FEBRUARY 2007

Universality Crossover of the Pinch-Off Shape Profiles of Collapsing Liquid Nanobridges in Vacuum and Gaseous Environments

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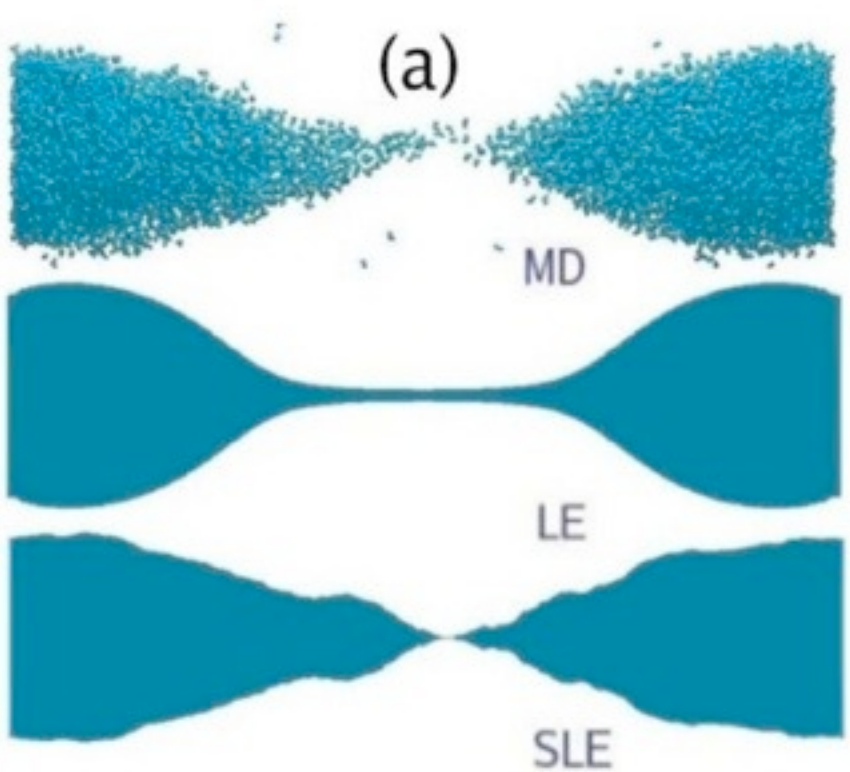
(Received 5 June 2006; published 7 February 2007)

Liquid propane nanobridges were found through molecular dynamics simulations to exhibit in vacuum a symmetric break-up profile shaped as two cones joined in their apexes. With a surrounding gas of sufficiently high pressure, a long-thread profile develops with an asymmetric shape. The emergence of a long-thread profile, discussed previously for macroscopic fluid structures, originates from the curvature-dependent evaporation-condensation processes of the nanobridge in a surrounding gas. A modified stochastic hydrodynamic description captures the crossover between these universal break-up regimes.

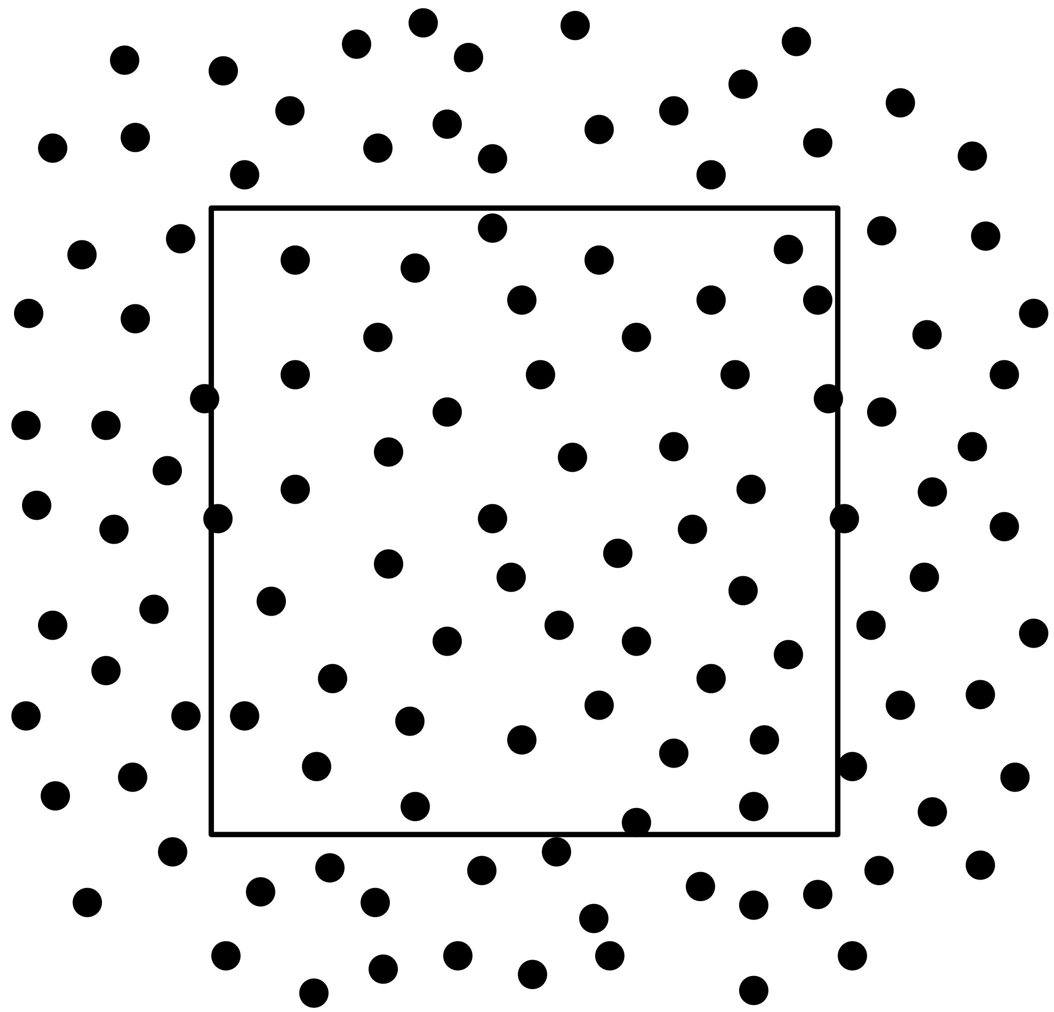


Molecular Dynamics

Lubrication Equation

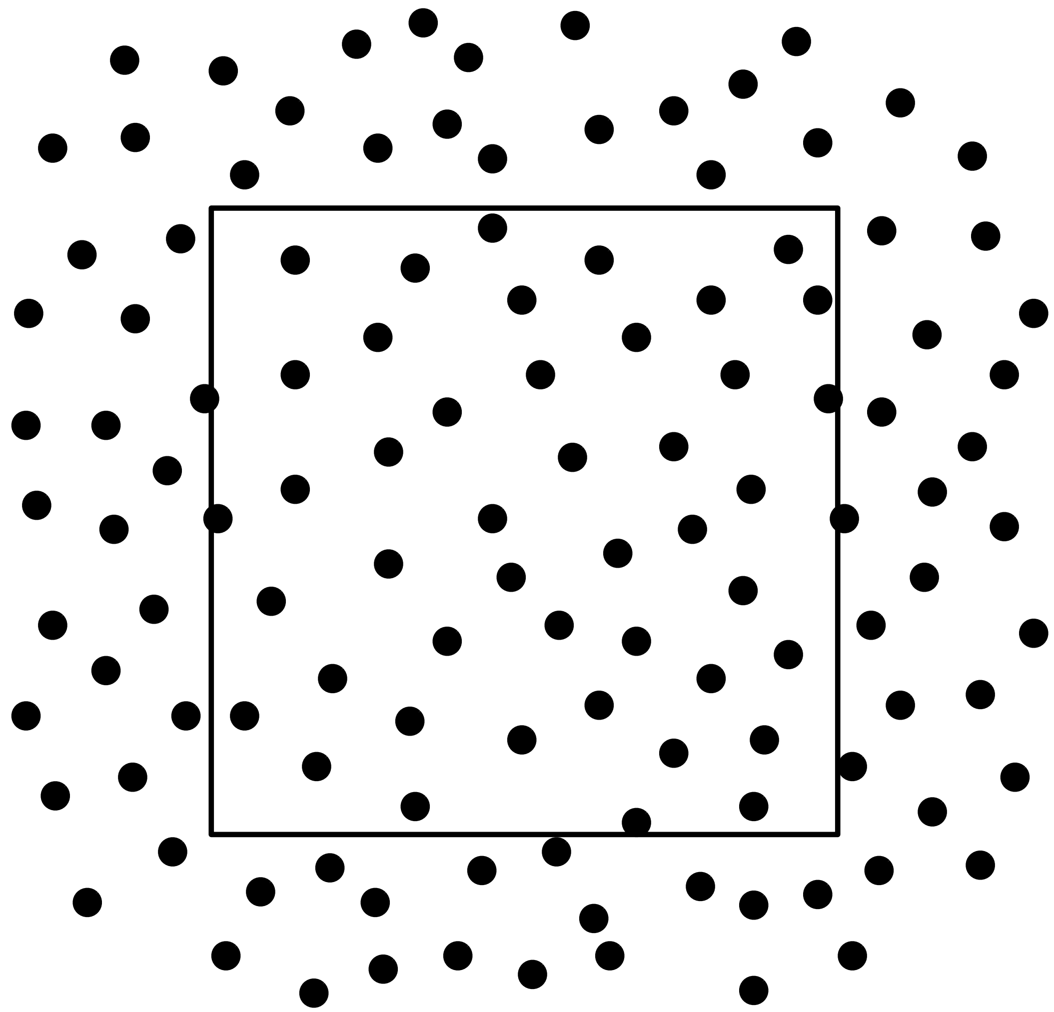
Stochastic Lubrication Equation

Particle number fluctuations

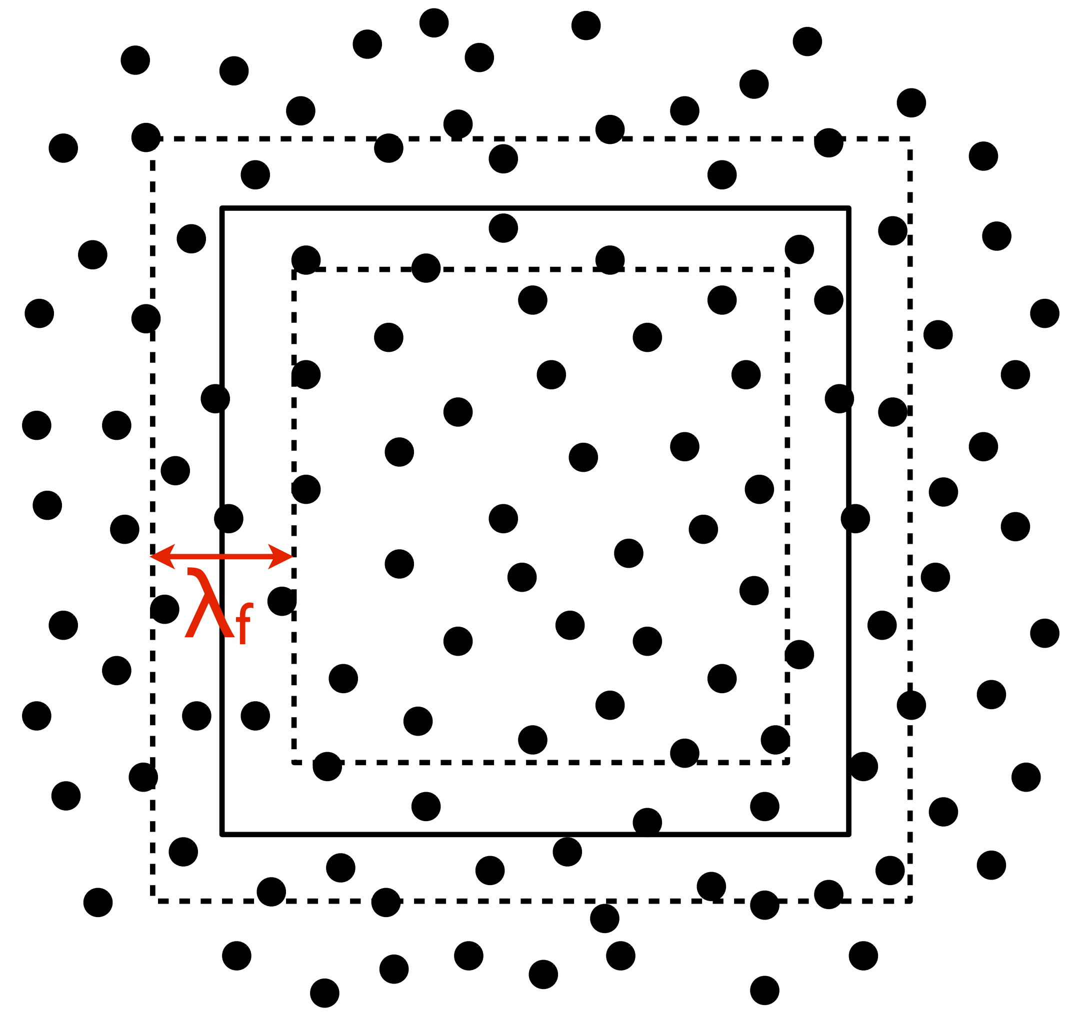


$(\Delta N)^2 \sim N \sim \rho V$?

Particle number fluctuations

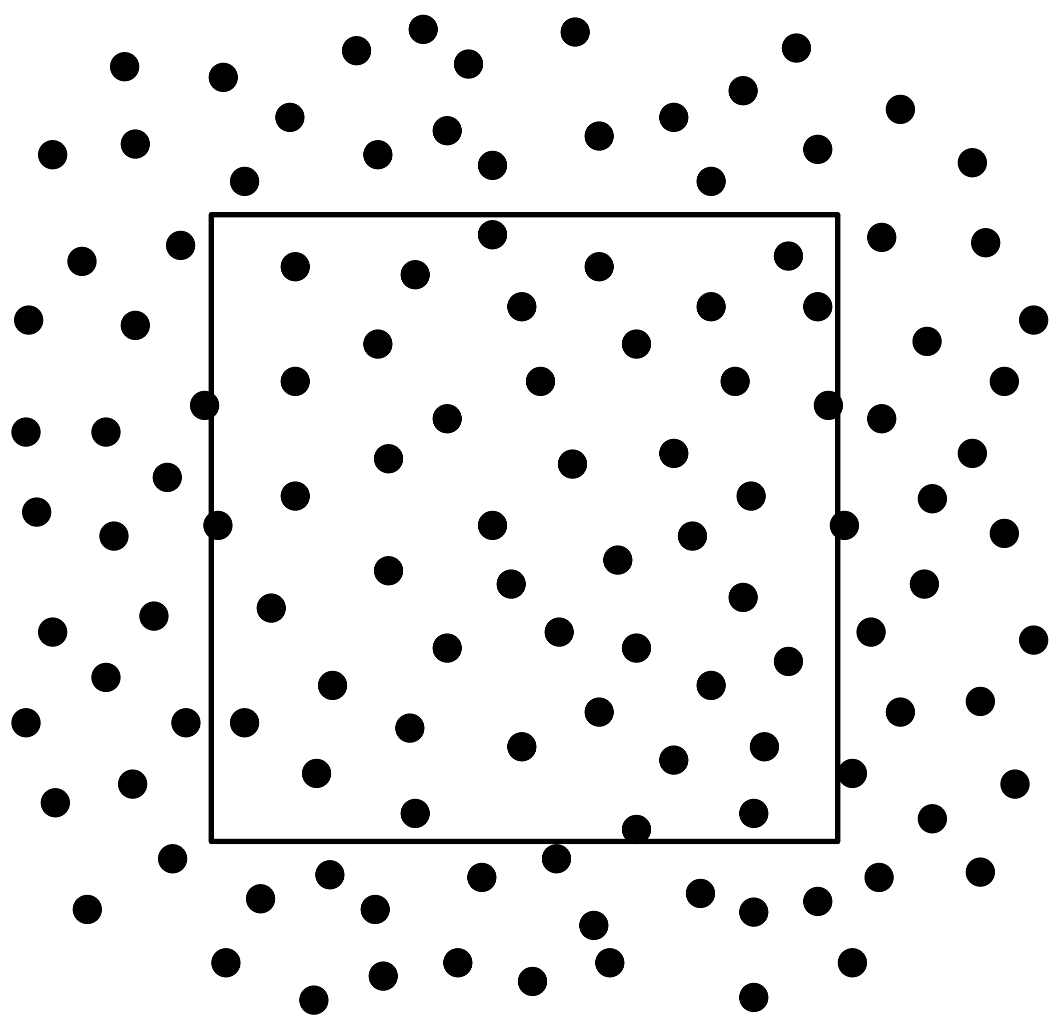


$(\Delta N)^2 \sim N \sim \rho V$?

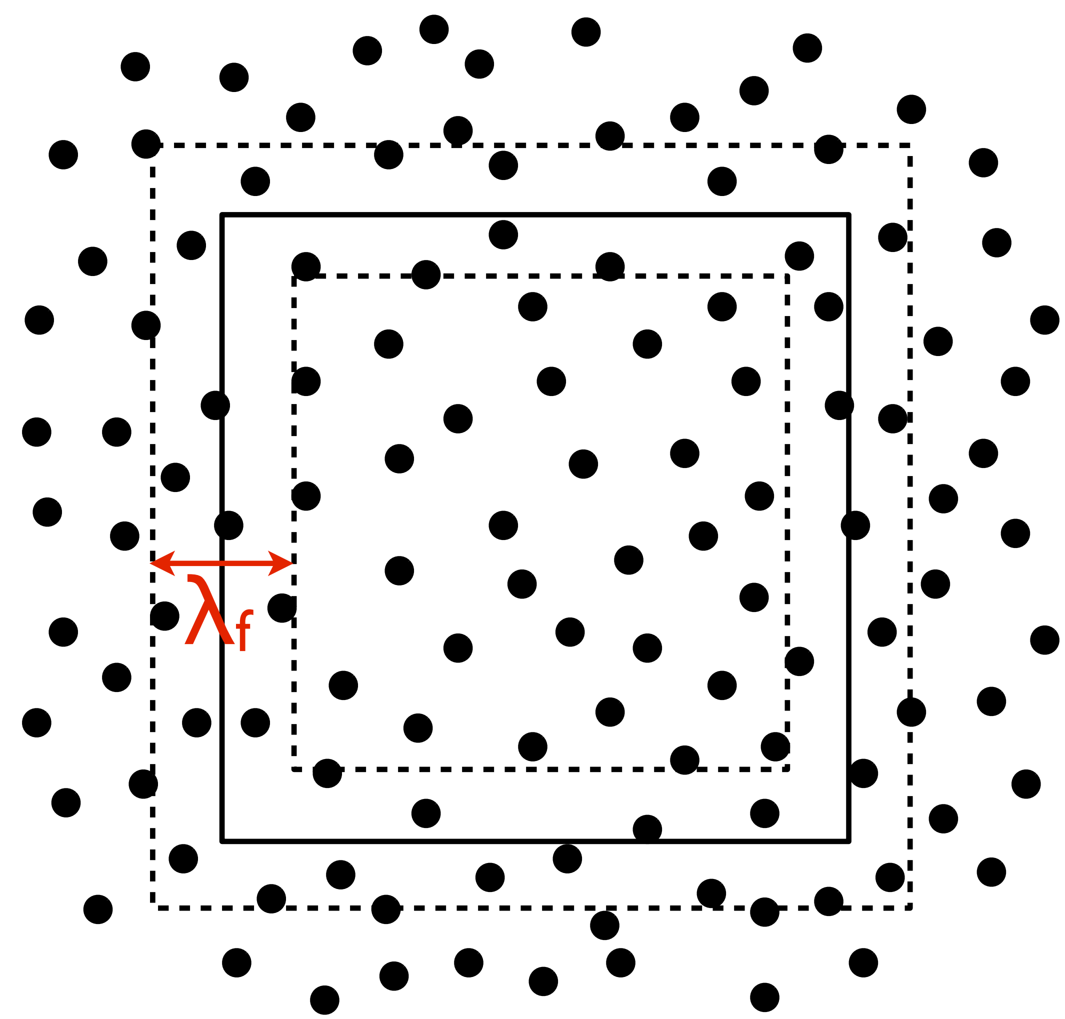


$(\Delta N)^2 \sim \rho\lambda_f$

Particle number fluctuations



$(\Delta N)^2 \sim N \sim \rho V$?



$(\Delta N)^2 \sim \rho\lambda_f$

$(\Delta p)^2 \sim \rho\lambda_f\, p \sim \eta$

Relativistic Dissipative Fluid Dynamics

$$T^{\mu\nu} = -Pg^{\mu\nu} + wu^{\mu}u^{\nu} + \Delta T^{\mu\nu}$$
$$J_B^{\mu} = n_B u^{\mu} + \Delta J_B^{\mu}$$

In the Landau-Lifshitz approach u^{μ} is the velocity of energy transport.

$$\Delta T^{\mu\nu} = \eta\left(\Delta^{\mu}u^{\nu} + \Delta^{\nu}u^{\mu}\right) + \left(\tfrac{2}{3}\eta - \zeta\right)H^{\mu\nu}\partial_{\rho}u^{\rho}$$

$$H^{\mu\nu} \equiv u^{\mu}u^{\nu} - g^{\mu\nu}, \quad \Delta_{\mu} \equiv \partial_{\mu} - u_{\mu}u^{\beta}\partial_{\beta}, \quad Q_{\alpha} \equiv \partial_{\alpha}T - Tu^{\rho}\partial_{\rho}u_{\alpha}$$

$$\Delta J_B^{\mu} = \chi\left(n_B T / w\right)^2 \Delta^{\mu}\left(\mu_B / T\right) \qquad s^{\mu} = su^{\mu} - \frac{\mu_B}{T}\Delta J_B^{\mu}$$

$$\partial_{\mu}s^{\mu} = \frac{\eta}{2T}\left(\partial_i u^j + \partial_j u^i - \tfrac{2}{3}\delta^{ij}\partial_k u^k\right)^2 + \frac{\zeta}{T}\left(\partial_k u^k\right)^2 + \frac{\chi}{T^2}\left(\partial_k T + T\dot{u}_k\right)^2 > 0$$

Landau's theory of hydrodynamic fluctuations

$$T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} + \Delta T^{\mu\nu} + S^{\mu\nu}$$

Stochastic source $$S^{\mu\nu} = S^{\mu\nu}_{\text{vis}} + S^{\mu\nu}_{\text{heat}}$$

Fluctuation - dissipation theorem: $S^{\mu\nu}$ is related to $\Delta T^{\mu\nu}$

$$\left\langle S^{\mu\nu}_{\text{vis}}(x) S^{\alpha\beta}_{\text{vis}}(y) \right\rangle = 2T\left[\eta\left(H^{\mu\alpha}H^{\nu\beta} + H^{\mu\beta}H^{\nu\alpha}\right) + \left(\zeta - \tfrac{2}{3}\eta\right)H^{\mu\nu}H^{\alpha\beta}\right]\delta^4(x-y)$$

$$\left\langle S^{\mu\nu}_{\text{heat}}(x) S^{\alpha\beta}_{\text{heat}}(y) \right\rangle = 2\chi T^2\left[H^{\mu\alpha}u^\nu u^\beta + H^{\nu\beta}u^\mu u^\alpha + H^{\mu\beta}u^\nu u^\alpha + H^{\nu\alpha}u^\mu u^\beta\right]\delta^4(x-y)$$

$$\left\langle S^{\mu\nu}_{\text{vis}}(x) S^{\alpha\beta}_{\text{heat}}(y) \right\rangle = 0$$

N. Salie, R. Wuffert, and W. Zimdahl, J. Phys. A **16**, 3533 (1983).
E. Calzetta, Class. Quant. Grav. **15**, 653 (1998).

Solution procedure

$$\partial_\mu T^{\mu\nu}_{\text{ideal}} + \partial_\mu \Delta T^{\mu\nu}_{\text{vis}} = -\partial_\mu S^{\mu\nu}$$

Choose initial conditions

Solve hydro equations for arbitrary sources $S^{\mu\nu}$

Calculate correlations / fluctuations of observables

Average over stochastic sources

Apply thermal freeze-out smearing (Cooper-Frye)

Bjorken scaling hydrodynamics

Bjorken variables: $\tau = \sqrt{t^2 - z^2}, \quad \xi = \tanh^{-1}(z/t), \quad t = \tau\cosh\xi, \quad z = \tau\sinh\xi$

$$u^0 = \cosh(\xi + \omega), \quad u^3 = \sinh(\xi + \omega)$$

Longitudinal flow fluctuations

$$T = T_0(\tau) + \delta T(\xi,\tau) \qquad \delta\varepsilon = c_V(T)\delta T$$

$$P = P_0(\tau) + \delta P(\xi,\tau) \qquad \delta P = s(T)\delta T \qquad w = \varepsilon + P$$

$$\varepsilon = \varepsilon_0(\tau) + \delta\varepsilon(\xi,\tau) \qquad \delta s = \delta\varepsilon / T \quad \Rightarrow \quad \rho = \delta s / s$$

Density fluctuations

$$\Delta T^{\mu\nu}_{\rm vis} = -\left(\tfrac{4}{3}\eta + \varsigma\right)(\partial\cdot u)h^{\mu\nu} \qquad S^{\mu\nu} = w(\tau)f(\xi,\tau)h^{\mu\nu}$$

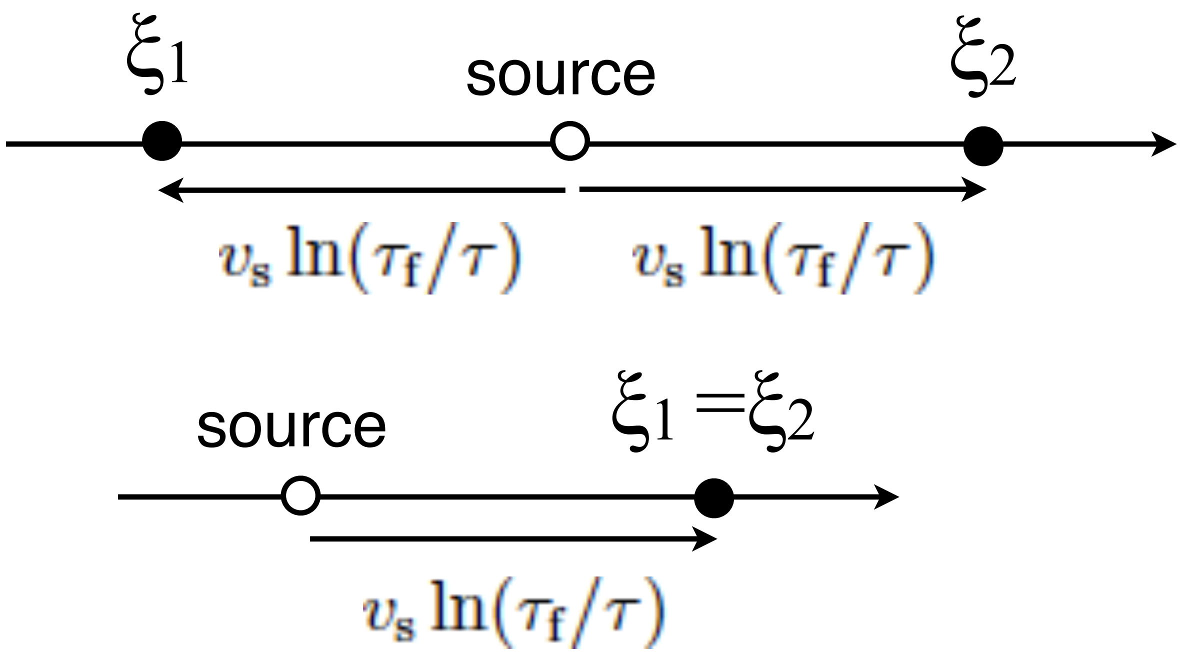
$$\langle f(\xi,\tau)f(\xi',\tau')\rangle = \frac{2T(\tau)}{A\tau w(\tau)^2}\left[\tfrac{4}{3}\eta(\tau) + \varsigma(\tau)\right]\delta(\tau - \tau')\delta(\xi - \xi')$$

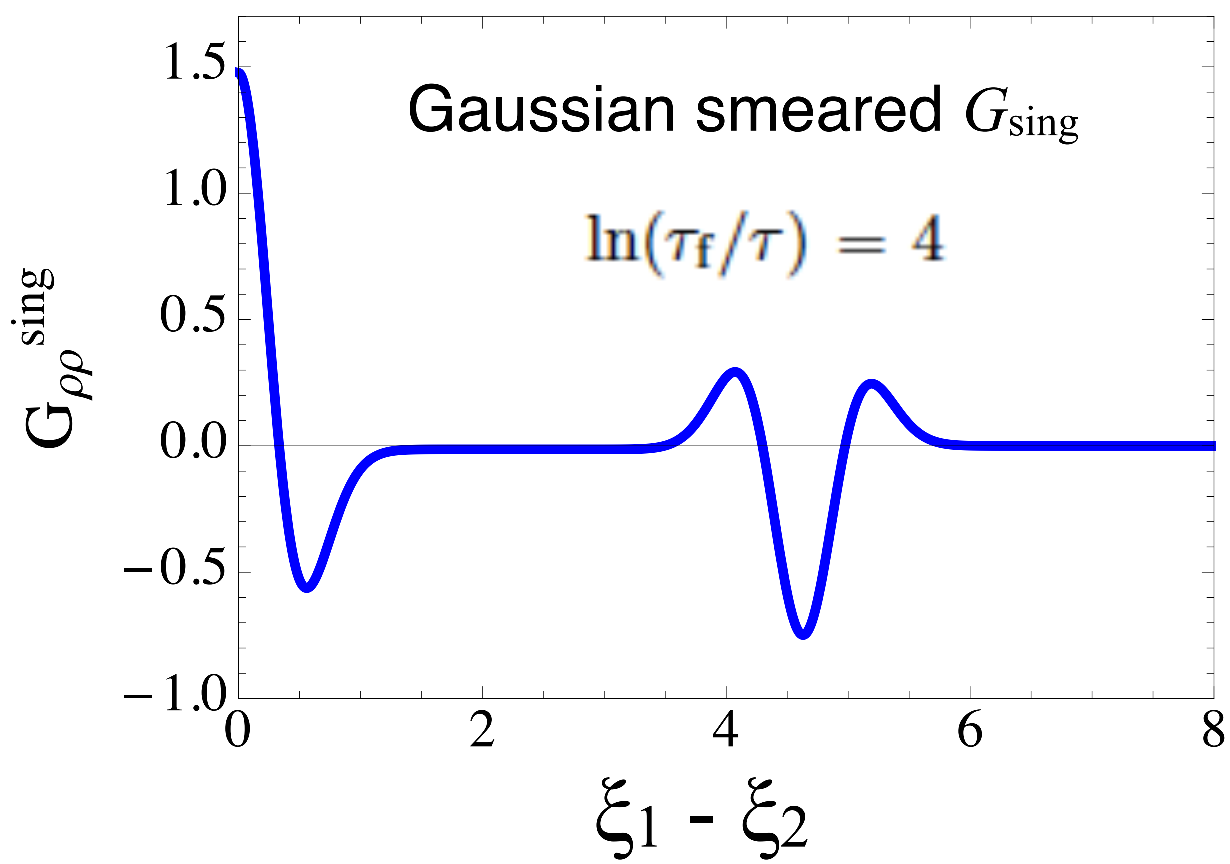
Boost invariant background suggests FT: $\tilde{X}(k,\tau) = \int_{-\infty}^{\infty} d\xi e^{-ik\xi} X(\xi,\tau)$

Sound horizon

Equations for small fluctuations can be solved analytically: Green's functions $G_{\omega/\rho}(k;\tau,\tau')$

$$\left\langle \tilde{X}(k_1,\tau_1)\,\tilde{Y}(k_2,\tau_2) \right\rangle = \frac{2\pi}{A}\delta(k_1+k_2) \int_{\tau_0}^{\min(\tau_1,\tau_2)} \frac{d\tau}{\tau^3}\frac{2\,\nu(\tau)}{w(\tau)}\,\tilde{G}_X(k_1;\tau_1,\tau)\tilde{G}_Y(k_2;\tau_2,\tau)$$





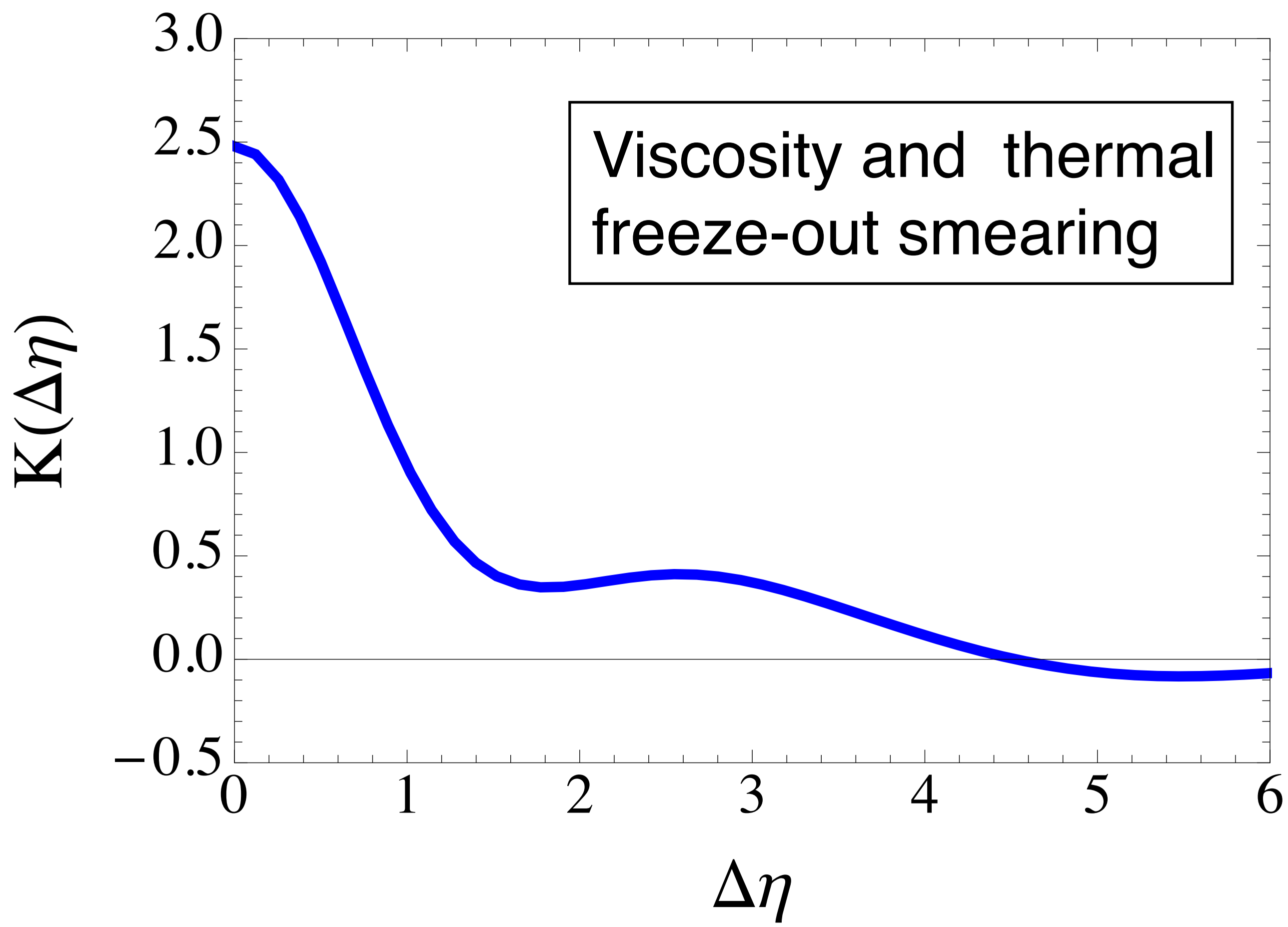
Sound horizon:

$$G_{XY}(\xi;\tau,\tau') = 0 \quad \text{when} \quad |\xi| > 2v_s \ln(\tau/\tau')$$

Rapidity fluctuations

Freeze-out with Cooper-Frye: $$p^0 \frac{dN_s}{d^3p} = \int_{\Sigma_f} d^3\sigma_\mu \, p^\mu \, \theta(\sigma \cdot p) \, d_s \, f_s(\boldsymbol{x}, \boldsymbol{p})$$

$$\left\langle \delta \frac{dN}{d\eta_1} \delta \frac{dN}{d\eta_2} \right\rangle \left\langle \frac{dN}{d\eta} \right\rangle^{-1} = \frac{45 d_s}{4\pi^4 N_{\text{eff}}(T_0)} \frac{\nu}{T_f \tau_f} \left(\frac{T_0^2}{T_f^2} \right)^{v_s^{-2}-2} K(\Delta\eta)$$



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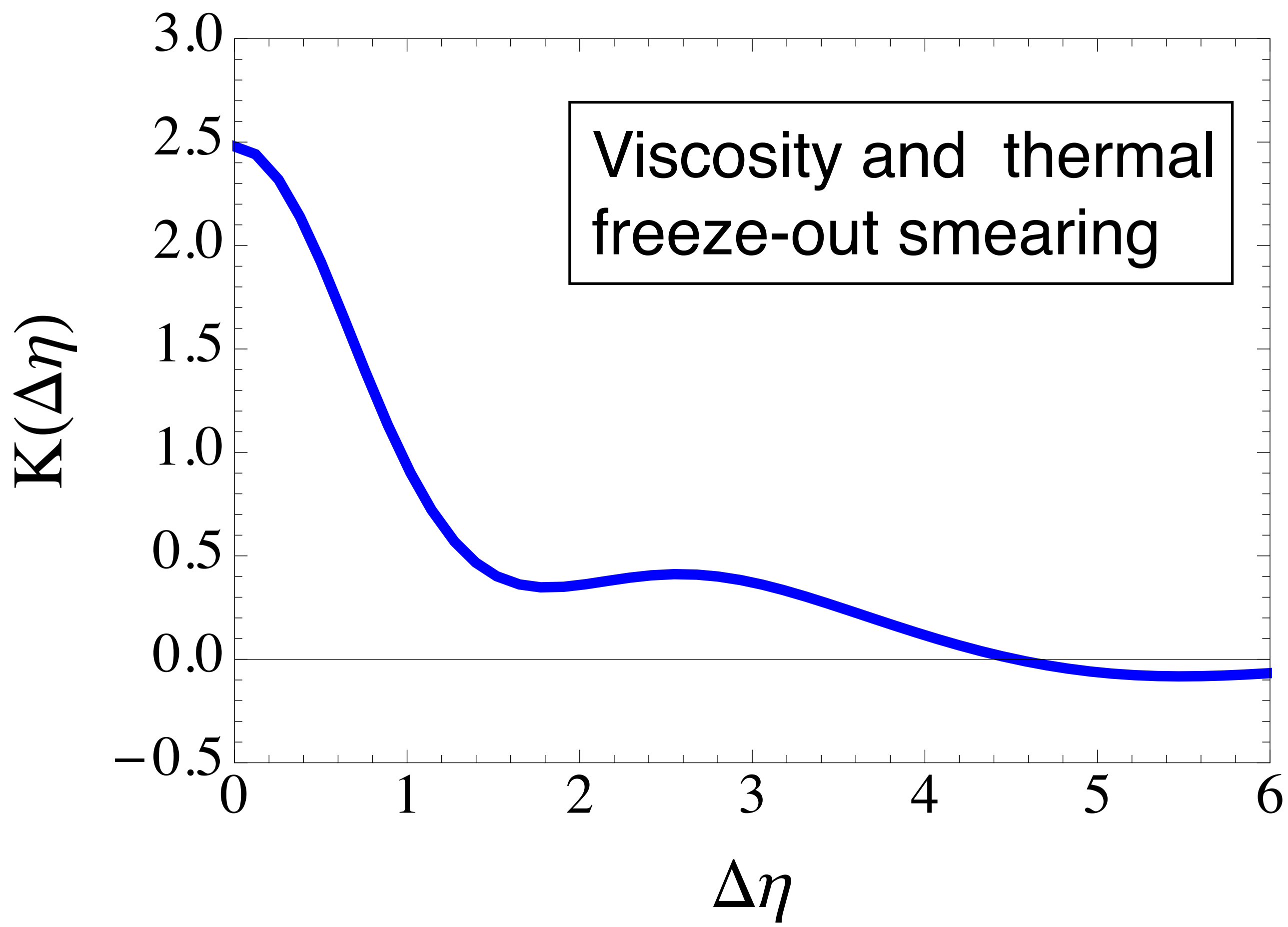

$\tau_0 = 0.16$ fm/c

$T_0 = 600$ MeV

$\tau_f = 10$ fm/c

$T_f = 150$ MeV

Summary

- Initial state fluctuations:
  - Approximate analytical form of the energy density fluctuations has been derived in a CGC inspired model.
  - The result is rather insensitive to details of gluon distribution in the colliding nuclei.
  - Energy density fluctuations are large: $\Delta\varepsilon/\varepsilon \sim 0.7$ at $\Delta\mathbf{x} = 0$

- Hydrodynamic fluctuations:
  - Can be model independently predicted given a hydro scenario.
  - Encode important information on transport coefficients and speed of sound.
  - Can generate rather long range rapidity correlations ($\Delta\eta \sim 3\text{-}4$).
  - Transverse flow under study by Stephanov et al.