Fluid dynamics beyond the gradient expansion: Divergence-type theories

Jerónimo Peralta Ramos

Nonequilibrium Quantum Field Theory Group
Departamento de Física
Facultad de Ciencias Exactas y Naturales
Universidad de Buenos Aires

International School of Nuclear Physics *34th Course*: *Probing the Extremes of Matter with Heavy Ions*

In this talk I'm going to

- present an **alternative** approach to (usual) relativistic fluid dynamics and compare it with second-order theory,
- *briefly* comment on it's derivation from kinetic theory, and
- discuss it's application to heavy ion collisions (within a *simple model*).

Work in collaboration with Esteban Calzetta.

Why an alternative?

- To use a fluid description in situations with large gradients (e.g. early time, shock waves) (see also Lublinsky and Shuryak, PRD **80**, 065026 (2009)).

- To provide an independent check of the gradient expansion for conditions prevailing in heavy ion collisions (see also talk by Vincenzo Greco; Bouras et al, PRC **82**, 024910 (2010)).

- To quantify uncertainty in extracted η/s that comes from differences in hydro formalisms (BRSSS,Muronga,Ottinger).

- To go beyond Grad's quadratic ansatz to dissipative corrections to f.

DTTs [Geroch 1990]

- Symmetries of underlying field theory are encoded in a single generating function χ.
- Introduce a new third-order tensor obeying $A^{\mu\nu\rho}_{;\mu} = I^{\nu\rho}$ and a symmetric and traceless tensor which vanishes in equilibrium $\xi_{\mu\nu}$.
- Once we know χ and $I^{\mu\nu}$, we know the equations of motion. Once a solution for $\xi_{\mu\nu}$ is obtained, $T^{\mu\nu}$ can be calculated.
- It does break down but not because gradients become large, rather because the dynamics cannot be described by few variables.

In some more detail:

$$S^\mu = \Phi^\mu - \beta_\nu T^{\mu\nu} - A^{\mu\nu\rho}\xi_{\nu\rho} \qquad S^\mu, \Phi^\mu \text{ are ultralocal in } \beta^\mu, \xi^{\mu\nu}$$

$$S^\mu_{;\mu} \geq 0 \text{ if } \frac{\partial \Phi^\mu}{\partial \beta_\nu} = T^{\mu\nu} \text{ and } \frac{\partial \Phi^\mu}{\partial \xi_{\nu\rho}} = A^{\mu\nu\rho}$$

$$\Phi^\mu = \frac{\partial \chi}{\partial \beta_\mu} \text{ since } T^{\mu\nu} = T^{\nu\mu}$$

* We developed a quadratic DTT that reduces, at 2nd order, to the 2nd order theory (*without vorticity*) of Baier et al, JHEP0804:100,2008.

* $\Pi^{\mu\nu}$ is given by

$$\Pi^{\mu\nu} = \eta\xi^{\mu\nu} - \frac{\lambda_1 \tau_\pi T^4}{3\eta}(\xi^{\mu\alpha}\xi^\nu_\alpha - \frac{1}{3}\Delta^{\mu\nu}\xi^{\alpha\gamma}\xi_{\alpha\gamma}) \ .$$

$$\partial_\tau \xi^{i\alpha} = -\frac{2}{3u^\tau}\xi^{i\alpha}\nabla_\mu u^\mu - \frac{1}{\tau_\pi u^\tau}\xi^{i\alpha} + \frac{1}{\tau_\pi u^\tau}\sigma^{i\alpha}$$
$$- \frac{\lambda_1}{3\tau_\pi \eta u^\tau}\xi_\mu^{<i}\xi^{\alpha>\mu} - \frac{u^i \xi_\mu^\alpha + u^\alpha \xi_\mu^i}{u^\tau} D u^\mu - \frac{u^j}{u^\tau}\partial_j \xi^{i\alpha}$$

JPR and EC, PRD **80**, 126002 (2009).
For comparison, in the 2nd order theory we have (without vorticity)

$$\partial_\tau \Pi^{i\alpha} = -\frac{4}{3u^\tau}\Pi^{i\alpha}\nabla_\mu u^\mu - \frac{1}{\tau_\pi u^\tau}\Pi^{i\alpha} + \frac{\eta}{\tau_\pi u^\tau}\sigma^{i\alpha}$$
$$- \frac{\lambda_1}{2\tau_\pi \eta^2 u^\tau}\Pi_\mu^{<i}\Pi^{\alpha>\mu} - \frac{u^i \Pi_\mu^\alpha + u^\alpha \Pi_\mu^i}{u^\tau} D u^\mu - \frac{u^j}{u^\tau}\partial_j \Pi^{i\alpha}$$

(Note: we neglect higher order corrections to transport coefficients;

see Denicol et al, 1206.1554 and PRD **85**, 114047 (2012)).

Application to heavy ion collision at $\sqrt{s} = 200$ GeV (just an example)

Simple model:

* 2+1 boost invariant dynamics.

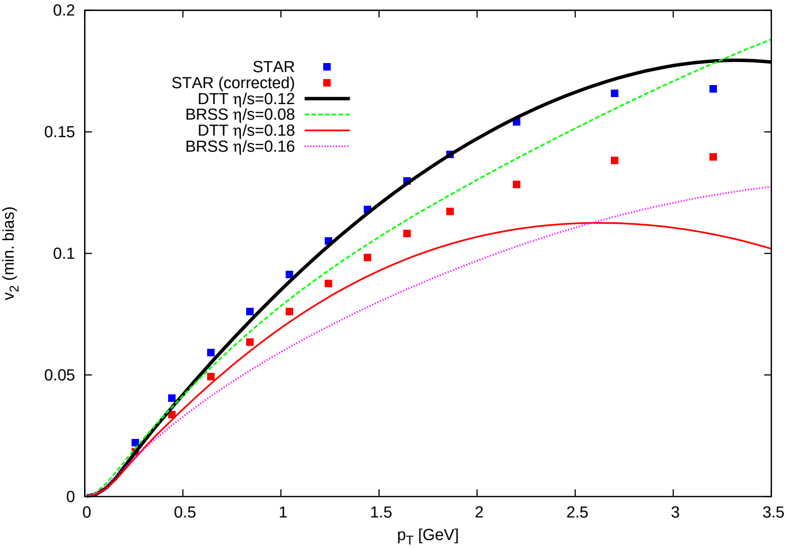
* *Smooth* Glauber initial conditions.

* Lattice QCD EOS.

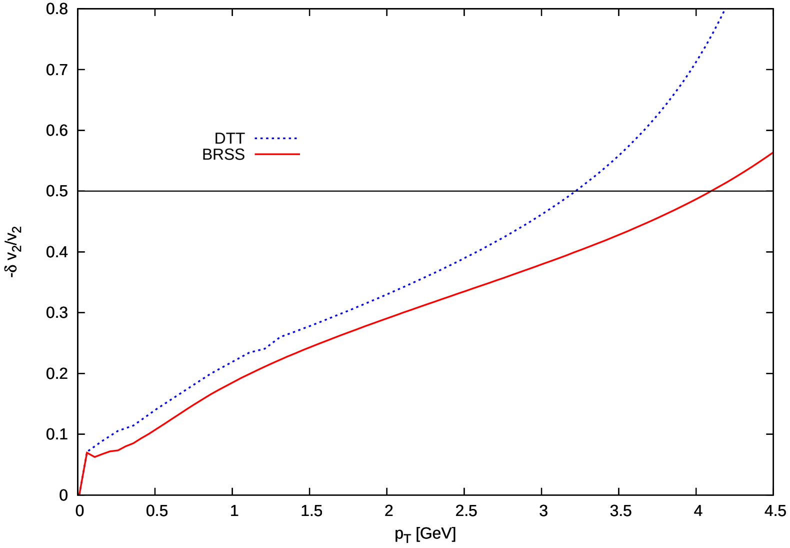
* We obtain $T_i = 332$ MeV and $T_f = 140$ MeV by matching Kaon total multiplicity and $\langle p_T \rangle$.

* Spectra of stable particles including feed-down contributions (AZHYDRO package - Kolb, Sollfrank, Heinz, Rapp).

JPR and EC, PRC **82**, 054905 (2010).



There is an uncertainty $\sim$ 30% in the extraction of η/s from RHIC data that comes from using 2nd order or DTT



Viscous corrections are larger than expected. Gradient expansion breaks down at $p_T \sim 3$ GeV/c.

* We derived the DTT from Boltzmann's equation in the r.t.a. by extremizing the entropy production: we get a **nonlinear** closure not based on gradient expansions.

$$\delta f = f_e G(\epsilon, T, \omega)\Pi_{\mu\nu} p^\mu p^\nu \left[1 + 3G(\epsilon, T, \omega) f_e \Pi_{\gamma\delta} p^\gamma p^\delta\right]$$

G depends on the collision kernel.

* At *linear order* and if $G = \frac{1}{2(\epsilon+p)T^2}$ (corresponding to e.g. $\lambda\phi^4$ but not to a hot gluon gas - see Dusling, Moore, Teaney PRC **81**, 034907 (2010)) we recover Grad's ansatz

$$\delta f_{\mathrm{Grad}} = f_e \frac{1}{2(\epsilon + p)T^2} \Pi_{\alpha\beta} p^\alpha p^\beta$$

EC and JPR, PRD **82**, 106003 (2010). See also Denicol et al, 1206.1554 and PRD **85**, 114047 (2012) and Vincenzo Greco's talk.

We are computing $v_4/(v_2)^2$ in the DTT

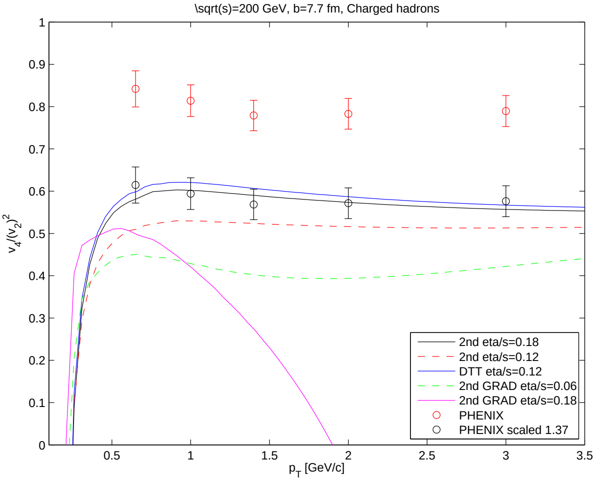
* $v^4/(v_2)^2$ very sensitive to (i) δf and (ii) IS fluctuations ($\sim$ just rescaling).

* Data favours $\sim p_T^a$ with $a \sim 1.5 - 2$ (Luzum and Ollitrault, PRC **82**, 014906 (2010); R. Lacey et al, PRC **82**, 034910 (2010)). $a_{\rm DTT} \sim 1 - 2$.

* $\lambda\Phi^4$: $a = 2$; Pure glue $2 \to 2$: $p^2/\ln(p)$; Pure glue collinear radiation: $a \sim 1.38$ (DMT PRC **81**, 034907 (2010)).

* The value of a is related to parton energy loss and $\hat{q}$ (see, e.g., Arnold, Moore and Yaffe, JHEP 0305, 051 (2003); DMT).

* (May be) Related to constituent quark scaling: baryons equilibrate faster (DMT).



Systematics and comparison to data are under way.

A. Adare et al, PRL **105**, 062301 (2010).

C. Gombeaud and J.Y. Ollitrault, PRC **81**, 014901 (2010).

M. Luzum, C. Gombeaud, and J.Y. Ollitrault, PRC **81**, 054910 (2010).

See also R. Lacey et al, PRC **82**, 034910 (2010).

Outlook:

* The closure picked out by the entropy production method is *nonlinear* and *generalizes Grad's quadratic ansatz*. It's use at freeze out leads to sizable modification of observables (e.g. $v_4/(v_2)^2$) which must be quantified in order to better constrain η/s (*in progress*). We must include IS fluctuations.

* A recent extension to include color within a DTT fluid dynamic description (derived from NeqQFT) may be of use to study **some aspects (long wavelength) of parton energy loss (JPR and EC, 1208.2715).**

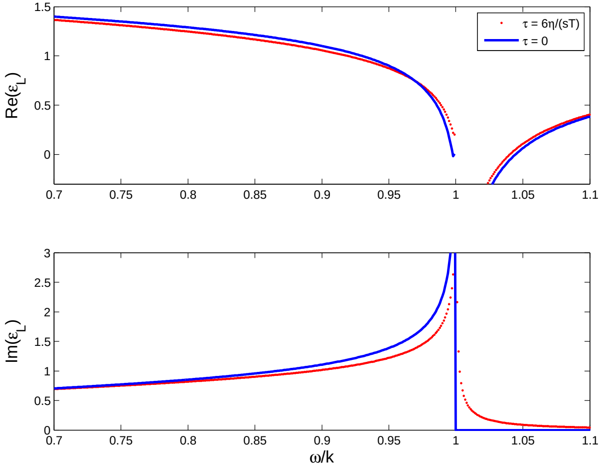
Viscous chromohydrodynamics:

* Scalar colored particles coupled to YM field.

* We apply the closure based on entropy production (we use r.t.a.) to Heinz transport equation.

* We obtain a set of coupled hydro equations for colored particles (the equations are similar to Wong equations but on a macroscopic scale). We plan to solve them.

* We compute the polarization tensor given the (hydro) response of the system to a perturbation.



τ = 6η/(sT)
τ = 0
Re(ε_L)
Im(ε_L)
ω/k
1.5
1
0.5
0
3
2.5
2
1.5
1
0.5
0
0.7
0.75
0.8
0.85
0.9
0.95
1
1.05
1.1

Thank you for your attention!