Comparing Lattice Results to Measurements from RHIC/LHC

S.P. PRL (2012)

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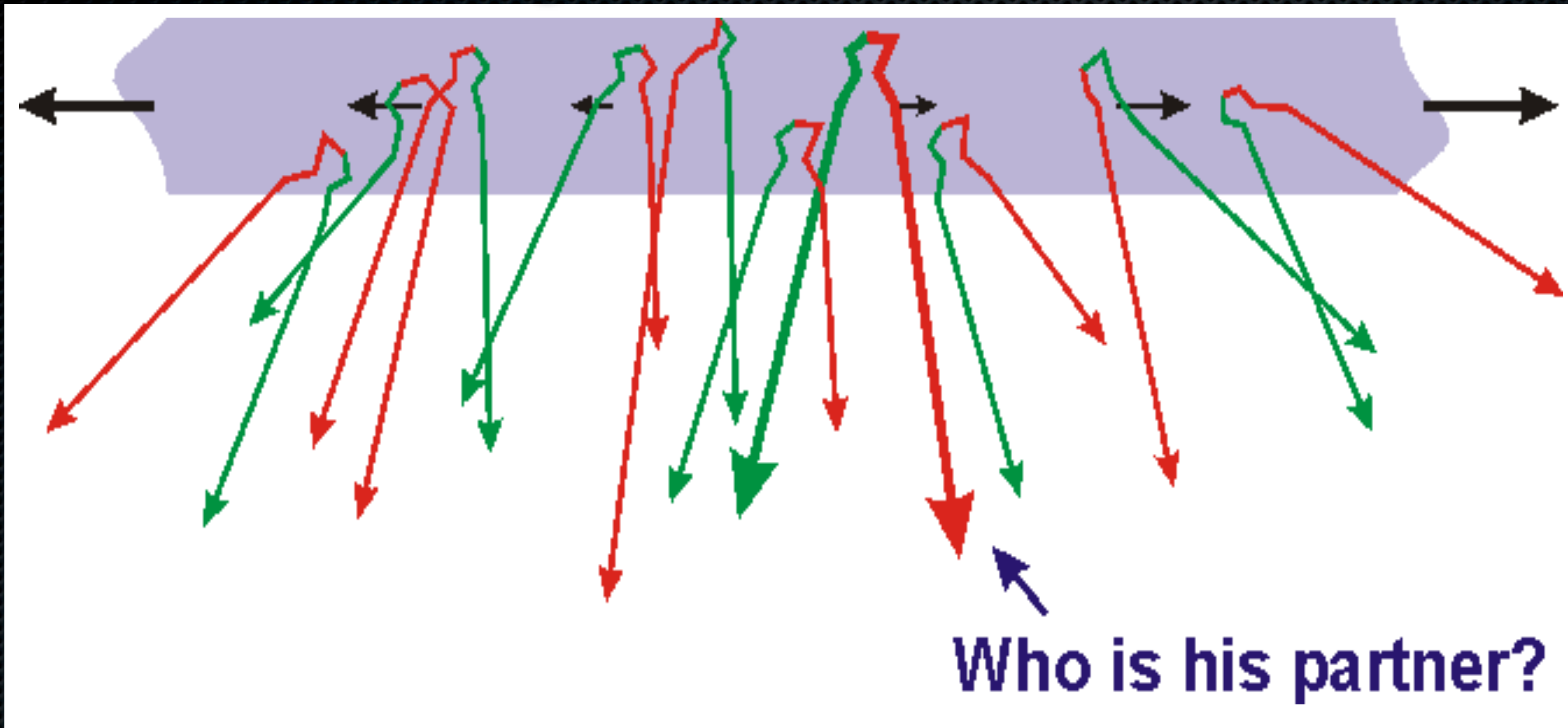
If a cat were to disappear in Pasadena and at the same time appear in Erice, that would be an example of global conservation of cats. This is not the way cats are conserved. Cats or charge or baryons are conserved in a much more continuous way. If any of these quantities begin to disappear in a region, then they begin to appear in a neighboring region. Consequently, we can identify a flow of charge out of a region with the disappearance of charge inside the region. This identification of the divergence of a flux with the time rate of change of a charge density is called a

***local conservation law*.**

A local conservation law implies that the total charge is conserved globally, but the reverse does not hold. However, relativistically it is clear that non-local global conservation laws cannot exist, since to a moving observer the cat will appear in Erice before it disappears in Pasadena.

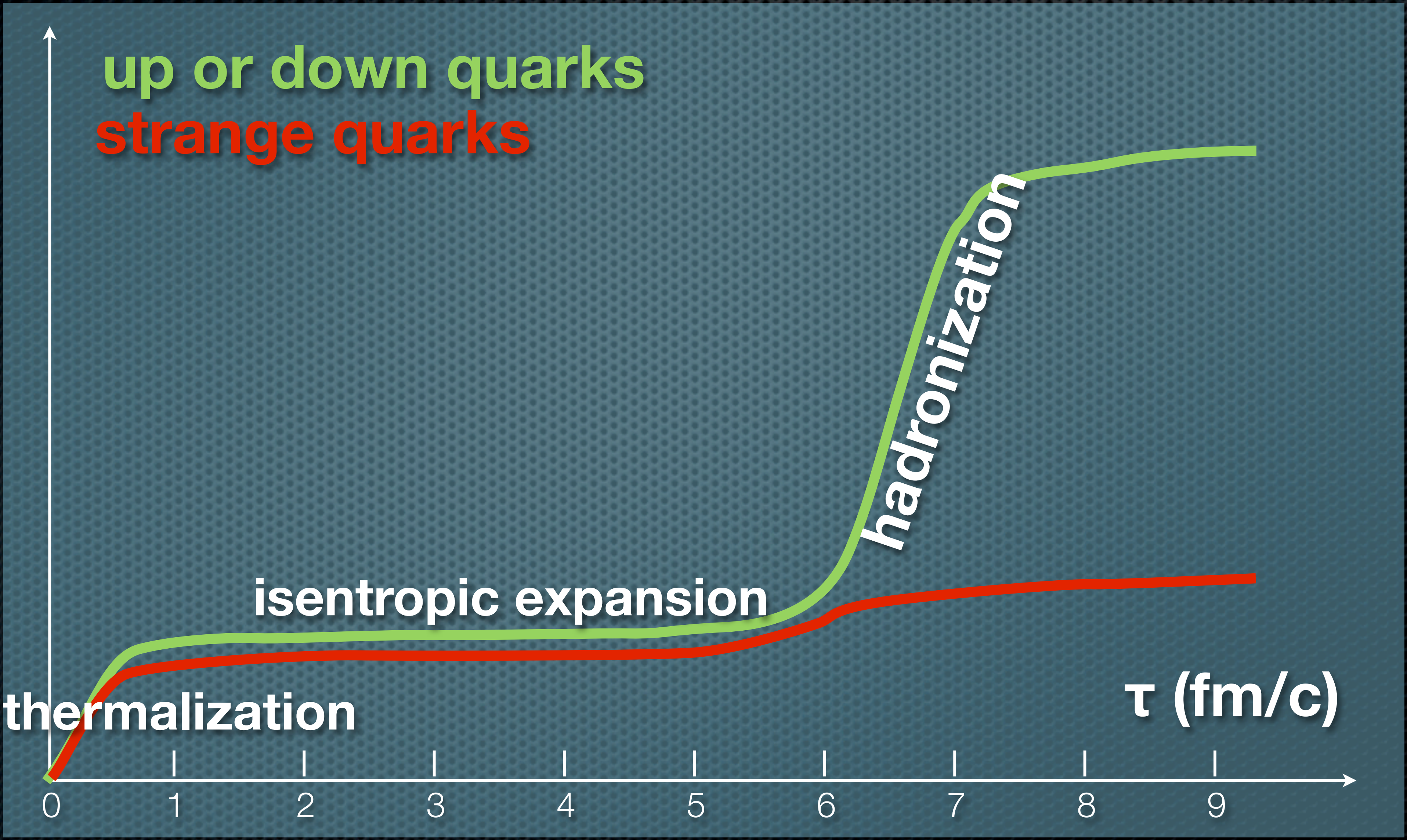
Charge Balance Functions



For each charge +Q, there is extra balancing charge −Q.

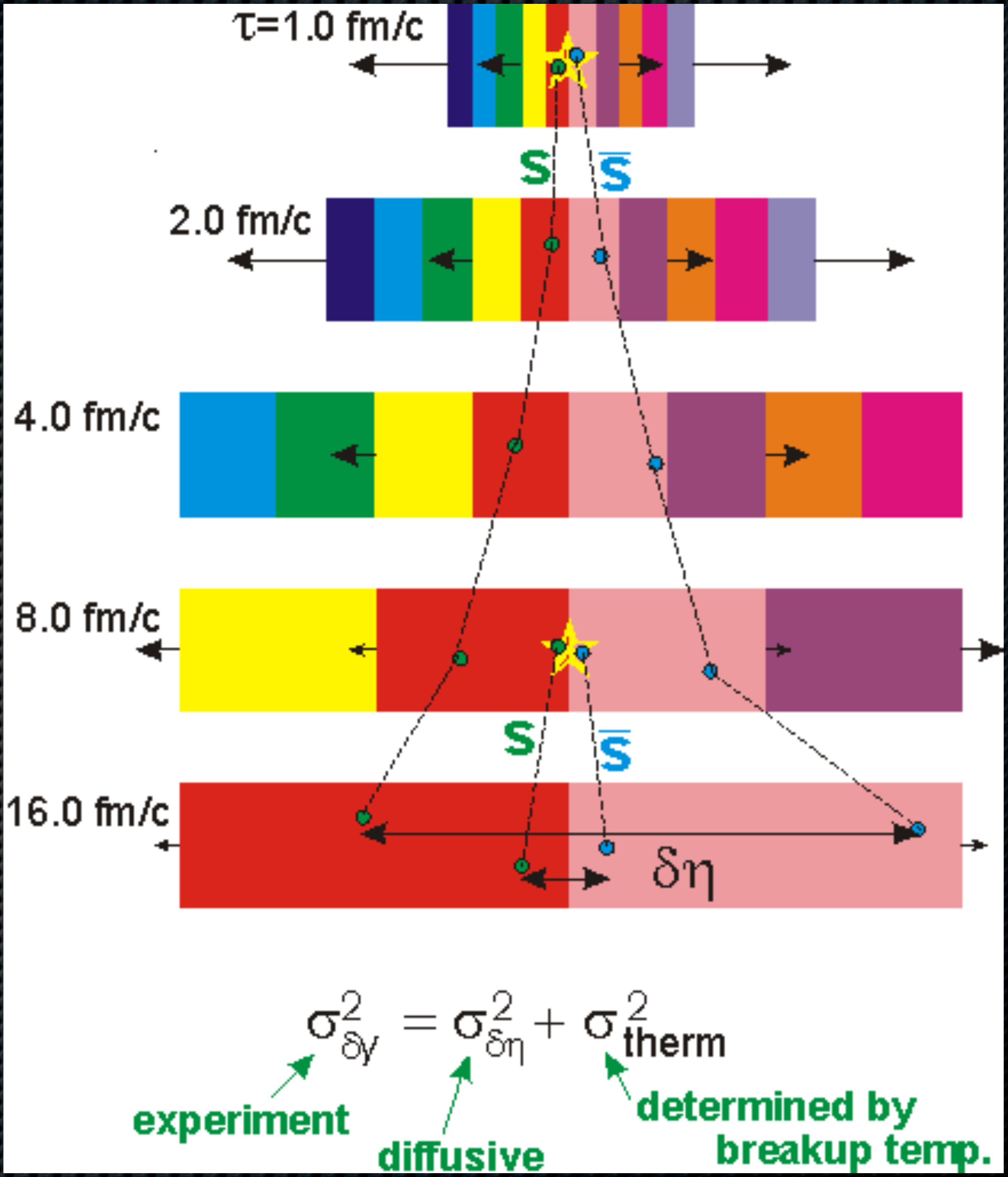
$$B(\Delta y) \equiv \frac{N_{+-}(\Delta y) - N_{++}(\Delta y)}{N_{+}}$$

Two waves of quark production



Balance Function is sensitive to when charge is created

S. Bass, P. Danielewicz & S.P., PRL 2000

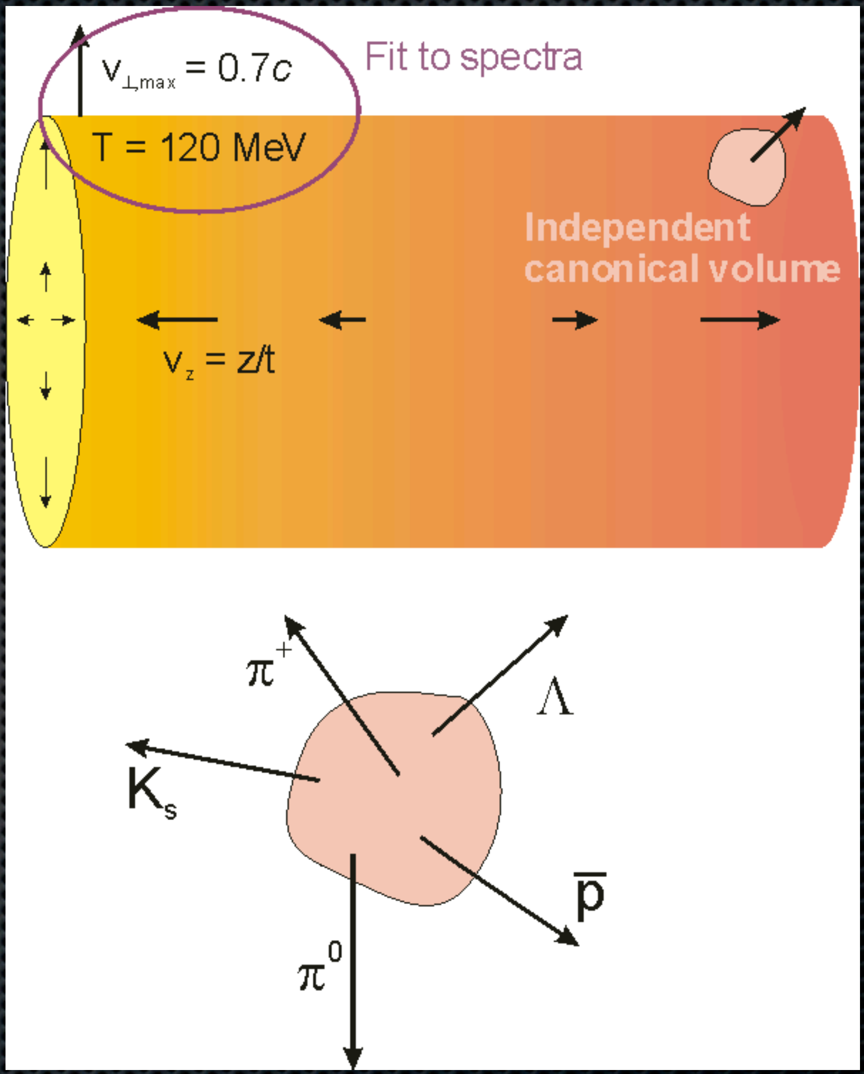


Blast Wave

Parameters:

$T, \varepsilon, v_{\perp x}, v_{\perp y}, \sigma_\eta, \sigma_\phi$

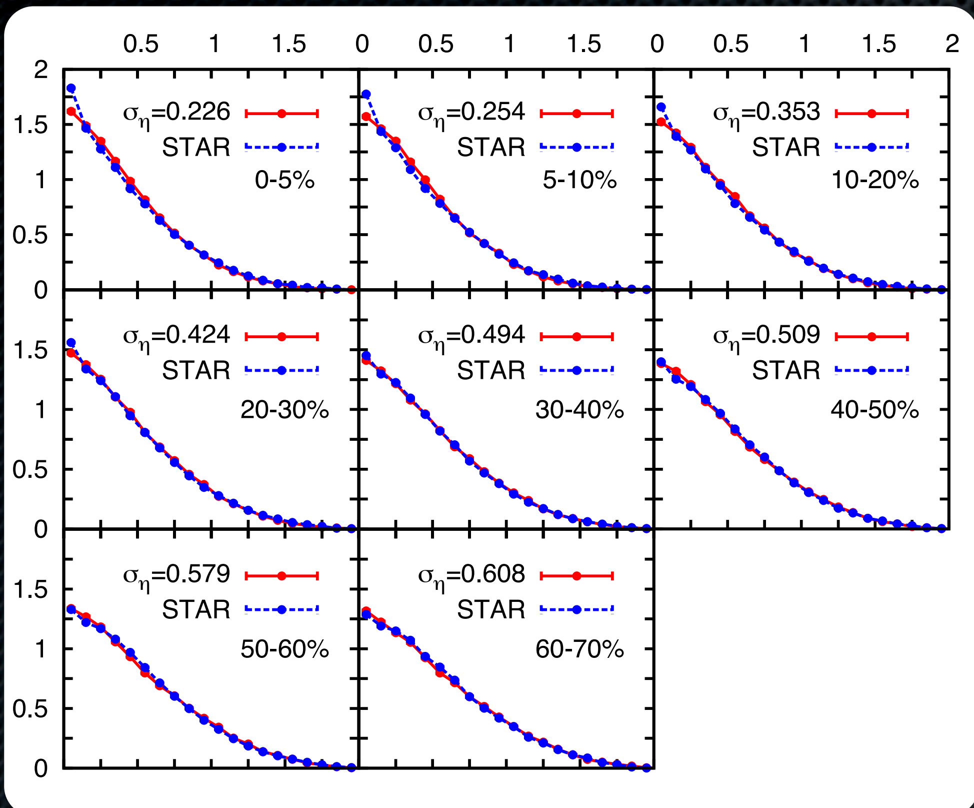
Relative spread of emission points of balancing charges



T and $v_\perp$ fixed by spectra (STAR fits)

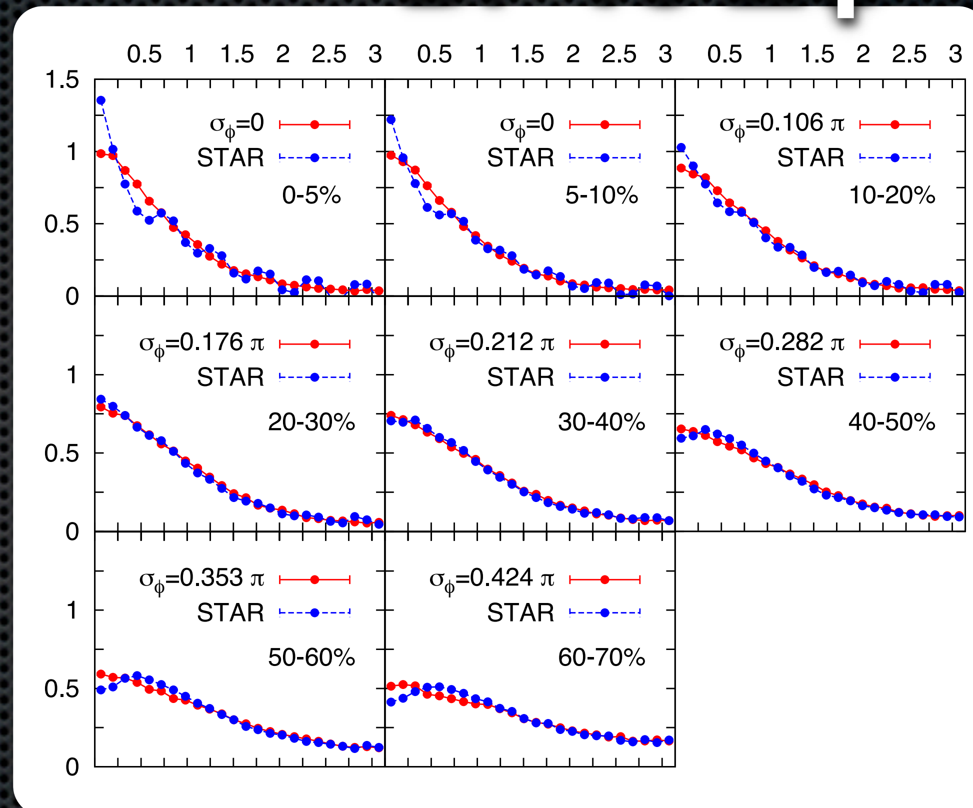
Canonical methods enforce conservation

BW vs. STAR



BF vs Δη

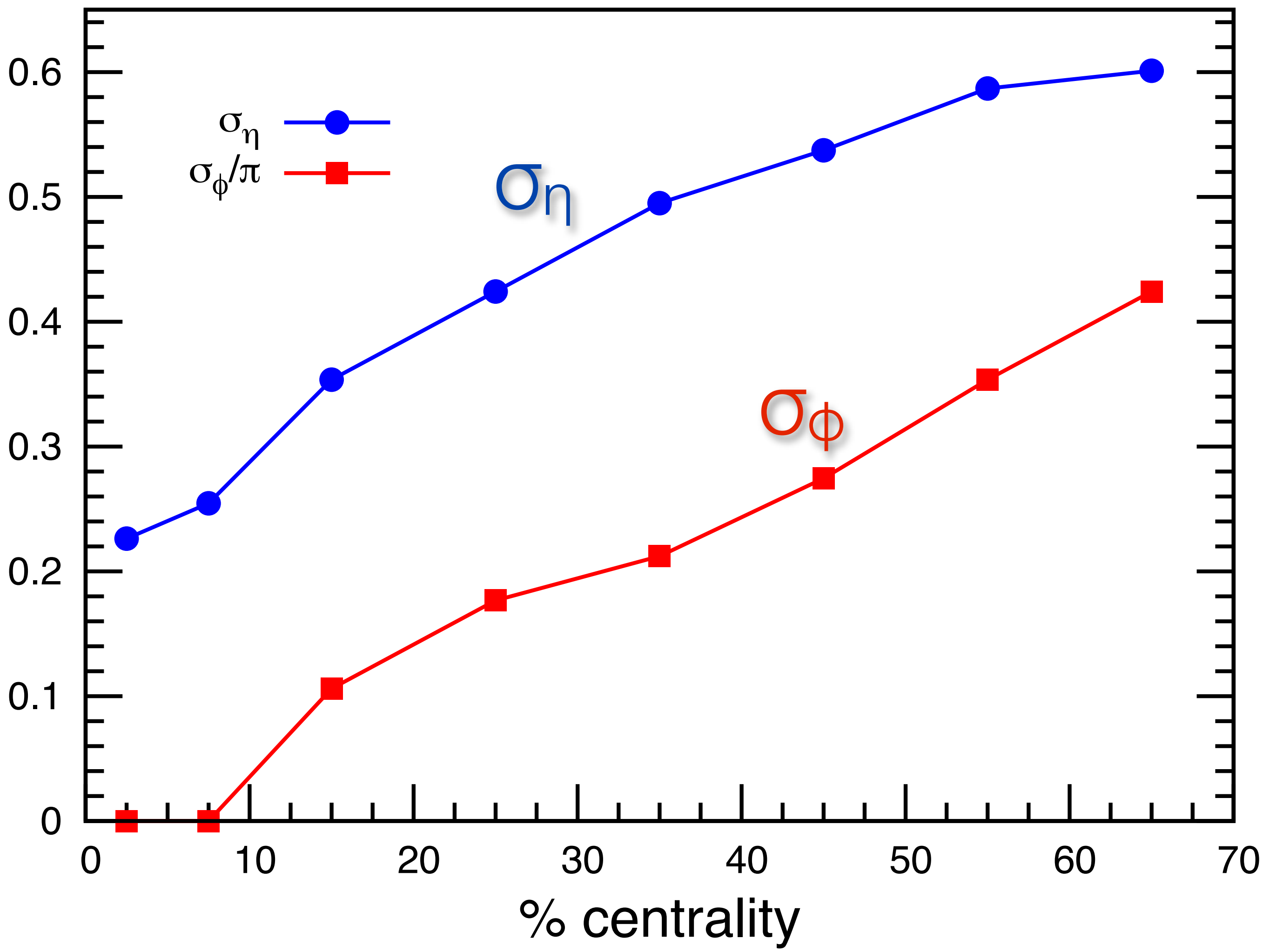
BF vs Δϕ



STAR's Blast Wave model (Lisa & Retierre) + Local Charge Conservation

also see P.Bozek, PLB(2005)

BF Widths



Narrowing B(Δη) *suggests* delayed hadronization

(Bass, Danielewicz and SP, PRL 2001)

Narrowing B(Δφ) signals radial flow

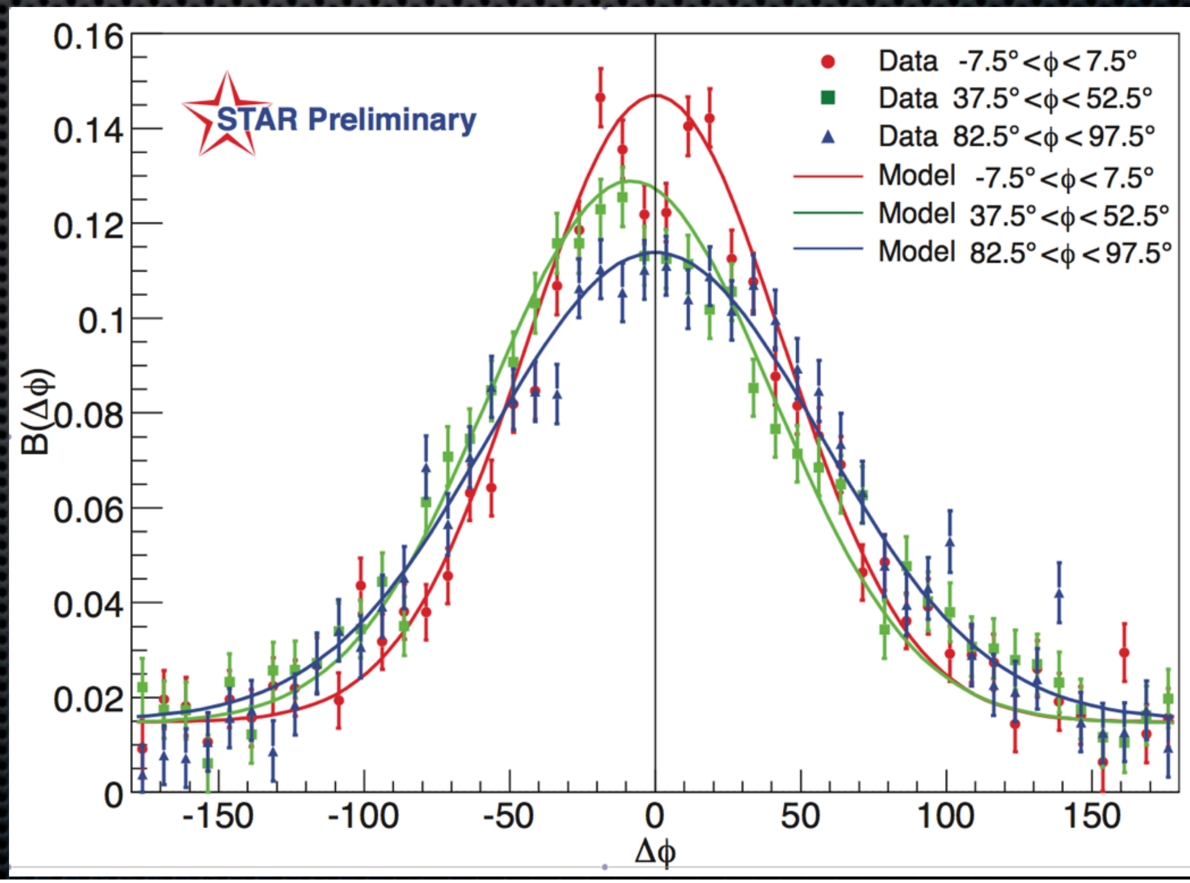
(Bozek, PLB 2005)

Balance function & "parity" Observable

$$\gamma = \langle \cos(\phi_1 + \phi_2) \rangle$$

$$\gamma_p = \gamma_{\mathrm{opp.sign}} - \gamma_{\mathrm{same\,sign}}$$

$$\gamma_p = \frac{2}{M^2} \int d\phi d\Delta\phi \frac{dM}{d\phi} B(\phi, \Delta\phi) \cos(2\phi + \Delta\phi)$$



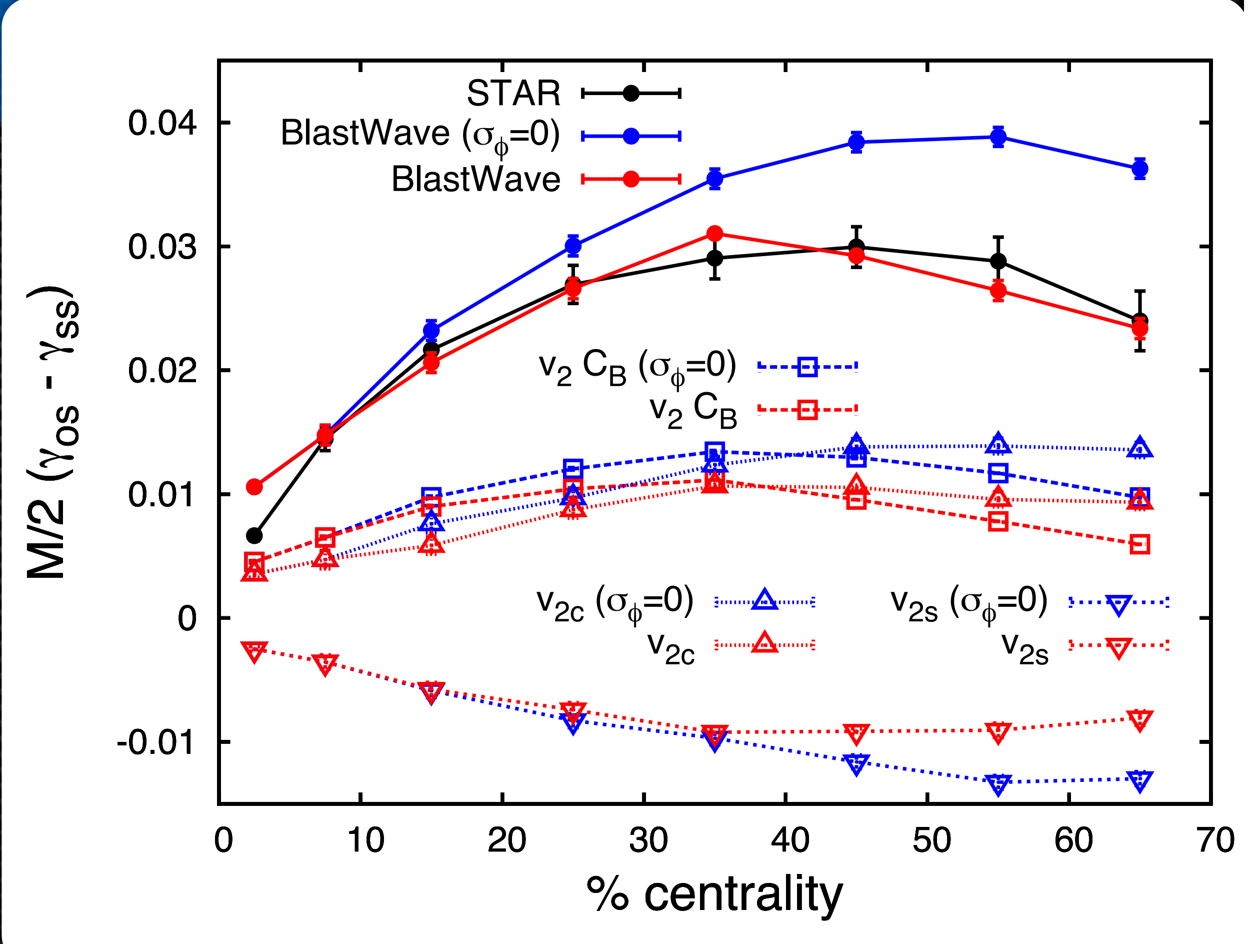
Use STAR's BW fit

Balance function & "parity" Observable

$$\gamma = \langle \cos(\phi_1 + \phi_2) \rangle = \langle \cos(2\phi + \Delta\phi) \rangle$$

$$= \langle \cos(2\phi) \rangle \langle \cos(\Delta\phi) \rangle$$
$$+ \langle \cos(2\phi)\cos(\Delta\phi) \rangle - \langle \cos(2\phi) \rangle \langle \cos(\Delta\phi) \rangle$$
$$- \langle \sin(2\phi)\sin(\Delta\phi) \rangle$$

$\gamma_p \approx v_2/M$ inevitable for low viscosity liquid & local charge conservation!!!!



Lattice uses charge correlations

$$\chi_{ab} \equiv \langle Q_a Q_b \rangle / V$$

a,b = uds

Parton gas:

$$\chi_{ab}^{\mathrm{QGP}} = (n_a + n_{\bar{a}})\delta_{ab}$$

Hadron gas:

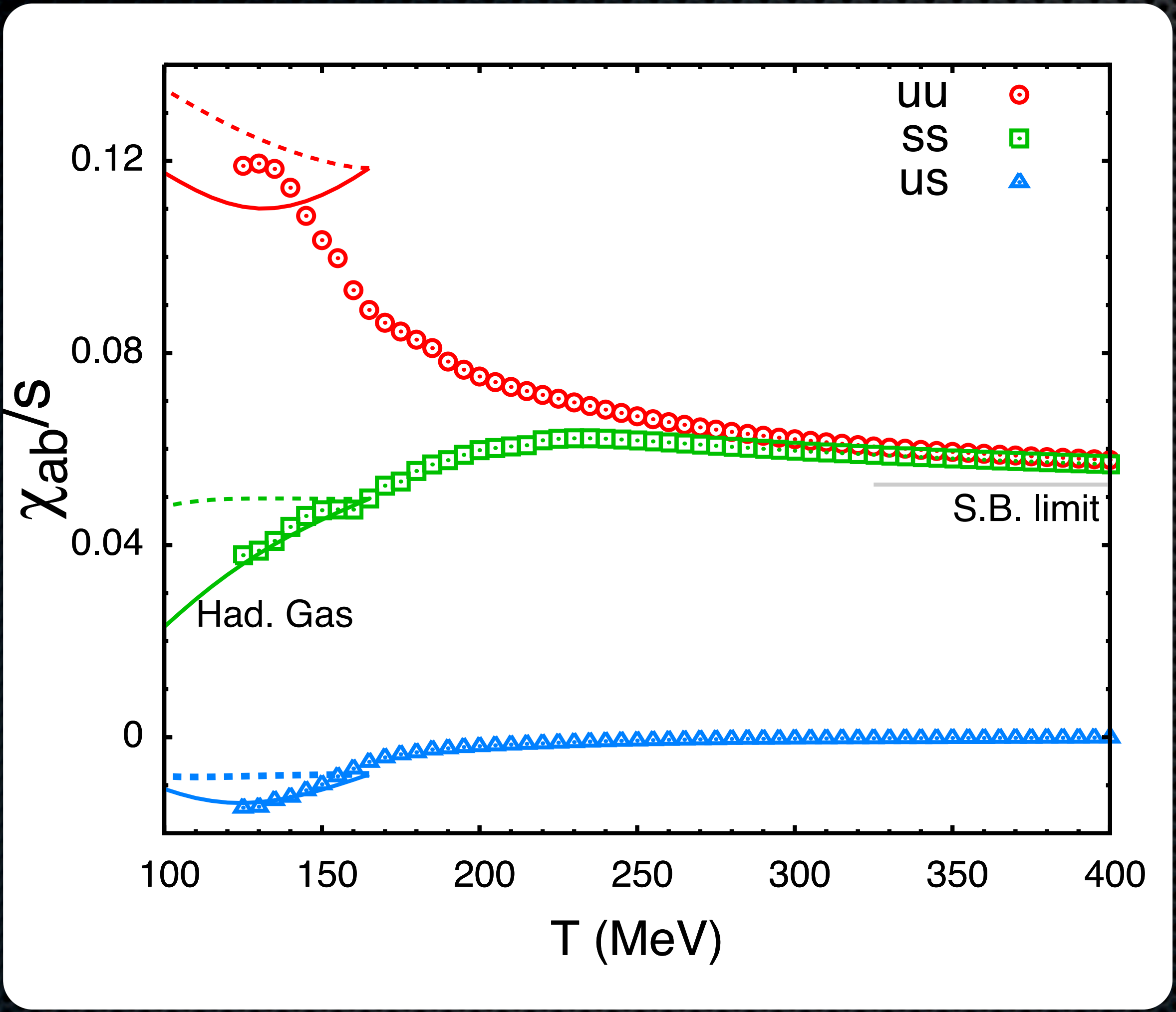
$$\chi_{ab}^{HAD} = \sum_{\alpha} n_\alpha q_{\alpha,a} q_{\alpha,b}$$

$\alpha = \pi^+, \pi^-, \pi^0, K^+ \ldots$

off-diagonal elements

Lattice results scaled by entropy

courtesy of Claudia Ratti



- Transformation not perfectly sharp
- Near Tc, up/down increase, strangeness slightly decreases

Problems with Comparing Experiment to Lattice

1. Lattice = Grand Canonical (Particle Bath)
   Experiment = Canonical (net charge = 0)

2. Charge created at hadronization

3. One measures hadrons -- not uds

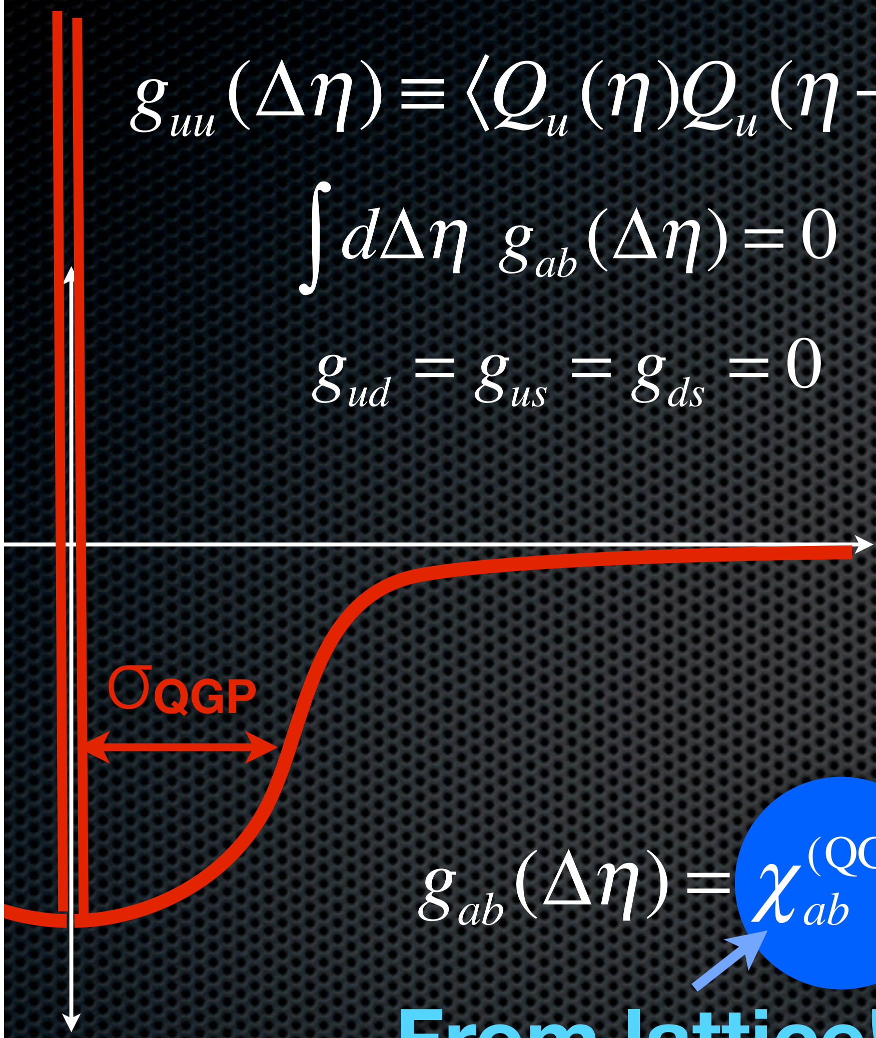
4. One measures momenta, not positions

1. Just before hadronization

$$g_{uu}(\Delta\eta) \equiv \langle Q_u(\eta) Q_u(\eta+\Delta\eta)\rangle$$

$$\int d\Delta\eta \;\, g_{ab}(\Delta\eta) = 0$$

$$g_{ud} = g_{us} = g_{ds} = 0$$



only extra parameter

$$g_{ab}(\Delta\eta) = \chi_{ab}^{(\mathrm{QGP})} \left\{ \delta(\Delta\eta) - \frac{\exp(-\Delta\eta^2 / 2\sigma_{(\mathrm{QGP})}^2)}{(2\pi\sigma_{(\mathrm{QGP})}^2)^{1/2}} \right\}$$

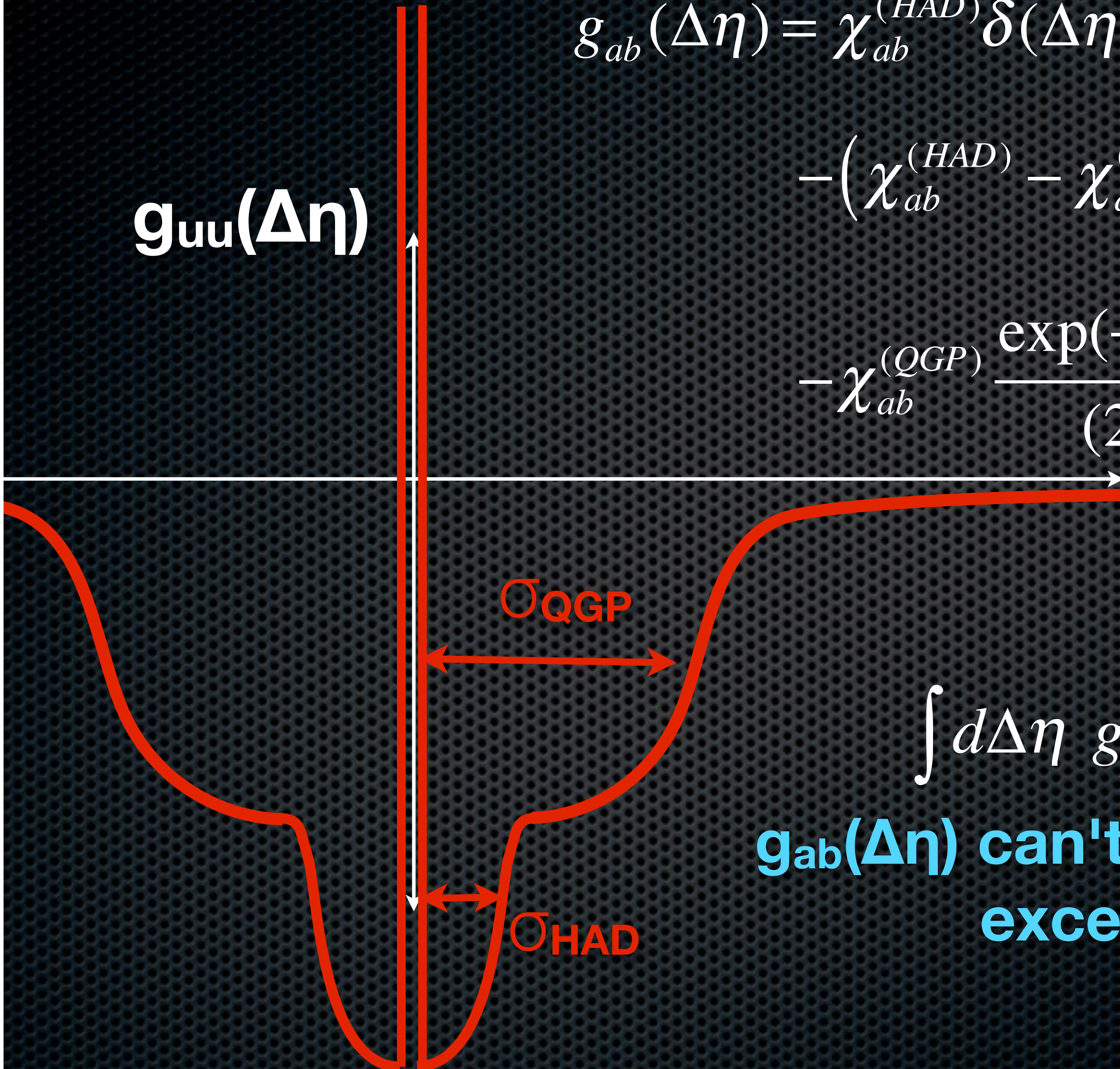
From lattice!

2. Just after hadronization

$$g_{ab}(\Delta\eta) = \chi_{ab}^{(HAD)}\delta(\Delta\eta)$$

$$-\left(\chi_{ab}^{(HAD)} - \chi_{ab}^{(QGP)}\right)\frac{\exp(-\Delta\eta^2/2\sigma_{(HAD)}^2)}{(2\pi\sigma_{(HAD)}^2)^{1/2}}$$

$$-\chi_{ab}^{(QGP)}\frac{\exp(-\Delta\eta^2/2\sigma_{(QGP)}^2)}{(2\pi\sigma_{(QGP)}^2)^{1/2}}$$



$$\int d\Delta\eta \; g_{ab}(\Delta\eta) = 0$$

$g_{ab}(\Delta\eta)$ can't change suddenly except at $\Delta\eta=0$

2. Just after hadronization
Summarizing...

$$-g'_{ab}(\Delta\eta) = \chi_{ab}^{(QGP)} \frac{e^{-\Delta\eta^2/2\sigma^2_{(QGP)}}}{\sqrt{2\pi\sigma^2_{(QGP)}}} + (\chi_{ab}^{(HAD)} - \chi_{ab}^{(QGP)}) \frac{e^{-\Delta\eta^2/2\sigma^2_{(HAD)}}}{\sqrt{2\pi\sigma^2_{(HAD)}}}$$

$$\chi_{ab}^{(HAD)} \equiv \sum_{\alpha\in\text{hadrons}} n_\alpha q_{\alpha,a} q_{\alpha,b}$$

$$\chi_{ab}^{(QGP)} \equiv 2n_a\delta_{ab}$$

3. But, we measure $G_{\alpha\beta}$ not g_{ab}!!!

$\alpha,\beta=\pi,p,K\ldots$ $a,b=u,d,s$

$$G_{\alpha\beta}(\Delta\eta) \equiv \langle [n_\alpha - n_{\bar{\alpha}}][n_\beta - n_{\bar{\beta}}] \rangle$$

$$e.g., \quad G_{pK^-} = \langle [n_p - n_{\bar{p}}][n_{K^-} - n_{K^+}] \rangle$$

Generalized Balance Function
(aside from factor of $\langle n_\beta \rangle$)

Analogous problem...

Given $\delta\rho_a$ and n_α, find δn_α

Solution: assign chemical potential

$$\delta n_\alpha = \langle n_\alpha \rangle \left(e^{\mu_a q_{\alpha,a}/T} - 1 \right)$$

$$\delta\rho_a = \sum_\alpha \delta n_\alpha q_{\alpha,a}$$

$$\frac{\mu_a}{T} = \frac{\delta\rho_a}{\sum_\alpha q_{\alpha,a} \langle n_\alpha \rangle q_{\alpha,b}} = \frac{\delta\rho_a}{\chi_{ab}^{had}}$$

3. Back to our problem...

Given: $$g'_{ab}(\Delta\eta) = \left\langle \delta\rho_a(0)\delta\rho_b(\Delta\eta)\right\rangle = \sum_{\alpha\beta}\left\langle n_\alpha(0)q_{\alpha,a}n_\beta(\Delta\eta)q_{\beta,b}\right\rangle$$

Assume: $$\left\langle n_\alpha(0)n_\beta(\Delta\eta)\right\rangle = \left\langle n_\alpha\right\rangle\left\langle n_\beta\right\rangle \exp\left\{\sum_{ab}\mu_{ab}(\Delta\eta)q_{\alpha,a}q_{\beta,b}\right\}$$

Solution: $$\mu_{ab}(\Delta\eta) = \chi_{ac}^{(HAD)-1}g'_{cd}(\Delta\eta)\chi_{db}^{(HAD)-1}$$

3. Putting this together

$$-G'_{\alpha\beta}(\Delta\eta) = w_{\alpha\beta}^{(\mathrm{QGP})} \frac{e^{-\Delta\eta^2/2\sigma^2_{(QGP)}}}{\sqrt{2\pi\sigma^2_{(QGP)}}} + w_{\alpha\beta}^{(\mathrm{HAD})} \frac{e^{-\Delta\eta^2/2\sigma^2_{(HAD)}}}{\sqrt{2\pi\sigma^2_{(HAD)}}}$$

$$w_{\alpha\beta}^{(\mathrm{QGP})} = -2\sum_{abcd} \langle n_\alpha \rangle q_{\alpha,a} \chi_{ab}^{-1(HAD)} \chi_{bc}^{(QGP)} \chi_{cd}^{-1(HAD)} \langle n_\beta \rangle q_{\beta,d}$$

$$w_{\alpha\beta}^{(HAD)} = -2\sum_{ab} \langle n_\alpha \rangle q_{\alpha,a} \chi_{ab}^{-1(HAD)} \langle n_\beta \rangle q_{\beta,b} - w_{\alpha\beta}^{(QGP)}$$

prefactors depend only only on yields and χ_{ab} from lattice

3. Prefactors...

(QGP,HAD)

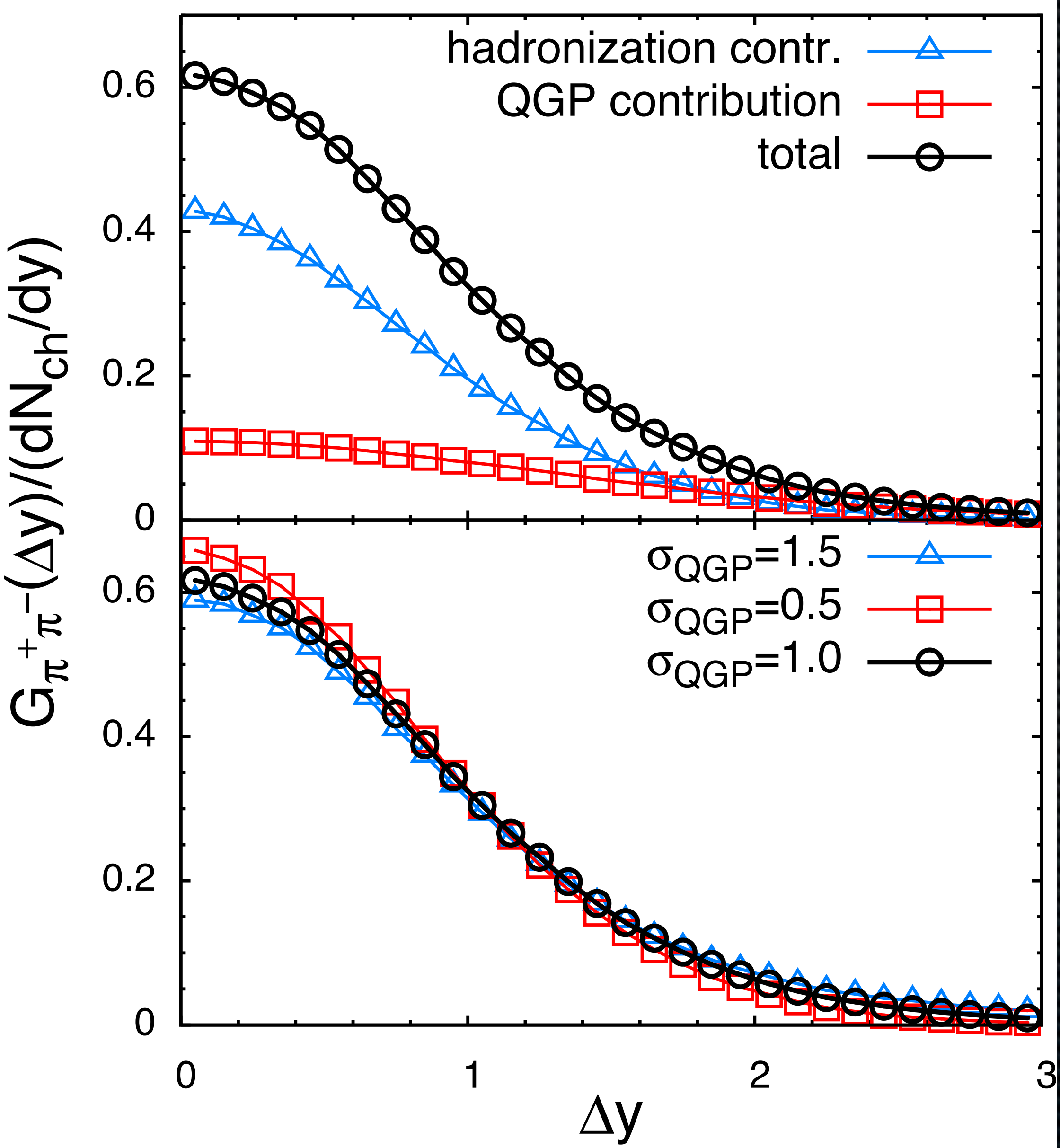
	p	Λ	Σ^+	Σ^-	Ξ^0	Ξ^-	Ω^-	π^+	K^+
$\bar{p}$	0.441,-0.066	0.485,-0.162	0.491,-0.146	0.479,-0.178	0.535,-0.242	0.529,-0.258	0.578,-0.338	0.006, 0.016	-0.044, 0.096
$\bar{\Lambda}$	0.183,-0.061	0.242,-0.094	0.242,-0.094	0.242,-0.094	0.302,-0.128	0.302,-0.128	0.361,-0.161	0.000,-0.000	-0.059, 0.033
$\bar{\Sigma}^-$	0.074,-0.022	0.097,-0.038	0.099,-0.033	0.095,-0.043	0.122,-0.049	0.120,-0.054	0.144,-0.064	0.002, 0.005	-0.023, 0.016
$\bar{\Sigma}^+$	0.072,-0.027	0.097,-0.038	0.095,-0.043	0.099,-0.033	0.120,-0.054	0.122,-0.049	0.144,-0.064	-0.002,-0.005	-0.025, 0.011
$\bar{\Xi}^0$	0.046,-0.021	0.069,-0.029	0.070,-0.028	0.069,-0.031	0.093,-0.036	0.092,-0.038	0.115,-0.045	0.001, 0.001	-0.023, 0.008
$\bar{\Xi}^+$	0.046,-0.022	0.069,-0.029	0.069,-0.031	0.070,-0.028	0.092,-0.038	0.093,-0.036	0.115,-0.045	-0.001,-0.001	-0.023, 0.007
$\bar{\Omega}^+$	0.009,-0.005	0.015,-0.007	0.015,-0.007	0.015,-0.007	0.021,-0.008	0.021,-0.008	0.027,-0.009	-0.000,-0.000	-0.006, 0.001
π^-	0.119, 0.318	0.000,-0.000	0.239, 0.636	-0.239,-0.636	0.119, 0.318	-0.119,-0.318	-0.000,-0.000	0.239, 0.636	0.119, 0.318
K^-	-0.175, 0.384	-0.627, 0.352	-0.603, 0.417	-0.651, 0.288	-1.055, 0.385	-1.079, 0.321	-1.507, 0.354	0.024, 0.064	0.452, 0.031

prefactors completely determined by χ_{QGP} and final-state hadronic yields

4. Use blast-wave to go from coordinate space η to momentum-space rapidity (Monte Carlo + decays)

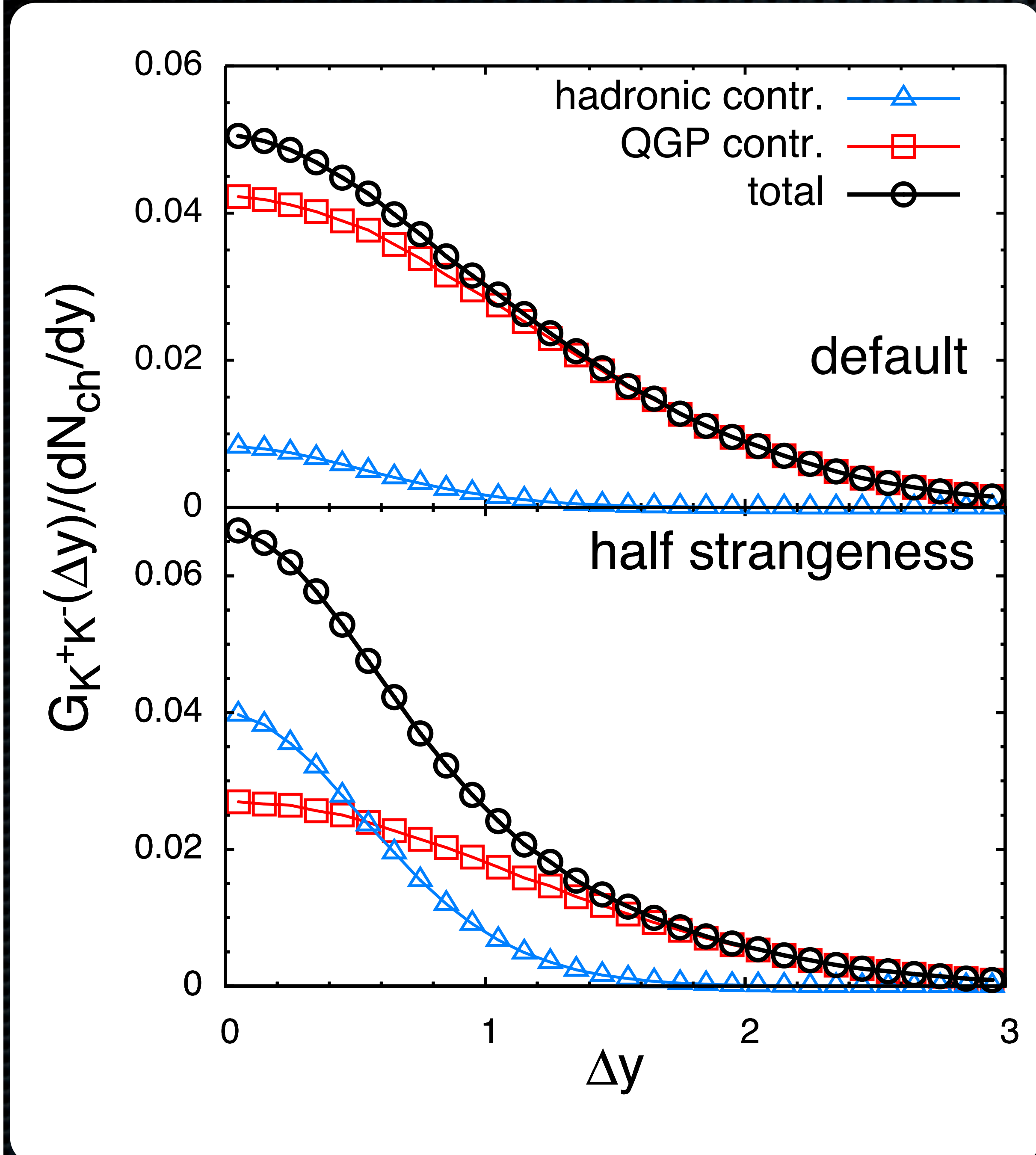
....

$\pi^+\pi^-$

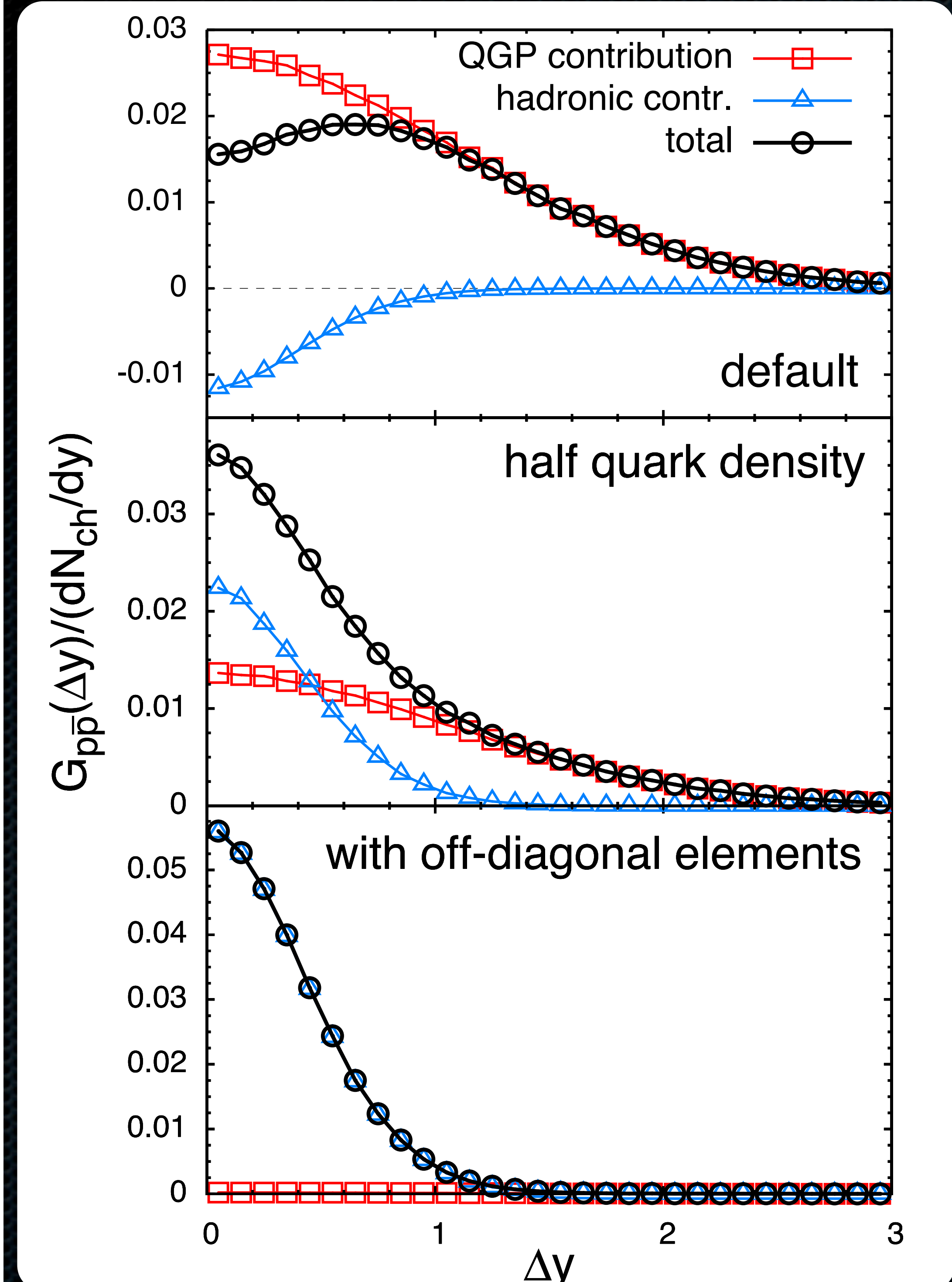


- **Hadronization part narrower**
- **Can't well separate components due to thermal smearing acceptance**
- **narrows with centrality**

K^+K^-

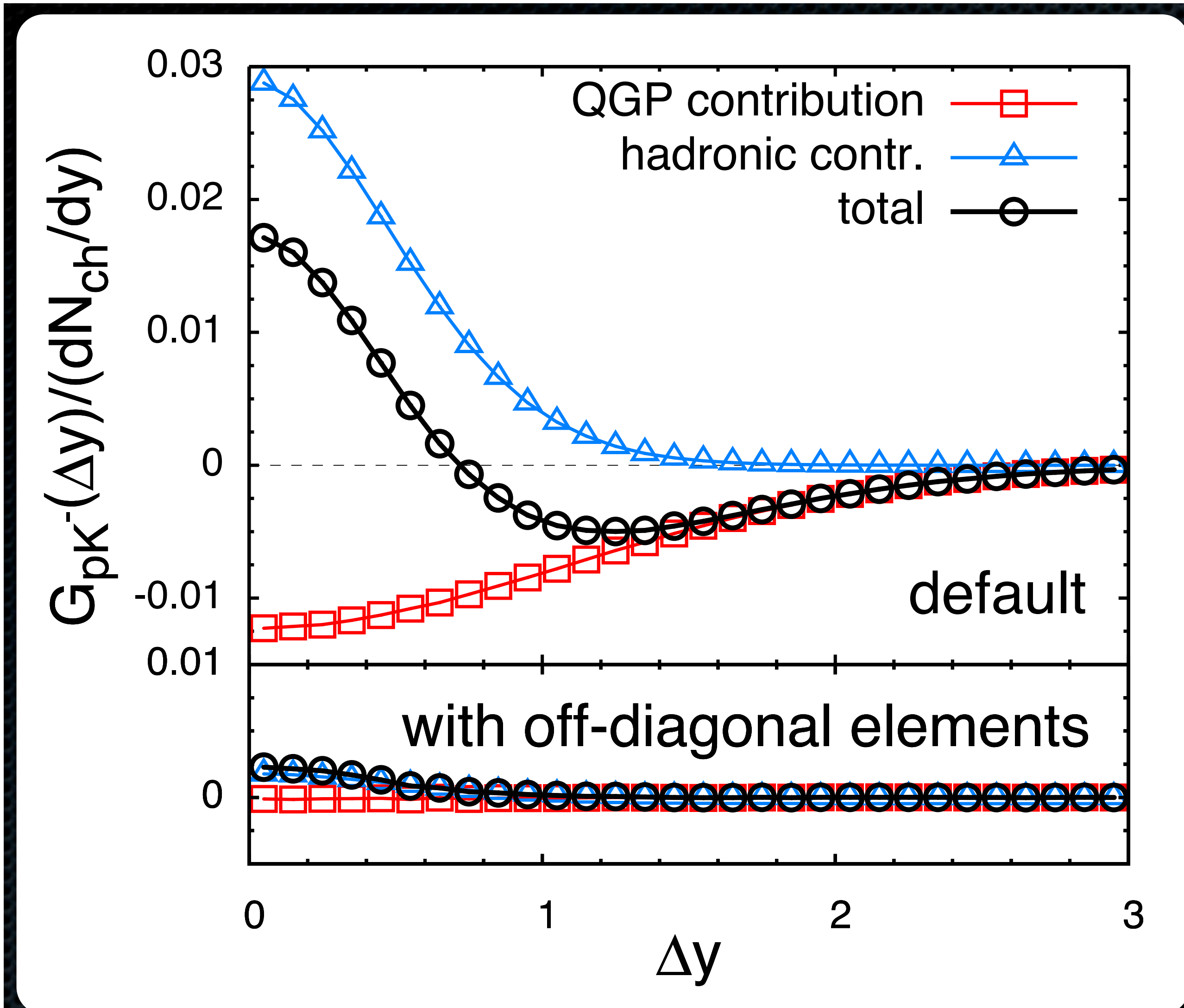


- **Little hadronic contribution**
- **Can test whether QGP is rich in strangeness**



p-pbar

- hadron contribution negative
- tests two-wave nature
- no narrowing with centrality
- sensitive to quark density of QGP



pK⁻

- QGP contribution negative
- dips negative
- too narrow for one source

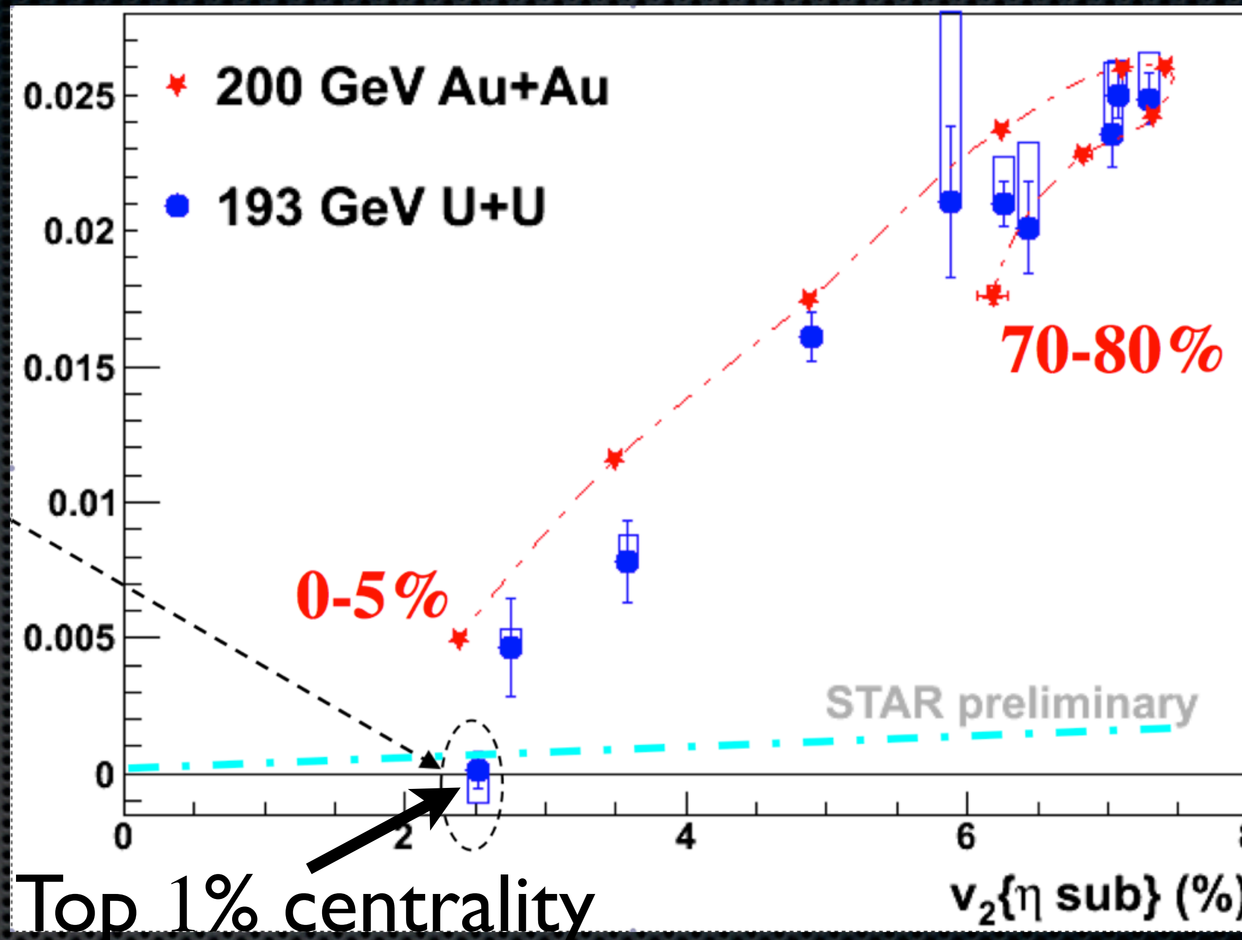
SUMMARY

Charge correlations provide hope...

- Clear test of χ_{ab} from lattice
  - quark density
  - strangeness in QGP
  - off-diagonal elements

- Can test 2-wave charge production

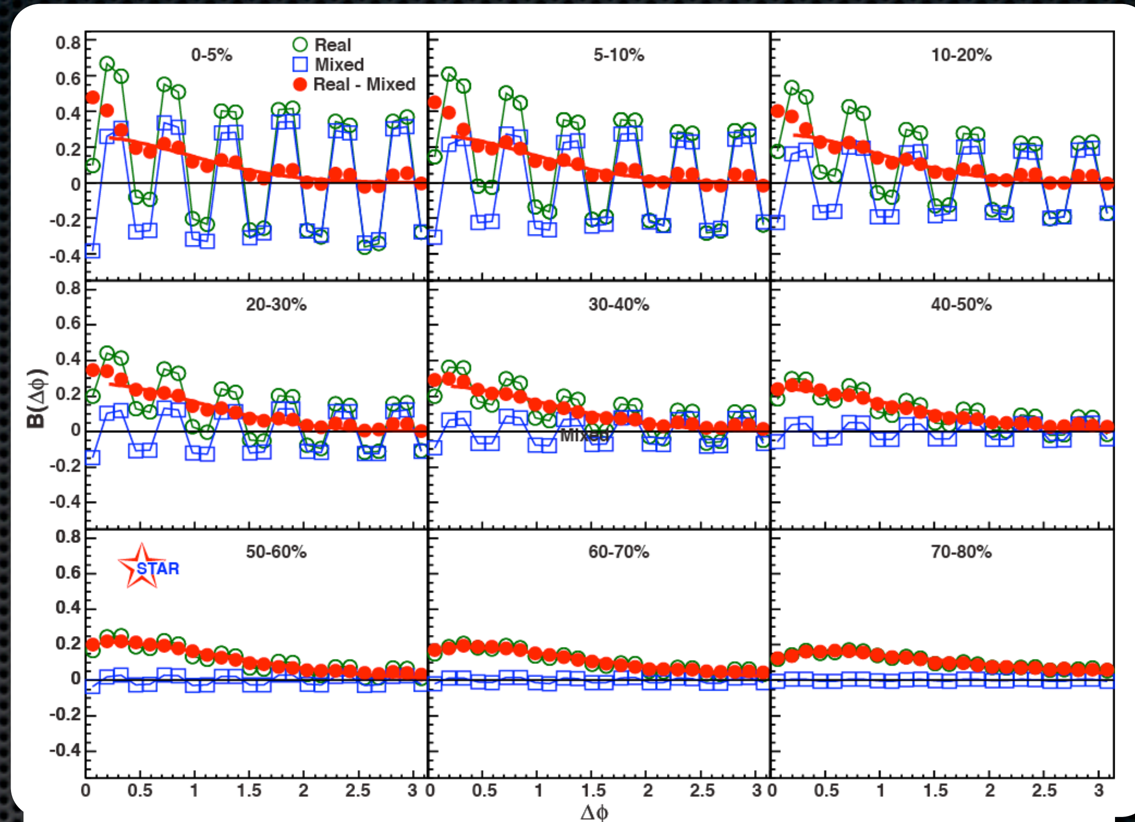
U+U Data

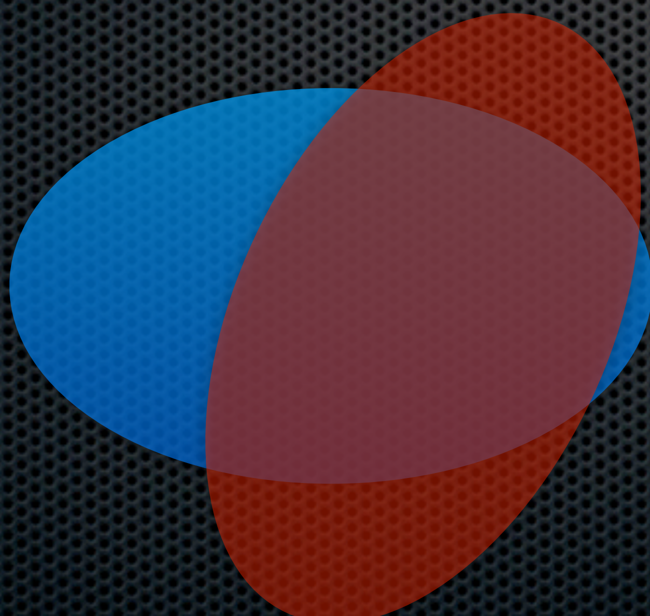


Conclusion: Since CC+Flow cannot go away for finite v2, effect must be due CME because CME should disappear for events with no anisotropy ?????

Problems with U+U

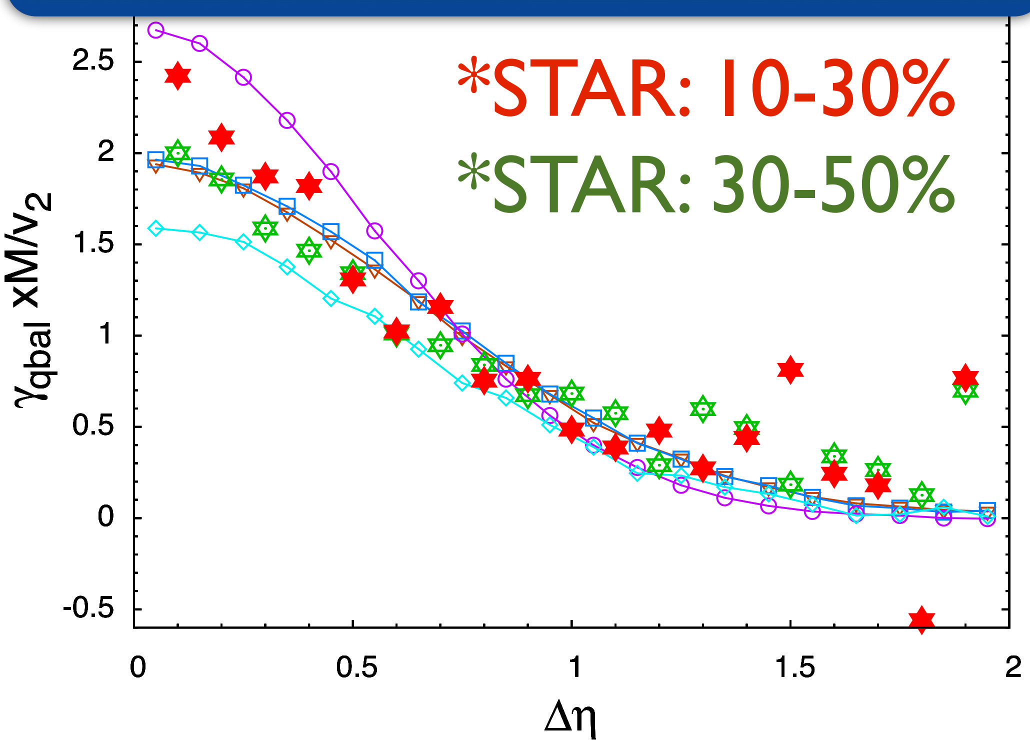
1. Detector effects are important at high mult.





2. E fields don't cancel in U+U -- can lead to charge separation

Pion cascade Δη dependence



Lines:
Different Calculations
- Default
- Double anisotropy
- Halve Size
- Double cross section

Pion Cascade Multiplicity Dependence