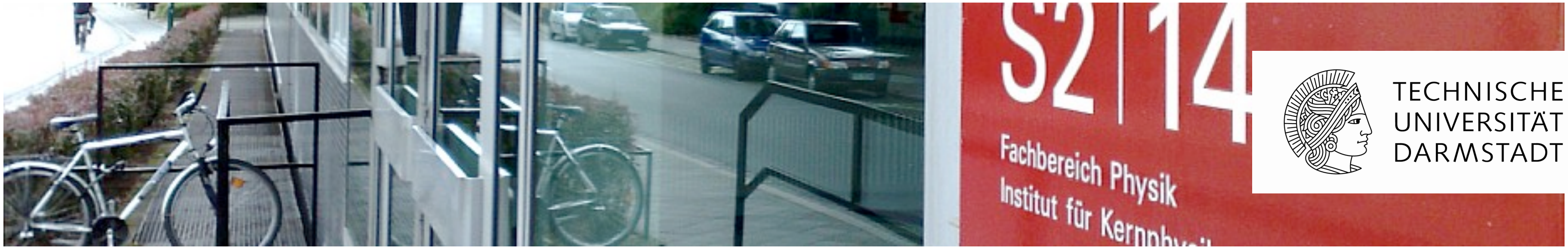


TECHNISCHE
UNIVERSITÄT
DARMSTADT

QCD-like theories at finite density

34th International School of Nuclear Physics

Probing the Extremes of Matter with Heavy Ions

Erice, Sicily, 23 September 2012

Lorenz von Smekal

Contents

- Introduction
- QCD with Isospin Chemical Potential ↔ Two-Color QCD

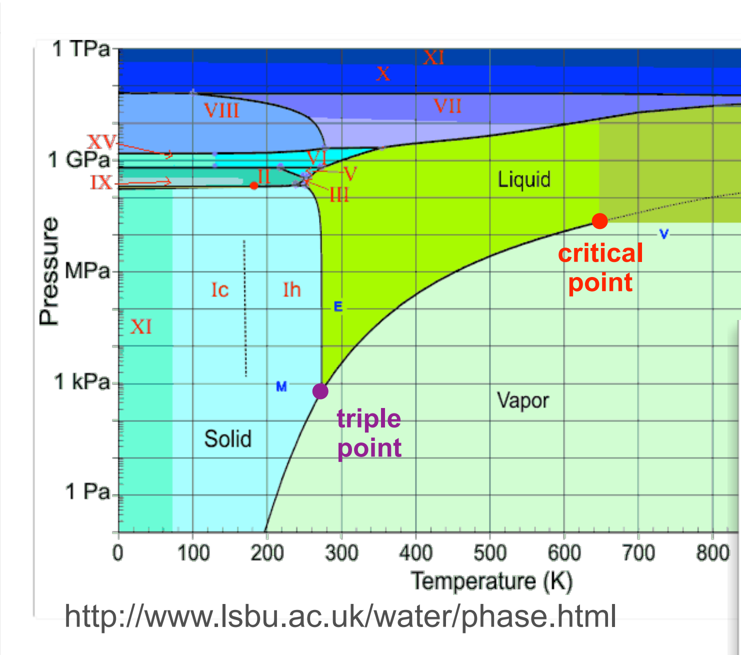
  N. Strodthoff, B.-J. Schaefer & L.v.S., Phys. Rev. D85 (2012) 074007

  K. Kamikado, N. Strodthoff, L.v.S. & J. Wambach, arXiv:1207.0400 [hep-ph]
- G_2 Gauge Theory at Finite Baryon Density

  A. Maas, L.v.S., B. Wellegehausen & A. Wipf, arXiv:1203.5653 [hep-lat]
- Summary and outlook

See also: L.v.S. in "Physics at all scales: The Renormalization Group," the 49th Schladming Winter School on Theoretical Physics, Nucl. Phys. B (PS) 228 (2012) pp. 179 - 220 [arXiv:1205.4205]

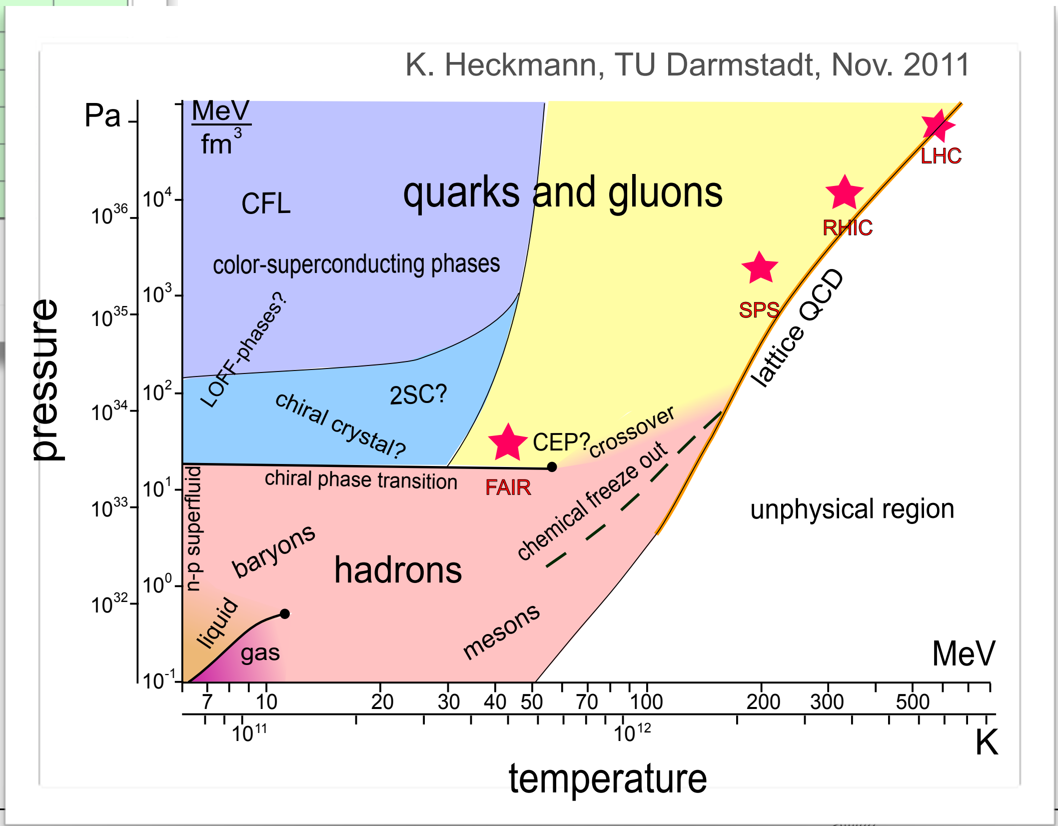
Phase Diagram



http://www.lsbu.ac.uk/water/phase.html

Water

QCD



K. Heckmann, TU Darmstadt, Nov. 2011

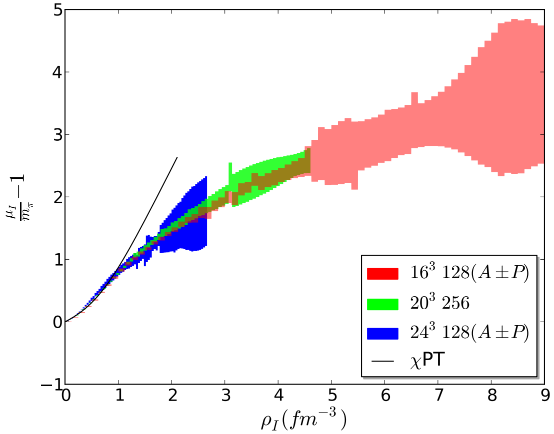
QCD-like Theories

Functional methods and effective models:

- **compare with lattice simulations**

  where there's no sign problem

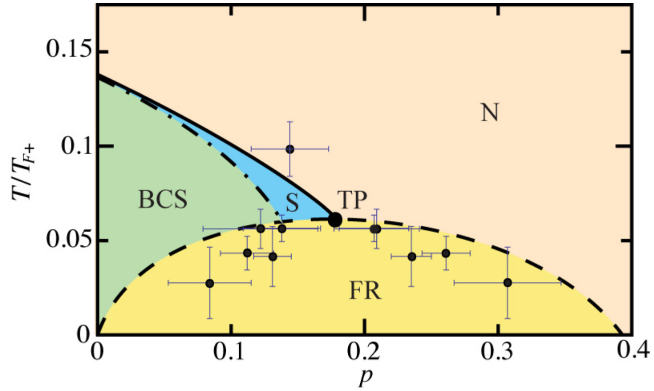
QCD at finite isospin density



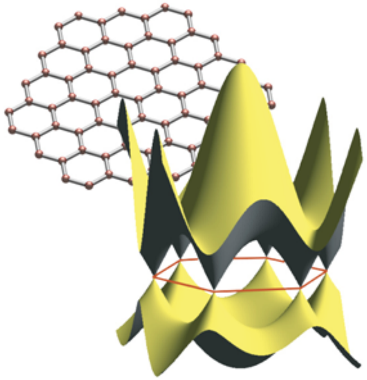
- **apply to ultracold fermi gases**

  exploit analogies and
  more experimental data

polarised fermi gas at unitarity



graphene



- **strongly correlated fermions in 2+1 dimensions**

  QED$_3$ (semimetal-insulator transition, $N_f < 4$),

  electronic properties of Graphene (half-filling, $N_f = 2$) – SFB 634

Fermion-Sign Problem

- **Dirac operator:** $D(\mu) = \gamma_\mu(\partial^\mu + iA^\mu) - \gamma^0\mu$

  $\nearrow$ anti-Hermitian $\nwarrow$ Hermitian

$$\Rightarrow \quad \gamma_5 D(\mu)^\dagger \gamma_5 = D(-\mu) \qquad \text{or} \qquad \big(\mathrm{Det}\, D(\mu)\big)^* = \mathrm{Det}\, D(-\mu)$$

- **in general, fermion-sign problem, except if:**

(a) anti-unitary symmetry $\quad TD(\mu)T^{-1} = D(\mu)^* \qquad T^2 = \pm 1$

fermion color representation:

Dyson index:

(i) pseudo-real $\quad T = \Sigma C \gamma_5\,,\ T^2 = 1 \qquad \beta = 1$

color, $\Sigma^2 = -1$ $\nearrow$

charge conjugation, $C^2 = -1$ $\uparrow$

two-color QCD

$\rightarrow$ B.-J. Schaefer's talk

(ii) real $\quad T = C\gamma_5\,,\ T^2 = -1 \qquad \beta = 4$

adjoint QCD, or G$_2$-QCD

$\rightarrow$ later this talk

Fermion-Sign Problem

...except if:

$$\big(\mathrm{Det}\, D(\mu)\big)^* = \mathrm{Det}\, D(-\mu)$$

Dyson index:

(b) two degenerate flavors with isospin chemical potential

$\beta = 2$

fermion determinant $\rightsquigarrow \mathrm{Det}\big(D(\mu_I)D(-\mu_I)\big)$

QCD at finite isospin density

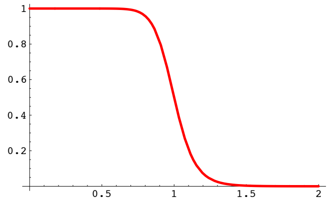
χ**PT:** Son & Stephanov, Phys. Rev. Lett. 86 (2001) 592

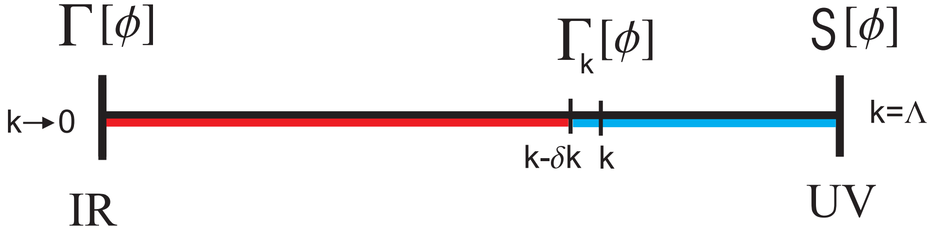
Silver Blaze: Cohen, Phys. Rev. Lett. 91 (2003) 222001

Lattice: Kogut & Sinclair, Phys. Rev. D 70 (2004) 094501; PoS LAT2006 147
de Forcrand, Stephanov & Wenger, PoS LAT2007 237
Detmold, Orginos & Shi, arXiv:1205.4224

NJL: He, Jin & Zhuang, Phys. Rev. D 71, (2005) 116001
Mu, He & Liu, Phys. Rev. D 82 (2010) 056006

Functional RG (Flow) Equations





Effective action: $\Gamma[\phi_j] = (j, \phi_j) - \ln Z[j]$

Legendre transform

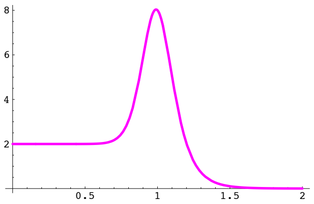
→ **1PI vertex functions**

$\Gamma^{(n)}(x_1, \ldots x_n)$

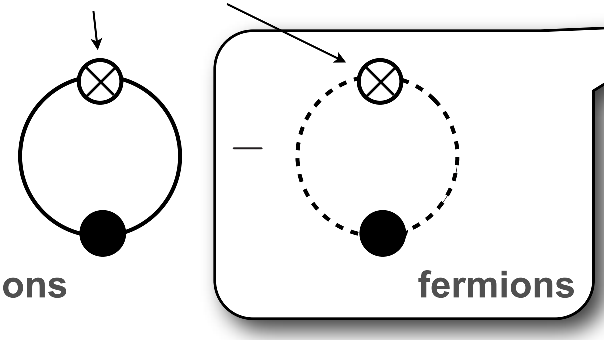
→ **grand potential**

$\Omega(T, \mu)$, at

$$\phi_{\min} = \langle \phi \rangle_{T,\mu}$$



$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}$$



extended mean field (eMF)

Wetterich, Phys. Lett. B 301 (1993) 90

QM Model with Isospin Chemical Potential

- **N_f = 2 quarks & mesons with Yukawa coupling:**

$$\begin{aligned}\mathcal{L} =& \bar{\psi}(\partial\!\!\!/ + g(\sigma + i\gamma^5\vec{\pi}\vec{\tau}) - \mu\gamma^0 - \mu_I\tau_3\gamma^0)\psi \\ &+ \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\pi_0)^2 + U(\rho^2, d^2) - c\sigma \\ &+ \frac{1}{2}\left((\partial_\mu + 2\mu_I\delta^0_\mu)\pi_+(\partial_\mu - 2\mu_I\delta^0_\mu)\pi_-\right)\end{aligned}$$

- **chemical potentials:**

$$\mu_u = \mu + \mu_I \qquad \mu_d = \mu - \mu_I$$

$\mu \gg \mu_I$: $\mu_I \rightsquigarrow$ imbalance between up and down

$\mu_I \gg \mu$: $\mu \rightsquigarrow$ imbalance between up and anti-down

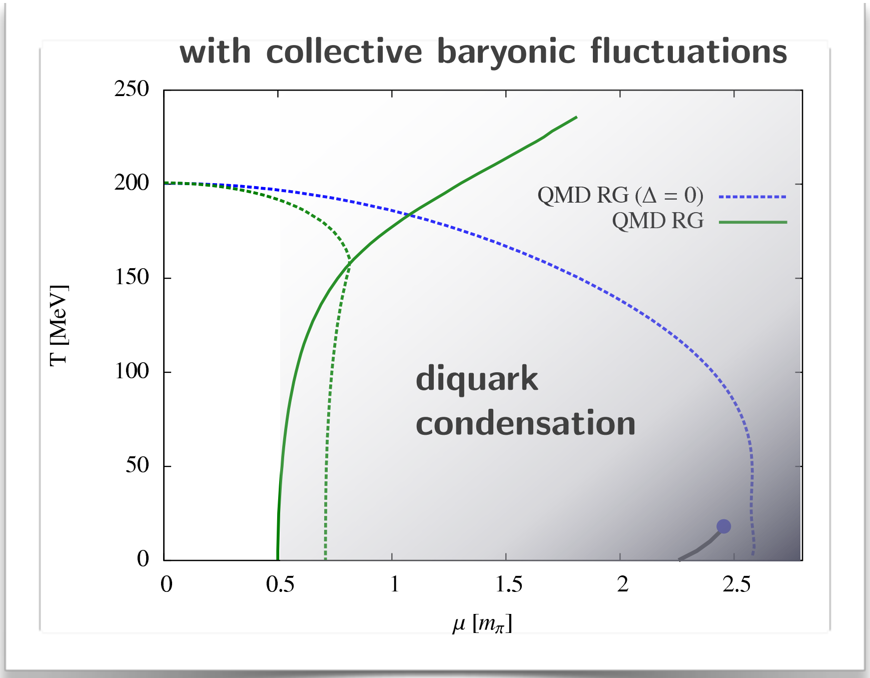
- **$\mu = 0$, map to QMD model for QC$_2$D:**

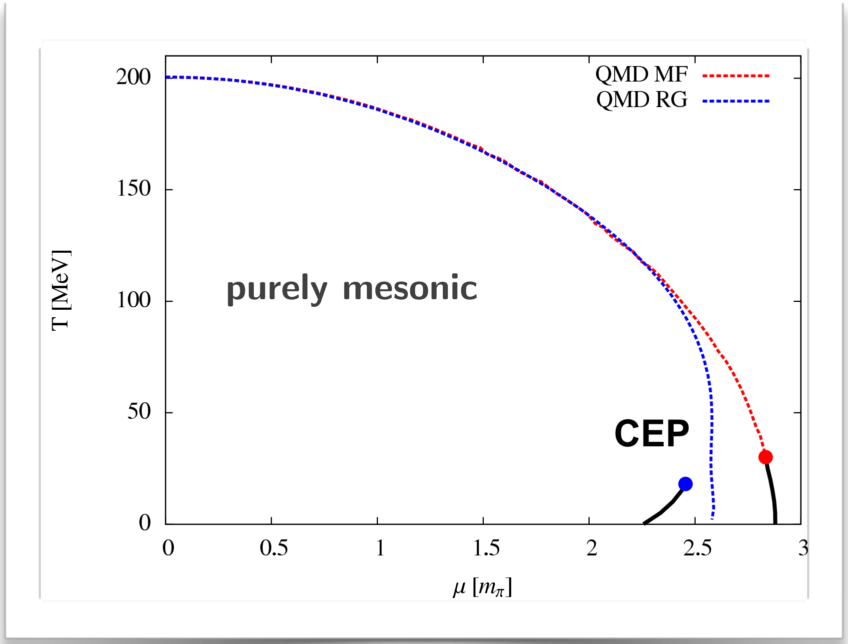
$$N_c\colon\ 3 \to 2 \qquad (\psi_u, \psi_d) \to (\psi_r, \tau_2 C\bar{\psi}_g) \qquad \mu_I \to \mu$$

$$\pi_+, \pi_- \to \Delta, \Delta^* \qquad \pi_0 \to \vec{\pi}$$

Two-Color QCD

- **QMD model phase diagram**





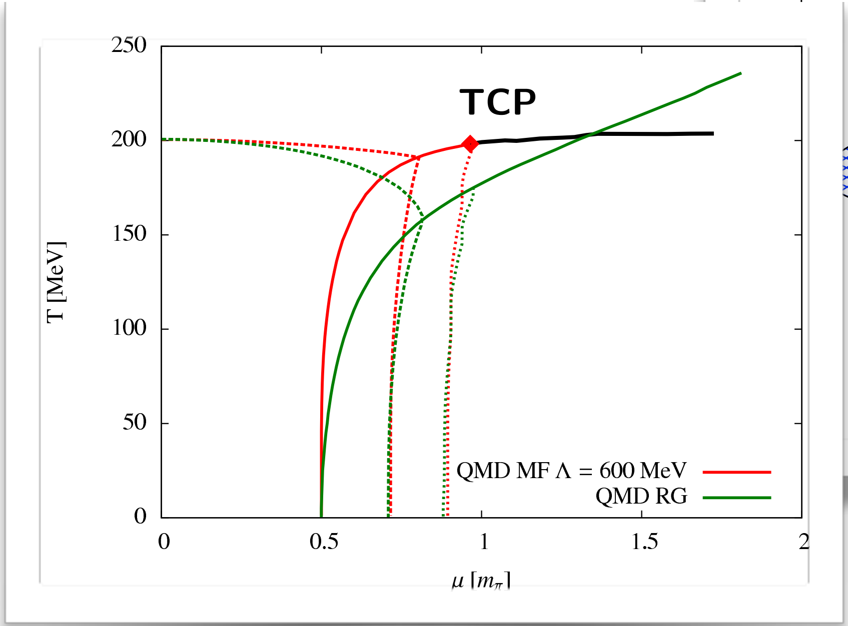
- 1^{st} order chiral transition and CEP at $\mu \approx 2.5\, m_\pi$

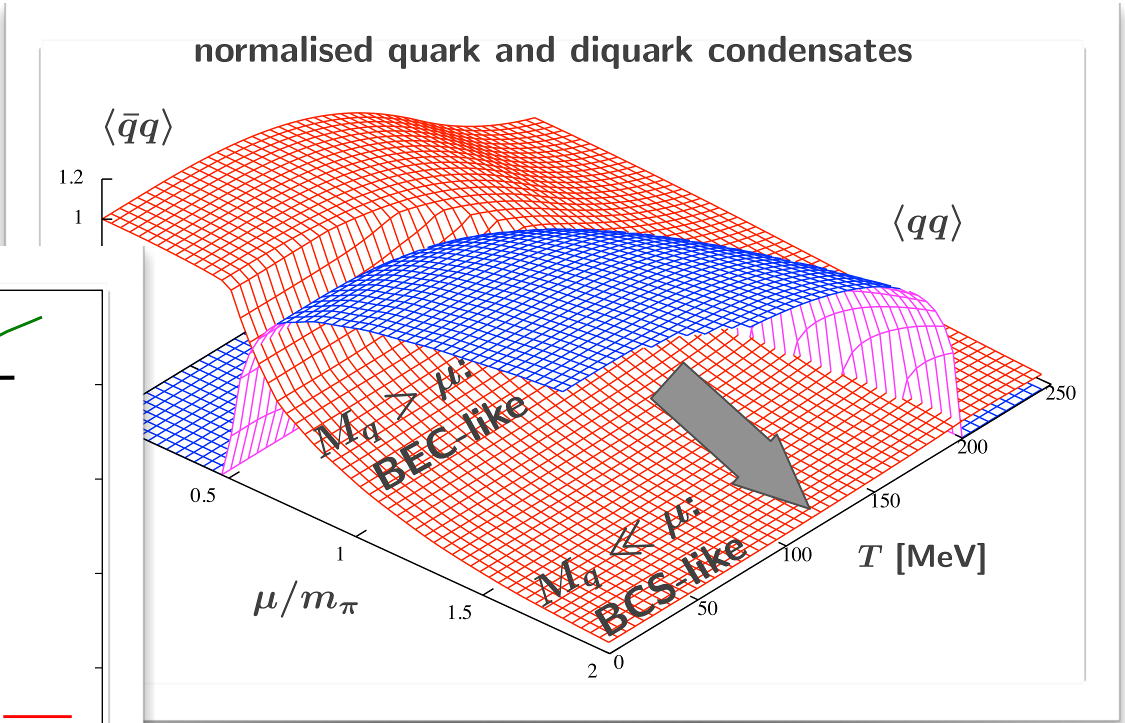
- no low-T $1^{\rm st}$ order transition,
  no CEP at $\mu \sim$ 2.5 m_π !

N. Strodthoff, B.-J. Schaefer & L.v.S., Phys. Rev. D85 (2012) 074007

Two-Color QCD

- **QMD model phase diagram**





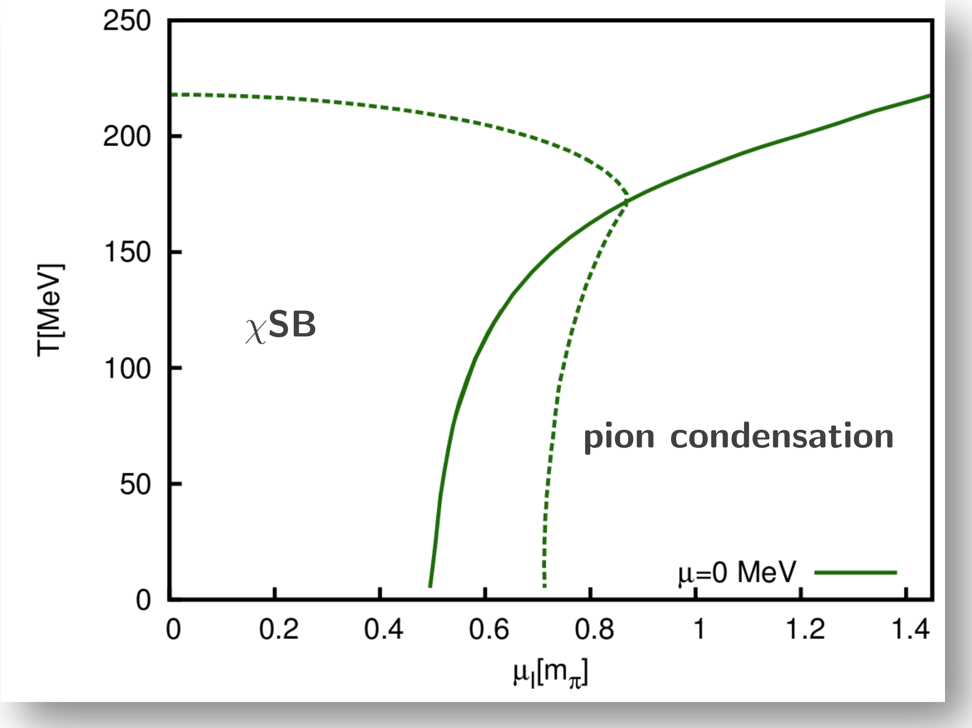
- Tricritical point predicted in:

  Splittorff, Toublan & Verbaarschot, Nucl. Phys. B 620 (2002) 290

N. Strodthoff, B.-J. Schaefer & L.v.S., Phys. Rev. D85 (2012) 074007

QCD with Isospin Chemical Potential

- **QM Model with fluctuating chiral & pion condensates**



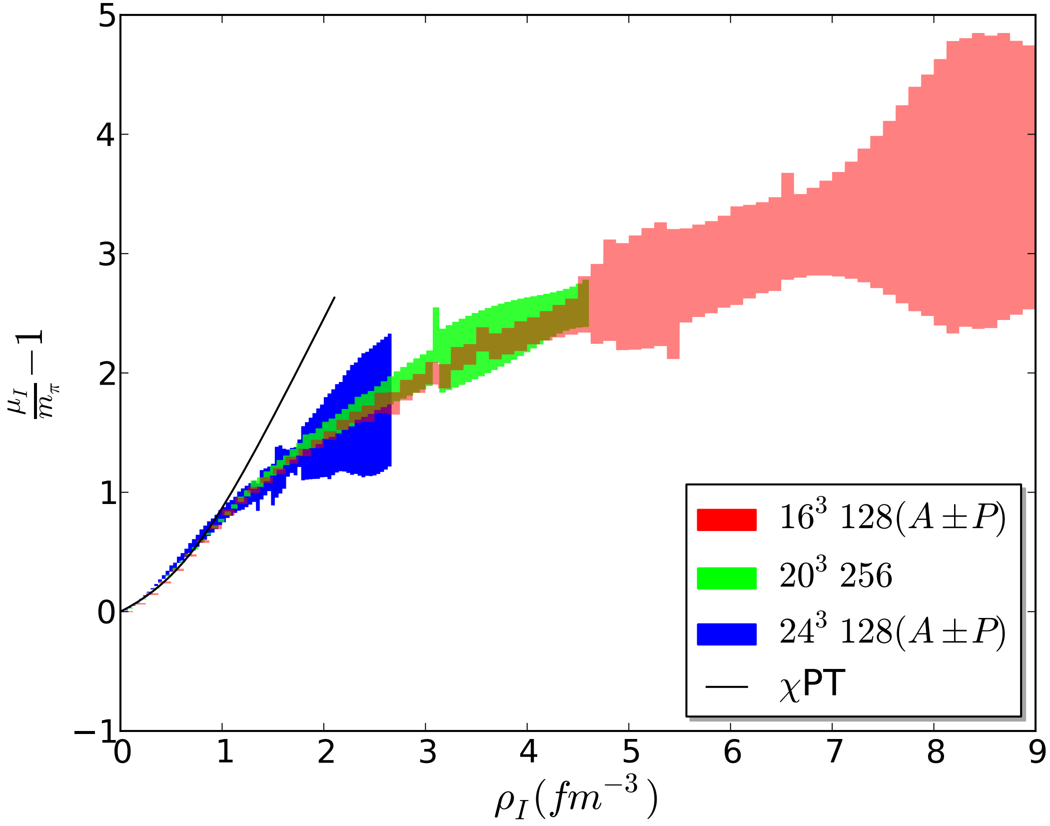
- **need 2 fields in effective potential**

$U = U(\rho^2, d^2)$, but replace $\rho^2 = \sigma^2 + \vec{\pi}^2$ and $d^2 = |\Delta|^2$
by $\rho^2 = \sigma^2 + \pi_0^2$ and $d^2 = \pi_1^2 + \pi_2^2 = \pi_+\pi_-$

K. Kamikado, N. Strodthoff, L.v.S. & J. Wambach, arXiv:1207.0400

QCD with Isospin Chemical Potential

- ***T* = 0 isospin density - lattice QCD:**



Detmold, Orginos & Shi, arXiv:1205.4224 [hep-lat]

QCD with Isospin Chemical Potential

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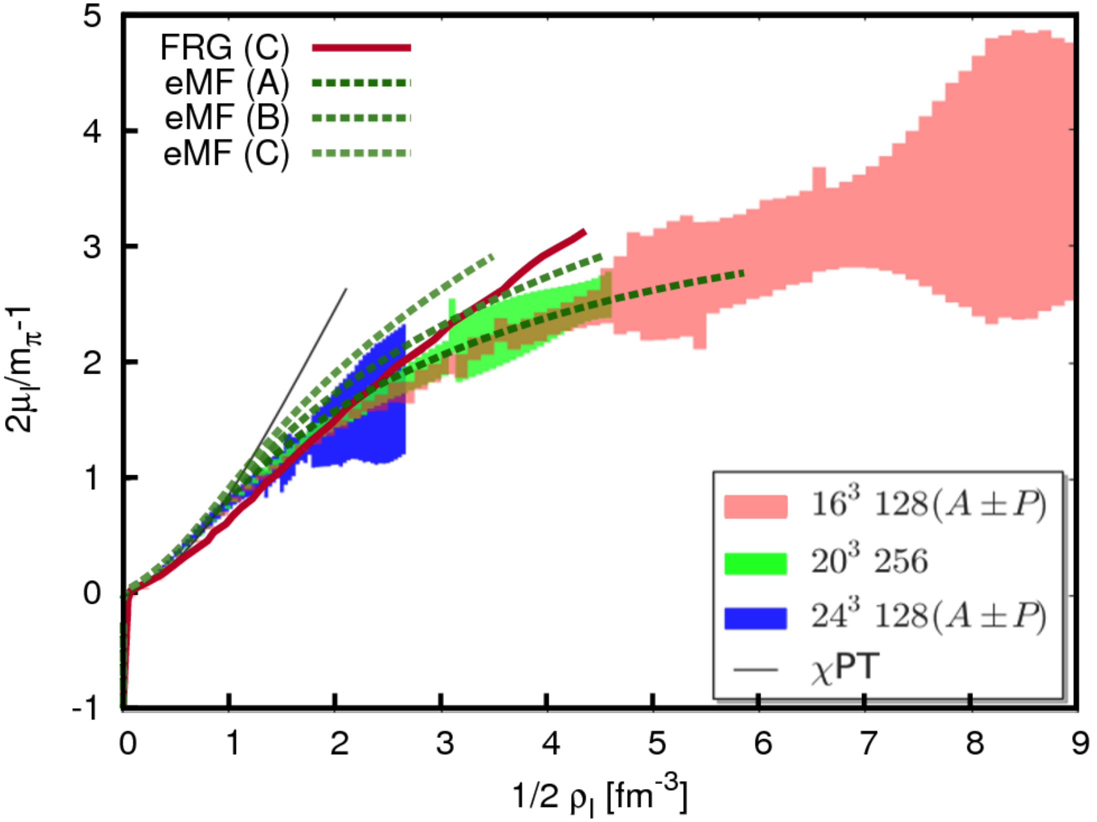
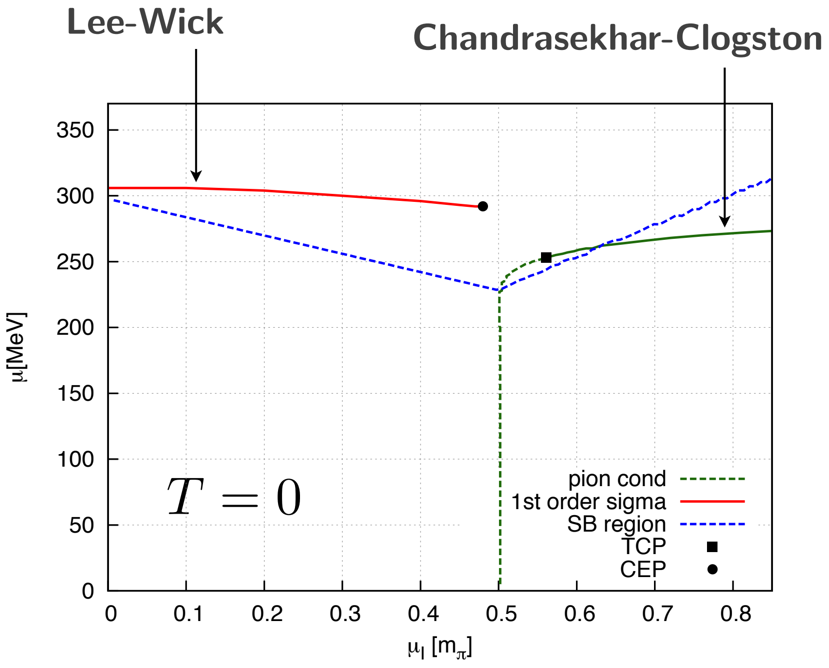
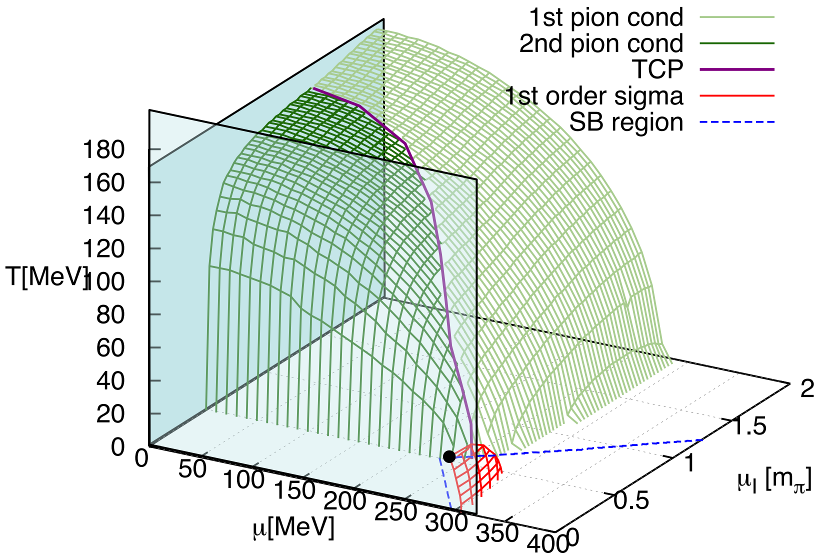


Detmold, Orginos & Shi, arXiv:1205.4224 [hep-lat]

Baryon & Isospin Chemical Potential

- **Fermionic flow (extended mean-field):**



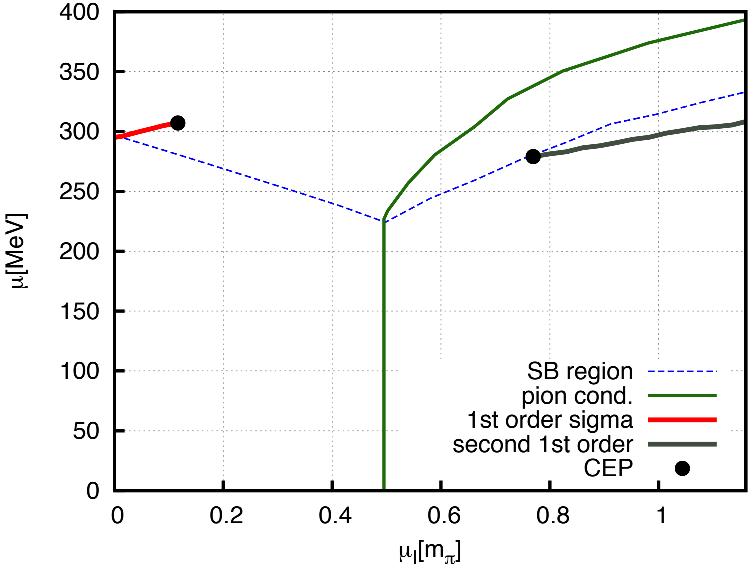


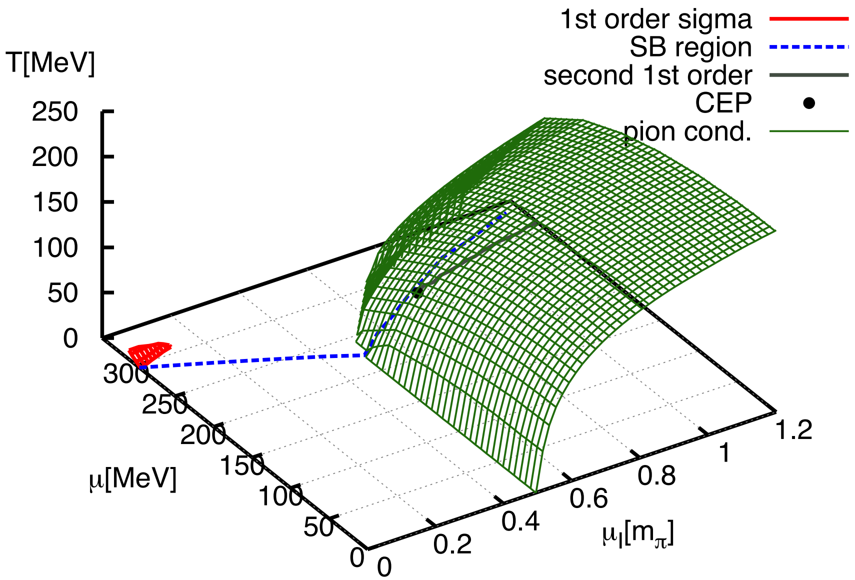
Kamikado, Strodthoff, LvS & Wambach, arXiv:1207.0400

Baryon & Isospin Chemical Potential

- **Full mesonic flow (2 dimensional):**

$T = 0$:





Kamikado, Strodthoff, LvS & Wambach, arXiv:1207.0400

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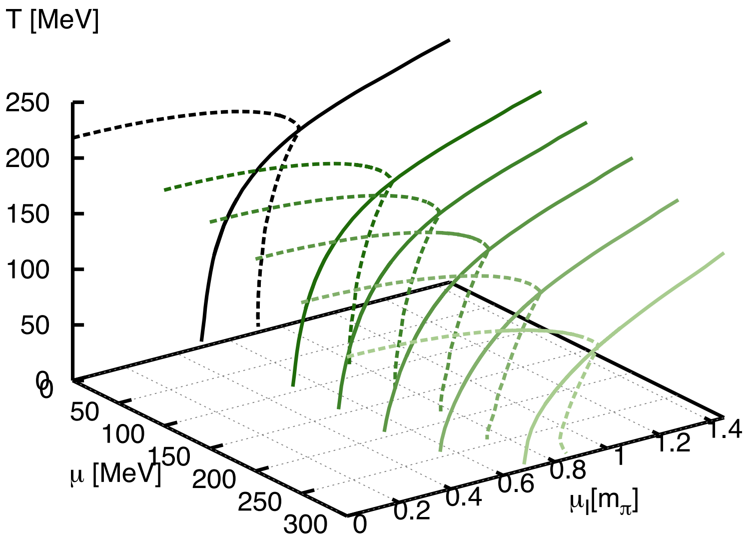

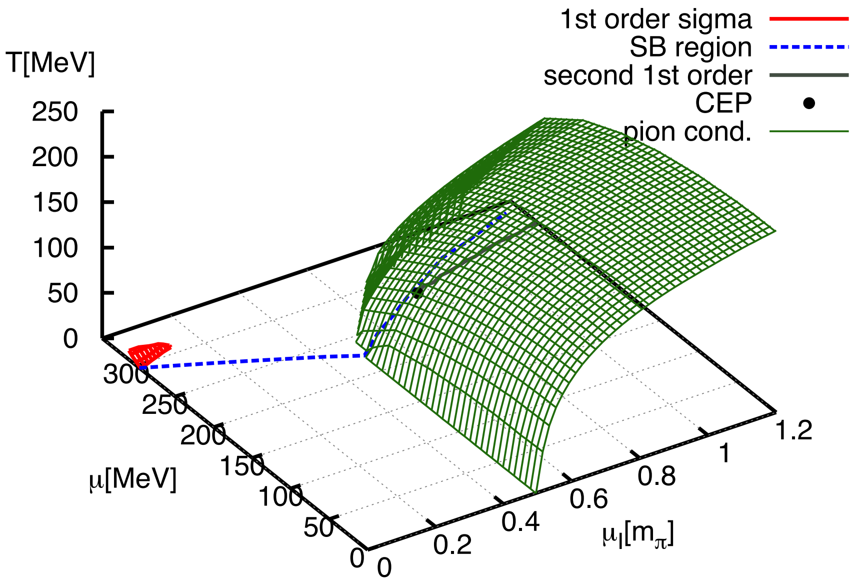

Kamikado, Strodthoff, LvS & Wambach, arXiv:1207.0400

Baryon & Isospin Chemical Potential

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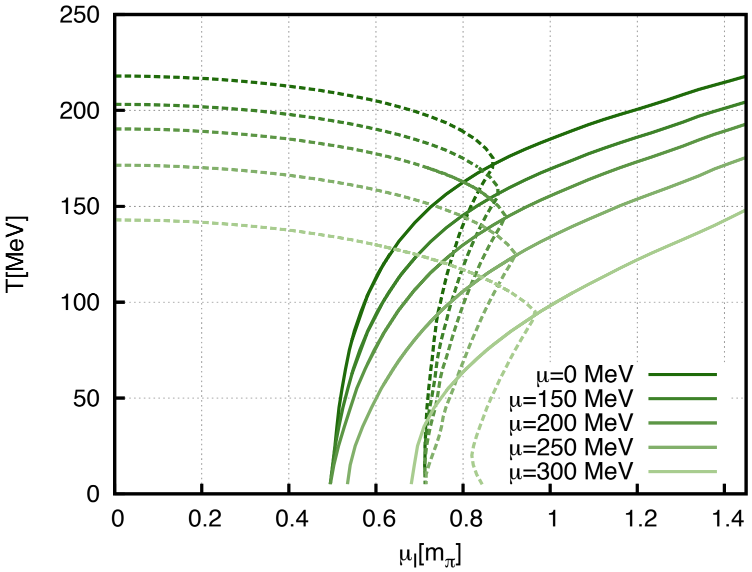
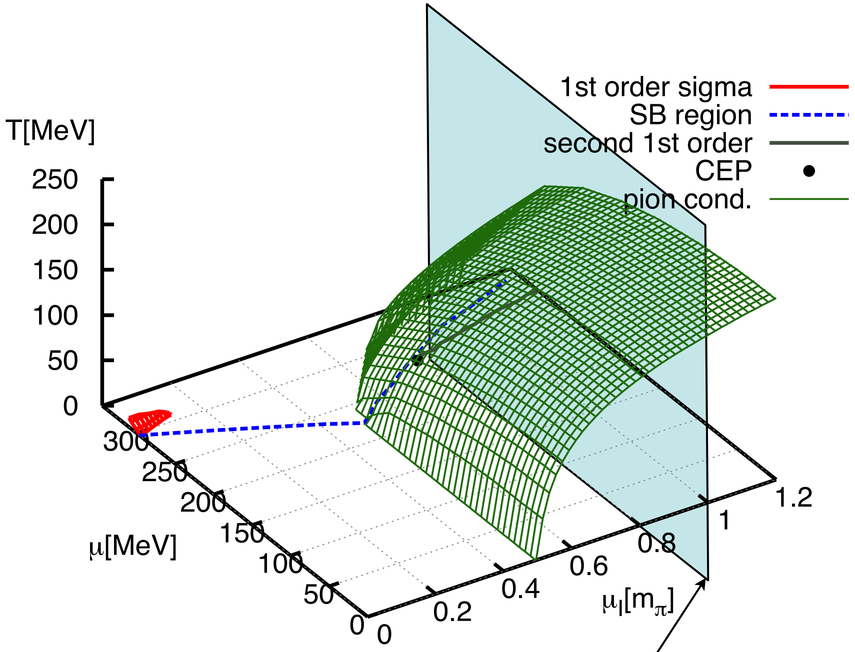


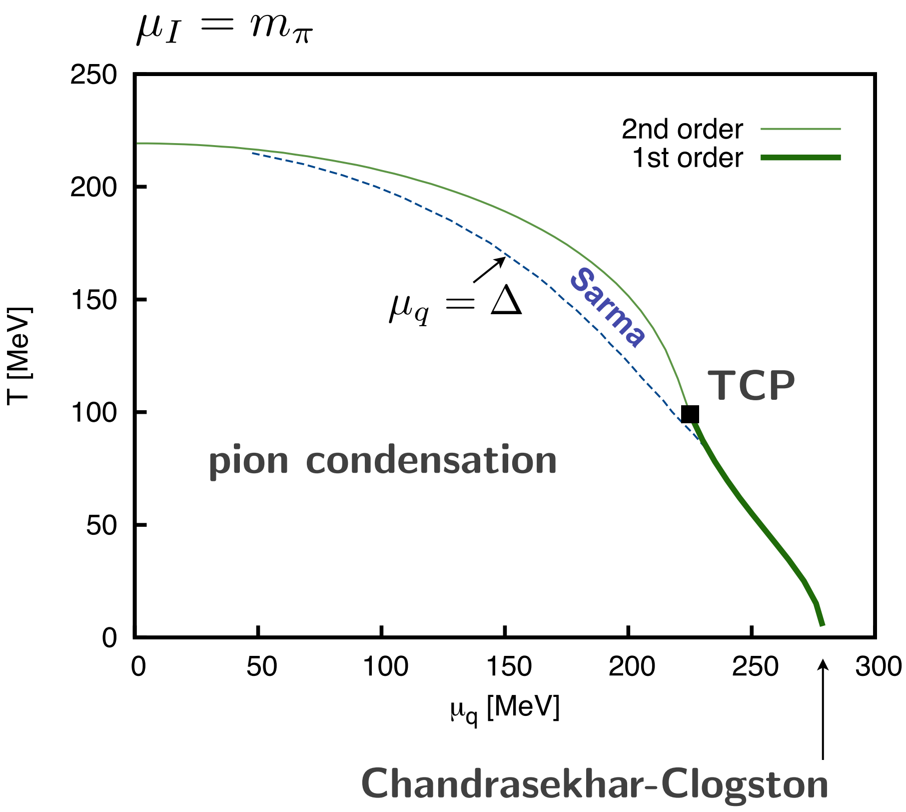


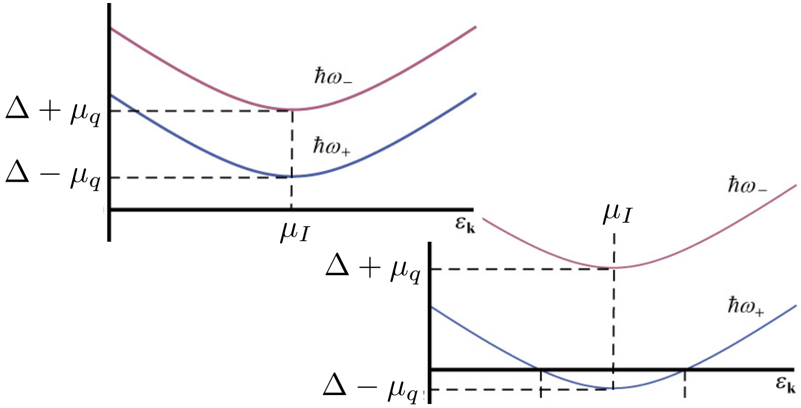
fixed $\mu_I = m_\pi$

μ for (up/anti-down) imbalance

Up-Antidown Population Imbalance

- **Fermionic flow (extended mean-field):**





Up-Antidown Population Imbalance

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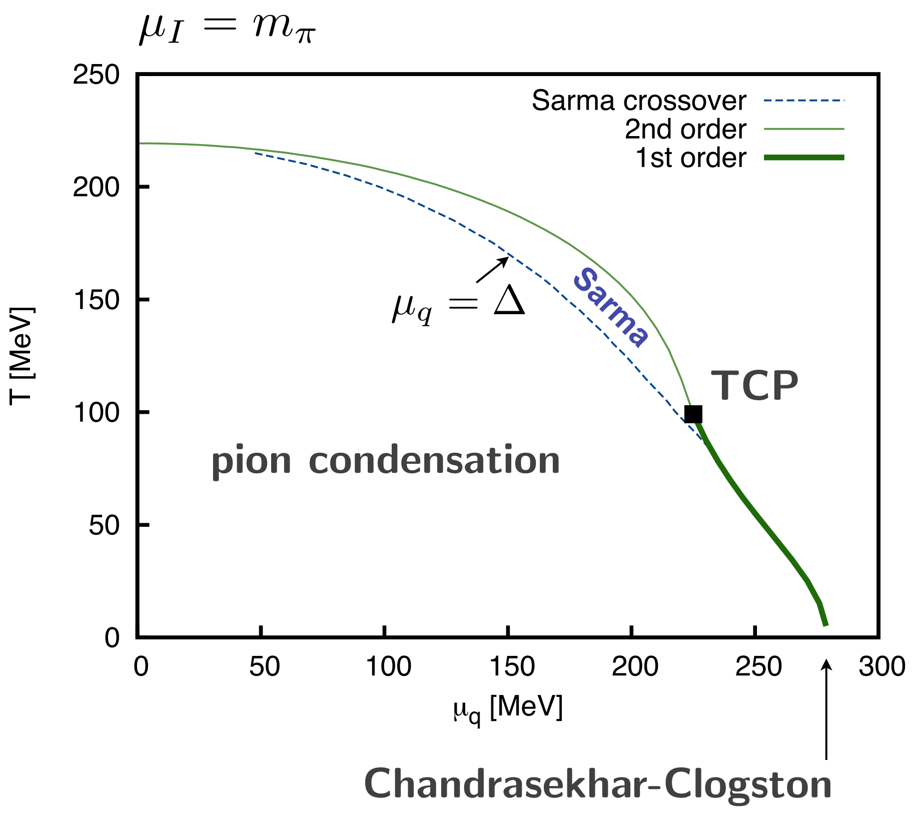
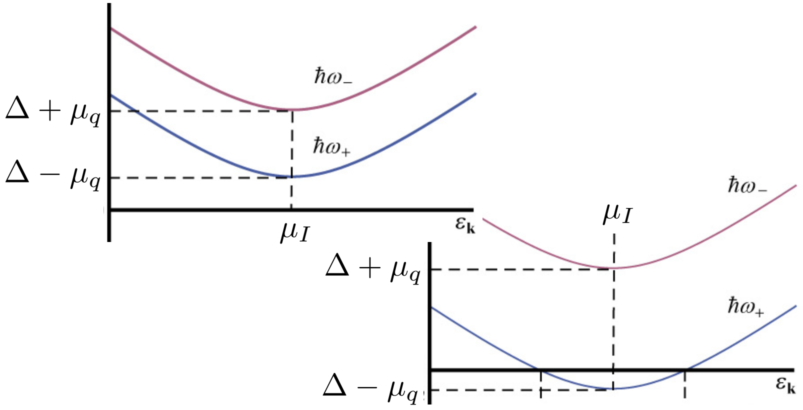
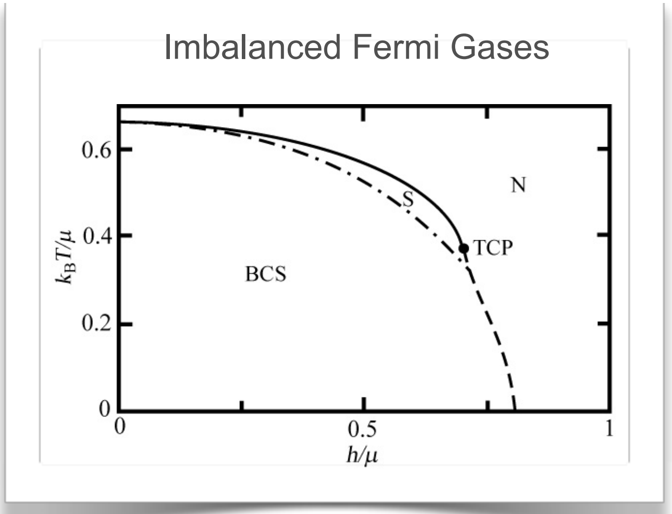




- **compare:**

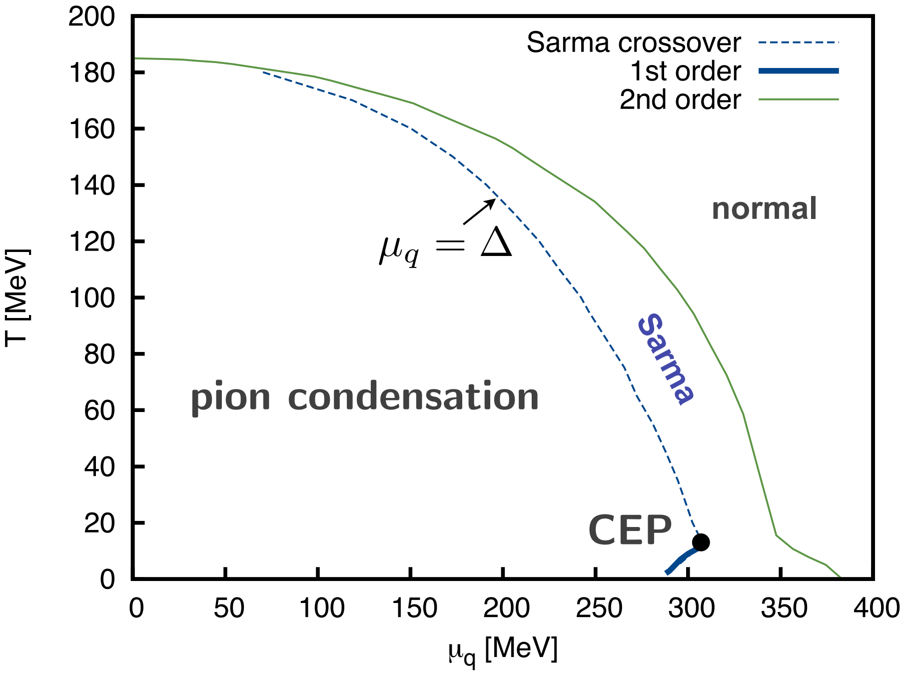


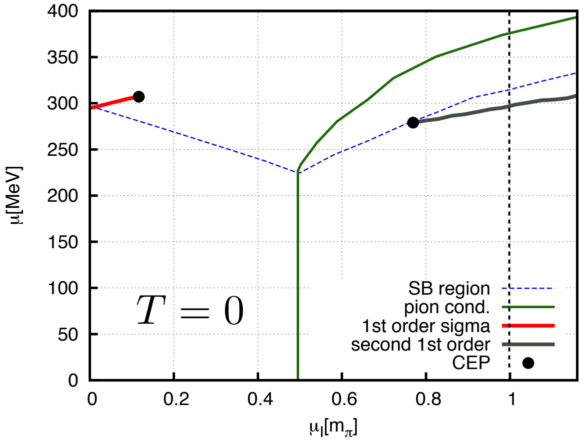
Gubbels, Stoof, arXiv:1205.0568

Up-Antidown Population Imbalance

- **Full flow with mesonic fluctuations:**

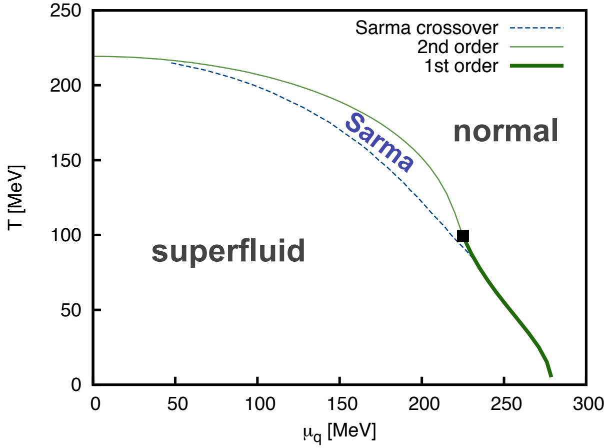
$\mu_I = m_\pi$





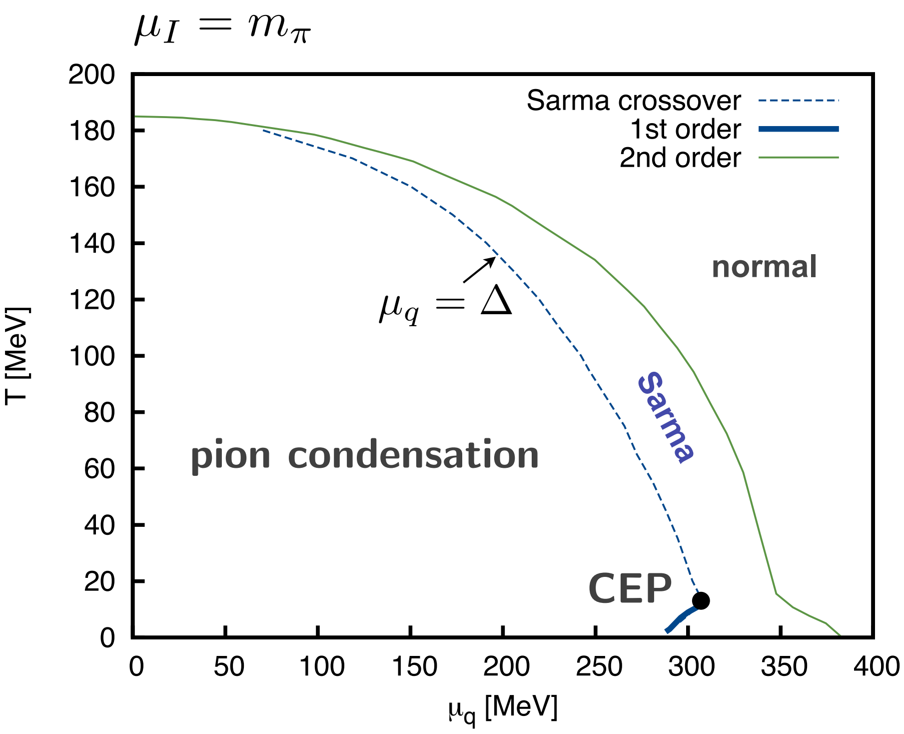
- **stable Sarma phase down to *T* = 0**

extended mean field



Up-Antidown Population Imbalance

- **Full flow with mesonic fluctuations:**

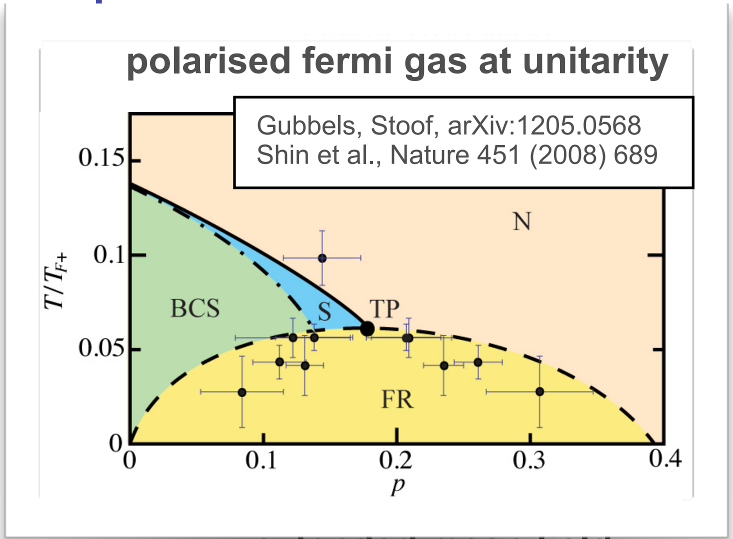


- **stable Sarma phase down to *T* = 0**

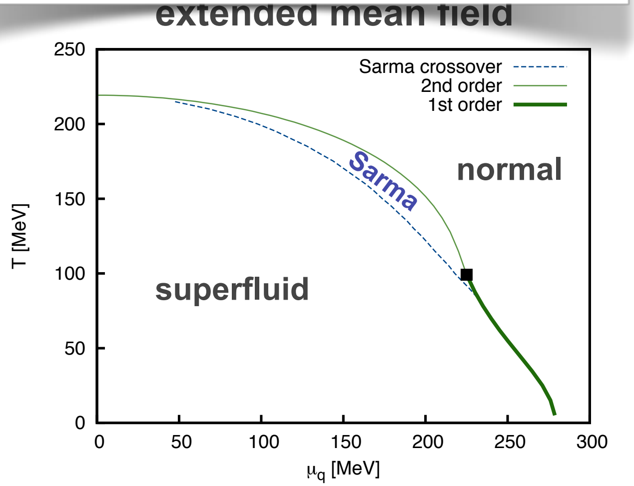
part. pol. superfluid near interface in fermi gases?

Strodthoff & LvS, in preparation

- **compare:**



Gubbels, Stoof, arXiv:1205.0568
Shin et al., Nature 451 (2008) 689



G_2 Gauge Theory at Finite Density

G_2 is real:

Dirac operator $\mathcal{D}$ has antiunitary symmetry S,

with $S^2 = -1$ (symplectic, $\beta = 4$).

Holland, Minkowski, Pepe & Wiese,
Nucl. Phys. B 668 (2003) 207
Wellegehausen, Wipf & Wozar, Phys.
Rev. D 83 (2011) 114502
Maas, LvS, Wellegehausen & Wipf, arXiv:1203.5653

- **no sign problem**

  real and positive for single flavor: $SU(2) \to U_B(1)$

  2 Goldstone bosons: scalar (anti)diquarks

- **O(3) symmetric effective potential**

  $U = U(\phi^2)$ where $\vec{\phi} = (\sigma, \mathrm{Re}\Delta, \mathrm{Im}\Delta)$

- **diquark condensation as in QC$_2$D**

- **but have fermionic baryons also**

  as QCD with adjoint quarks

- **breaks down to QCD**

$$G_2 \xrightarrow{\text{Higgs}} SU(3)$$

coset:

$$G_2/SU(3) \sim SO(7)/SO(6) \sim S^6$$

$$(7) \to (3) \oplus (\bar{3}) \oplus (1)$$

massive Higgs

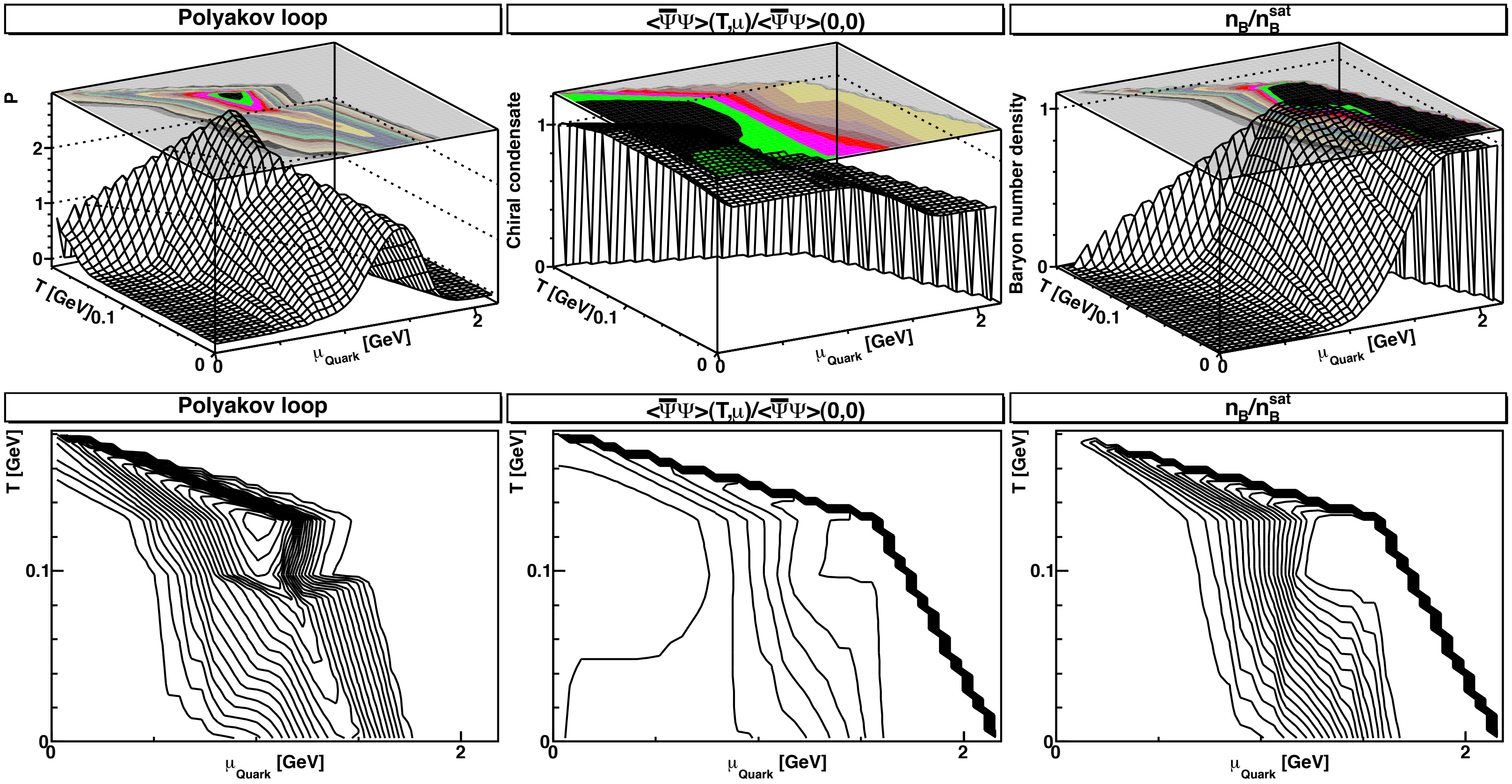
$$(14) \to (3) \oplus (\bar{3}) \oplus (8)$$

gluons

heavy gauge

G_2 Gauge Theory at Finite Density

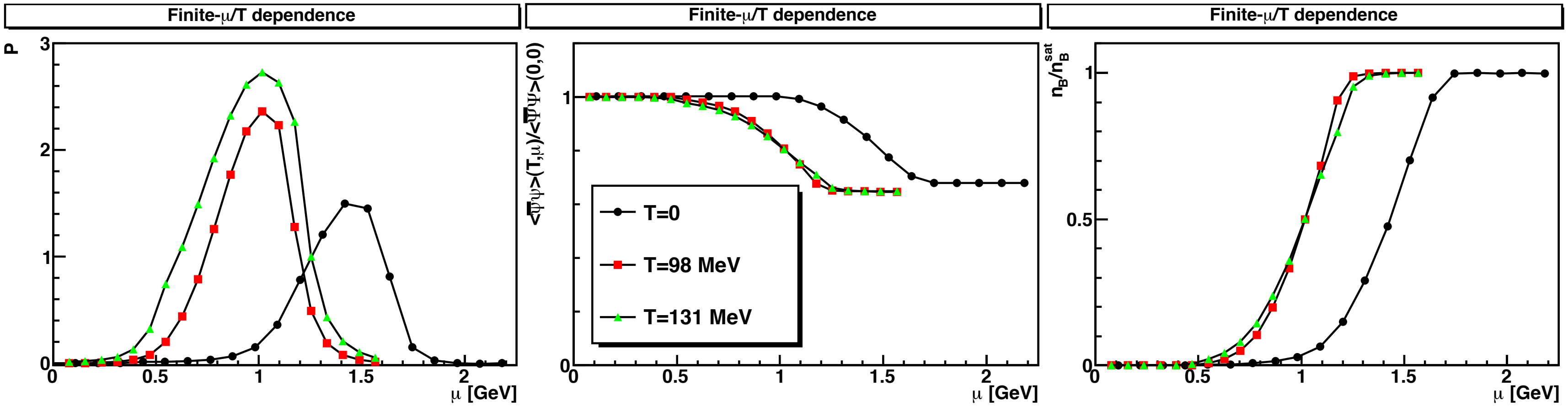
- **phase diagram with 1 flavor dynamcial Wilson fermion**



Maas, LvS, Wellegehausen & Wipf, arXiv:1203.5653.

G_2 Gauge Theory at Finite Density

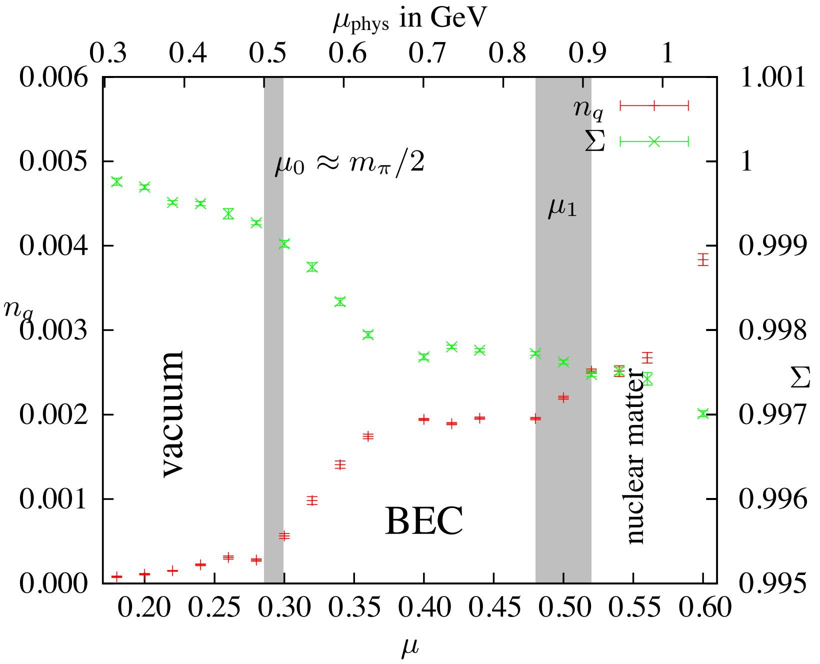
- **finite baryon density (bosonic and fermionic)**

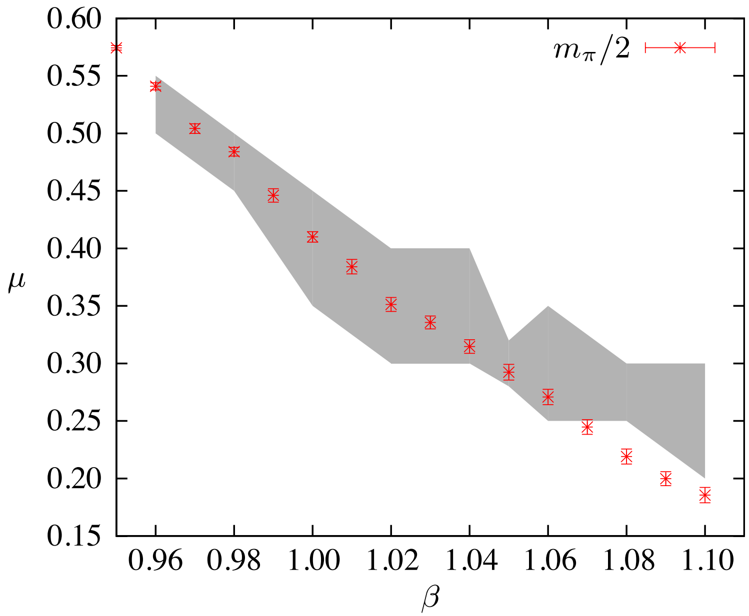


Maas, LvS, Wellegehausen & Wipf, arXiv:1203.5653.

G_2 Gauge Theory at Finite Density

- **onset of diquark condensation:**

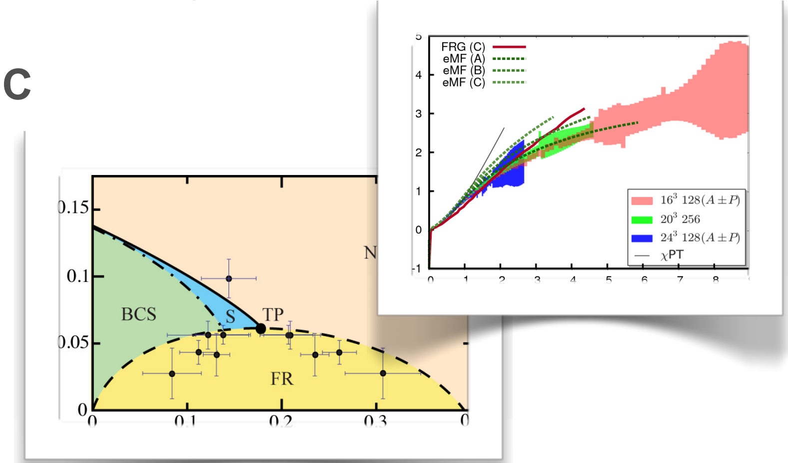




Bjoern Wellegehausen, PhD thesis, Jena 2012.

Summary & Outlook

- **Finite Isospin Density in QCD and Baryon Density in Two-Color QCD**
  - detailed understanding of phase diagram
  - functional methods and models vs. lattice MC
  - analogies with ultracold fermi gases BEC-BCS crossover, population imbalance with universal phase diagram...



- **Phase Diagram of G_2 Gauge Theory**
  - no sign problem − fermionic baryons
- **Fermions in 2+1 Dimensions**
  - quantum phase transitions, transport properties, topological aspects...
- **QCD Phase Diagram**
  - refined functional methods & models, baryonic dofs, finite volume...

Thank You for Your Attention!

European Commission

HIC for FAIR
Helmholtz International Center

DFG Deutsche Forschungsgemeinschaft