Entropy Production in high-energy Heavy Ion Collisions

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in collaboration with Berndt Müller (Duke) and many more

- The problem
- 1. approach: Kolmogorov-Sinai entropy, Husimi transformation and Wehrl entropy
- 2. approach: AdS/CFT
- Conclusions

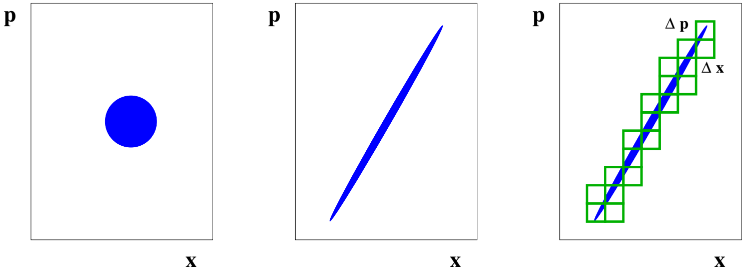
The aim is to provide input for hydrodynamics, plus to address a fundametal theory problem

Thermalization requires massive entropy production at early stages (< 1 fm/c)

But QCD is (basically) time-reversal invariant. Entropy is typically produced by measurements $\Rightarrow$ Coarse-graining.

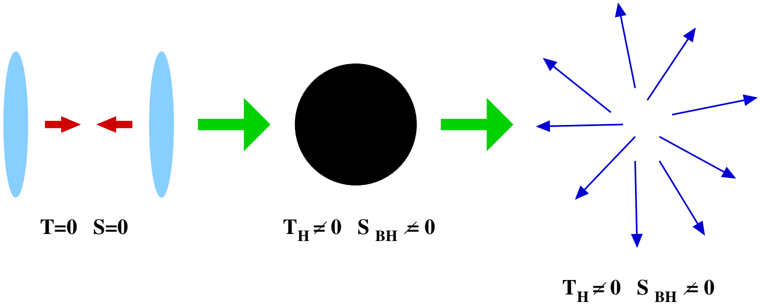
How can entropy be produced at all before any measurement takes place **?**
How can it be produced so fast **?**

Coarse graining in non-linear mechanics



- Under time evolution the phase space volume is conserved
- The finite resolution of any measurement implies an increase in phase space volume

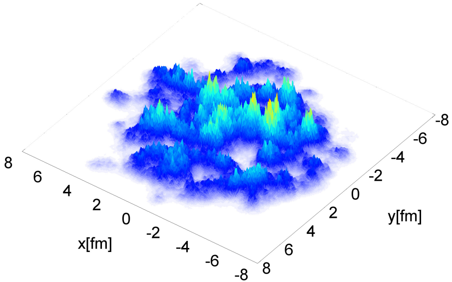
A related issue: The information problem of black-hole physics



If the S-Matrix is unitary $S_{inital} = S_{final}$.
But the Bekenstein-Hawking entropy is $S_{BH} = k_B A/(4G)$.
Entropy production corresponds to information loss. Within AdS/CFT entropy generaton in the boundary theory (QFT) and information loss in 5-dim are equivalent.

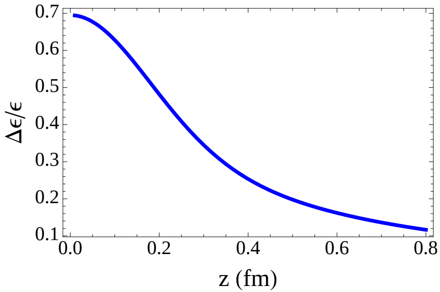
Question: Does thermalization of gauge theories depend crucially on the initial state ?

If Yes: We would have a problem because fluctuations are large according to theory $\Rightarrow$ Berndt Müller's talk



B. Schenke, Tribedy, Venugopalan 1206.6805

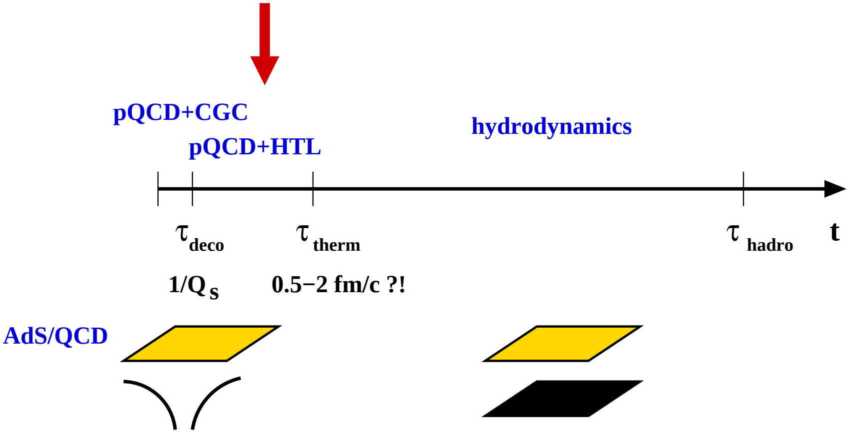
energy density at 0.2 fm/c



B.M. & A.S. 1111.3347

If No: Generic calculations should give sensible results $\Rightarrow$ classical field theory and/or AdS/CFT could catch the essential physics

Different stages of entropy production in a HIC



1. approach: Kolmogorov Sinai entropy etc.

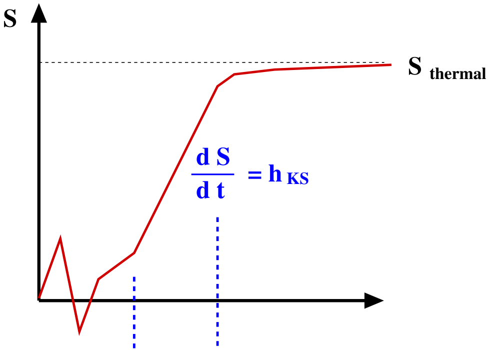
Non-linear Dynamics and Quantum Decoherence

The Lyapunov exponents of a classical theory are determine numerically. The Kolmogorov-Sinai entropy is defined as

$$h_{KS} = \sum_{i, \lambda_i > 0} \lambda_i$$

The “Kolmogorov-Sinai entropy” is no entropy, but an entropy growth rate.

a generic picture



initial linear equilibrium phase

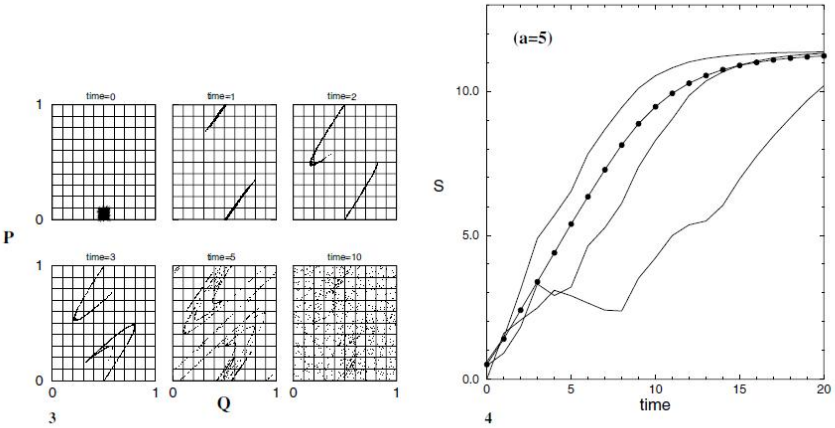
In the linear phase: $\frac{dS}{dt} = h_{KS}$

h_{KS} seems to play the same role in quantum theories !

Example: Standard map

M. Baranger, Chaos, Solitons and Fractals 13(2002)471

$$q, p \to Q, P : \qquad P = p + \frac{q \sin(2\pi q)}{2\pi} \text{ [mod 1]}; \qquad Q = q + P \text{ [mod 1]}$$



Husimi function and Wehrl entropy

The Husimi function

$$H_\Delta(p, x; t) = \int \frac{dp'\,dx'}{\pi\hbar} \exp\left(-\frac{1}{\hbar\Delta}(p-p')^2 - \frac{\Delta}{\hbar}(x-x')^2\right) W(p', x'; t)$$

The Wehrl entropy

$$S_{H,\Delta}(t) = -\int \frac{dp\,dx}{2\pi\hbar} H_\Delta(p, x; t) \ln H_\Delta(p, x; t); \qquad \lim_{t\to\infty} \frac{dS_{H,\Delta}}{dt} = h_{KS}$$

Crucial assumption: Classical and quantum system have similar h_{KS}

Classical YM theory

$$\begin{aligned} H &= \frac{1}{2}\sum_{x,a,i} E_i^a(x)^2 + \frac{1}{4}\sum_{x,a,i,j} F_{ij}^a(x)^2 \\ F_{ij}^a(x) &= \partial_i A_j^a(x) - \partial_j A_i^a(x) + \sum_{b,c} f^{abc} A_i^b(x) A_j^c(x) \end{aligned}$$

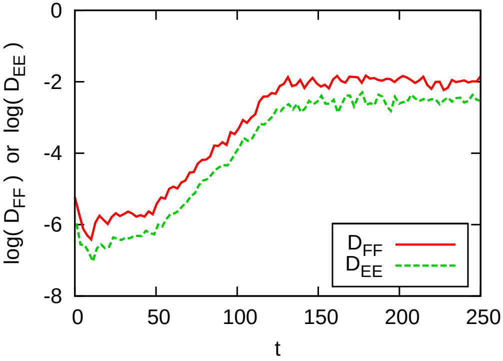
$$\delta\dot{X}(t) = \mathcal{H}\delta X(t)$$

$$\begin{aligned} \dot{A}_i^a(x) &= E_i^a(x) \\ \dot{E}_i^a(x) &= \sum_j \partial_j F_{ji}^a(x) + \sum_{b,c,j} f^{abc} A_j^b(x) F_{ji}^c(x) \end{aligned}$$

Different distance measures give the same result.

$$D_{EE} = \sqrt{\sum_x \{\sum_{a,i} E_i^a(x)^2 - \sum_{a,i} E'^a_i(x)^2\}^2}$$

$$D_{FF} = \sqrt{\sum_x \{\sum_{a,i,j} F_{ij}^a(x)^2 - \sum_{a,i,j} F'^a_{ij}(x)^2\}^2}$$



After many detailed studies we concluded

$$\tau_{\rm eq} \approx 2\ {\rm fm/c}$$

with substantial theoretical uncertainties

and that a value below 1 fm/c is very unlikely

Presently we (i.e. our Japanese collegues) study the dependence on initial conditions

2. approach: Equilibration times from AdS/CFT ?

Maldacena **Conjecture**:
Solving Einstein's equations in five dimensions with negative constant curvature
is dual to
Solving SU(N) supersymmetric ($\mathcal{N} = 4$), conformal gauge theory at strong coupling for N$\rightarrow \infty$
How can this work ?

- QCD is nearly conformal at large temperature $T > \Lambda_{QCD}$
- Fermionic degrees of freedom are less important at high temperature
- Thermodynamic quantities show only a weak N dependence

Two possible approaches:

- Try to find the dual of QCD $\Rightarrow$ IHQCD Gürsoy, Kiritsis et al.
  Fix parameters by comparison with lattice
- Analyse quantities which are insensitive to details

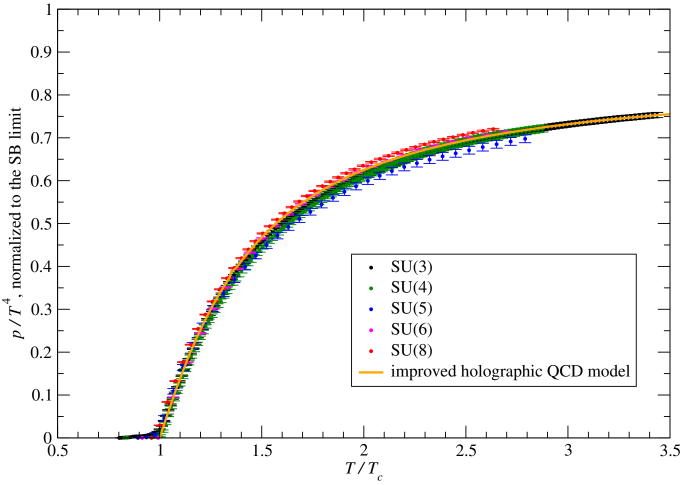
We do both

We analyse different SU(N) groups and 1+3 as well as 1+2 dimensions ($AdS_4 \times S^6$; solid state physics, quantized Hall effect, superconductivity, etc.)

We have a large lattice group (SFB/TR-55, STRONGnet, S. Kovalevskaja group (P. Buividovich, ...)

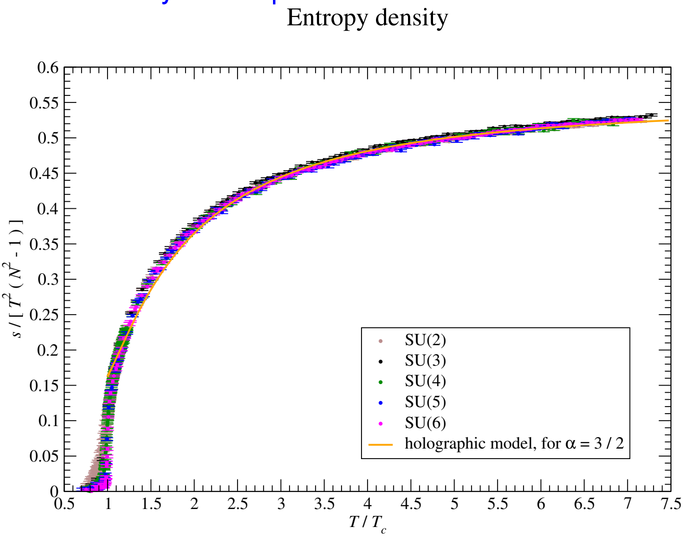
Thermodynamic quantities for 1+3 dimensions

Pressure



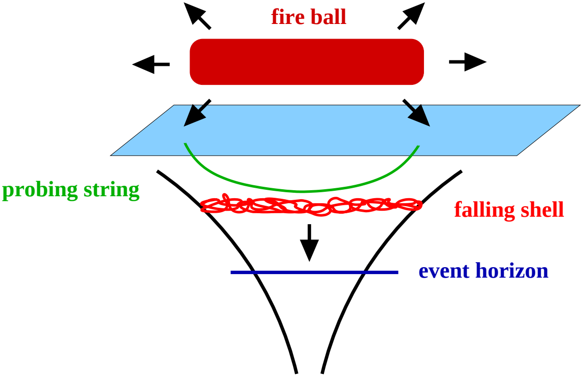
M. Panero, 0907.3719

Thermodynamic quantities for 1+2 dimensions



M. Caselle, Castagnini, Feo, Gliozzi, Gürsoy, Panero, AS, 1102.0723

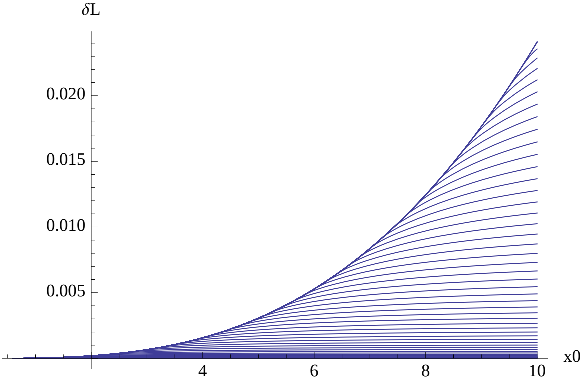
Earlier work: Probe black brane formation with a string or membrane.



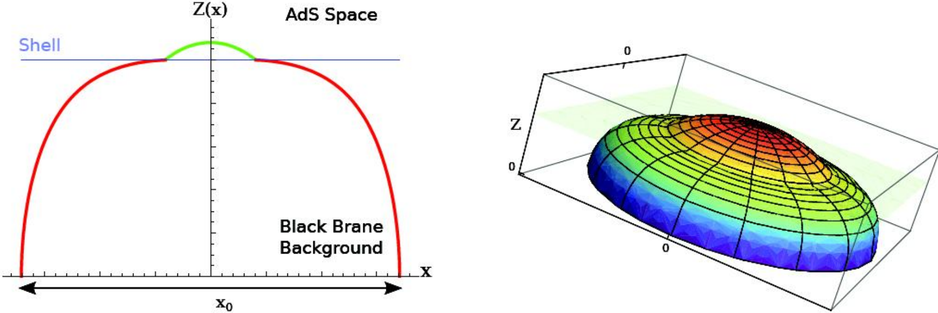
with de Boer, Craps, Keski-Vakkuri, Bernamonti, Staessens, Balasubramanian, Shigemori, Copland
results from 1012.4753 and 1103.2683

The change in geodetic length is sensitive to equal time correlators of high dimension gluonic operators which are in turn sensitive to thermalization.

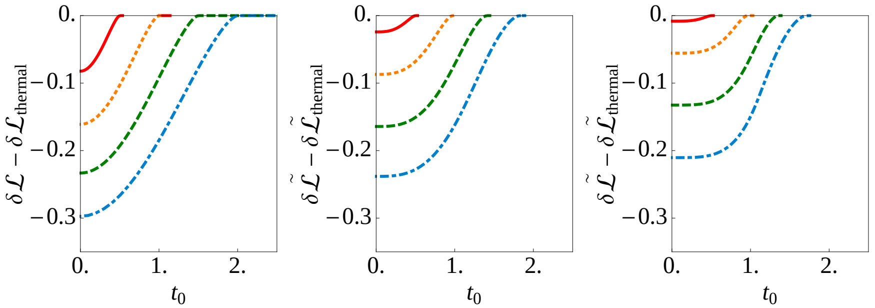
$$\frac{\langle \mathcal{O}(t_{shell}, x)\mathcal{O}(t_{shell}, 0)\rangle_{\text{shell}}}{\langle \mathcal{O}(t_{shell}, x)\mathcal{O}(t_{shell}, 0)\rangle_{\text{AdS}}} \approx e^{-\Delta \delta L(t_{shell}, x)}$$



We solved analytically and numerically different cases:
$AdS_3 \sim CFT(1+1)$, $AdS_4 \sim CFT(1+2)$, $AdS_5 \sim CFT(1+3)$
and analyzed how the length of the geodesic/the area of the surface approaches its thermal value, as a function of ℓ and t_0.



(A) (B)



$\delta\tilde{\mathcal{L}} - \delta\tilde{\mathcal{L}}_{\text{thermal}}$ ($\tilde{\mathcal{L}} \equiv \mathcal{L}/\ell$) for $d = 2, 3, 4$ (left,right, middle) and $\ell = 1, 2, 3, 4$ (top to bottom curve).

Observations

- Thermalization is approached as fast as compatible with causality.
  For heavy ion collisions this implies
  $\tau \sim 1/(2Q_s) \sim 0.1 fm/c$
- Short distances thermalize first, top-down rather than bottom-up thermalization
  Unavoidable in the AdS dual theory. A fundamental difference between strong and weak coupling ???
- Confirmed by completely different holographic investigations
  i.e. S. Caron-Huot, P.M. Chesler D. Teaney 1102.1073

A new project: QCD in a background magnetic field

Extensively studies in AdS/CFT in view of solid state applications

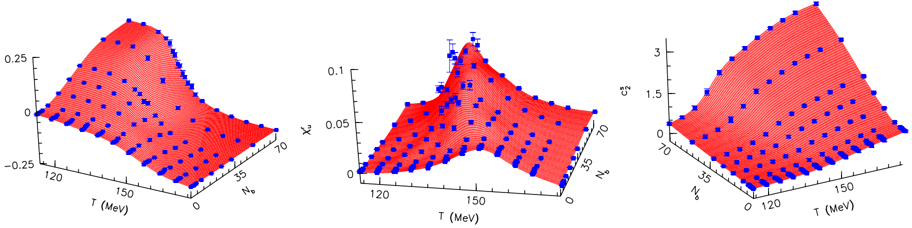
Easy to implement on the lattice: Modify gauge links according to

$$u_y(n) \; = \; e^{\mathrm{i} a^2 qBn_x} \qquad ect.$$

Of interest in view of the strong magnetic fields produced in HICs (CME etc.)

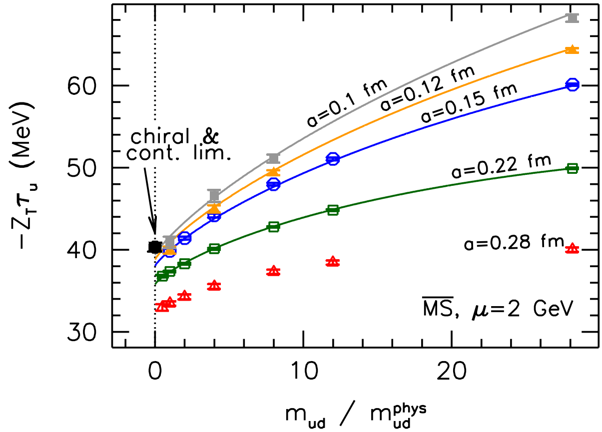
QCD phase transition in a background *B* field

Only simulation with small quark masses (G. Endrodi (Regensburg); collaboration with Budapest-Wuppertal)



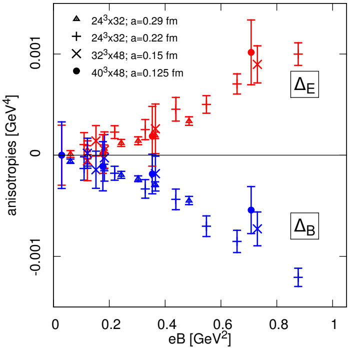
renormalized up quark condensate, its susceptibility and the strange susceptibility

$$\langle \bar{\psi}_f \sigma_{\mu\nu} \psi_f \rangle = q_f F_{\mu\nu} \cdot \tau_f$$



The quarks in the finite T vacuum are diamagnetic (as expected) tbp

$$\Delta_E = (\langle E_\perp^2(QCD)\rangle_B - \langle E_\perp^2(QCD)\rangle_0) - (\langle E_\parallel^2(QCD)\rangle_B - \langle E_\parallel^2(QCD)\rangle_0)$$



The gluon field strength components in a magnetic field at $T = 0$ tbp

This has many interesting aspects

Bosons are paramagnetic, fermions are diamagnetic (was related to the different signs in the QCD β function). Effective models with only bosonic degrees of freedom tend to fail.

We see a drop of T_c as function of B. It was suggested that the physics might be: Large B leads to 1+1 dimensional dynamics $\Rightarrow$ Mermin-Wagner-theorem $\Rightarrow$ no chiral symmetry breaking

Conclusions

- Understanding entropy production during thermalization in HICs is a problem of fundamental importance.
- Thermalization via non-linear dynamics and coarse graining with $\hbar$ needs $\tau \approx 2\mathrm{fm/c}$.
- Thermalization for strong coupling as described by AdS/CFT is top-down and very fast $\tau \approx 0.1\mathrm{fm/c}$.
- AdS/CFT and LQCD could form a powerful team
- Projects
  - Classical YM dynamics: Influence of initial conditions
  - Equilibration time for fluctuations from AdS/CFT
  - QCD in a constant B field $\Rightarrow$ non-leading AdS corrections
  - A new way to determine transport coefficients from lattice QCD:
    Fit AdS parameters to $\langle T_{\mu\nu} T_{\mu\nu} \rangle_{\mathrm{LATTICE}} \Rightarrow \eta/s > 1/(4\pi)$

Papers

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