Inhomogeneous chiral symmetry breaking phases

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International School of Nuclear Physics
34^{th} Course
"Probing the Extremes of Matter with Heavy Ions"
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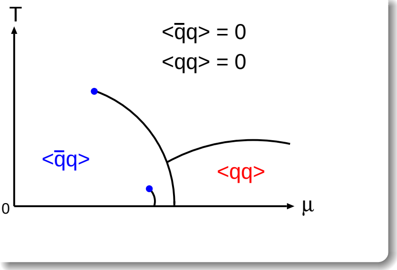
Motivation

QCD phase diagram (schematic):



- frequent assumption:
  $\langle \bar{q}q \rangle$, $\langle qq \rangle$ constant in space
- How about inhomogeneous phases ?

Inhomogeneous phases: (incomplete) historical overview

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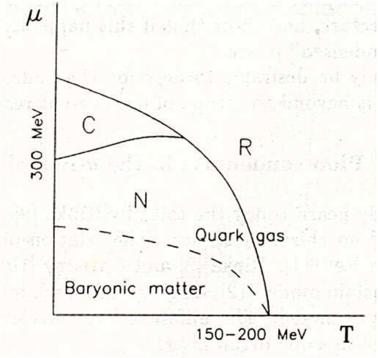
- 1960s:
  - spin-density waves in nuclear matter (Overhauser)
  - crystalline superconductors (Fulde, Ferrell, Larkin, Ovchinnikov)
- 1970s – 1990s:
  - p-wave pion condensation (Migdal)
  - chiral density wave (Dautry, Nyman)
- after 2000:
  - 1+1 D Gross-Neveu model (Thies et al.)
  - crystalline color superconductors (Alford, Bowers, Rajagopal)
  - quarkyonic matter (Kojo, McLerran, Pisarski, ...)



Broniowski et al. (1991)

Model

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- NJL model:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G_S\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right]$$

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- bosonize: $\sigma(x) = \bar{\psi}(x)\psi(x), \quad \vec{\pi}(x) = \bar{\psi}(x)i\gamma_5\vec{\tau}\psi(x)$

$$\Rightarrow \quad \mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m + 2G_S(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\right)\psi - G_S\left(\sigma^2 + \vec{\pi}^{\,2}\right)$$

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- mean-field approximation:

$$\sigma(x) \to \langle\sigma(x)\rangle \equiv S(\vec{x}), \quad \pi_a(x) \to \langle\pi_a(x)\rangle \equiv P(\vec{x})\,\delta_{a3}$$

  - $S(\vec{x})$, $P(\vec{x})$ time independent classical fields
  - retain space dependence !

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- mean-field thermodynamic potential:

$$\Omega_{MF}(T,\mu) = -\frac{T}{V}\ln\int\mathcal{D}\bar{\psi}\mathcal{D}\psi\,\exp\left(\int_{x\in[0,\frac{1}{T}]\times V}(\mathcal{L}_{MF} + \mu\bar{\psi}\gamma^0\psi)\right)$$

Mean-field model

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- mean-field Lagrangian:

$$\mathcal{L}_{MF} = \bar{\psi}(x)\,\mathcal{S}^{-1}(x)\,\psi(x) - G_S\left[S^2(\vec{x}) + P^2(\vec{x})\right]$$

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  - $\mathcal{H}_{MF}$ hermitean $\Rightarrow$ can (in principle) be diagonalized (eigenvalues E_λ)
  - $\mathcal{H}_{MF}$ time-independent $\Rightarrow$ Matsubara sum as usual

Mean-field thermodynamic potential

- thermodynamic potential:

$$\Omega_{MF}(T, \mu; S, P) = -\frac{T}{V}\mathbf{Tr}\ln\left(\frac{1}{T}(i\partial_0 - \mathcal{H}_{MF} + \mu)\right) + \frac{G_S}{V}\int_V d^3x\ \left(S^2(\vec{x}) + P^2(\vec{x})\right)$$

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- remaining tasks:
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- general case: extremely difficult!

Periodic structures

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- crystal with a unit cell spanned by vectors $\vec{a}_i$, $i = 1, 2, 3$

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- mean-field Hamiltonian in momentum space:

$$\mathcal{H}_{\vec{p}_m,\vec{p}_n} = \begin{pmatrix} -\vec{\sigma}\cdot\vec{p}_m\,\delta_{\vec{p}_m,\vec{p}_n} & \sum_{\vec{q}_k} M_{\vec{q}_k}\,\delta_{\vec{p}_m,\vec{p}_n+\vec{q}_k} \\ \sum_{\vec{q}_k} M^*_{\vec{q}_k}\,\delta_{\vec{p}_m,\vec{p}_n-\vec{q}_k} & \vec{\sigma}\cdot\vec{p}_m\,\delta_{\vec{p}_m,\vec{p}_n} \end{pmatrix}$$

  - different momenta coupled by $M_{\vec{q}_k}$ $\Rightarrow$ $\mathcal{H}$ is nondiagonal in momentum space!
  - $\vec{q}_k$ discrete $\Rightarrow$ $\mathcal{H}$ is still block diagonal

Periodic structures: minimum free energy

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- general procedure:
  - choose a unit cell $\{\vec{a}_i\} \;\Rightarrow\; \{\vec{q}_k\}$
  - choose Fourier components $M_{\vec{q}_k}$
  - diagonalize $\mathcal{H}_{MF} \quad \rightarrow \quad \Omega_{MF}$
  - minimize Ω_{MF} w.r.t. $M_{\vec{q}_k}$
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$\rightarrow$ further simplifications necessary

One dimensional modulations

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- consider only one-dimensional modulations: $M(\vec{x}) = M(z) = \sum_{q_k} M_k \, e^{ikqz}$

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- popular choice: $M(z) = M_1 \, e^{iqz}$ (chiral density wave)
  - $\Leftrightarrow \quad S(\vec{x}) = \Delta \cos(qz) \,, \quad P(\vec{x}) = \Delta \sin(qz)$
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- 1 + 1D solutions known analytically: [M. Thies, J. Phys. A (2006)]

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- remaining task:
  - minimize w.r.t. 2 parameters: Δ, ν
  - (almost) as simple as CDW, but more powerful
  - $m \neq 0$: 3 parameters

Phase diagram (chiral limit)

[D. Nickel, PRD (2009)]

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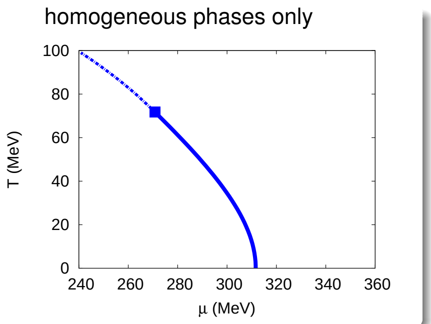

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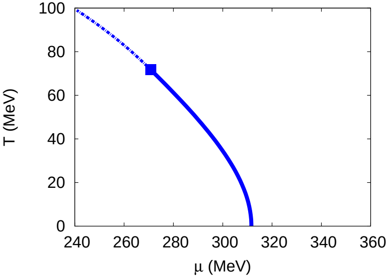
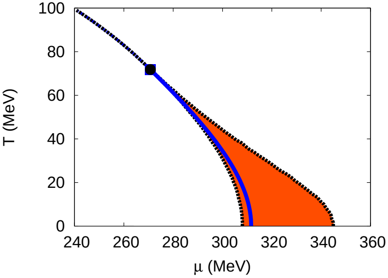
homogeneous phases only



including inhomogeneous phase



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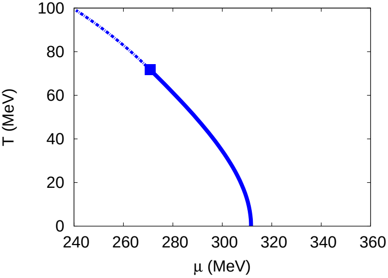

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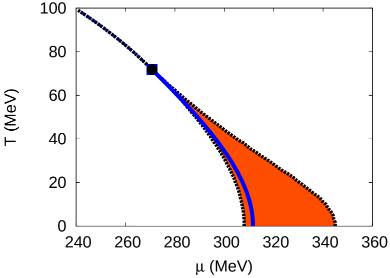

- 1st-order line completely covered by the inhomogeneous phase!
- all phase boundaries 2nd order
- critical point coincides with Lifshitz point

Mass functions and density profiles ($T = 0$)

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- $M(z) = \sqrt{\nu}\Delta\, \mathrm{sn}(\Delta z|\nu) \quad \rightarrow \quad \begin{cases} \Delta \tanh(\Delta z) & \text{for} \quad \nu \to 1 \\ \sqrt{\nu}\Delta \sin(\Delta z) & \text{for} \quad \nu \to 0 \end{cases}$

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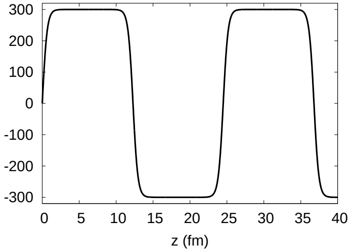
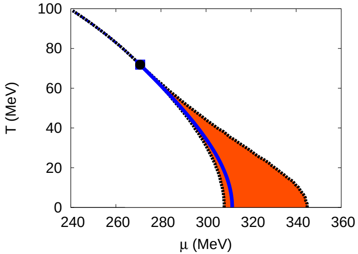
$M(z)$ (μ = 307.5 MeV)





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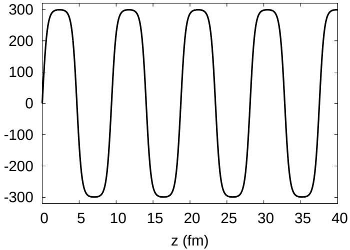

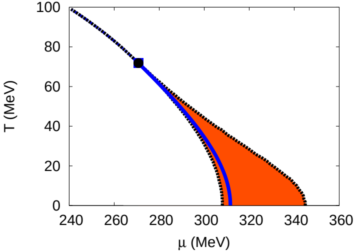

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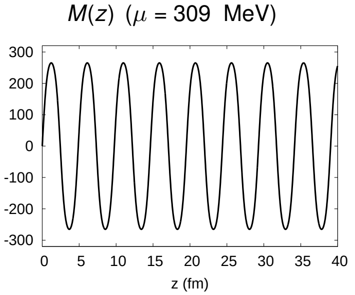

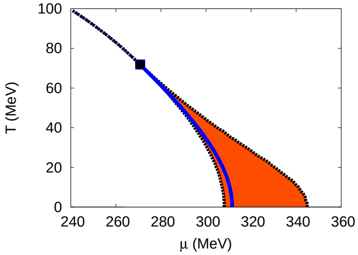

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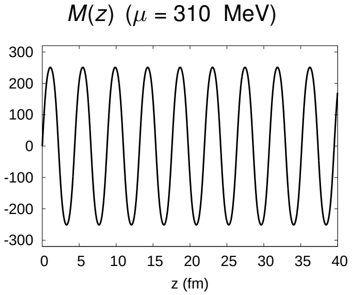

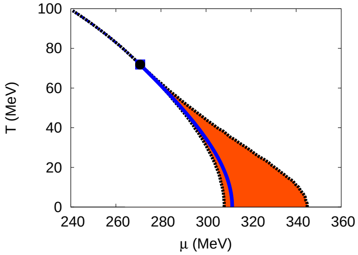

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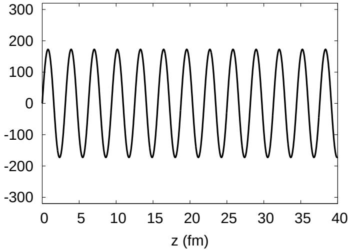

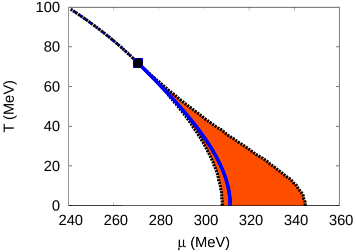

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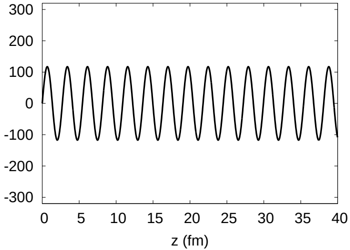

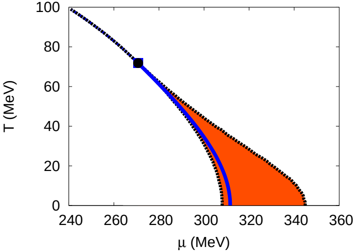

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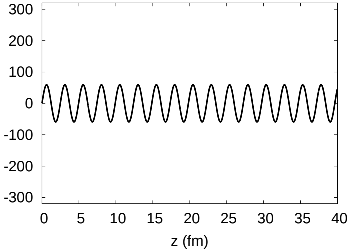
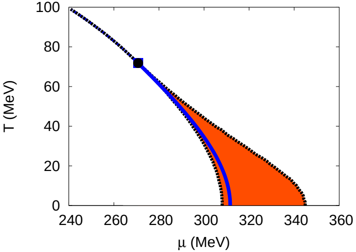
- $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z|\nu) \quad \to \quad \begin{cases} \Delta \tanh(\Delta z) & \text{for} \quad \nu \to 1 \\ \sqrt{\nu}\Delta \sin(\Delta z) & \text{for} \quad \nu \to 0 \end{cases}$

$M(z)$ (μ = 340 MeV)





Mass functions and density profiles ($T = 0$)

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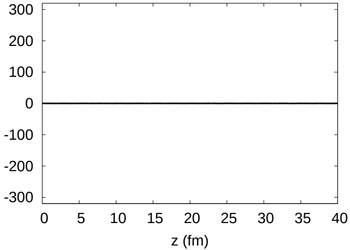

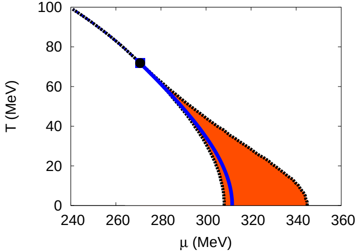

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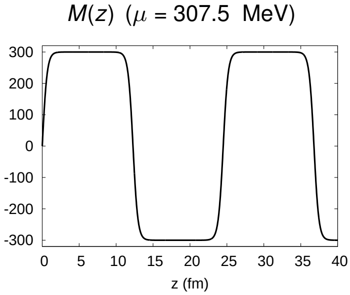

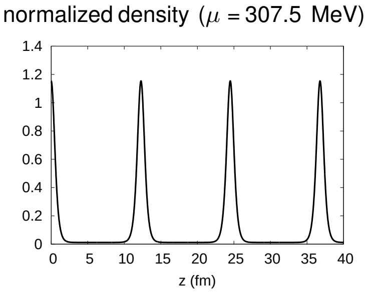

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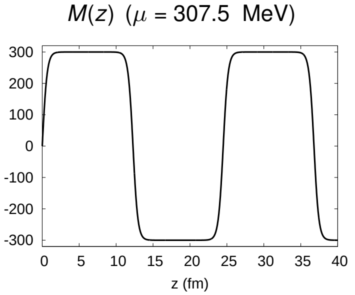

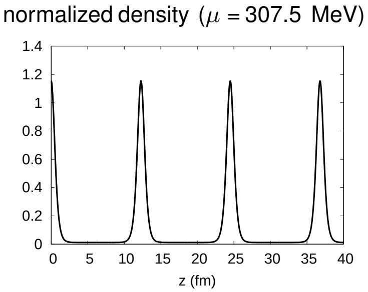

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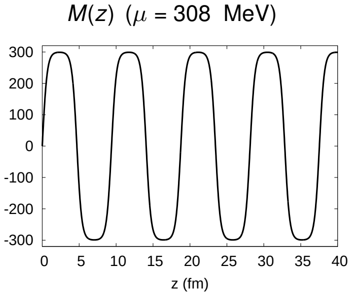

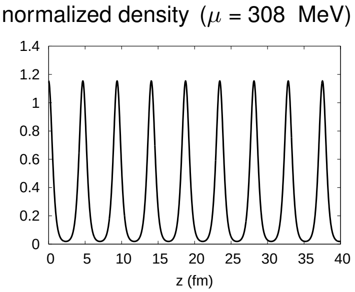

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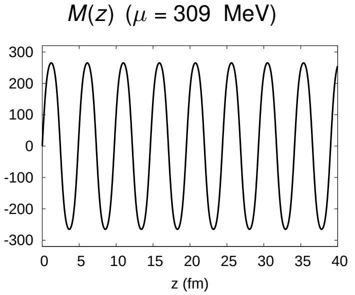

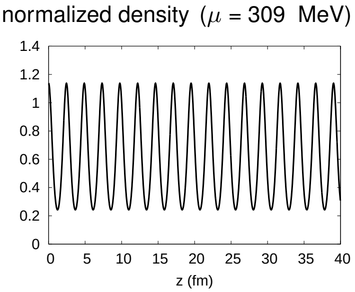

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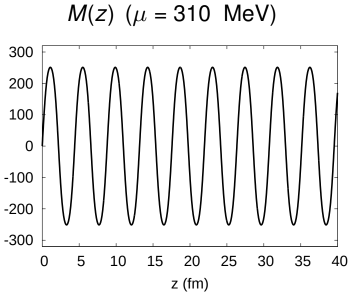

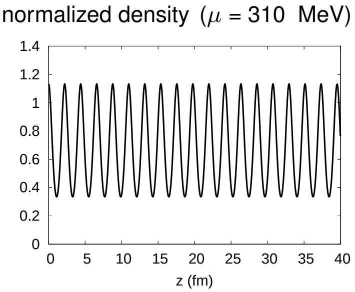

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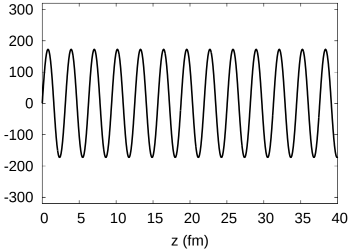
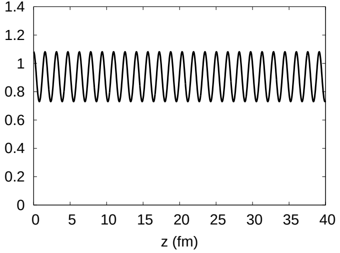
$M(z)$ (μ = 320 MeV)



normalized density (μ = 320 MeV)



- Quarks reside in the chirally restored regions.
- Density gets smoothened with increasing μ and T.

Mass functions and density profiles ($T = 0$)

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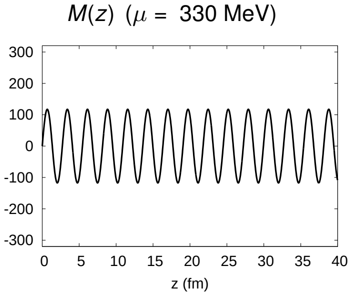

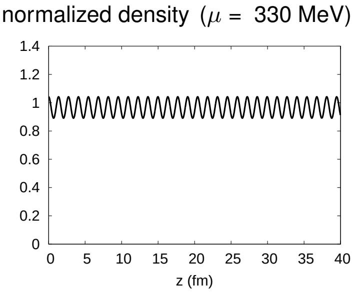

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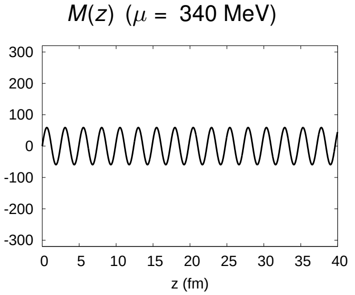

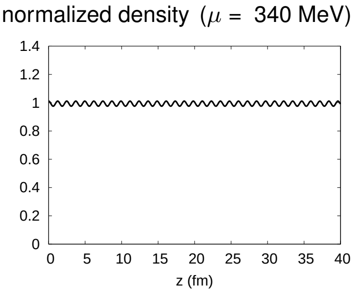

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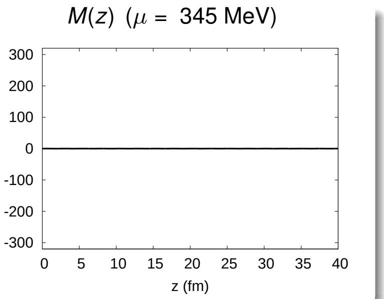

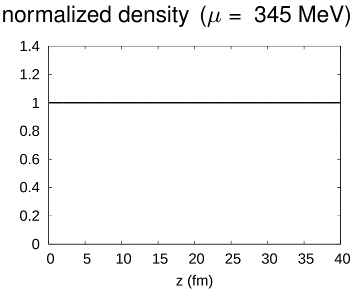

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Including vector interactions

[S. Carignano, D. Nickel, M.B., PRD (2010)]

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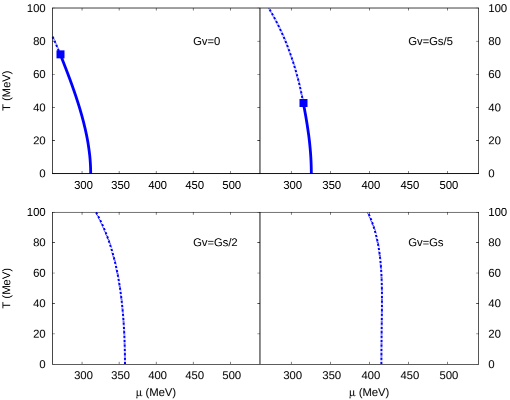

- additional interaction term:

  $\mathcal{L}_V = -G_V(\bar{\psi}\gamma^\mu\psi)^2$

- homogeneous phases: strong G_V-dependence of the critical point

Including vector interactions

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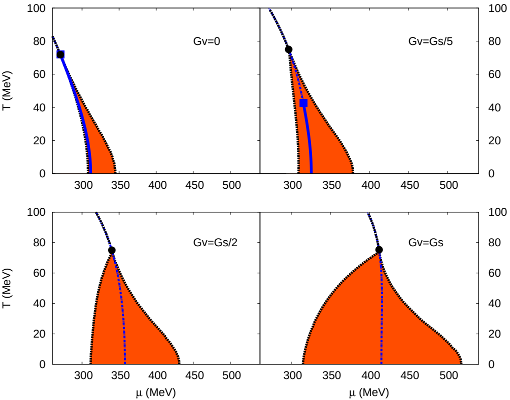

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- inhomogeneous regime: stretched in μ direction, Lifshitz point at constant T

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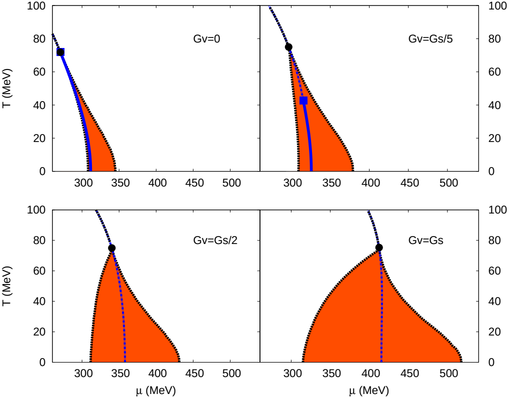

- additional interaction term:

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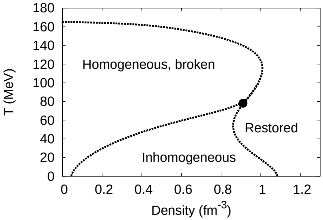
  T-$\langle n \rangle$ phase diagram:



- independent of G_V!

- homogeneous phases: strong G_V-dependence of the critical point
- inhomogeneous regime: stretched in μ direction, Lifshitz point at constant T

Susceptibilities

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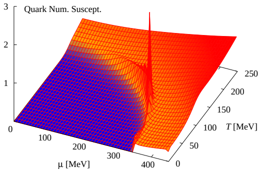
- signature of the critical point: divergent susceptibilities
- e.g., quark number susceptibility:

$$\chi_{nn} = -\frac{\partial^2 \Omega}{\partial \mu^2} = \frac{\partial n}{\partial \mu}$$

homogeneous phases only:



[K. Fukushima, PRD (2008)]

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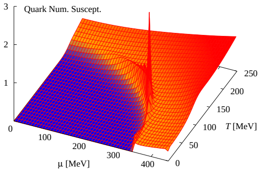

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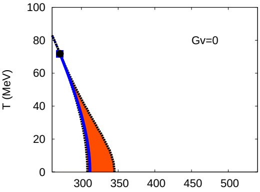

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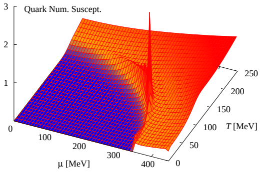

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- $\underline{G_V = 0:}$
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  $\rightarrow$ no qualitative change

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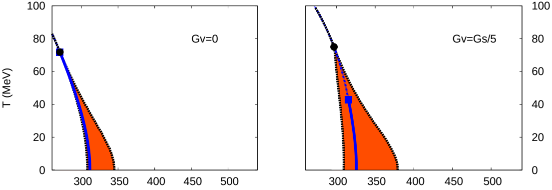

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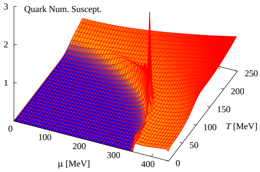

[K. Fukushima, PRD (2008)]

- $G_V = 0$:
  CP = Lifshitz point
  $\rightarrow$ no qualitative change
- $G_V > 0$:
  no CP $\rightarrow$ no divergence

Susceptibilities

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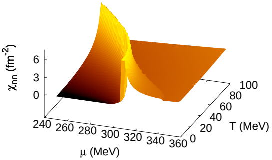
- results:



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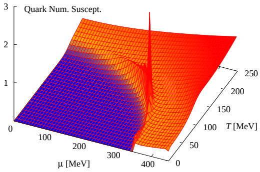

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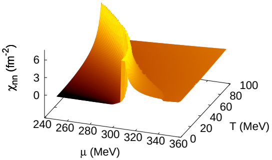

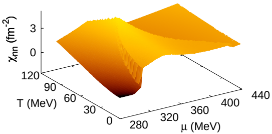

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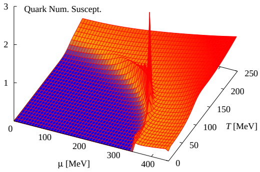

[K. Fukushima, PRD (2008)]

- $G_V = 0$:

  χ_{nn} diverges at phase boundary (hom. broken - inhom.)

- $G_V > 0$:

  no divergence

Two-dimensional modulations

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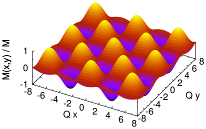
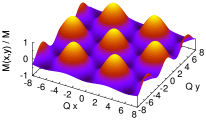
- consider two shapes:
  - square lattice (“egg carton”)

    $$M(x,y) = M\cos(Qx)\cos(Qy)$$



  - hexagonal lattice

    $$M(x,y) = \frac{M}{3}\left[2\cos(Qx)\cos\left(\frac{1}{\sqrt{3}}Qy\right) + \cos\left(\frac{2}{\sqrt{3}}Qy\right)\right]$$



- minimize both cases numerically w.r.t. M and Q

Two-dimensional modulations: results

[S. Carignano, M.B., arXiv:1203.5343]

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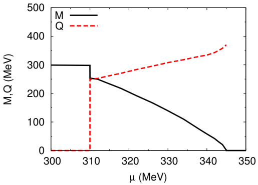
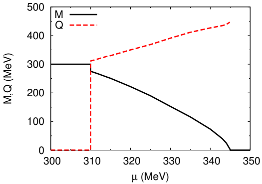
- amplitudes and wave numbers:
  - egg carton:



  - hexagon:



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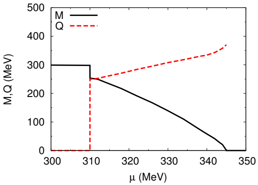

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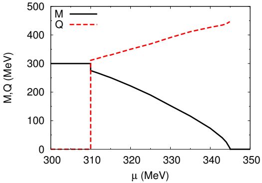
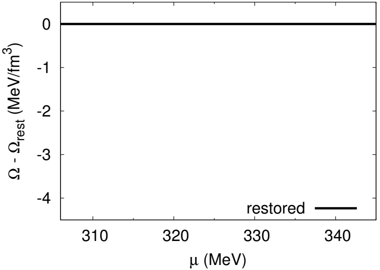


free-energy gain at $T = 0$:



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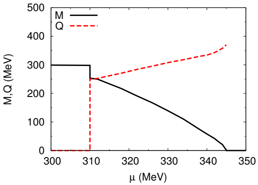

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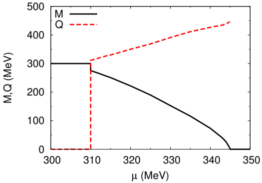

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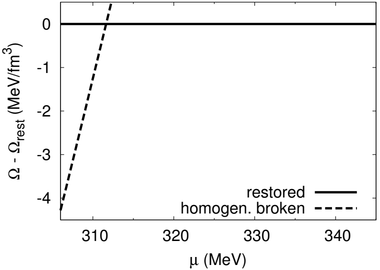

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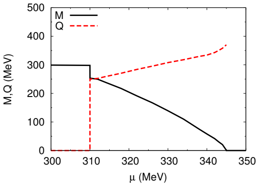

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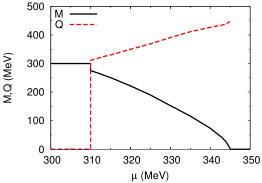

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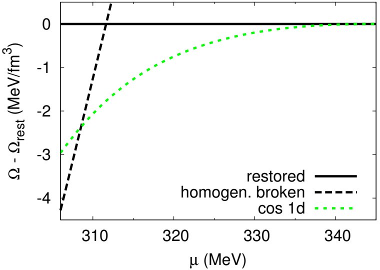

Two-dimensional modulations: results

[S. Carignano, M.B., arXiv:1203.5343]

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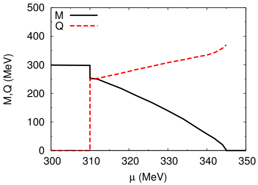
  - egg carton:



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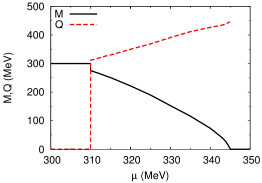

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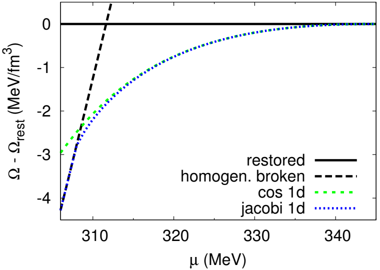

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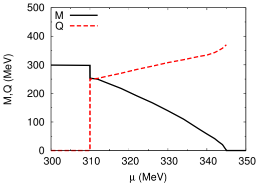

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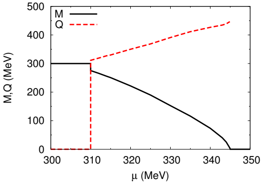

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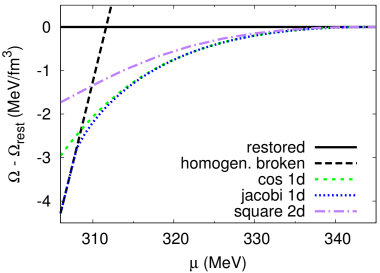

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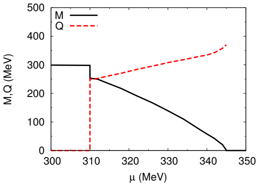

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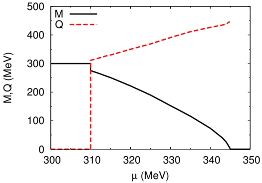

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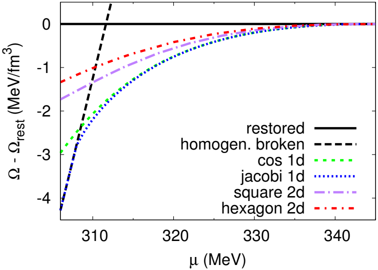

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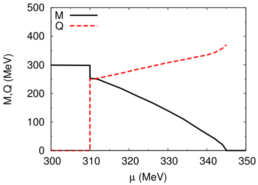

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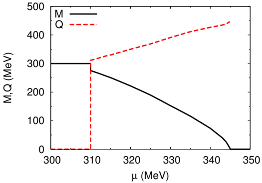

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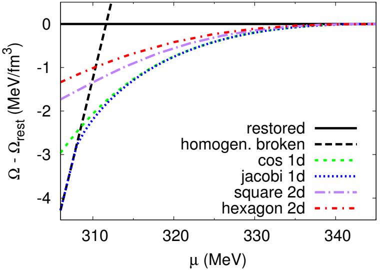

- 2d not favored over 1d in this regime

Two-dimensional modulations: further results

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- rectangular lattice:

  $M(x, y) = M \cos(Q_x x) \cos(Q_y y)$

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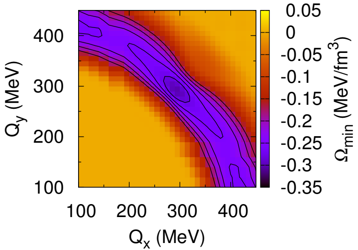

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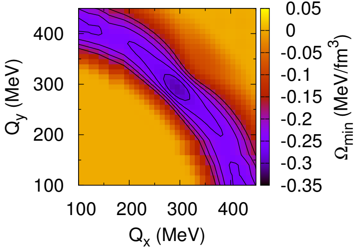

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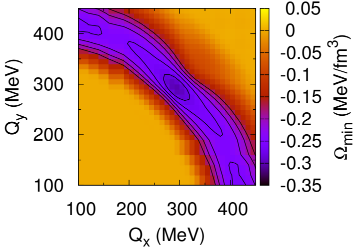

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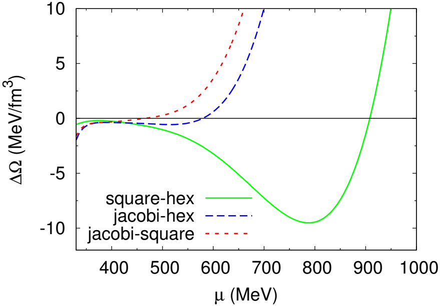
- higher chemical potentials



- 450 MeV $< \mu <$ 900 MeV: egg carton favored
- $\mu >$ 900 MeV: hexagon favored

From quark droplets to solitonic lasagne

[M.B., S. Carignano, in prep.]

TECHNISCHE UNIVERSITÄT DARMSTADT

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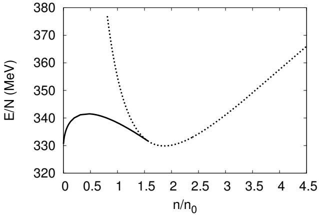

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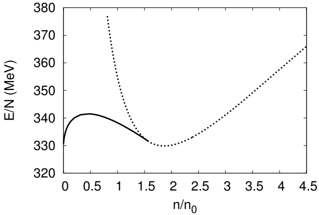

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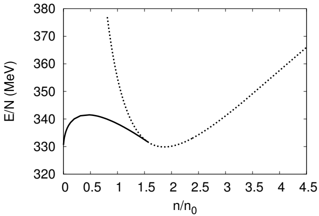

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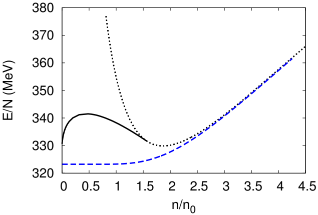

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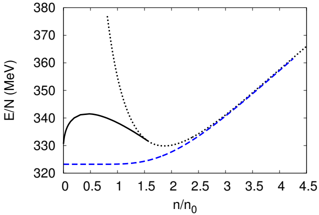

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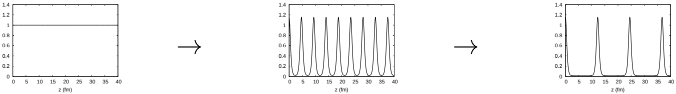

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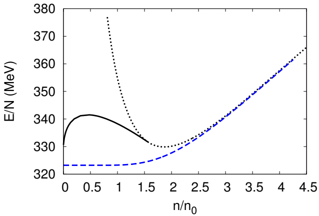

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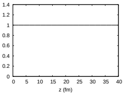

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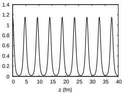

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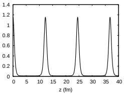
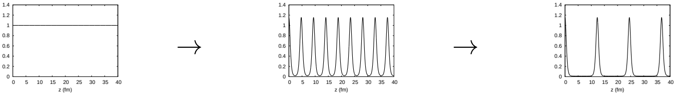


  - lowest energy at $\bar{n} = 0$ $\hat{=}$ single soliton limit

Interpretation?



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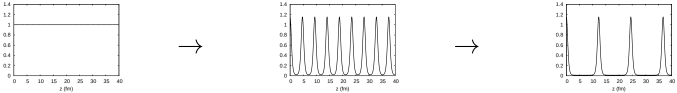

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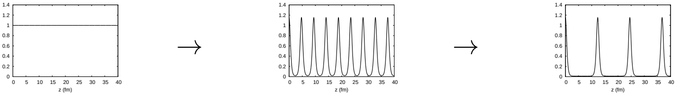

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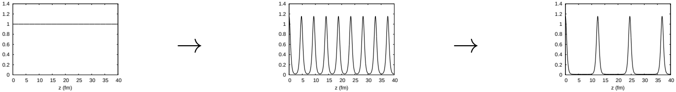

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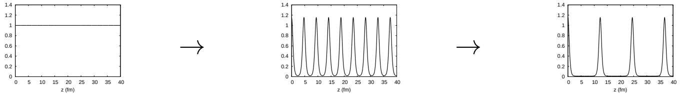

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  - measurable effects on fireball expansions?

Conclusions

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Collaborators

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Dominik Nickel
(INT Seattle → Siemens)

Stefano Carignano
(TU Darmstadt)