Chiral matter

Alessandro Drago - Ferrara

Luca Bonanno

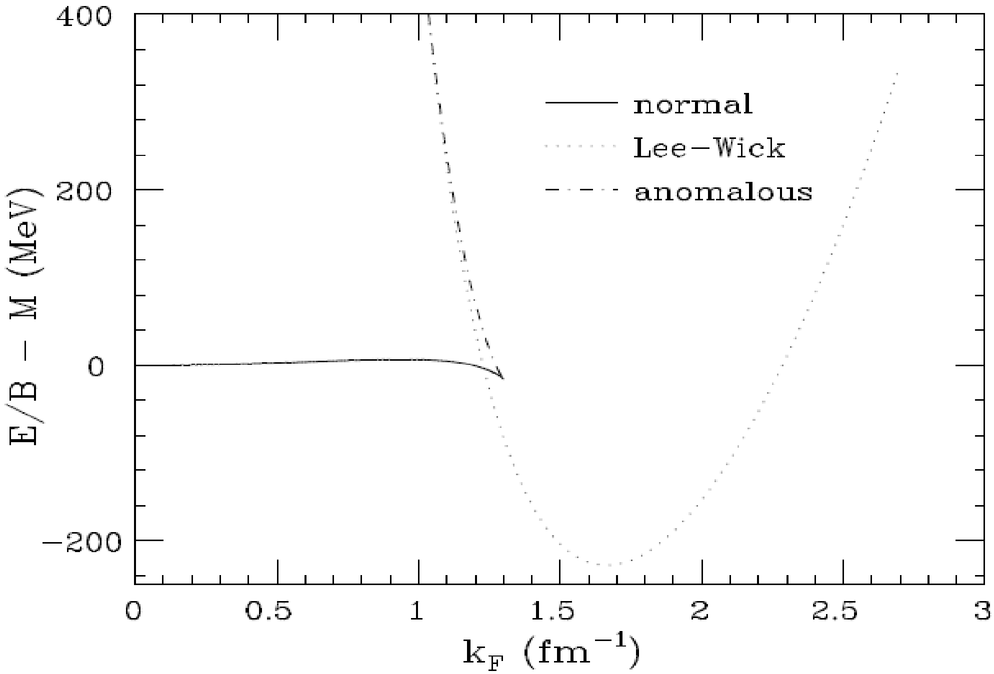
Valentina Mantovani Sarti

- Chiral lagrangian for nuclear matter
  - Linear σ-model fails
  - Non-linear σ-model (but with a scalar field…)
  - Scaled linear models
- Scaled chiral nuclear model at finite ρ and T

  Phys. Rev.Lett. 99 (2007) 242301; Phys.Rev.C 79 (2009) 045801
  - Similarities with McLerran-Pisarski large N_c scheme
- Solitons from a scaled chiral quark model
- Wigner-Seitz lattice of solitons Phys.Rev.C86(2012)015211
  - Linear σ-model fails
  - Non-linear σ-model (without a scalar field, Glendenning)
  - Scaled linear models: without and with vector mesons

Linear σ-models based on the "Mexican-hat" potential

$$\mathcal{L} = \overline{\psi}[i\gamma_\mu \partial^\mu - g_v \gamma_\mu V^\mu - g_\pi(\sigma + i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi})]\psi$$
$$+ \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi}\cdot\partial^\mu \boldsymbol{\pi}) - \frac{1}{4}\lambda(\sigma^2 + \boldsymbol{\pi}^2 - v^2)^2$$
$$- \frac{1}{4}(\partial_\mu V_\nu - \partial_\nu V_\mu)^2 + \frac{1}{2}m_v^2 V_\mu V^\mu + \epsilon\sigma \; .$$



Boguta NPA501(1989)637
Furnstahl, Serot, Tang NPA598 (1996) 539

The ground state is not the normal solution, but the Lee-Wick one, having effective nucleon mass M* = 0

Non-linear σ-model with a scalar field Furnstahl, Serot, Tang 1996

- Furnstahl and Serot 1993 conclude that the failure of many chiral models is due to the restrictions on the scalar field dynamics imposed by the "Mexican hat" potential. The problem is not alleviated by introducing scaled versions of the "Mexican hat".

- FST 1996 use a non-linear realization of chiral symmetry in which a scalar-isoscalar effective field is introduced, as a chiral singlet, to simulate intermediate range attraction.

- The dynamics of the chiral singlet field is no-more regulated by the "Mexican hat" potential.

The chiral-dilaton model

Carter, Ellis, Heide, Rudaz (Univ. Minnesota)

- On scale and Chiral Symmetry in Nuclear Matter PLB 282 (1992) 271
- Implications of a Modified Glueball Potential for Nuclear Matter PLB 293 (1992) 870
- An Effective Lagrangian with Broken Scale and Chiral Symmetry:
  - Applied to Nuclear Matter and Finite Nuclei NPA 571 (1994) 713
  - Pion phenomenology NPA 603 (1996) 367
  - Mesons at Finite Temperature NPA 618 (1997) 317
  - Nucleons and Mesons at Finite Temperature NPA 628 (1998) 325

Other approaches to linear chiral lagrangians in nuclear physics developed by the Frankfurt group, also extending to $SU(3)_f$

Chiral lagrangians in Nuclear Physics

Problem: the linear sigma model fails to yeald saturation.
It provides chiral symmetry restoration ($m_N = 0$) already at low density
due to the form of the meson self-interaction.

Some physical ingredient is missing

In QCD, scale symmetry is broken by trace anomaly.

This mechanism is responsible for the existence of Λ_{QCD} parameter,
which sets the scale of hadron masses and radii.

In an effective model, the **QCD trace anomaly** is reproduced at a mean field level,

by introducing a scalar field, **the dilaton field** (Schechter 1980)**,** so that

$$\Theta_\mu^\mu = 4\varepsilon(\phi/\phi_0)^4$$

Heide, Rudaz and Ellis 1992 modifies the dilaton potential by including chiral fields

$$\mathcal{V} = B\phi^4\left(\ln\frac{\phi}{\phi_0} - \frac{1}{4}\right) - \frac{1}{2}B\delta\phi^4\ln\frac{\sigma^2 + \boldsymbol{\pi}^2}{\sigma_0^2}$$

The chiral dilaton model

An effective lagrangian with broken scale and chiral symmetry

(Nucl.Phys.A628:325,1998)

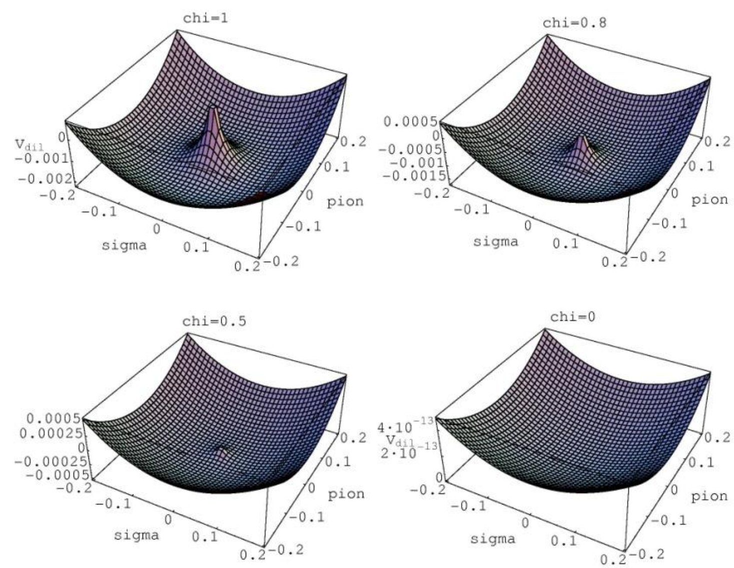
$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \tfrac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} + \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \tfrac{1}{2}G_{\omega\phi}\phi^2\omega_\mu\omega^\mu$$

$$+[(G_4)^2\omega_\mu\omega^\mu]^2 + \bar{N}\left[\gamma^\mu(i\partial_\mu - g_\omega\omega_\mu) - g\sqrt{\sigma^2+\boldsymbol{\pi}^2}\right]N - \mathcal{V}\,,$$

$$\mathcal{V} = B\phi^4\left(\ln\frac{\phi}{\phi_0} - \frac{1}{4}\right) - \tfrac{1}{2}B\delta\phi^4\ln\frac{\sigma^2+\boldsymbol{\pi}^2}{\sigma_0^2} + \tfrac{1}{2}B\delta\zeta^2\phi^2\left[\sigma^2+\boldsymbol{\pi}^2 - \frac{\phi^2}{2\zeta^2}\right]$$

$$-\tfrac{1}{4}\epsilon_1'\left(\frac{\phi}{\phi_0}\right)^2\left[\frac{4\sigma}{\sigma_0} - 2\left(\frac{\sigma^2+\boldsymbol{\pi}^2}{\sigma_0^2}\right) - \left(\frac{\phi}{\phi_0}\right)^2\right] - \tfrac{3}{4}\epsilon_1'\,.$$

Dilaton potential with $\varepsilon'=0$



$$\Theta^\mu_\mu = 4\varepsilon(\phi/\phi_0)^4$$

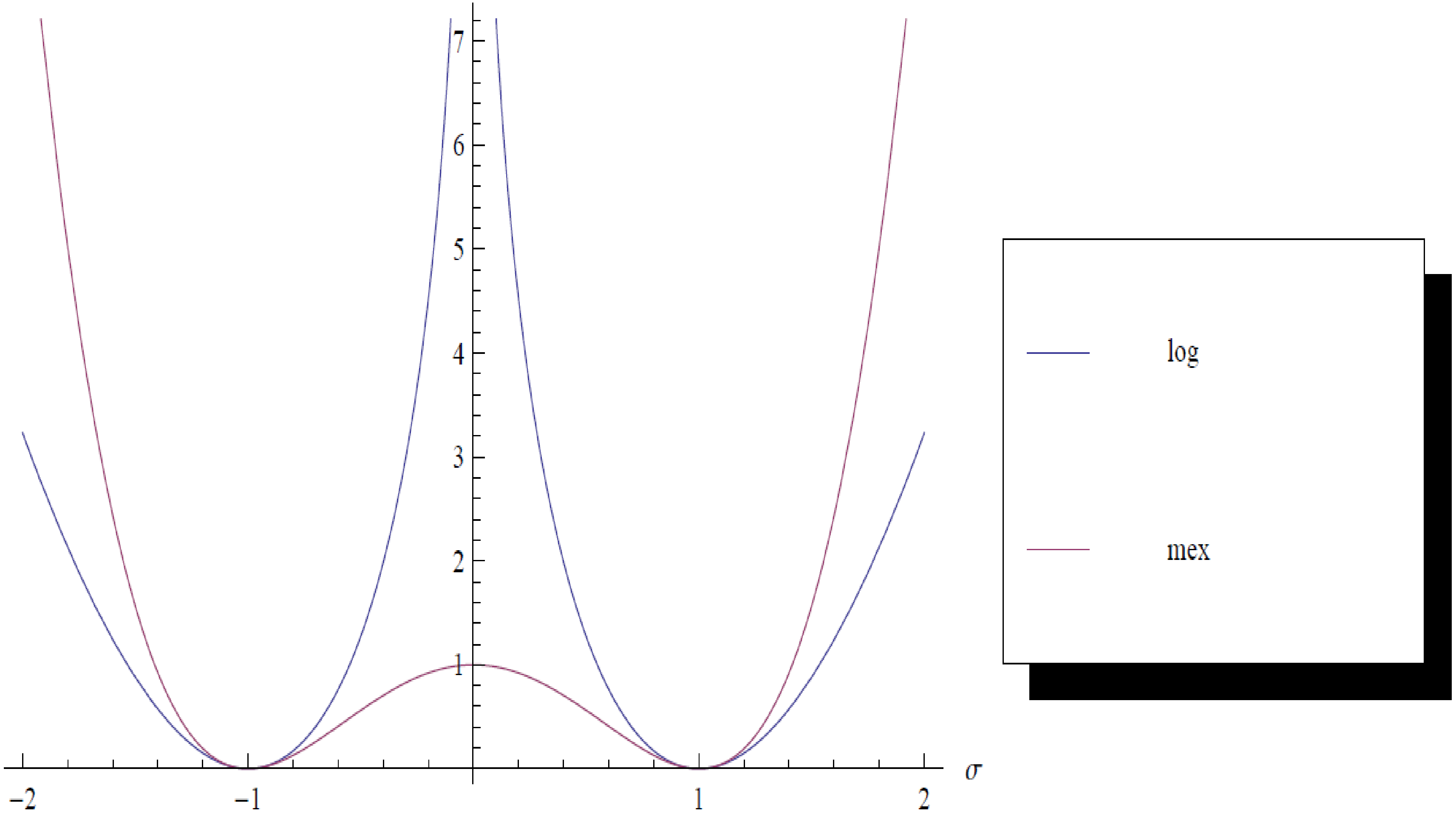
$$\varepsilon = -B\phi_0{}^4(1-\delta)/4$$

$\delta=4/33$ from QCD β function

When scale invariance is restored also

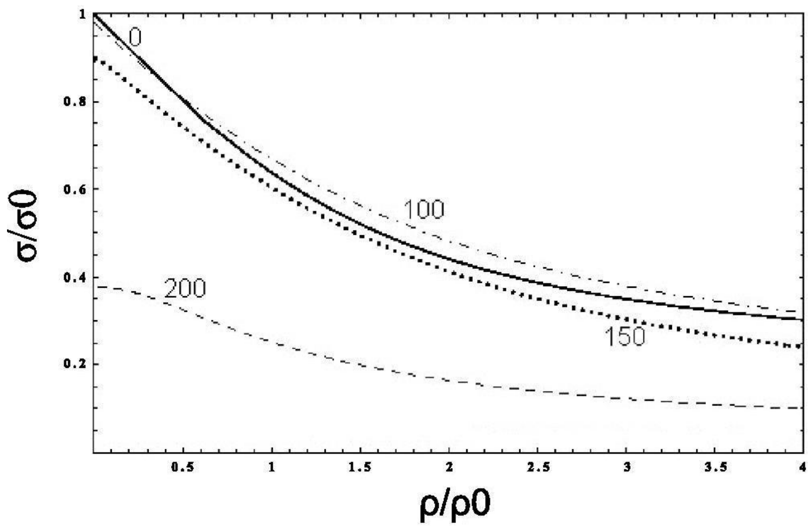
chiral simmetry is restored

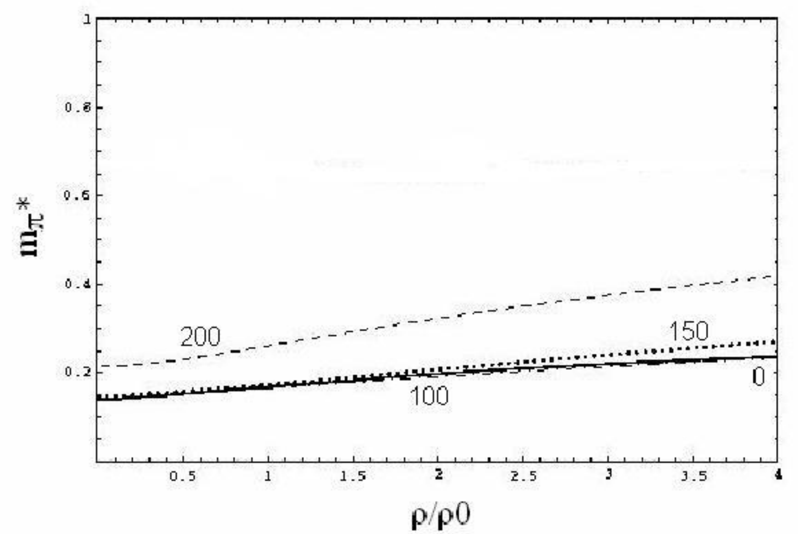
Mexican hat vs Log potential



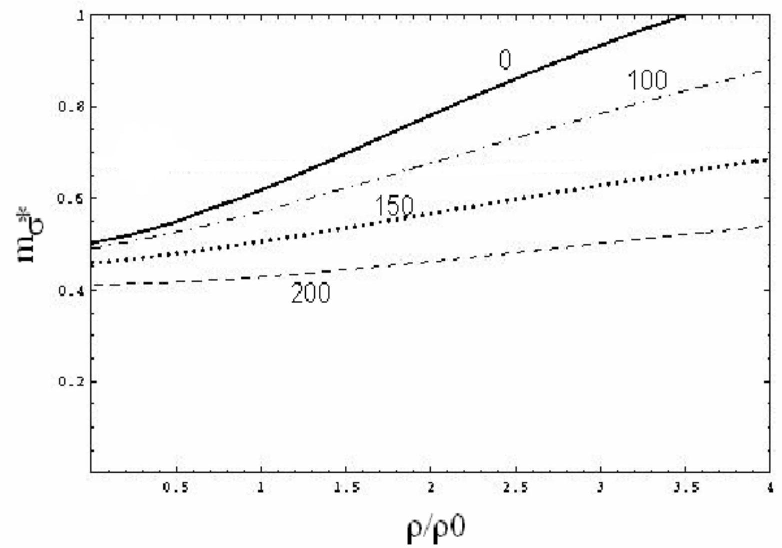
Fields and masses at finite density and temperature

Symmetry broken case (m_π= 138 MeV)



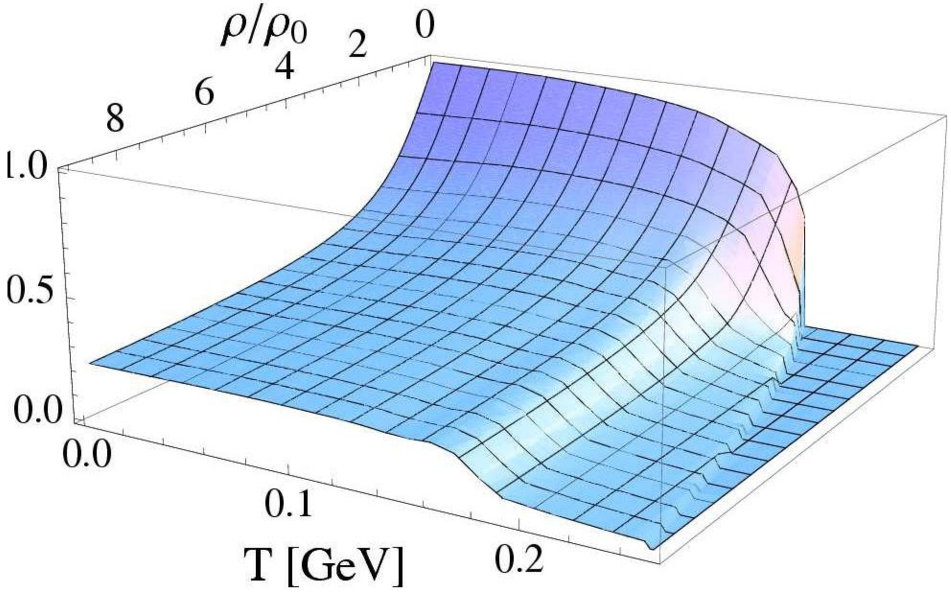


If an explicit breaking of chiral symmetry is introduced, there is not a phase transition but a **crossover**.

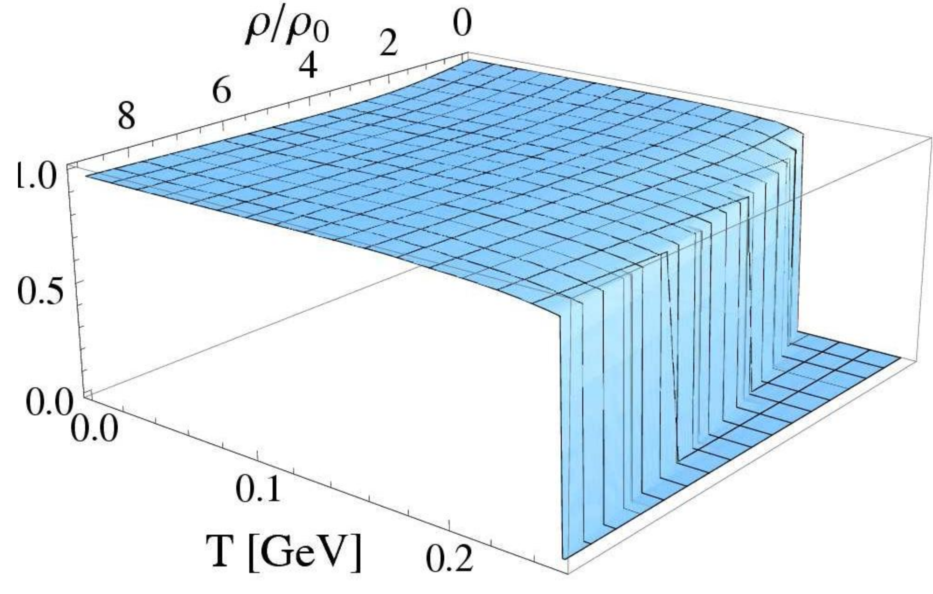


Luca Bonanno, 0909.0924

chiral condensate and dilaton at finite ρ and T



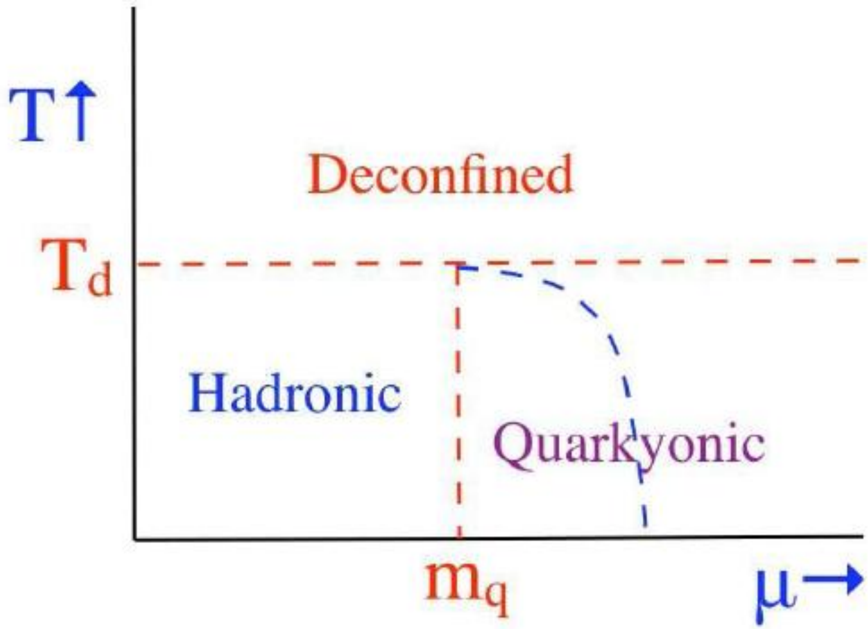
σ/σ_0

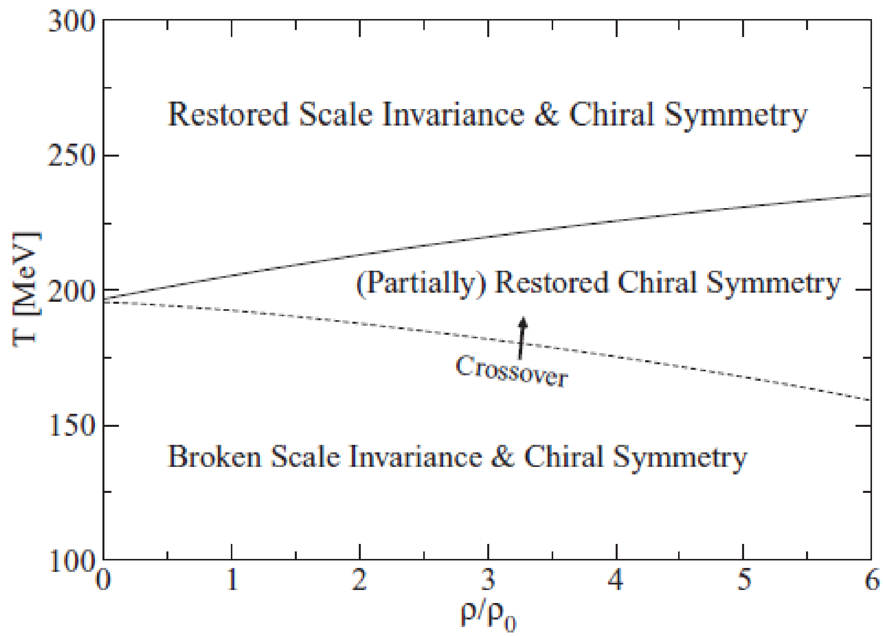


ϕ/ϕ_0

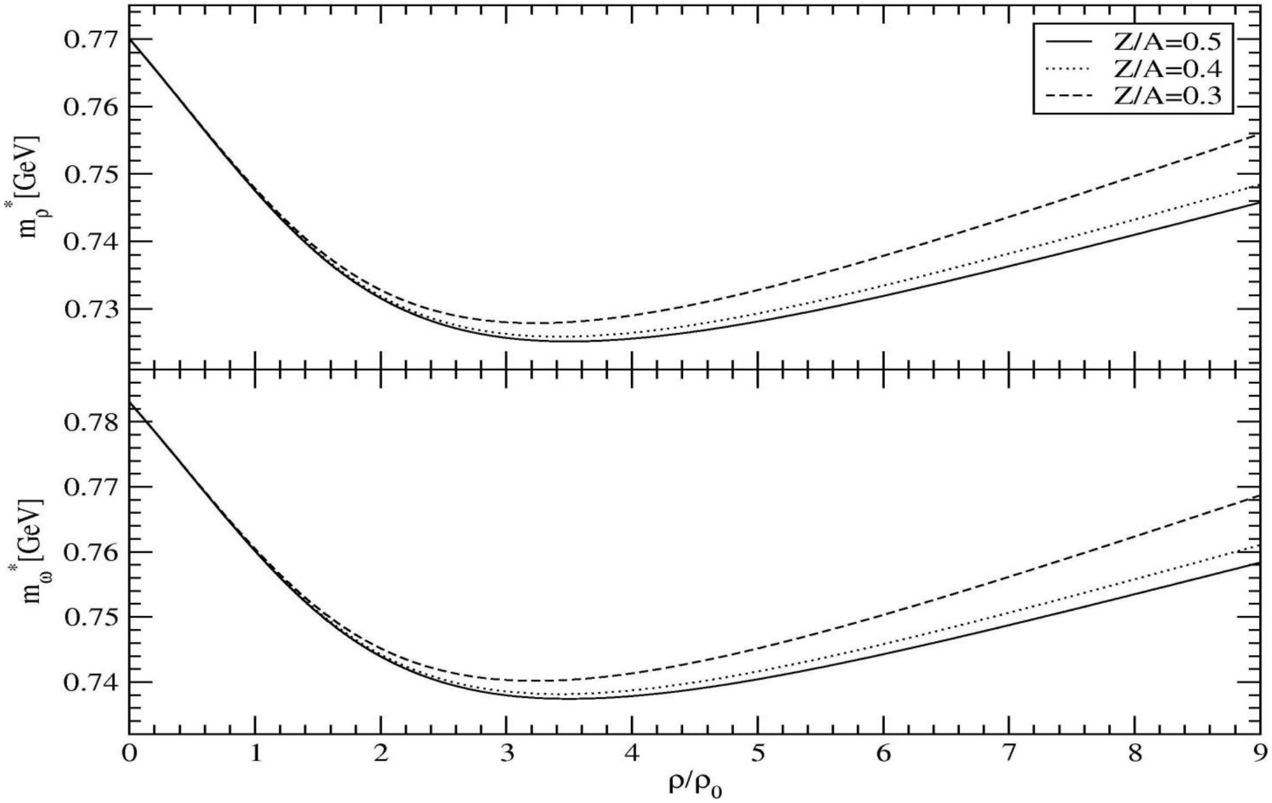
McLerran and Pisarski 2007

Phases of dense quarks at large N_c





Vector Meson effective masses



The mass of the ρ does not reduce with density
No Brown-Rho scaling

From nucleons to quarks

Main idea: to use as a quark Lagrangian the same used as a nucleon lagrangian, but **intepreting now the fermions as quarks**.

The nucleons should now emerge as chiral solitons from the dynamics of the quarks.

Points to be checked:

- Do solitonic solutions exist at all?
- Are those solitons able to provide a reasonable description of nucleons?
- Can a lattice of solitons be built? Can it describe nuclear matter saturation?

It has been done before using the σ-model, but:

- At nucleons' level the σ-model does not provide a good description for nuclear matter, Lee-Wick phase is the ground state already at low densities
- At quarks' level the σ-model does not allow to describe nuclear saturation, solutions disappear at low densities

Chiral-dilaton solitons

- Non-topological chiral solitons
- Baryon number provided by the quarks
- First discussed by Kahana-Ripka-Soni 1984 using a non-linear σ-model
- Broniowski and Banerjee 1986 include vector mesons in the linear σ-model soliton
- Mean field solutions based on the so-called hedgehog ansatz, **G** = **J**+**I** = **0**
- Projection on spin and isospin in order to describe a nucleon
- **New ingredient: log potential** for dilaton and chiral fields

Field equations and hedgehog

$$[i\gamma^\mu \partial_\mu - g_\pi(\sigma + i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\gamma_5) + g_\rho \gamma^\mu \frac{\boldsymbol{\tau}}{2}\cdot(\boldsymbol{\rho}_\mu + \gamma_5 \boldsymbol{A}_\mu) - \frac{g_\omega}{3}\gamma^\mu \omega_\mu]\psi = 0,$$

$$\partial_\mu D^\mu \sigma = -g_\rho \boldsymbol{A}_\mu \cdot \boldsymbol{\pi} - g\bar{\psi}\psi - \frac{\partial V}{\partial \sigma},$$

$$\partial_\mu D^\mu \boldsymbol{\pi} = g_\rho(-\boldsymbol{\rho}_\mu \times D^\mu \boldsymbol{\pi} + \boldsymbol{A}_\mu D^\mu \sigma) - ig\bar{\psi}\boldsymbol{\tau}\gamma_5\psi - \frac{\partial V}{\partial \boldsymbol{\pi}},$$

$$-\partial^\mu \boldsymbol{\rho}_{\mu\nu} = g_\rho \boldsymbol{v}_\nu + m_\rho^2 \boldsymbol{\rho}_\nu,$$

$$-\partial^\mu \boldsymbol{A}_{\mu\nu} = g_\rho \boldsymbol{a}_\nu + m_\rho^2 \boldsymbol{A}_\nu,$$

$$-\partial^\mu \omega_{\mu\nu} = -\frac{1}{3} g_\omega \bar{\psi}\gamma_\nu\psi + m_\omega^2 \omega_\nu.$$

Here $\boldsymbol{v}_\nu$ and $\boldsymbol{a}_\nu$ are the vector and the axial-vector currents:

$$\boldsymbol{v}_\nu = \boldsymbol{\rho}_\mu \times \boldsymbol{\rho}_{\mu\nu} + \boldsymbol{A}_\mu \times \boldsymbol{A}_{\mu\nu} + \boldsymbol{\pi} \times D^\nu \boldsymbol{\pi} + \bar{\psi}\gamma_\nu \frac{\boldsymbol{\tau}}{2}\psi,$$

$$\boldsymbol{a}_\nu = \boldsymbol{\rho}_\mu \times \boldsymbol{A}_{\mu\nu} + \boldsymbol{A}_\mu \times \boldsymbol{\rho}_{\mu\nu} + \boldsymbol{\pi} \times D^\nu \sigma - \sigma D_\nu \boldsymbol{\pi} + \bar{\psi}\gamma_5\gamma_\nu \frac{\boldsymbol{\tau}}{2}\psi$$

$$\psi = \frac{1}{\sqrt{4\pi}} \begin{pmatrix} u(r) \\ iv(r)\sigma\cdot\hat{\boldsymbol{r}} \end{pmatrix} \frac{1}{\sqrt{2}}(|u\downarrow\rangle - |d\downarrow\rangle)$$

$$\langle\hat{\sigma}\rangle = \sigma(r),$$

$$\langle\hat{\boldsymbol{\pi}}_a\rangle = r_a \pi(r),$$

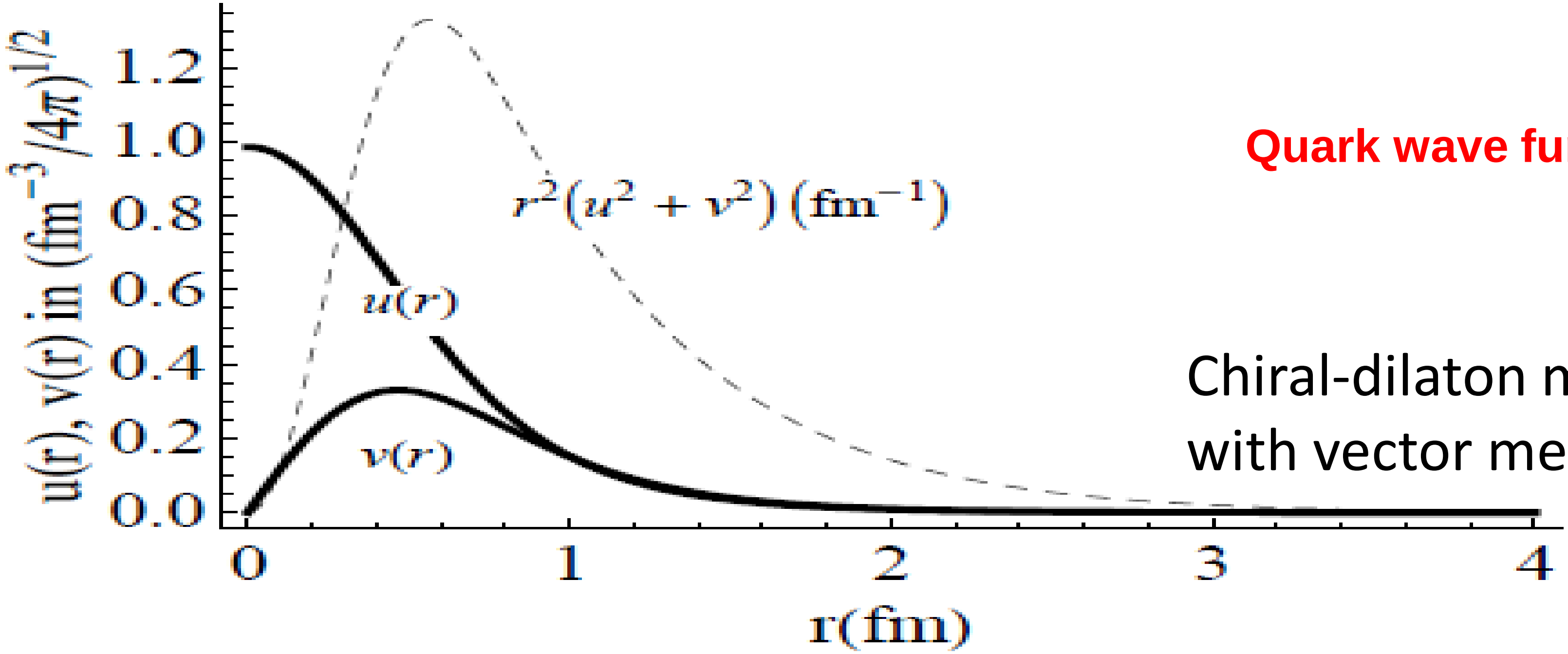
$$\langle\hat{\omega_0}\rangle = \omega(r),$$

$$\langle\hat{\boldsymbol{\rho}}^i_a\rangle = -\epsilon^{ika} r^k \rho(r),$$

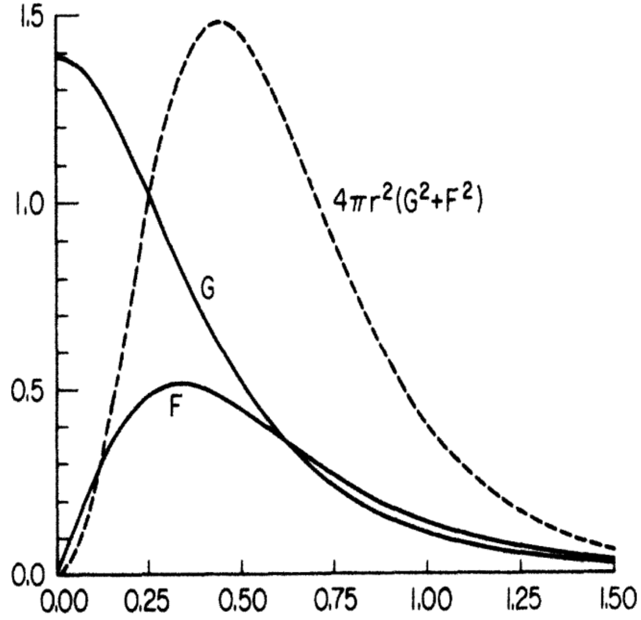
$$\langle\hat{\boldsymbol{A}}^i_a\rangle = A_S(r)\delta_{ia} + A_T(r)(\hat{r}_i\hat{r}_a - \frac{1}{3}\delta_{ia})$$

Hedgehog ansatz

Quark wave function



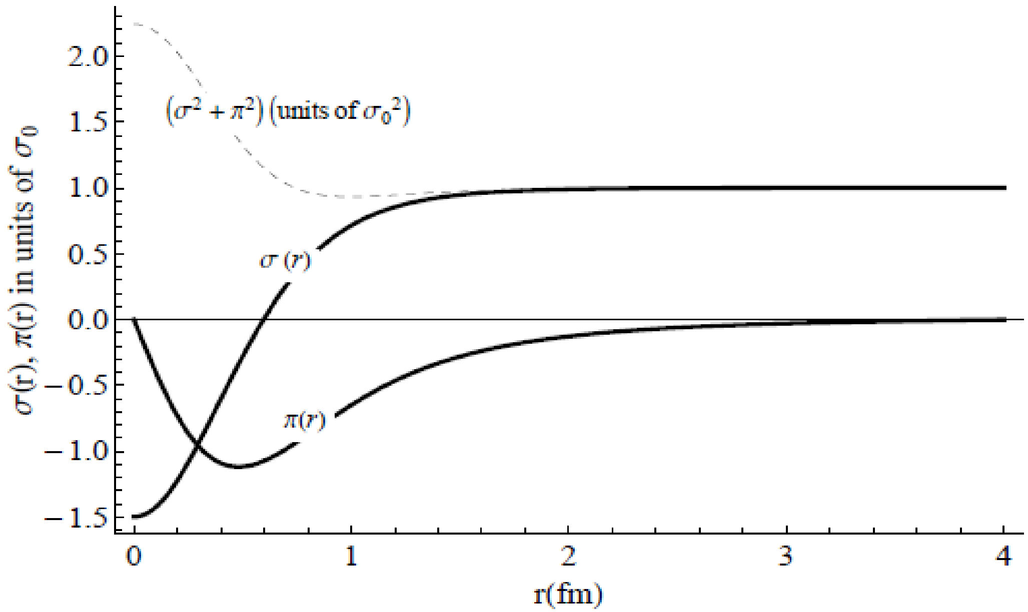
Chiral-dilaton model
with vector mesons



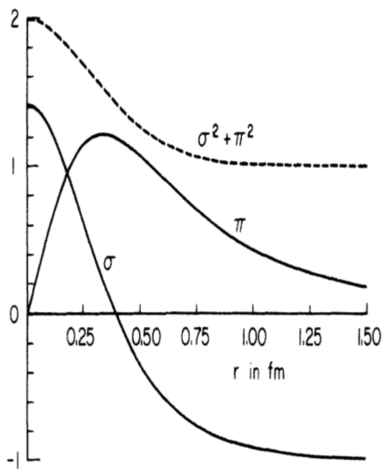
Linear σ-model with vector mesons

Broniowski and Banerjee PRD34 (1986) 849

Chiral fields



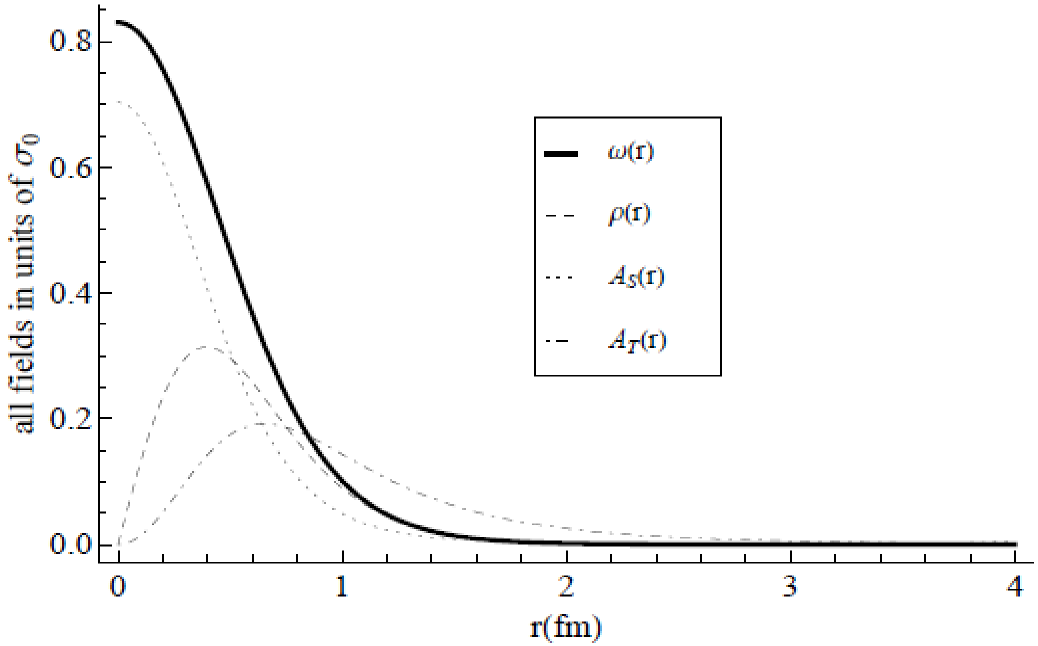
Chiral-dilaton model with vector mesons



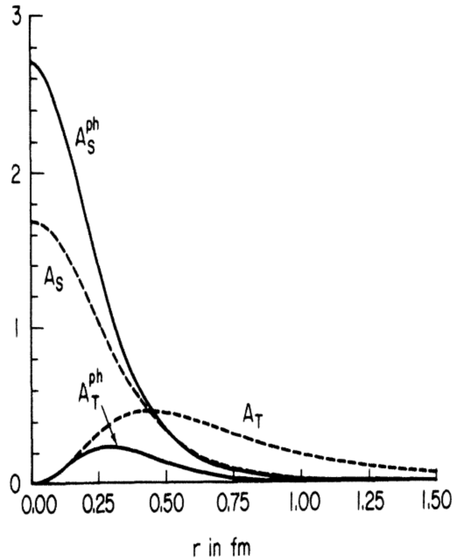
Linear σ-model with vector mesons

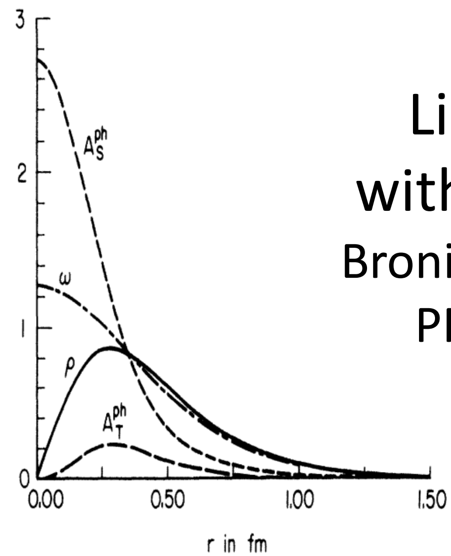
Broniowski and Banerjee PRD34 (1986) 849

Vector mesons



Chiral-dilaton model with vector mesons





Linear σ-model with vector mesons

Broniowski and Banerjee PRD34 (1986) 849

Single nucleon results

Model without VM :

Quantity	Log. Model	σ-Model	Exp.
$E_{1/2}\,(MeV)$	1075	1002	
$M_N\,(MeV)$	960	894	938
$E_{3/2}\,(MeV)$	1140	1075	
$M_\Delta\,(MeV)$	1032	975	1232
$\langle r_E^2\rangle_p\,(fm^2)$	0.55	0.61	0.74
$\langle r_E^2\rangle_n\,(fm^2)$	-0.02	-0.02	-0.12
$\langle r_M^2\rangle_p\,(fm^2)$	0.7	0.72	0.74
$\langle r_M^2\rangle_n\,(fm^2)$	0.72	0.75	0.77
$\mu_p\,(\mu_N)$	2.25	2.27	2.79
$\mu_n\,(\mu_N)$	-1.97	-1.92	-1.91
g_a	1.52	1.10	1.26

Model with VM, SET I:

Quantity	Log. Model	σ-Model	Exp.
$E_{1/2}\,(MeV)$	1020	1008	
$M_N\,(MeV)$	926	912	938
$E_{3/2}\,(MeV)$	1148	1147	
$M_\Delta\,(MeV)$	1066	1063	1232
$\langle r_E^2\rangle_p\,(fm^2)$	0.67	0.66	0.74
$\langle r_E^2\rangle_n\,(fm^2)$	-0.05	-0.05	-0.12
$\langle r_M^2\rangle_p\,(fm^2)$	0.77	0.76	0.74
$\langle r_M^2\rangle_n\,(fm^2)$	0.78	0.77	0.77
$\mu_p\,(\mu_N)$	2.63	2.64	2.79
$\mu_n\,(\mu_N)$	-2.37	-2.38	-1.91
g_a	1.58	1.46	1.26

Going to **finite density**

- Approximating nuclear matter by a lattice of nucleons
- Further approximation by using a Wigner-Seitz lattice (spherical cell of radius R) instead of a cubic lattice
- Imposing periodic boundary conditions

Crucial point to be discussed: formation of bands

- How is a band defined?
- Physical meaning of the band
- Crossing of bands and deconfinement

Periodic boundary conditions

Bloch theorem

The states in the band have w.f.s defined as $\phi_{\mathbf{k}} = e^{i\mathbf{kr}}\, u_{\mathbf{k}}(\mathbf{r})$,

which are obtained by solving the equation

$$(H_D + \boldsymbol{\alpha}\cdot\mathbf{k})u_{\mathbf{k},c0^+} = \epsilon_k u_{\mathbf{k},c,0^+} \quad \text{with} \quad u_{\mathbf{k}=0,\,c,\,0^+} = u_{c,0^+}$$

for all **k** in the first Briouillin zone $\quad -\pi/2R \le k_x, k_y, k_z \le \pi/2R = k_B$

The **bottom of the band** is defined as the state satisfying the following periodic boundary conditions, dictated by symmetry arguments

$$v(R) = \pi(R) = \rho(R) = 0$$

$$u'(R) = \sigma'(R) = \omega'(R) = A'_S(R) = A'_T(R) = 0$$

How to define the band

In our work we use two different methods to estimate the band width:

- A (rather crude) approximation to the width of a band can be obtained by using (Glendenning, Banerjee PRC 34(1986)):

$$\Delta = \sqrt{\epsilon_0^2 + \left(\frac{\pi}{2R}\right)^2} - |\epsilon_0|,$$

$$\epsilon_{top} = \epsilon_0 + \Delta.$$

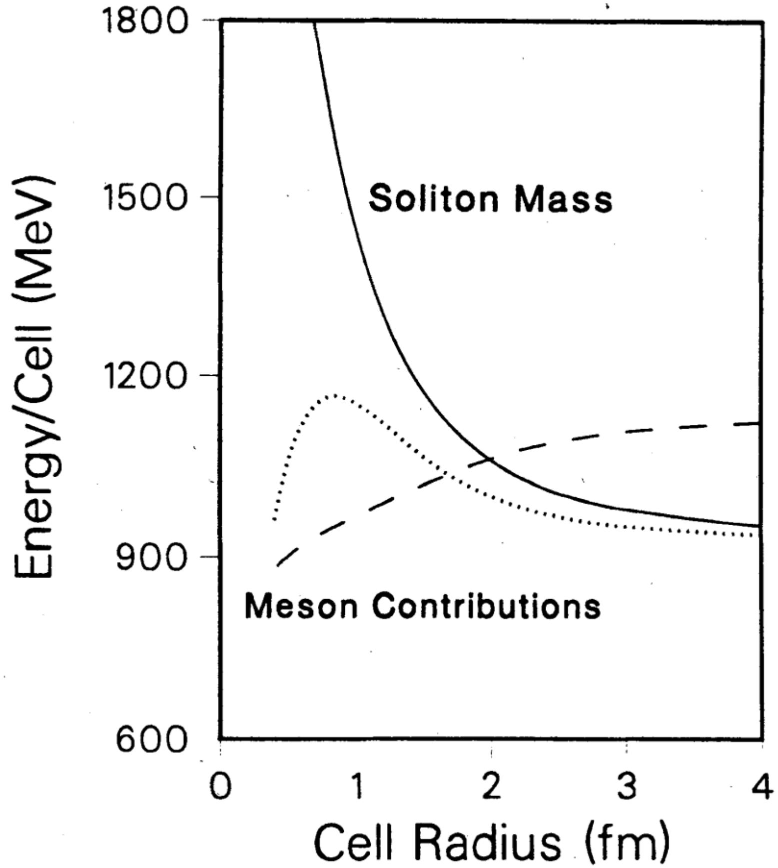
- An alternative approximation is obtained by imposing that the upper Dirac component vanishes at the boundary(Birse,Rehr,Wilets PRC38 (1988)):

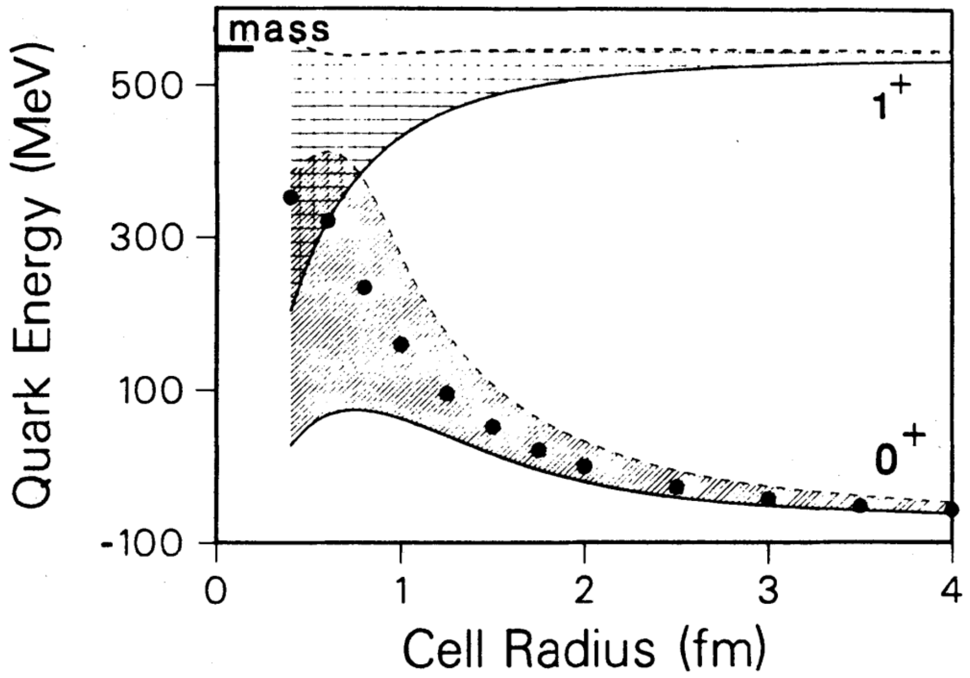
$$u(R) = 0$$

- the eigenvalue obtained imposing this boundary condition represents an upper limit to the top and the true top would be about half way between this upper limit and the bottom of the band Weber and McGovern 1997
- uniform filling of the band → lower band has $G = 0$, color is the only degeneracy left → 3 quarks per soliton completely fill the band

Results in the non-linear σ-model
Hahn and Glendenning 1987

No saturation!

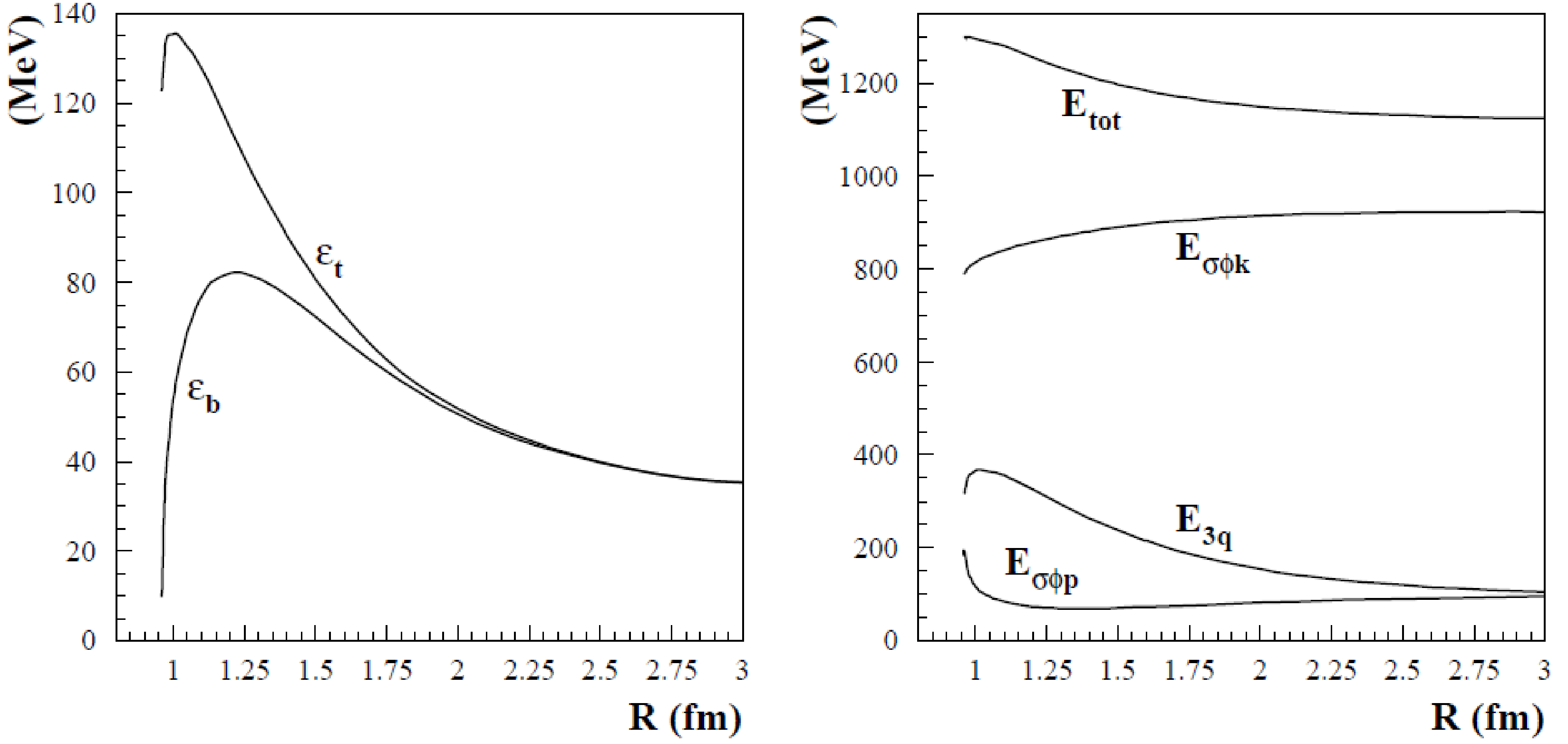




ρ=1/(4 π R^3/3)

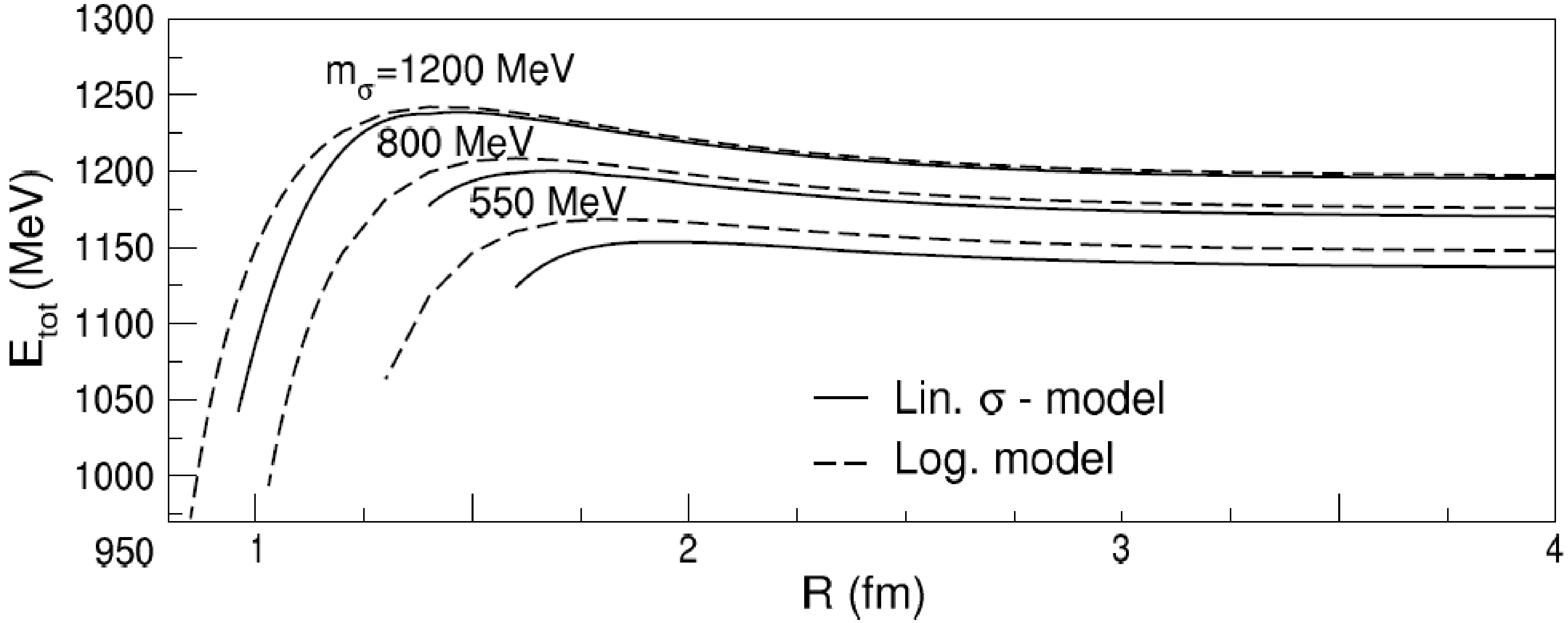
Results in the linear σ-model
Weber and McGovern 1997

No saturation! No solutions at moderate densities!



Role of logaritmic potential in stabilizing the soliton

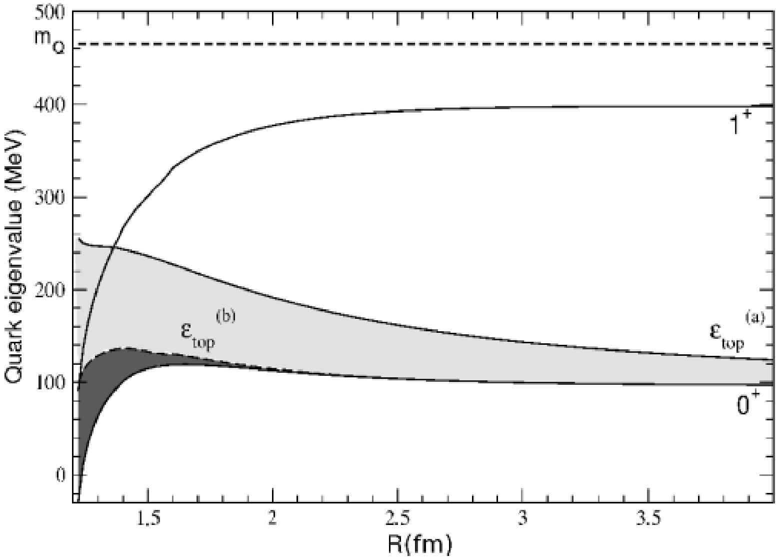
No vector mesons in this Figure



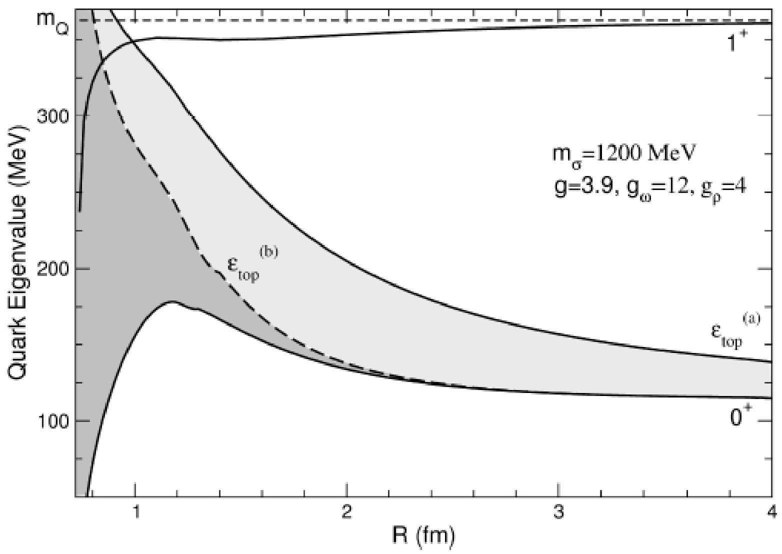
For every value of mσ the log.potential provides solitons which remain stable at larger densities

Role of vector mesons in obtaining saturation

Model without VM

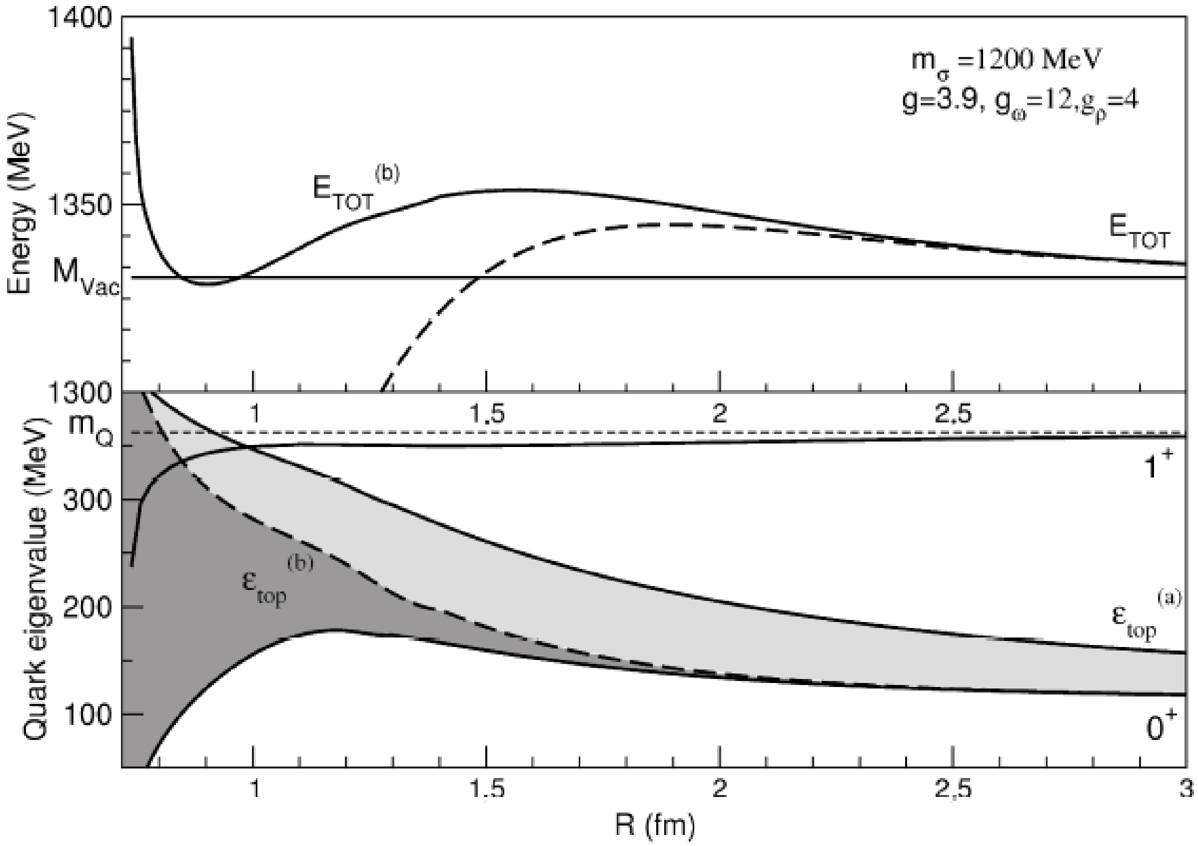


Model with VM

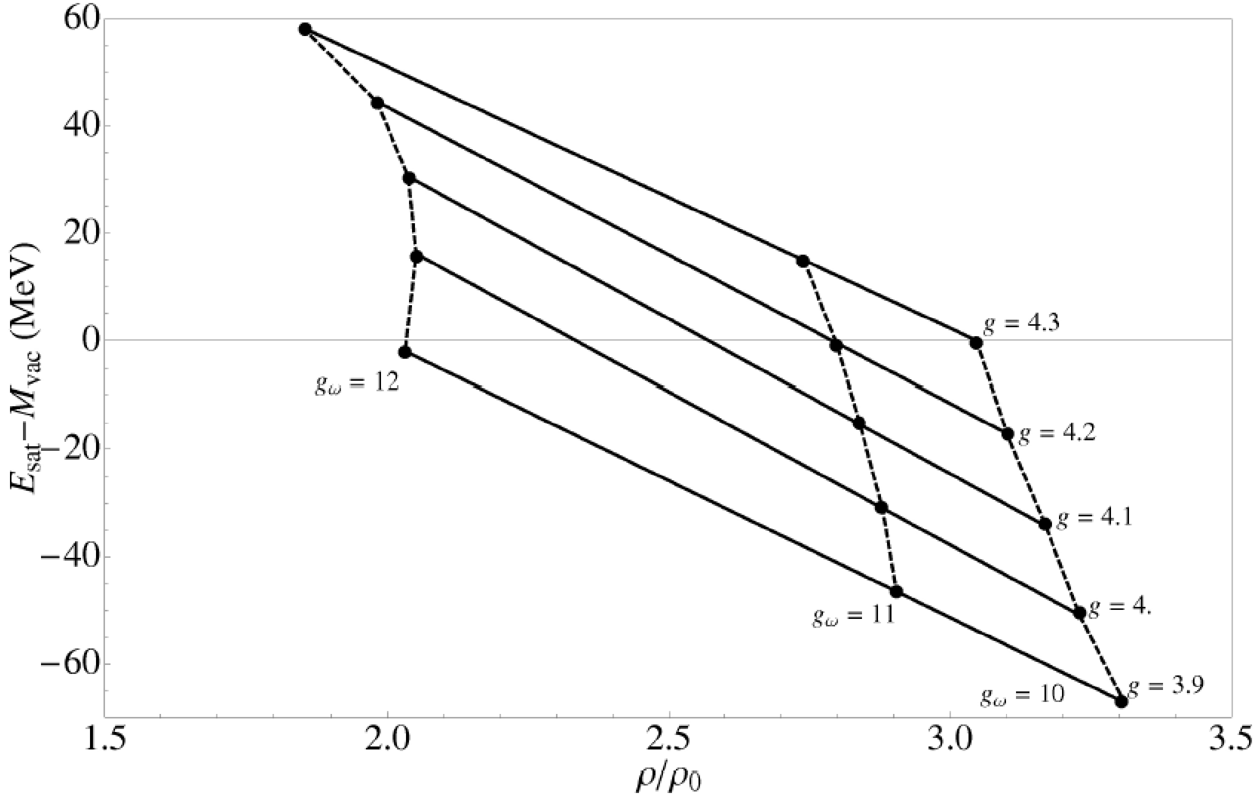


- in absence of VM → saturation can never be obtained
- with VM → significant increase of the top of the band at high densities → saturation

Saturation and «deconfinement»



Parameter space for saturation



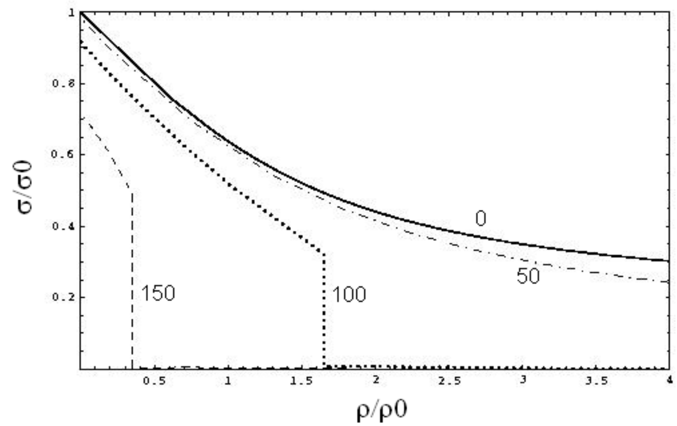
- the model admits "saturation" for different sets of parameters → partial overlap with parameters for the single nucleon (Broniowski, Banerjee PRD34 (1986))

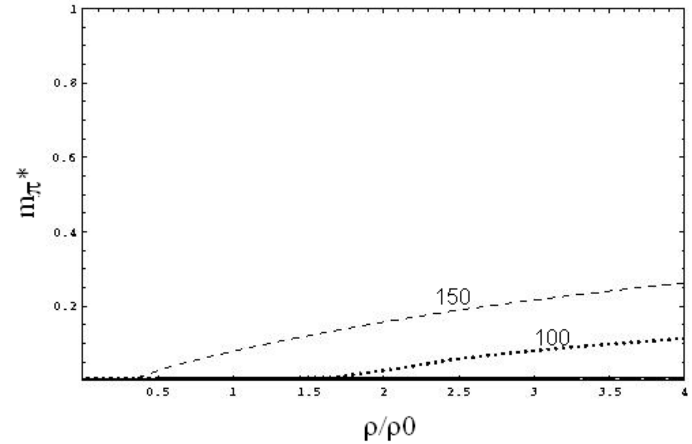
Conclusions and outlook

- A chiral-dilaton model can be used to describe nuclear matter (and nuclei). It provides a description of the phase space in the ρ-T plane not too different from the McLerran-Pisarski scenario.
- The same chiral-dilaton model can be used to describe the quark dynamics. Nucleons emerge as chiral solitons.
- A Wigner-Seitz lattice can be built and **saturation takes place**, *via* the interplay between vector mesons and chiral fields. Band formation plays a role in obtaining saturation.
- The extension to finite temperature is very difficult but also extremely interesting: dynamics of the dilaton field, critical end-point…
- To go beyond Wigner-Seitz appriximation in order to study more complicated chiral structure (half-Skyrmion and similars, collaboration with V. Vento and B. Y. Park)

Chiral symmetry restoration at high ρ and finite T

Simmetric nuclear matter in the chiral limit ➡ $m_\pi = 0$

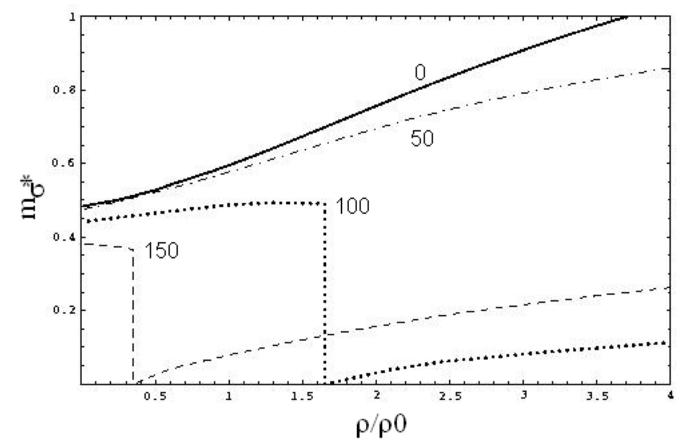




Chiral Simmetry restoration:

$$\langle \sigma \rangle = 0$$

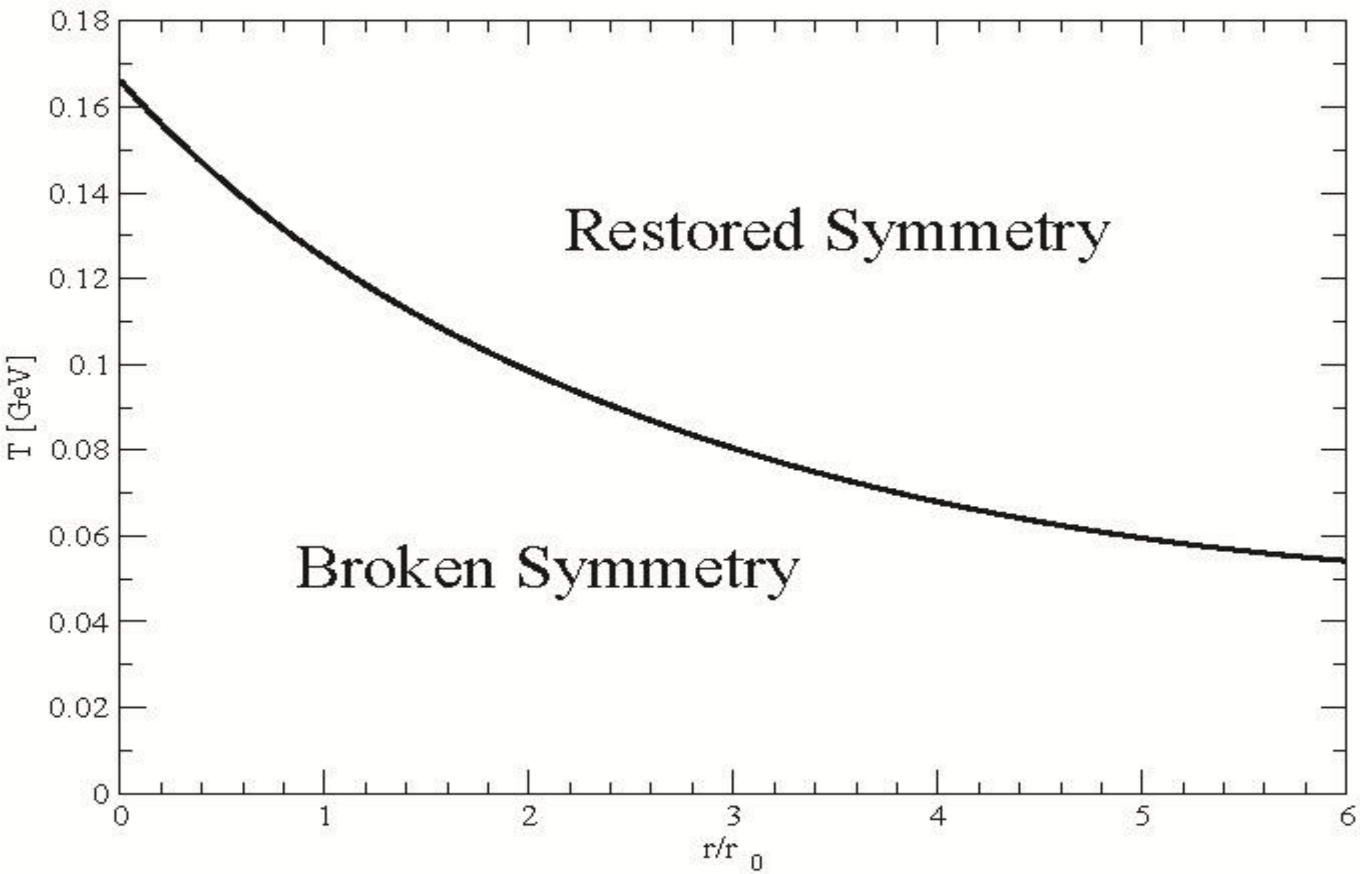
$$m_\sigma^{\ *} = m_\pi^{\ *}$$



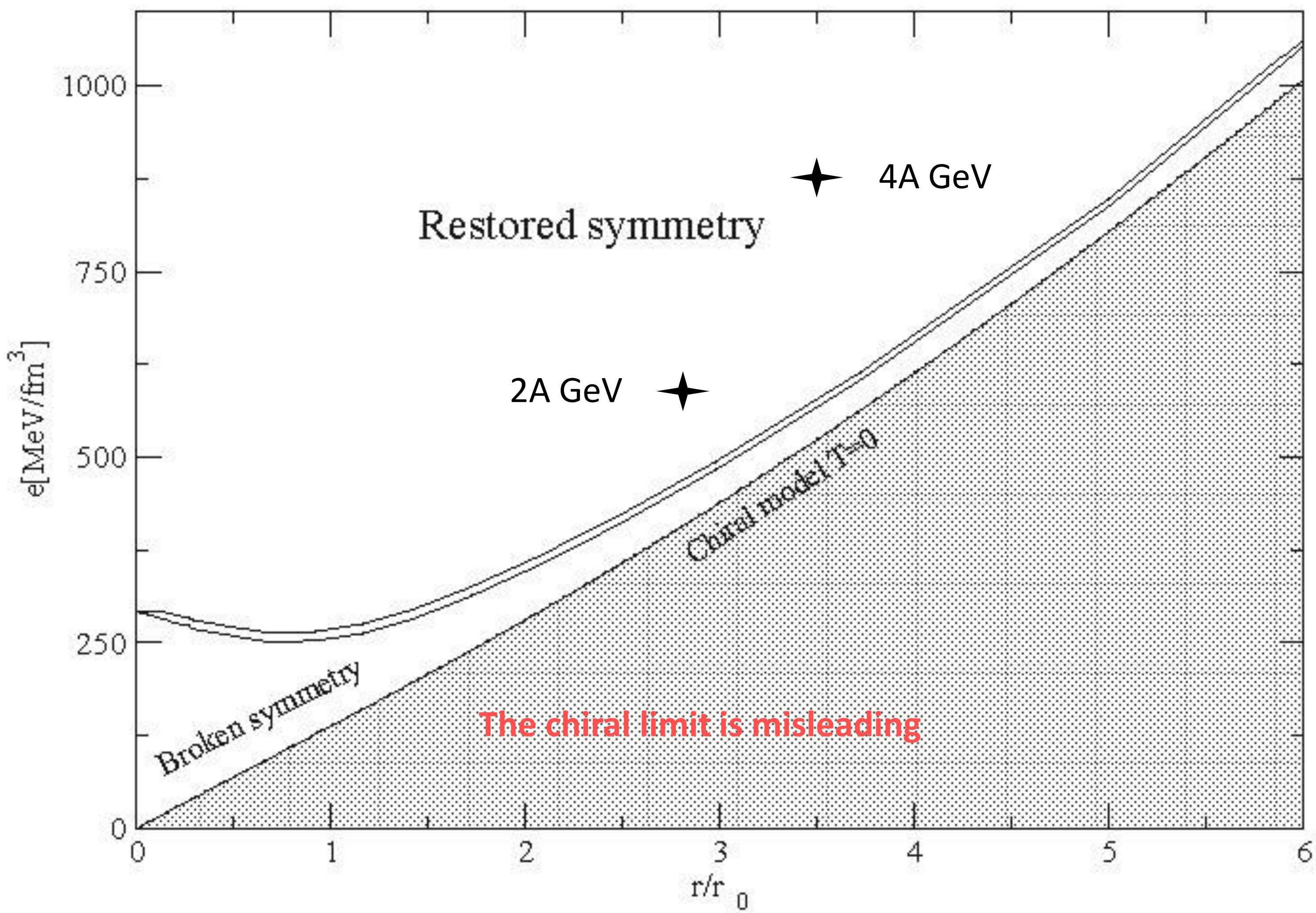
First order transition, σ discontinous

Chiral transition in the ρ-T plane

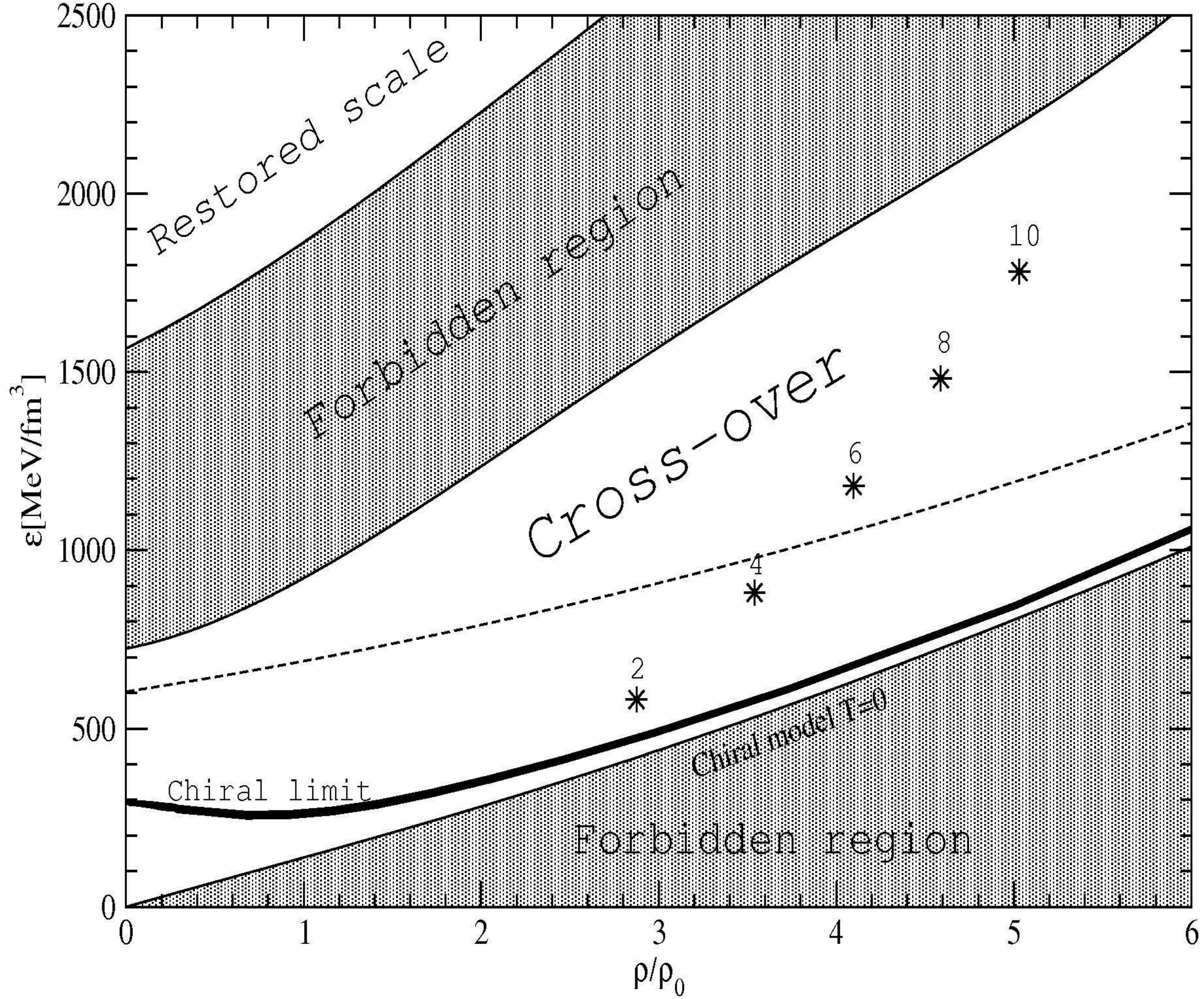
chiral limit $m_\pi = 0$



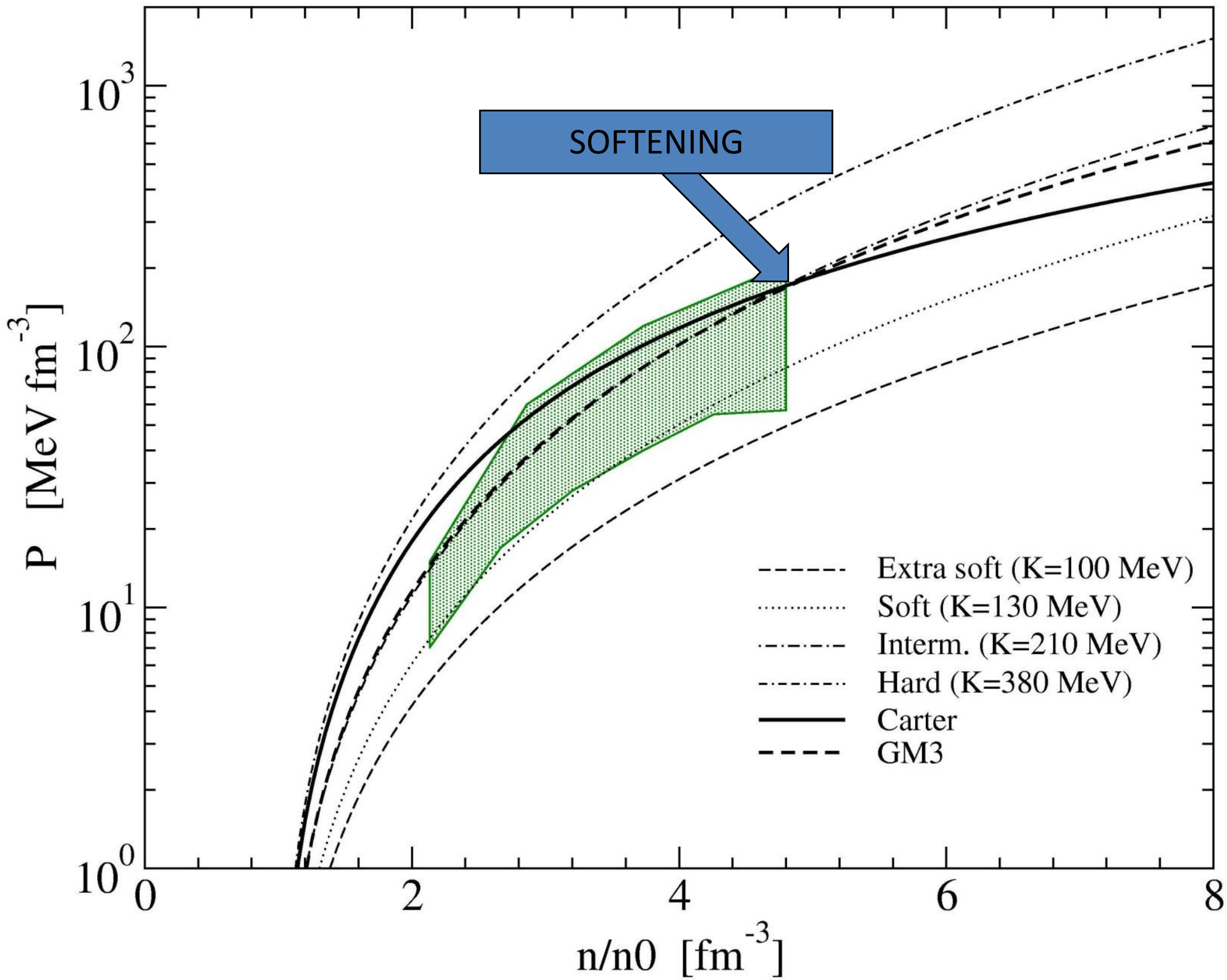
Chiral transition in the $\rho-\epsilon$ plane
chiral limit $m_\pi = 0$



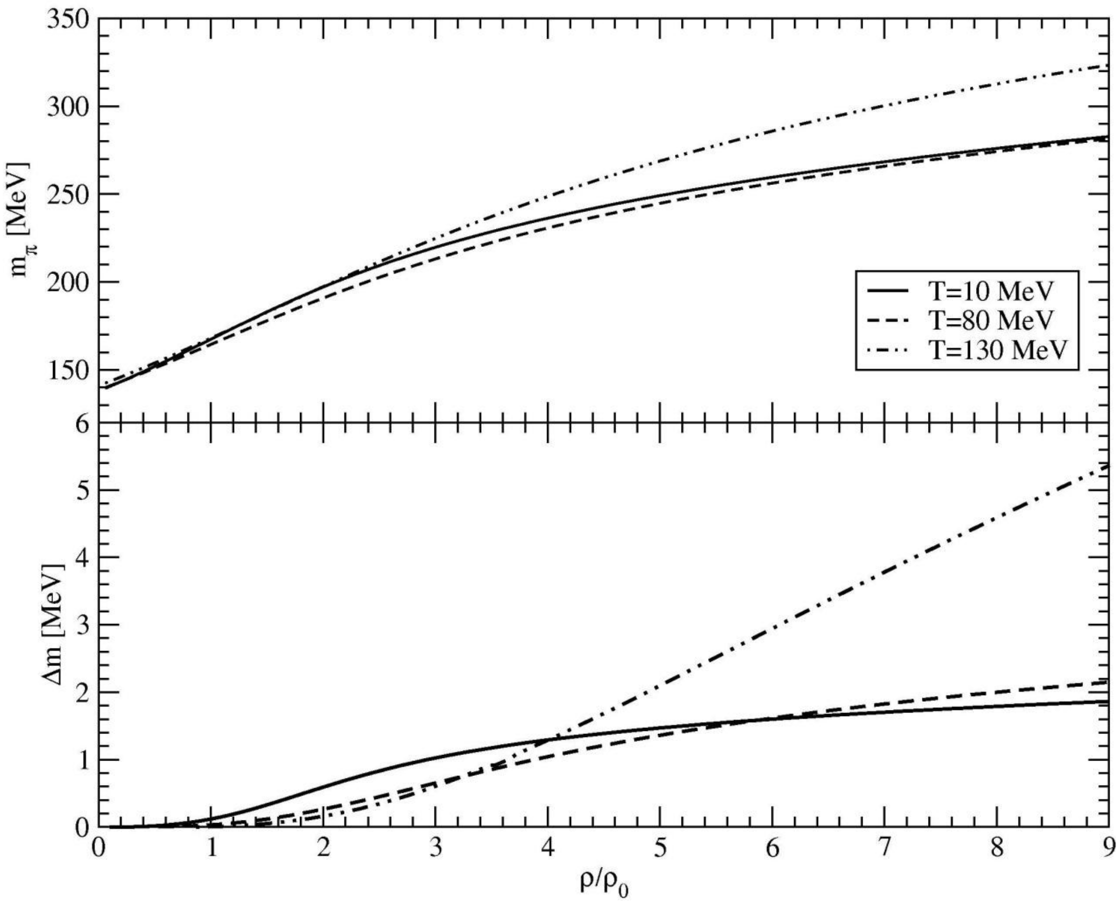
Chiral symmetry broken case (m_π= 138 MeV)



EOS Symmetric nuclear matter at T=0



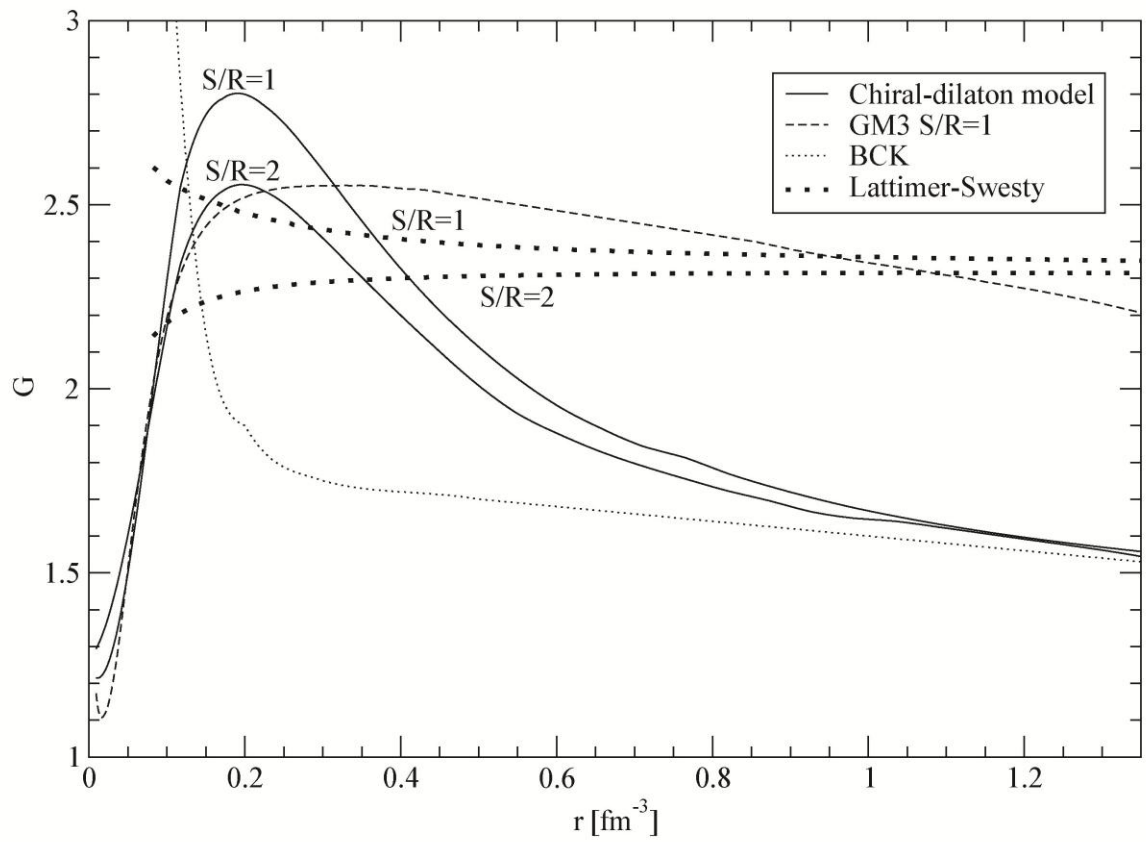
Effective pion mass



Still preliminary experiments on deeply bound 1s and 2p states in pionic atoms indicate an increase of about 30 MeV at saturation density.

Friedman and Gal, Phys .Lett. B432, 235 (1998)

Adiabatic index in pre-supernova matter



Chiral symmetry restoration dramatically reduces the adiabatic index,

but at too high densities to allow a prompt explosion.

Interesting effects could be observed in neutron stars merging.

Mass-radius relation

from Klahn et al. 2006, adapted