Transport approach at fixed $\eta/s(T)$ & Chemical Composition of the QGP

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Outline

- Transport Theory at fixed η/s :
  - Motivations
  - Green-Kubo vs Chapmann-Enskog & Relax time Approx.

  What is $\eta \leftrightarrow \sigma(\theta)$, ρ, M, T, … ?

- First application to HIC:
  - RHIC = LHC ? $v_{2,4}$ sensitivity to $\eta/s(T)$ of QGP
  - v_2 up to p_T 12 GeV
  - v_n <-> η/s + microscopic details?

- Transport Theory with Mean Field:
  - Dynamics associated to Quasi-Particle Model: (Kaempfer on Friday)

  Chemical equilibration and quark dominance

Transport approach

$$\left\{p^{*\mu}\partial_\mu + \left[p^*_\nu F^{\mu\nu} + m^*\partial^\mu m^*\right]\partial^{p^*}_\mu\right\} f(x,p^*) = C_{2\leftrightarrow 2} + C_{2\leftrightarrow 3} + \ldots$$

Free streaming | Field Interaction -> ε≠3P | Collisions -> η≠0

$$\mathcal{C}_{22} = \frac{1}{2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int\frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 |\mathcal{M}_{1'2'\to 12}|^2 (2\pi)^4\delta^{(4)}(p'_1+p'_2-p_1-p_2)$$

$$\sigma_{22} = \frac{1}{2s}\frac{1}{\nu}\int\frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2}|\mathcal{M}_{12\to 1'2'}|^2(2\pi)^4\delta^{(4)}(p_1+p_2-p'_1-p'_2)$$

- Provide a measure of η/s other than viscous hydro
- Microscopic scale has some relevance?
  Can we know more about QGP?
- Can we link the effective $\mathcal{L}$ <-> transport dynamics in HIC
  Thermodynamics <-> phenomenology of HIC

$$L = \bar{\Psi}\left(i\gamma_\mu D^\mu - m\right)\Psi + \frac{G}{2}\left(\bar{\Psi}\Psi\right)^2 + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + U(\Phi) + \ldots$$

Wigner Tranforms +
semiclassical approx.

Motivation for Transport approach

$$\left\{ p^{*\mu}\partial_\mu + \left[p^*_\nu F^{\mu\nu} + m^*\partial^\mu m^* \right] \partial^{p^*}_\mu \right\} f(x, p^*) = C_{2\leftrightarrow 2} + C_{2\leftrightarrow 3} + \ldots$$

Free streaming Field Interaction -> $\varepsilon \neq 3P$ EoS Collisions -> $\eta \neq 0$

- valid also at intermediate & high p_T out of equilibrium: region of modified hadronization at RHIC
- valid also at high η/s -> LHC - $\eta/s(T)$, cross-over region
- Relevant at LHC due to large amount of minijet production
- Appropriate for heavy quark dynamics
- CGC p_T non-equilibrium phase (beyond the difference in ε_x):

A unified framework against a separate modelling with a wide range of validity in η, ζ, p_T + microscopic level

Simulate a fixed shear viscosity!?

Usually input of a transport approach are *cross-sections and fields*, but here we reverse it and start from η/s with aim of creating a more direct link to viscous hydrodynamics

Relax. Time Approx. (RTA)

$$\frac{\eta}{s} = \frac{1}{15}\frac{\bar{p}}{\sigma_{tr} n} = \frac{1}{5}\frac{T}{\sigma_{tr} n} = \text{cost.}$$

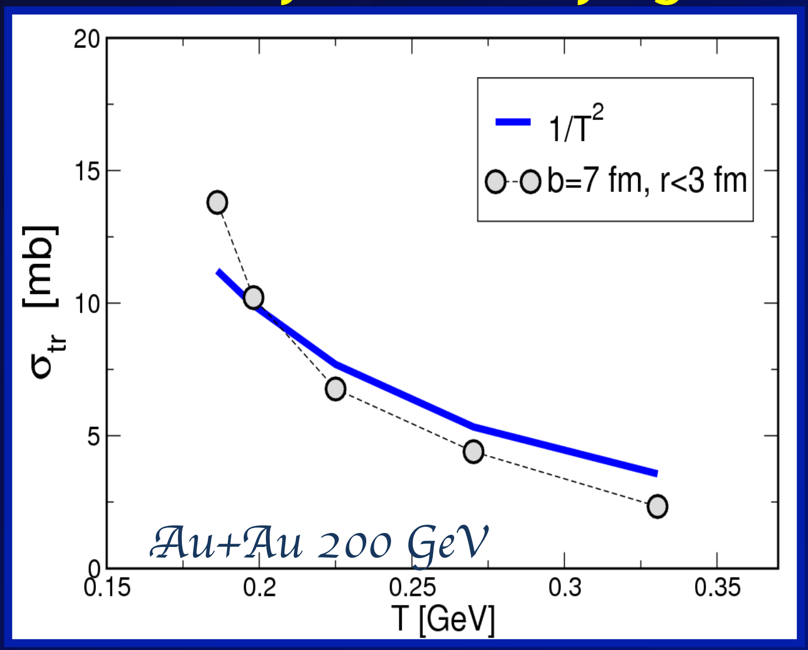
Cascade code

$$\sigma_{tr}(\rho(\vec{r}),T) = \sigma_{tr,\alpha} = \frac{1}{15}\frac{\bar{p}_\alpha}{n_\alpha}\frac{1}{\eta/s}$$

Space-Time dependent cross section evaluated locally

α=cell index in the r-space

Viscosity fixed varying σ



In other words

$$\sigma^* = K\sigma_{pQCD}$$

K fixed by η/s

G. Ferini et al., PLB670 (09)

V. Greco at al., PPNP 62 (09)

Part I

Do we really have the wanted shear viscosity η with the relax. time approx.?

- Check η with the Green-Kubo correlator

Shear Viscosity in Box Calculation

Green-Kubo correlator

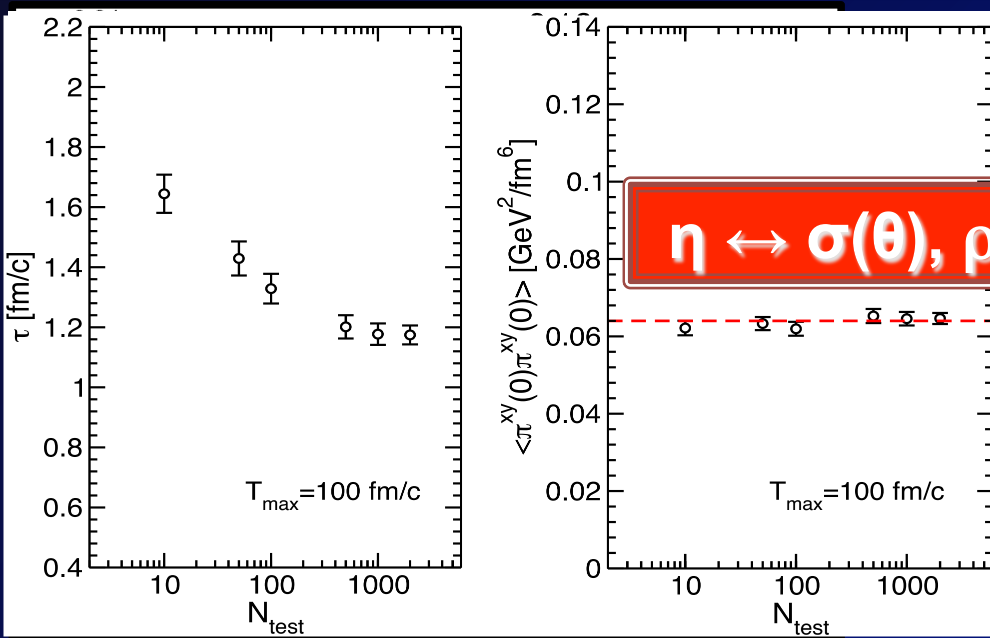
$$\eta = \frac{1}{T}\int_0^\infty dt \int_V d^3x \left\langle \Pi^{xy}(\vec{x},t)\,\Pi^{xy}(0,0)\right\rangle$$

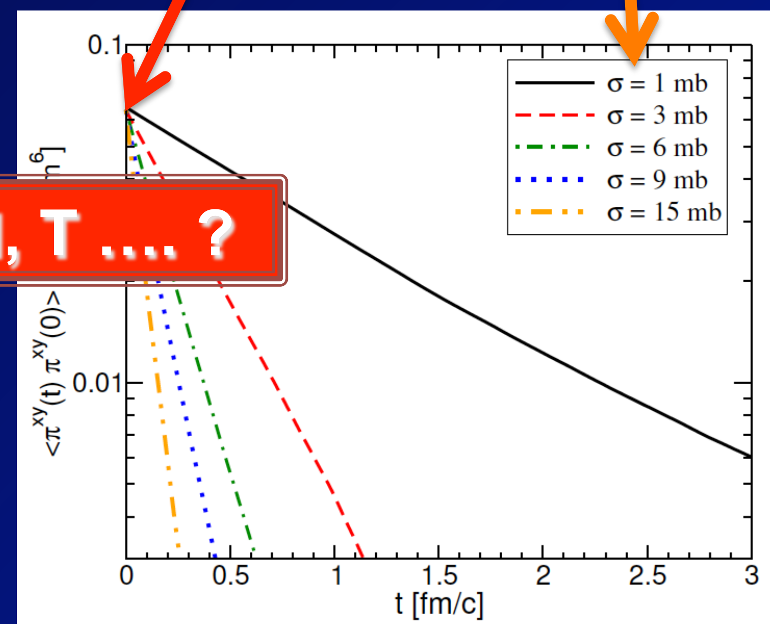
$$\left\langle \Pi^{xy}(\vec{x},t)\Pi^{xy}(0,0)\right\rangle = \left\langle \Pi^{xy}(0,0)\Pi^{xy}(0,0)\right\rangle \cdot e^{-t/\tau}$$

$$\eta = \frac{V}{T}\langle \pi^{xy}(0)\,\pi^{xy}(0)\rangle\,\tau$$

microscopic details (τ)

macroscopic observables: $\langle \pi^{xy}(0)\,\pi^{xy}(0)\rangle = \frac{4}{15}\frac{\varepsilon T}{V}$





$\eta \leftrightarrow \sigma(\theta), \rho, M, T$ .... ?

S. Plumari et al., arxiv:1208.0481;see also:
Wesp et al., Phys. Rev. C 84, 054911 (2011);
Fuini III et al. J. Phys. G38, 015004 (2011).

Needed very careful tests of convergency vs. N_{test}, Δx_{cell}, # time steps !

Isotropic Cross Section

Relax. Time Approx. - Gavin NPA(1985); Kapusta, PRC82(10); Redlich and Sasaki, PRC79(10), NPA832(10)…

$$\eta = \frac{1}{15T}\int_0^\infty \frac{d^3p_a}{(2\pi)^3}\frac{|p_a|^4}{E_a^2}\frac{1}{w_a(E_a)} f_a^{eq}$$

$$w_a \tau_a^{-1} = w_a(E_a) = \rho\,\sigma_{tot}\,\langle v_{rel}\rangle \quad f_b^{eq}\,\sigma_{tot}$$

$$\eta_{RTA} = 0.8\frac{T}{\sigma_{TOT}\langle v_{rel}\rangle}$$

Usual as relax. time approx. – Israel Stewart

$\sigma_{tot} \to \sigma_{tr} = 2/3\,\sigma_{tot}$ for isotropic cross section

$$\eta_{RTA}^{IS} = 0.8\frac{T}{\sigma_{tr}\langle v_{rel}\rangle} = 1.2\frac{T}{\sigma_{TOT}\langle v_{rel}\rangle}$$

Molnar-Huovenin PRC(2009), G. Ferini PLB(2009), Khvorostukhin PRC (2010) …

Chapmann-Enskog expansion

Isotropic Cross section – massless particles

1st order CE

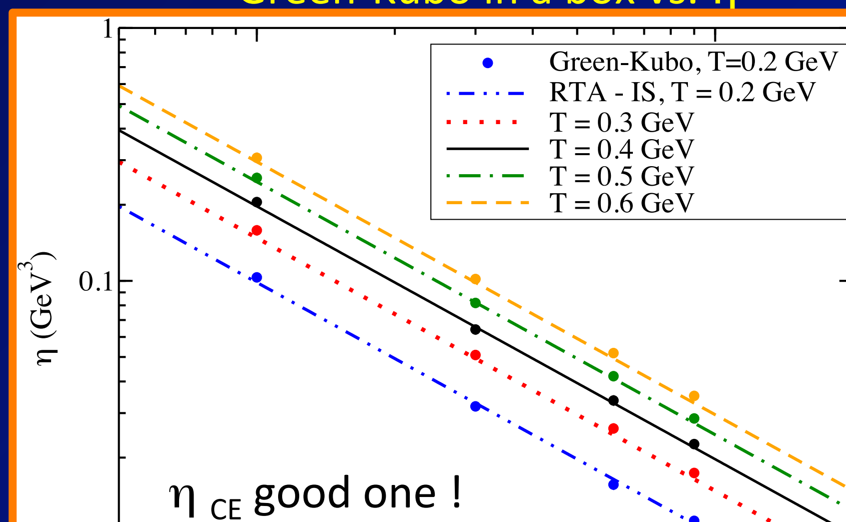
$$\eta_{CE}^{I} = 1.2\frac{T}{\sigma_{TOT}\langle v_{rel}\rangle}$$

16th order CE

$$\eta_{CE}^{XVI} = 1.267\frac{T}{\sigma_{TOT}\langle v_{rel}\rangle}$$

Prakash et al., arxiv:1203.0281 [nucl-**th**]

Green-Kubo in a box vs. η^{IS}



pQCD-like cross section are not isotropic!
as well as hadronic ones!

Non Isotropic Cross Section - $\sigma(\theta)$

Relaxation Time Approximation – non-isotropic cross section

$$\eta_{RTA}^{IS} = 0.8\frac{T}{\sigma_{tr}} = 0.8\frac{T}{\sigma_{TOT}\langle f(a)\rangle}$$

$$\sigma_{tr} = \int d\Omega_{cm}\sin^2\theta_{cm}\frac{d\sigma}{d\Omega_{cm}} = \sigma_{TOT} f(a) \leq \frac{2}{3}\sigma_{TOT}$$

$$f(a) = 4a(1+a)\left[(2a+1)\ln(1+a^{-1})-2\right] \quad , a = m_D^2/s$$

m_D regulates the angular dependence

m_D -> ∞ isotropic cross section

for pQCD-like cross section:

G.Ferini, PLB(2009); D. Molnar, JPG35(2008); V.Greco, PPNP(2009); Khvorostukhin PRC (2010)…

$$\frac{d\sigma}{d\Omega} = \frac{9\pi\alpha_s^2}{2}\frac{1}{\left(q^2(\theta)+m_D^2\right)^2}\left(1+\frac{m_D^2}{s}\right)$$

I° Chapmann-Enskog - anisotropic σ

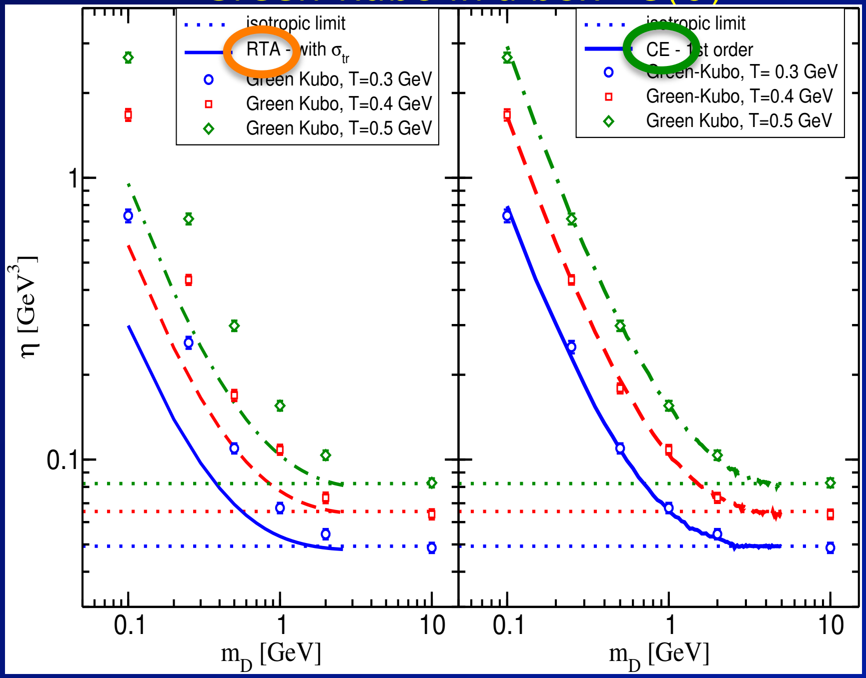
$$[\eta]_{CE}^{1st} = 10T\left[\frac{K_3(z)}{K_2(z)}\right]^2\frac{1}{c_{00}} = 0.8\frac{T}{\sigma_{TOT}\, g(m_D,T)}$$

$$c_{00} = 16\left[\omega_2^{(2)} - z^{-1}\omega_1^{(2)} + (3z^2)^{-1}\omega_0^{(2)}\right]$$

~ σ_{tr}

$$\omega_i^{(2)} = \frac{z^3}{[K_2(z)]^2}\int_1^\infty dy(y^2-1)^3 y^i K_2(2zy)\int_0^\pi d\Omega\frac{d\sigma}{d\Omega}sen^2\theta$$

Green-Kubo in a box - $\sigma(\theta)$

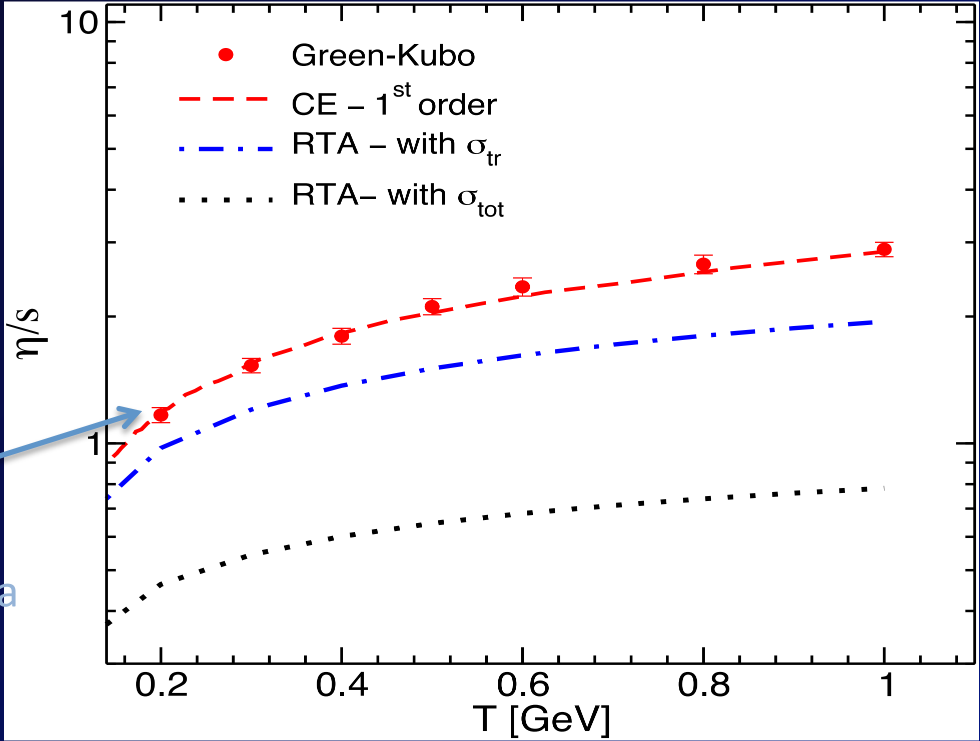


- CE and RTA can differ by about a factor 2-3
- Green-Kubo agrees with CE

S. Plumari et al., arXiV:1208.0481

Example:Viscosity of a pQCD gluon plasma

Only t+ u channel with simplified HTL propagator



close to AMY result JHEP(2003), but there is a significant simplification in the propagator

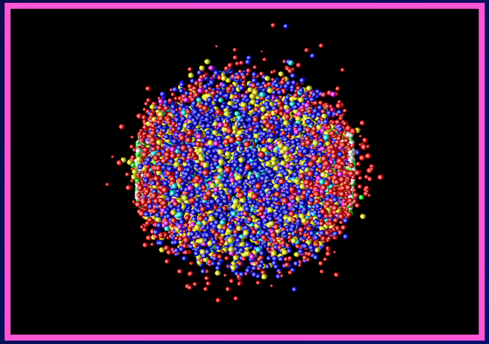
$$\frac{d\sigma^{gg\to gg}}{dq^2} = \frac{9\pi\alpha_s^2}{2}\frac{1}{(q^2+m_D^2)^2}.$$

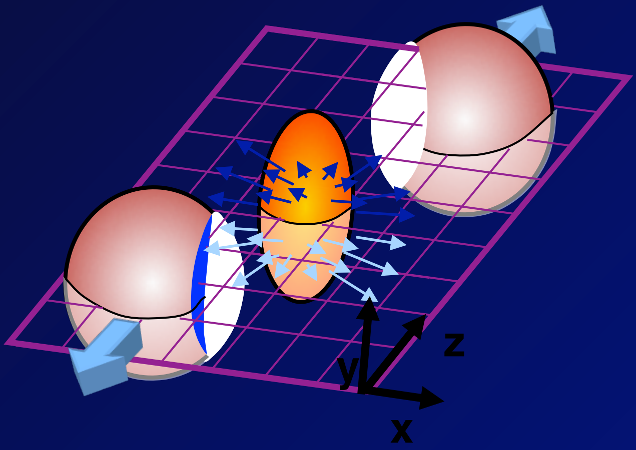
$$\alpha_s(T) = \frac{4\pi}{11\ln\left(\frac{2\pi T}{\Lambda}\right)^2} \quad m_D = T\sqrt{4\pi\alpha_s}$$

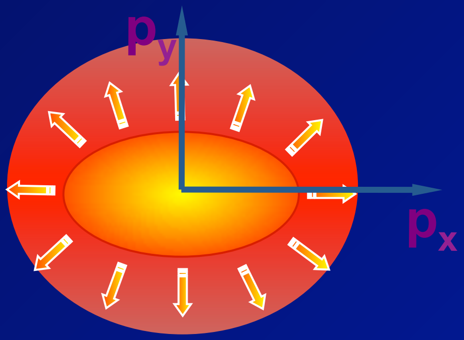
- ✧ We know how to fix locally $\eta/s(T)$
- ✧ We have checked the Chapmann-Enskog:
  - CE good already at I° order ≈ 5% (≈ 3% at II° order)
  - RTA even with σ_{tr} severely underestimates η

Part II

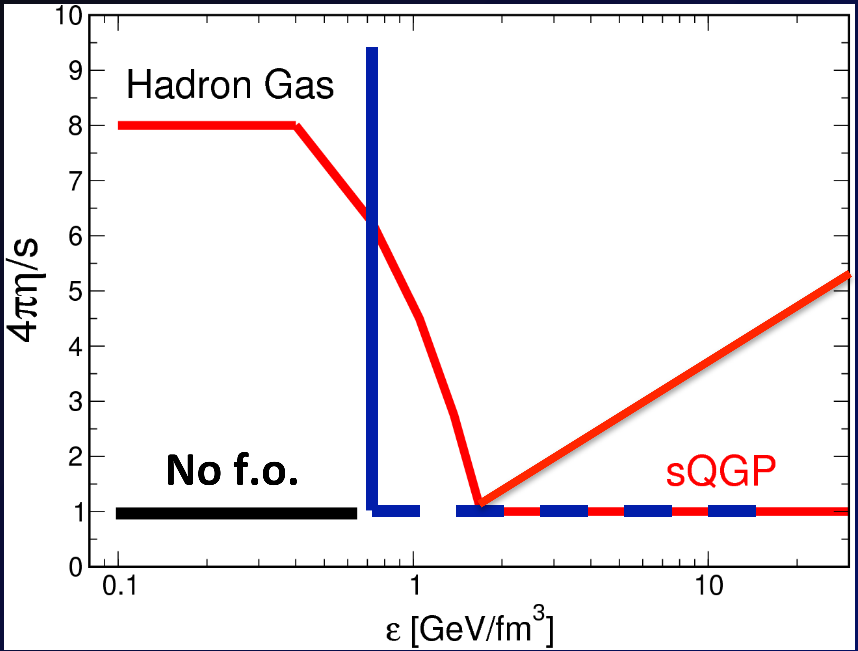
Let's apply the transport code to A+A Collisions....







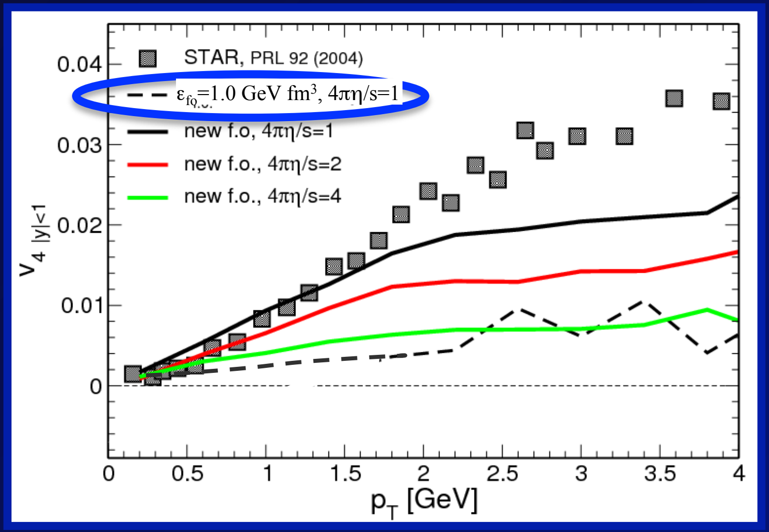
Terminology about freeze-out



a) collisions switched off for $\varepsilon<\varepsilon_c=0.7$ GeV/fm^3

b) η/s increases in the cross-over region, faking the smooth transition in the cross-over region: small s -> natural f.o.

$$\sigma_{tr} = \frac{1}{15}\frac{\bar{p}}{n}\frac{1}{\eta/s}$$

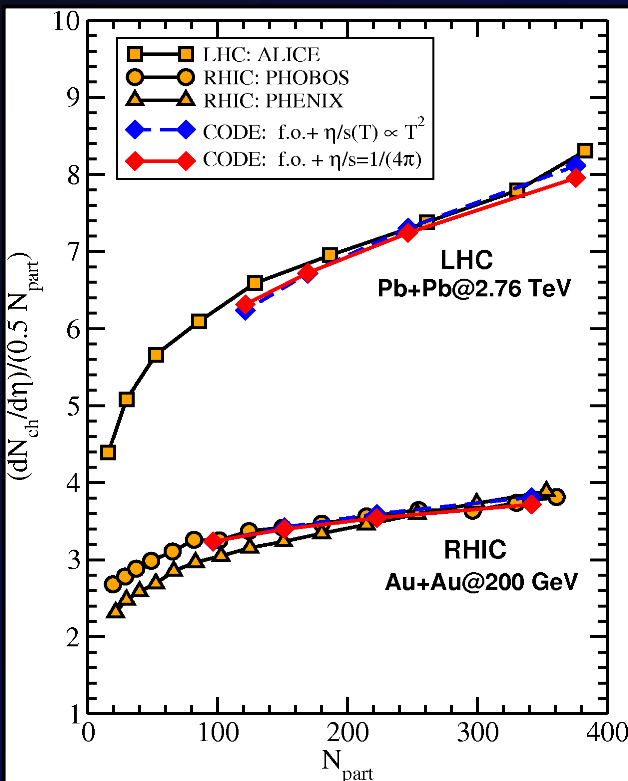


A sudden f.o. tend to kill v_4!

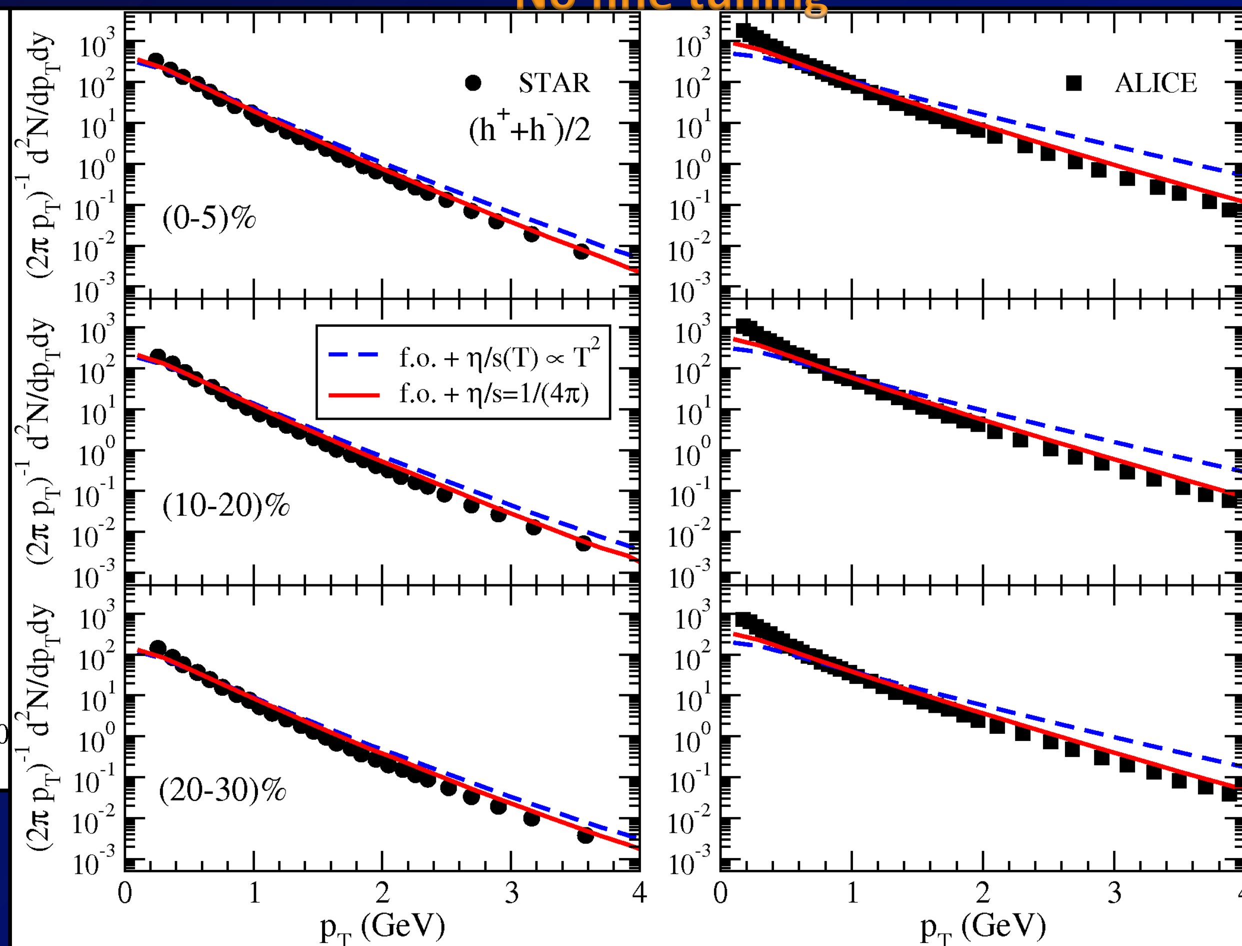
V. Greco et al., PPNP(2009)

Multiplicity & Spectra

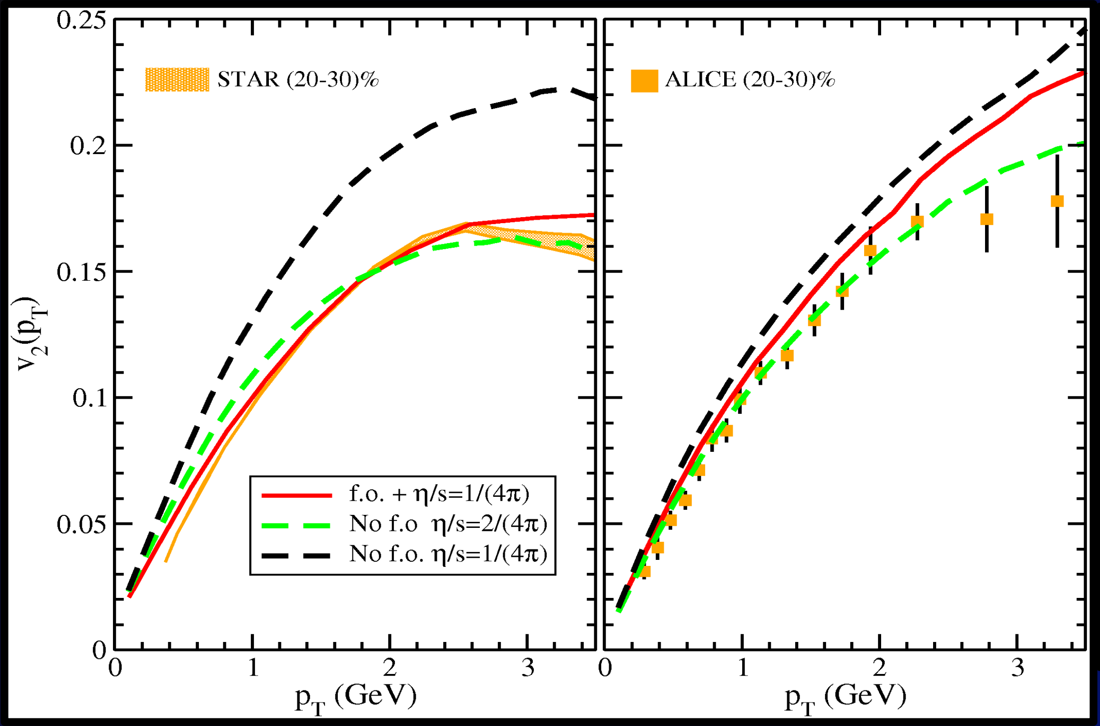
- ✧ r-space: standard Glauber condition
- ✧ p-space: Boltzmann-Juttner T_{max}=2(3) T_c [p_T<2 GeV]+ minijet [p_T>2GeV]

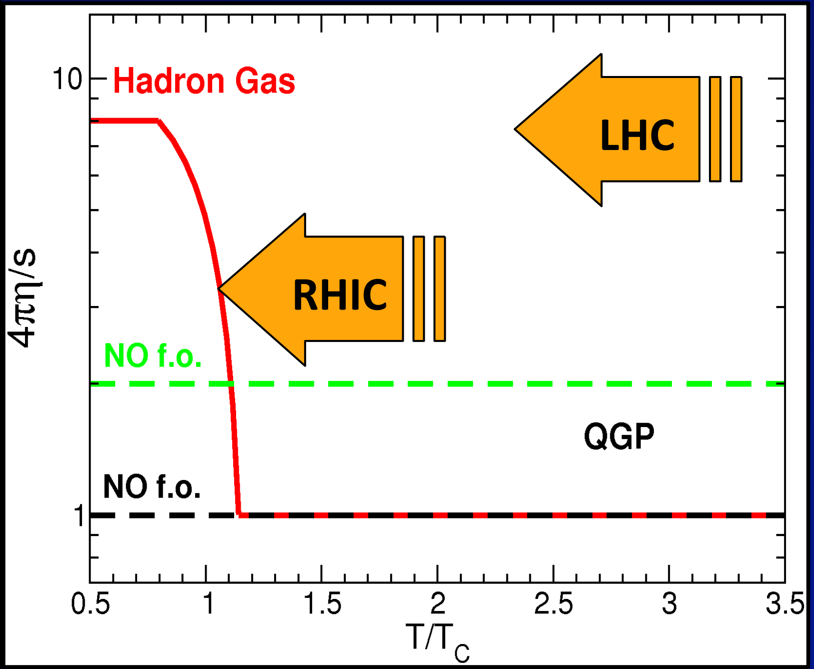


No fine tuning



First application: f.o. at RHIC & LHC

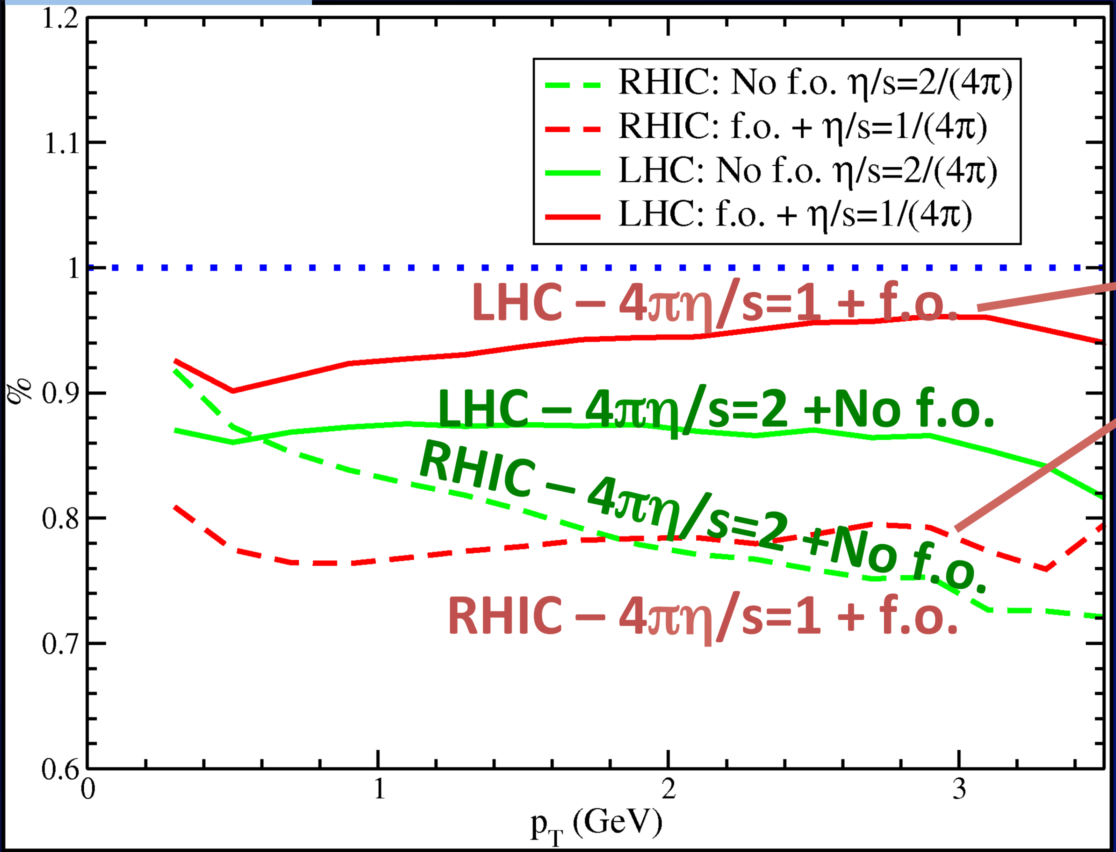


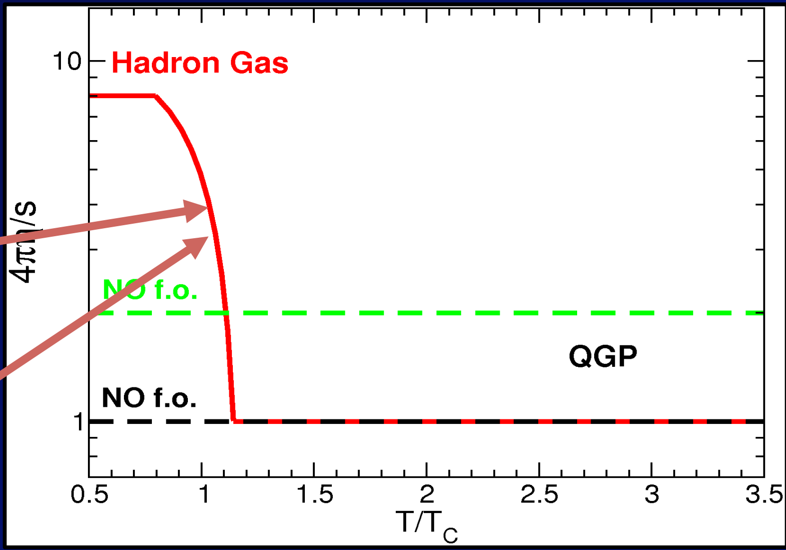


- RHIC: η/s increase in the cross-over region equivalent to double η/s in the QGP
- LHC: almost insensitivity to cross-over (≈ 5%) : v_2 from pure QGP,
  but at LHC less sensitivity to T-dependence of η/s? *see later*
- Without $\eta/s(T)$ increase T≤Tc we would have v_2(LHC)<v_2(RHIC)

Effect of η/s of the hadronic phase at LHC

$\frac{v_2(*)}{v_2(4\pi\eta/s=1+no\ f.o.)}$ Suppression of v_2 respect the ideal **$4\pi\eta/s$=**1 at all T





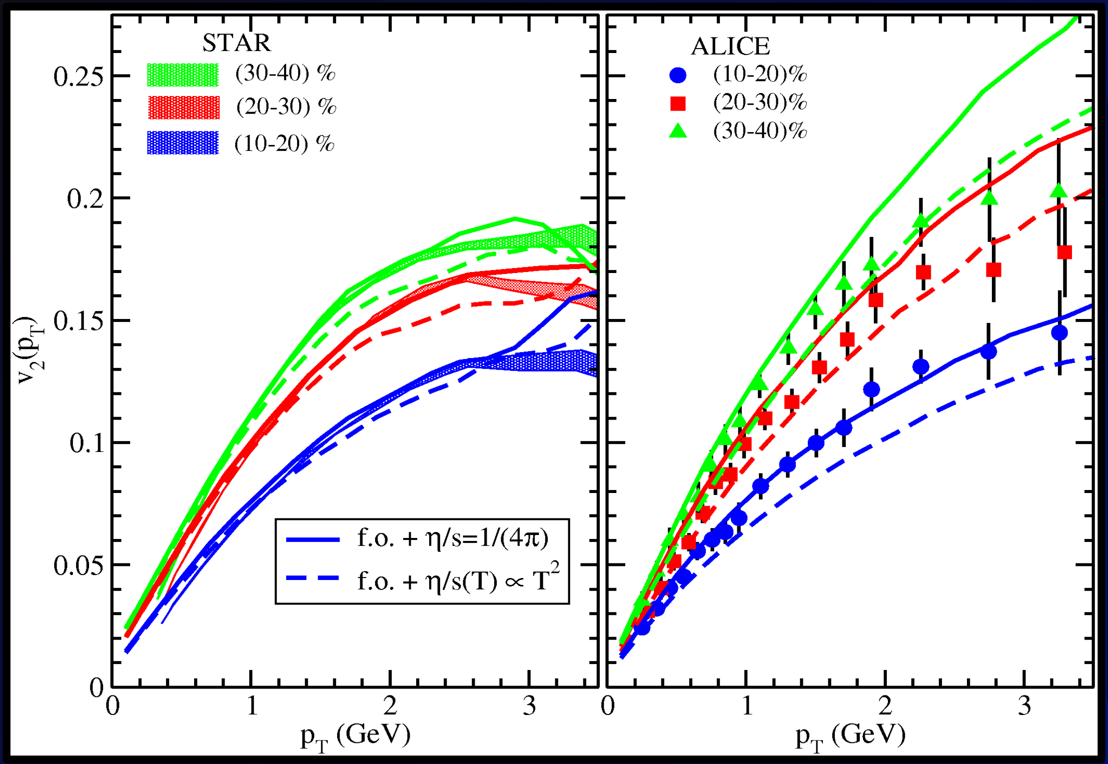
At LHC the contamination of cross-over & hadronic phase becomes negligible

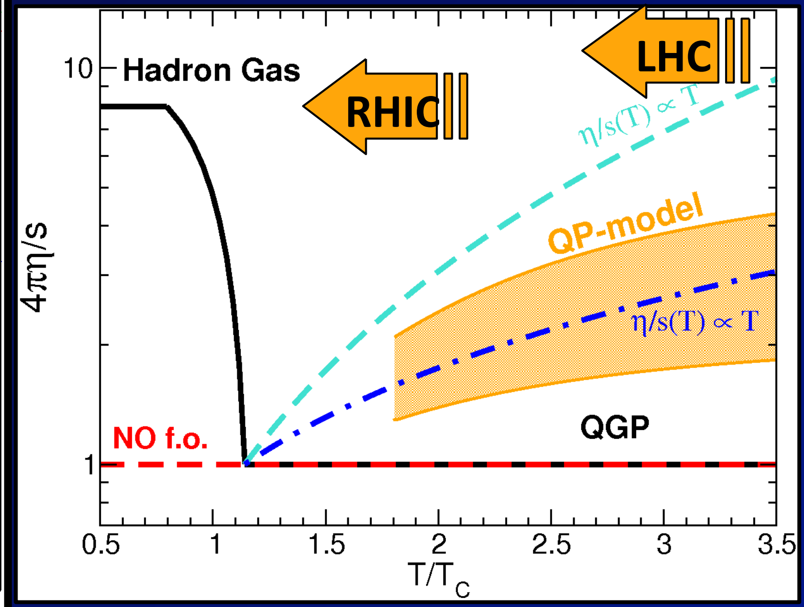
Longer lifetime of QGP ($t \approx$ 10-12 fm/c)

-> v_2 completely developed in the QGP phase

S. Plumari and VG, EPIC@LHC, AIP1422(2012)- arXiV:1110.4138 [hep-ph]

Sensitivity to Temperature dependent η/s(T)





Plumari et al., arxiv:1103.5611[hep-ph].

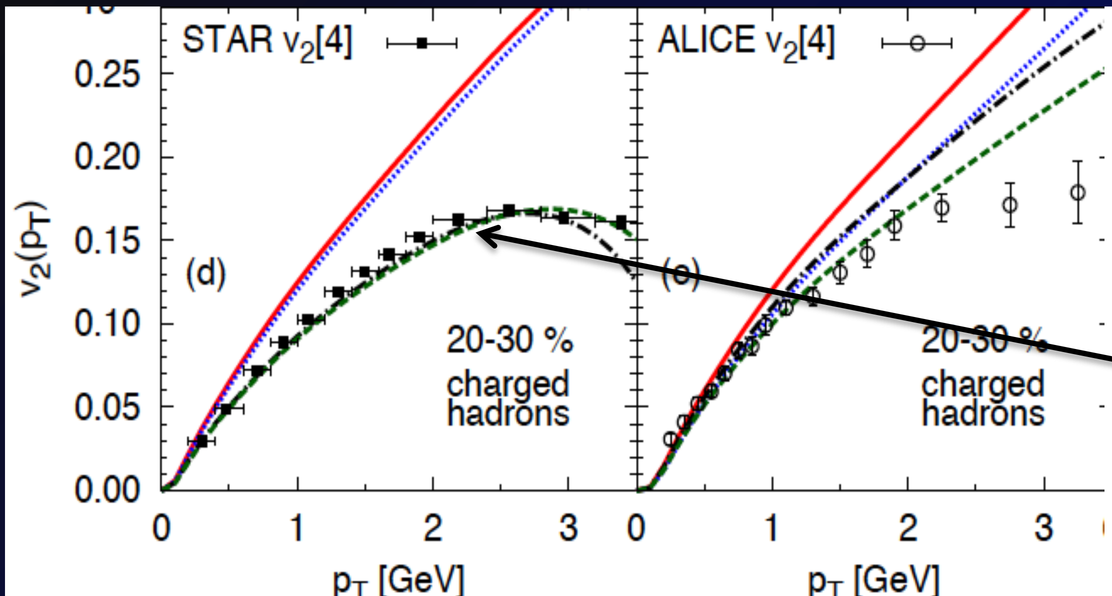
- The v_2 is nearly insensitive to the value of $\eta/s(T)$ in the QGP phase at RHIC, some sensitivity at LHC
- [[$\eta/s \sim T^2$ cannot account for the v_2 decrease for $p_T > 2.5$ GeV.]]

- Transport prediction for $v_2(p_T)$ is RHIC ~ LHC up to p_T 2-3 GeV

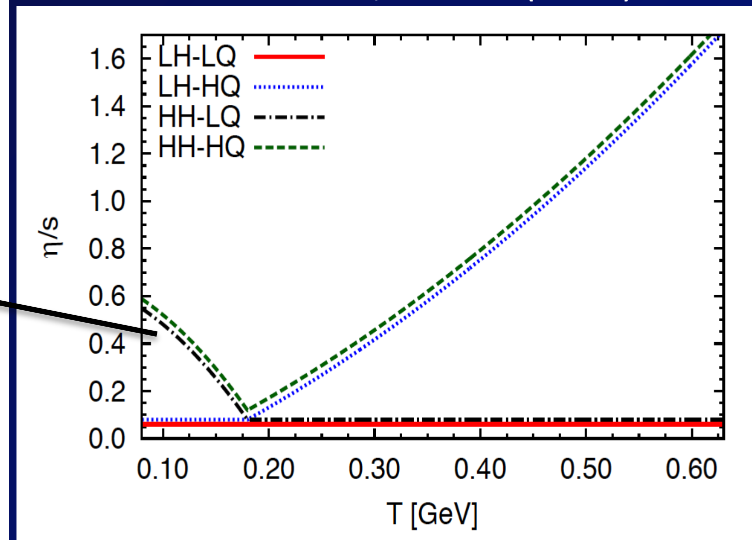
  [do not believe what is written in ALICE - PRL105(2010)]

  important to have a f.o. and a correct η/s as coming from Chapmann-Enskog

Sensitivity η /s(T) in Hydro – Niemi et al.



Niemi et al., PRL106(2011)



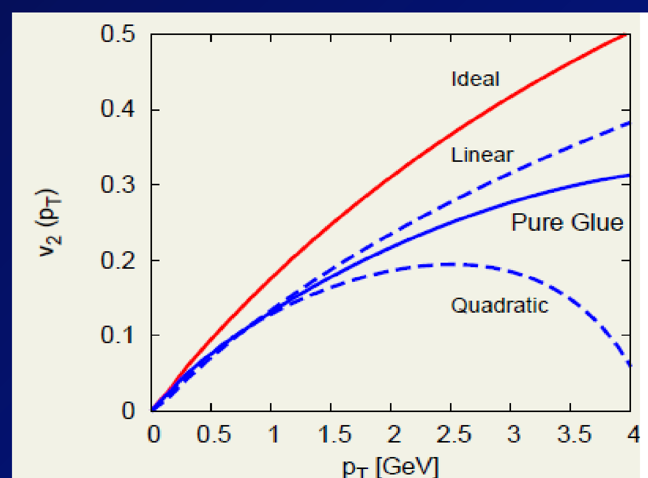
$$T^{\mu\nu}_{eq} + \delta T^{\mu\nu} \Leftarrow f_{eq} + \delta f$$

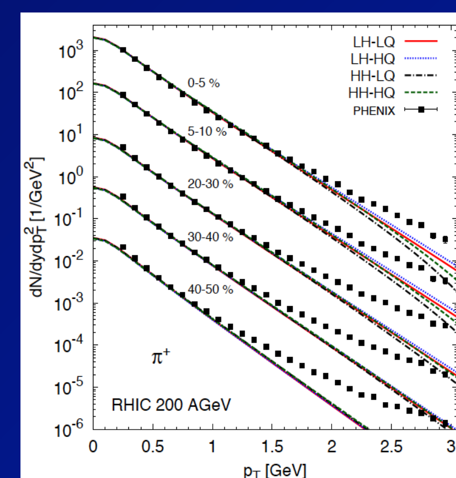
There is no one to one correspondence!

An Asantz (Grad)

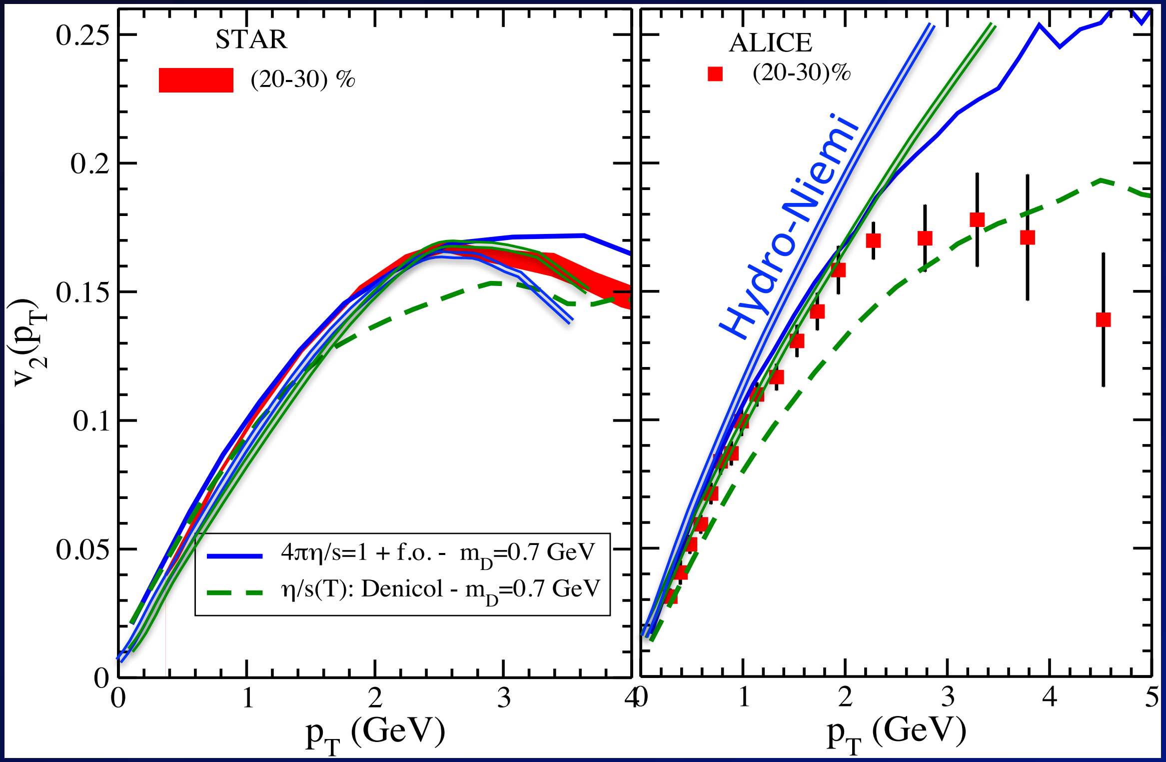
$$\delta f = \frac{\pi^{\mu\nu}}{\varepsilon + P}\frac{p_\mu p_\nu}{T^2} f_{eq} \approx \frac{\eta}{3s}\frac{p_T^2}{\tau T^2} f_{eq}$$

- this implies RTA and not CE
- Hydro make sense up to p_T~3 GeV !? $\delta f/f \approx 5$





Sensitivity in transport using same η /s(T)

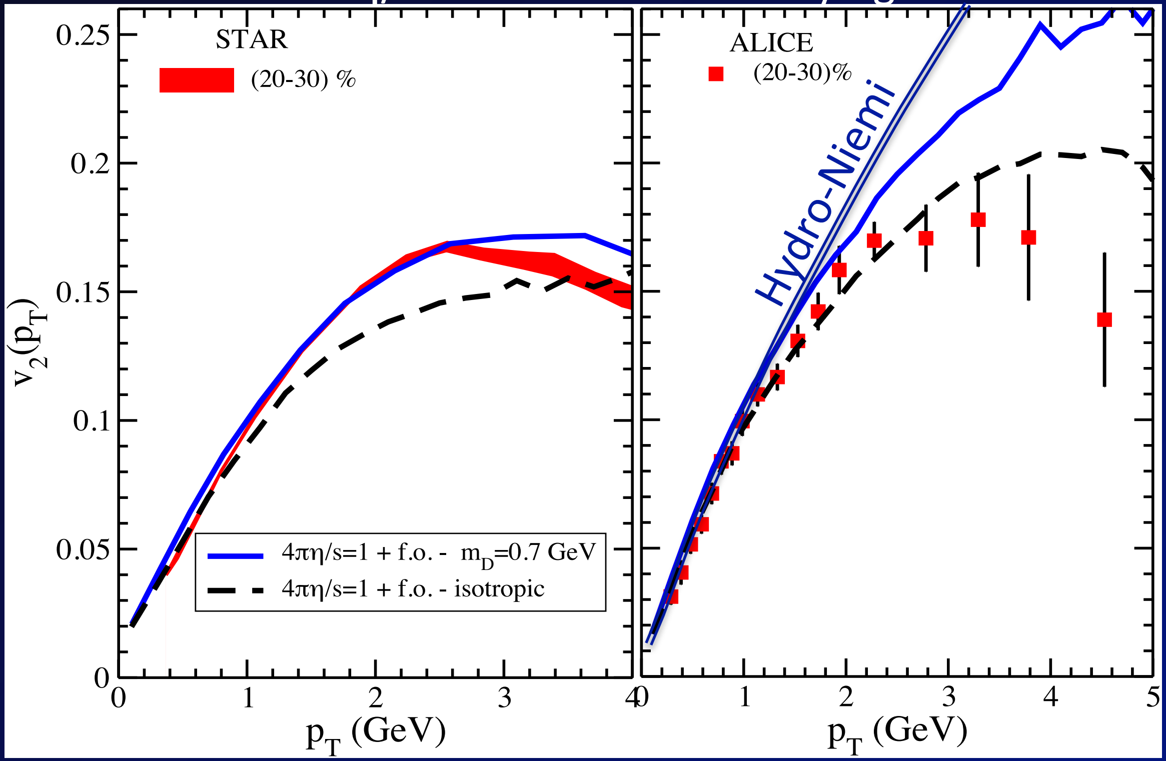


- ✓ Larger sensitivity on η/s (T) at LHC
- ✓ Effect larger respect to viscous hydro, but this depends also on δf
- ✓ An estimate of η/s is more meaningful with transport approach

Does it depends also on some detail of the cross section?

Relevance of microscopic scale: $\sigma(\theta)$

Same η/s but different underlying σ



Out of pure hydro language!

- Microscopic details of the cross section matter at p_T> 2 GeV
- Larger screening mass at LHC!?
- v_n at p_T> 2 GeV can provide more information on the QGP micro details

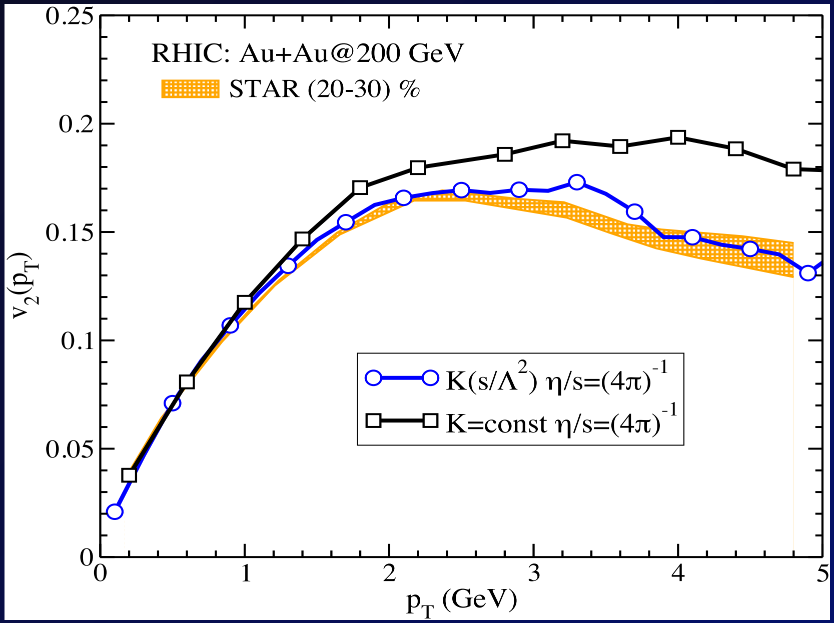
From low to high p_T

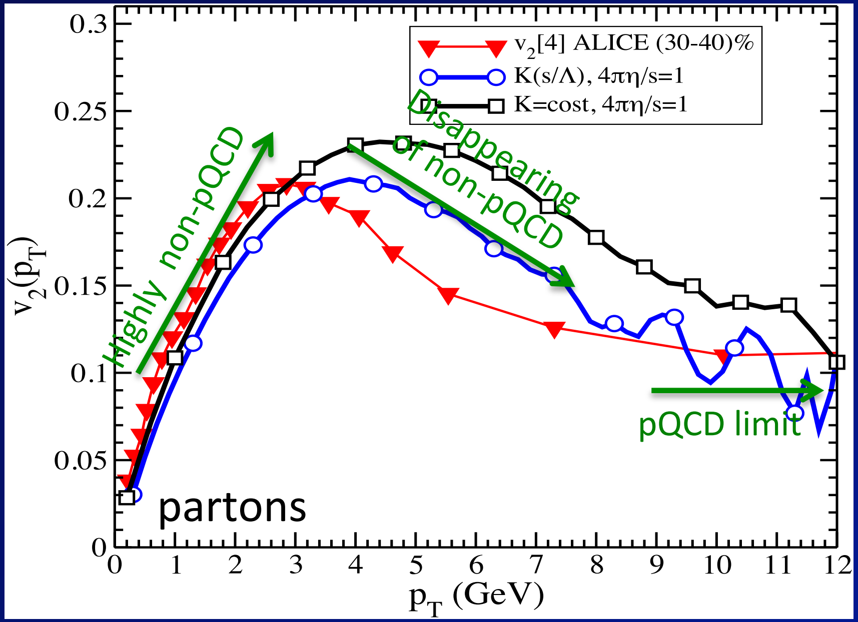
$$\sigma^*(s) = K\sigma_{pQCD}(s) >> \sigma_{pQCD}(s)$$

Takes into account the non-perturbative physics -> $4\pi\eta/s=1$, but at all p_T !

Including the obvious: $\sigma^*(s) = K(s/\Lambda^2)\sigma_{pQCD}(s)$ $K(s) = 1 + \gamma e^{-s/\Lambda^2}$

γ not a parameter it is fixed by η/s)





Only approach that describe $v_2(p_T)$ up to 12 GeV in a unified framework!

- Allow to extend the agreement to larger p_T, but does not affect the low p_T

No Fine tuning! Done employing the relaxation time approximation for K (γ)!

S. Plumari and VG, EPIC@LHC, AIP1422(2012)- arXiV:1110.4138 [hep-ph]

Part III

<u>Quasiparticle model-> finite mass excitations:</u>

- Study quark-gluon chemical equilibrium
- Imply a transport evolution with EoS-lQCD

$$\left\{ p^{*\mu}\partial_\mu + m^*(x)\partial^\mu m^*(x)\,\partial_\mu^{p^*} \right\} f(x,p^*) = C[f]$$

Field Interaction -> ε≠3P

$$\frac{\partial B}{\partial m_i} + d_i \int \frac{d^3p}{(2\pi)^3} \frac{m_i(x)}{E_i(x)} f(x,p) = 0$$

Using a simple QP-model

U.Heinz and P. Levai, PRC (1998)…. *Kampfer TALK*

$$P(T) = \sum_{i=g,q,\bar{q}} \frac{D_i}{(2\pi)^3} \int_0^\infty d^3k \frac{k^2}{3\omega_i(k)} f_i(k) - B(T)$$

$$\varepsilon(T) = \sum_{i=g,q,\bar{q}} \frac{d_i}{(2\pi)^3} \int_0^\infty d^3k\ \omega_i(k)\ f_i(k) + B(T) + \widetilde{W}_B(T)$$

Plumari, Alberico, Greco, Ratti, PRD84 (2011)

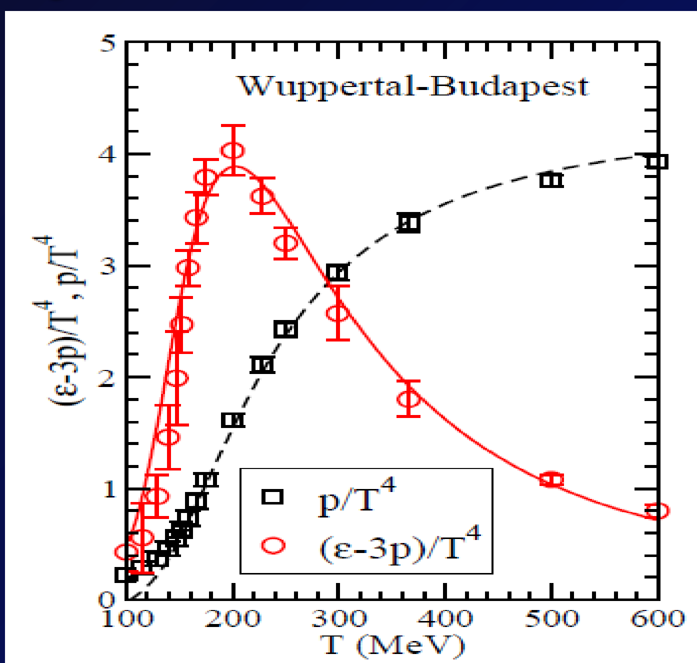
$W_B=0$ guarantees Thermodynamicaly consistency

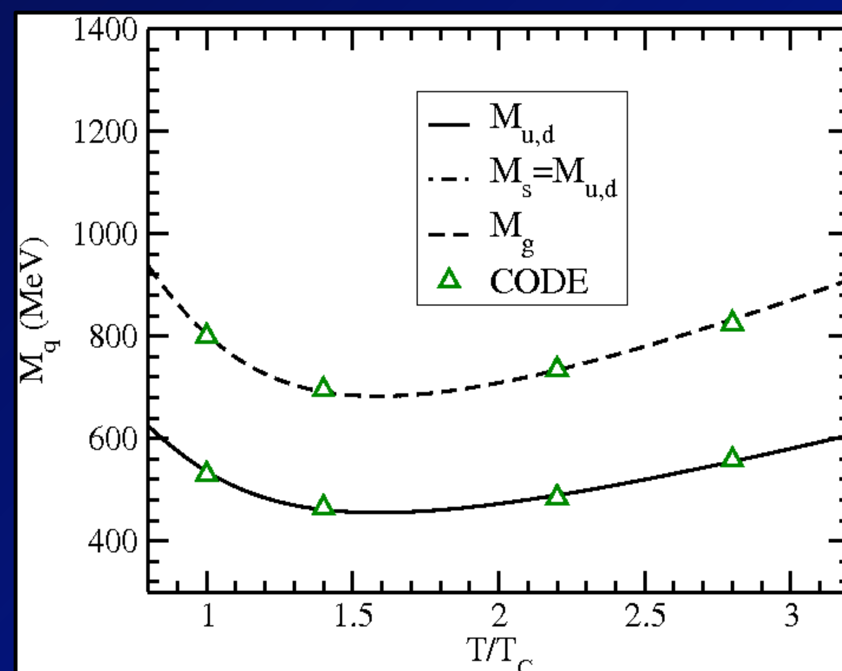
$$B(T) = B(T^*) - \sum_{i=g,q,\bar{q}} \frac{D_i}{(2\pi)^3} \int_{T^*}^{T} dT' M_i(T') \frac{dM_i(T')}{dT'} \int_0^\infty \frac{d^3k}{\omega_i} f_i(k)$$

$$\omega^2 = k^2 + M^2(T)$$

$M_g(T)=3/2\ g(T)T$

$M_g(T)$ from a fit to ε from lQCD -> good reproduction of P, e-3P, c_s



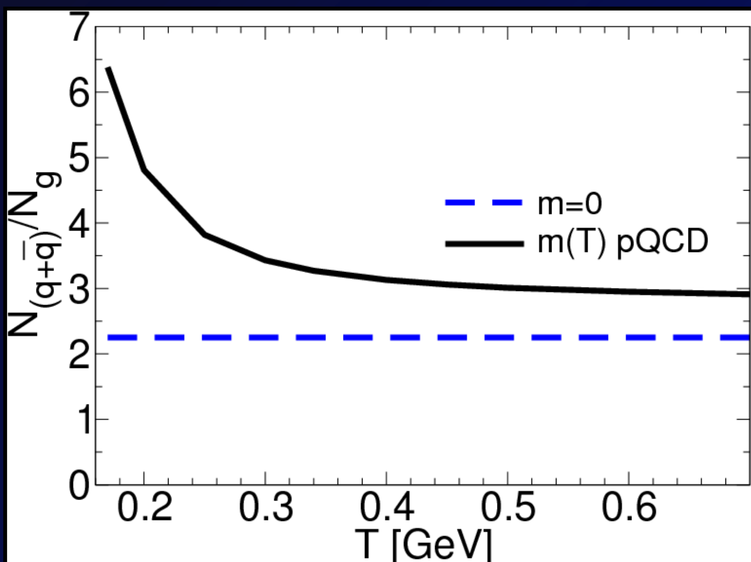


QP-model: implications for chemical composition

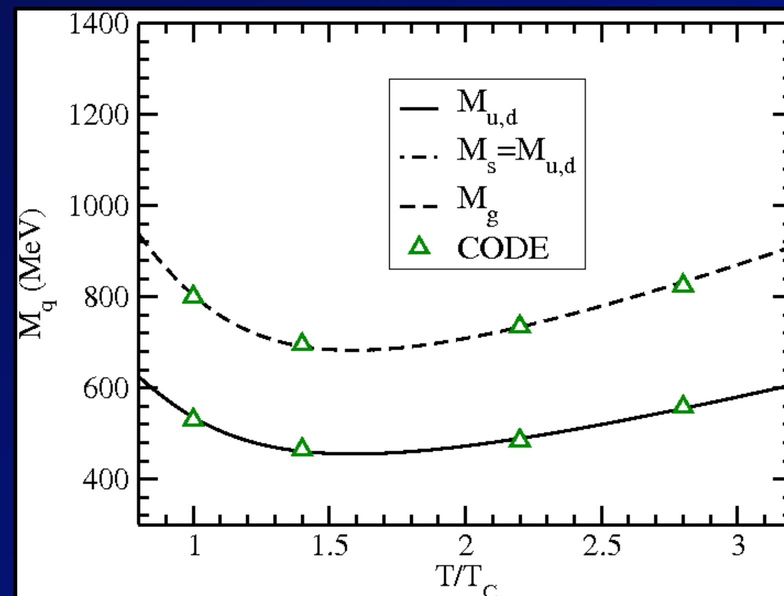
Passed several numerical test on the box. We reproduce the lQCD EoS .

$$
\begin{cases}
p^{\mu}\partial_{\mu}f(x,p)+m_i(x)\partial_{\mu}m_i(x)\partial_p^{\mu}f(x,p)=C_{22}[f] \\
\dfrac{\partial B}{\partial m_i}+d_i\displaystyle\int\frac{d^3p}{(2\pi)^3}\frac{m_i(x)}{E_i(x)}f(x,p)=0
\end{cases}
$$

, i=g,u,d,s



$$\frac{N_{q+\bar{q}}}{N_g}=\frac{d_{q+\bar{q}}}{d_g}\frac{m_q^2(T)\,K_2(m_q/T)}{m_g^2(T)\,K_2(m_g/T)}$$



Polyakov loop should modify it,
see M. Ruggieri et al., PRD(2012) arxiv:1204.5995
for Yang-Mills case

Using the QP-model: q<-> g conversion

Inelastic cross section with massive parton gg->qq

$$|M_t|^2 = \alpha_s^2 \pi^2 \frac{8}{3} \frac{(m_q^2 + m_g^2 - t)(m_q^2 + m_g^2 - u) - 2m_q^2(m_q^2 + t) - m_g^2 s - 4m_q^2 m_g^2}{(t - \mu_q^2)^2}$$

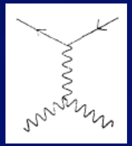
$$|M_u|^2 = \alpha_s^2 \pi^2 \frac{8}{3} \frac{(m_q^2 + m_g^2 - t)(m_q^2 + m_g^2 - u) - 2m_q^2(m_q^2 + u) - m_g^2 s - 4m_q^2 m_g^2}{(u - \mu_q^2)^2}$$

$$|M_s|^2 = \alpha_s^2 \pi^2 12 \frac{(m_q^2 + m_g^2 - t)(m_q^2 + m_g^2 - u) - 3m_g^2 s + 2m_q^2 m_g^2}{(s - \mu_g^2)^2}$$

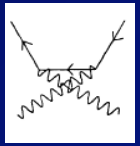
See also: Combridge, NPB151 (1979); Biro-Levai-Muller, PRD42(1990)

→

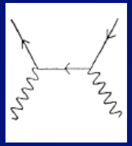
$$|M_t|^2 = \alpha_s^2 \pi^2 \frac{8}{3} \frac{t}{u}$$

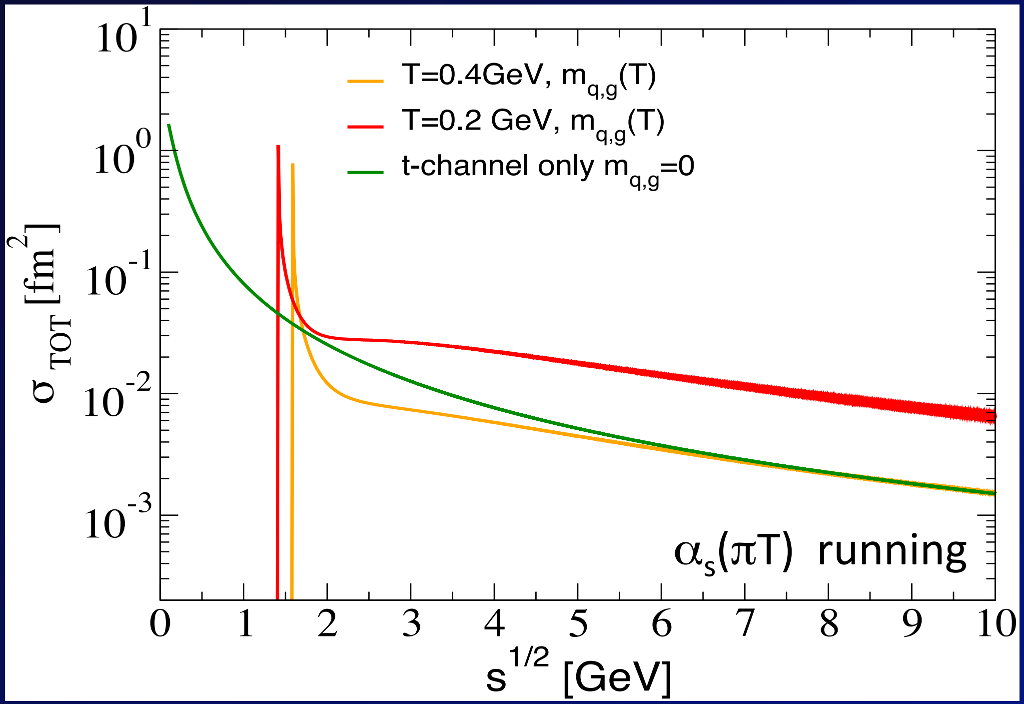


$$|M_u|^2 = \alpha_s^2 \pi^2 \frac{8}{3} \frac{u}{t}$$

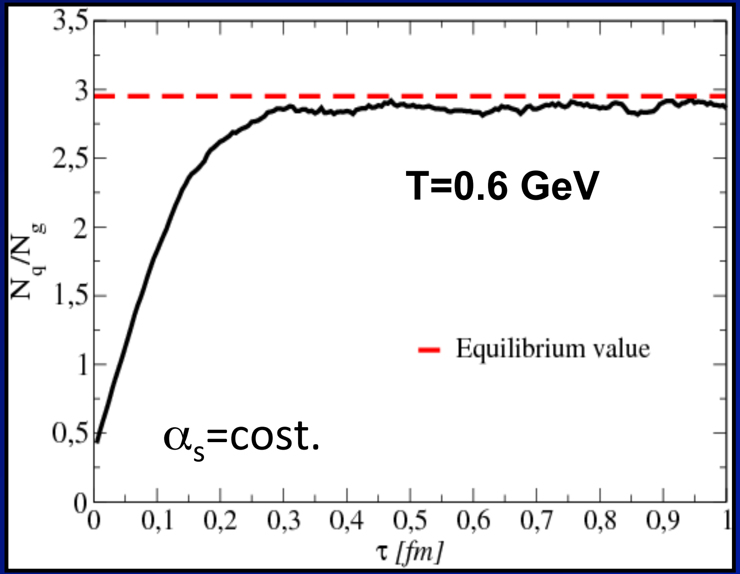


$$|M_s|^2 = \alpha_s^2 \pi^2 12 \frac{u \cdot t}{s^2}$$



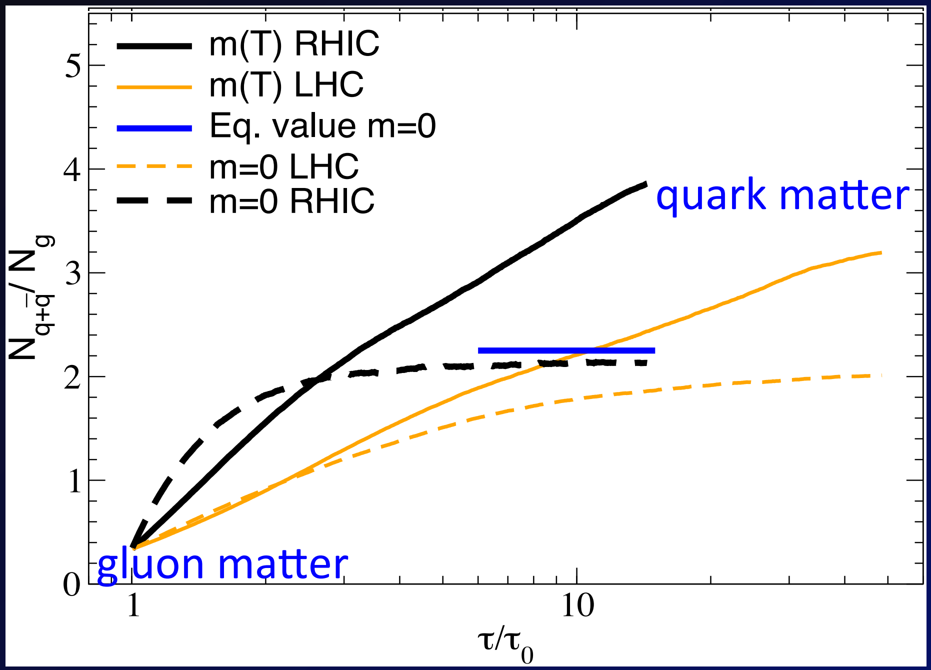


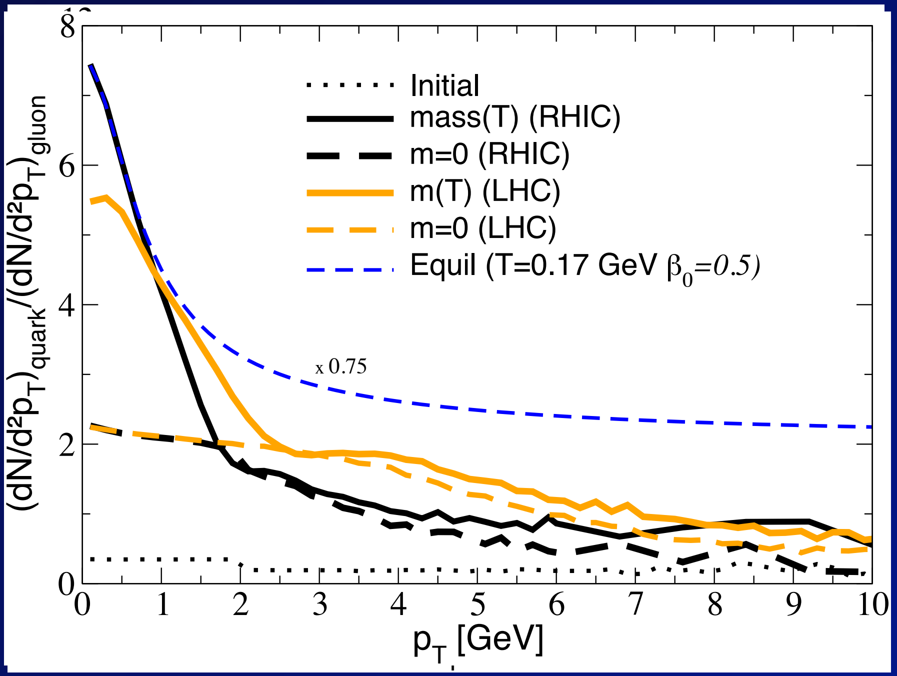
Evolution in a Box



QP-model:QGP composition in HIC

F. Scardina et al., arXiv:1202.2262 [nucl-th].





- Quark dominance clode to T_c : ~ 80% of total partons composed of q + anti-q even starting from a 80% gluon matter
- LHC matter less “chemically”equilibrated
- Massive quarks close to T_c -> coalescence
- Should one rivisit jet quenching, quarkonia suppression…

Summary

Calculating η/s :

- Chapmann-Enskog I°order agree with Green-Kubo
- Relax. Time Approx can severely underestimate η/s

Developing a Boltzmann-Vlasov partonic transport:

➢ Agreement with data for $4\pi\eta/s \approx 1$

similarly to hydro (but wider p_T-range):

- $p_T < 2$ GeV same $v_2(p_T)$ at RHIC and LHC
- $p_T > 2$ GeV microscopic $\sigma(\theta)$ matters
- v_n more sensitive to both $\eta/s(T)$ and $\sigma(\theta)$

➢ **Quasi-particle in transport theory:**

- **Massive quark plasma close to T_c (less equil. at LHC)**

Outlook

- Include initial state fluctuations - >v_2, v_3, v_4,v_5
- Include hadronization
- Study Mean Field in Transport vs EoS in hydro

Chapmann-Enskog vs Green Kubo:massive case

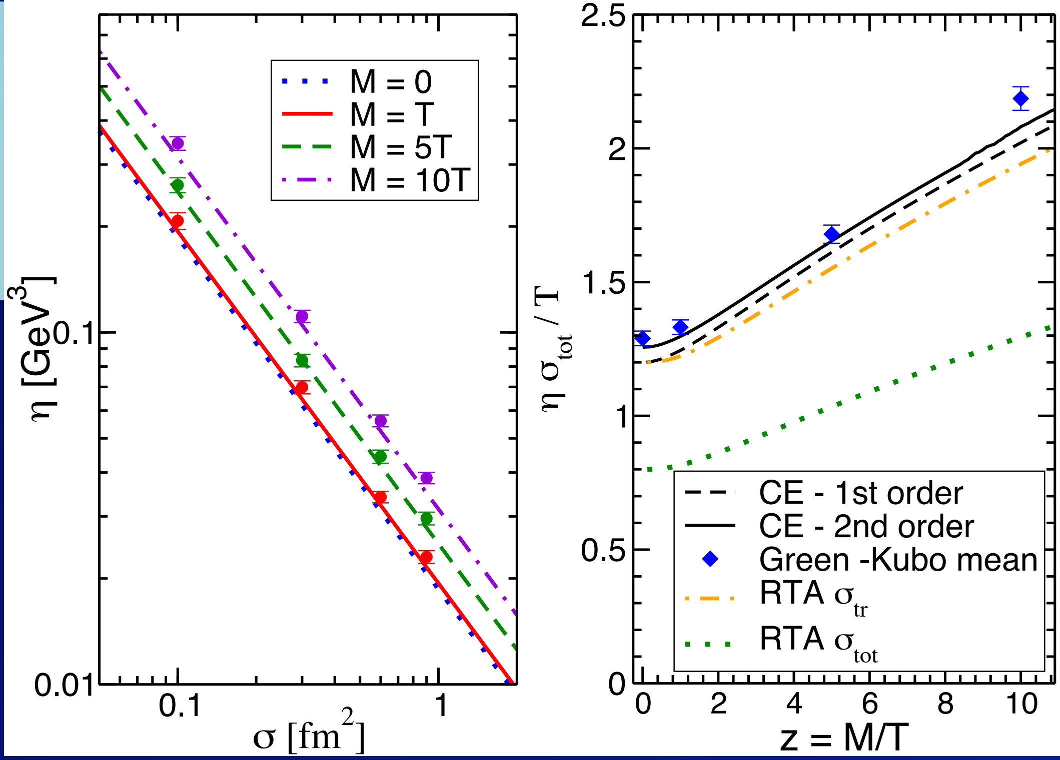
Massive case is relevant in quasiparticle models where $M_{q,g}(T)=g(T)T$
Hence we need it to extend the approach to a Boltzmann-Vlasov transport

Again good agreement with CE 1st order for $\sigma(\theta)$=cost.

Isostropic σ – massive particles

$$[\eta]_{1st}^{CE} = f(z)\frac{T}{\sigma_{tot}} \qquad z=M/T$$

$$f(z)=\frac{15}{16}\frac{z^4 K_3^2(z)}{(15z^2+2)K_2(2z)+(3z^3+49z)K_3(2z)}$$

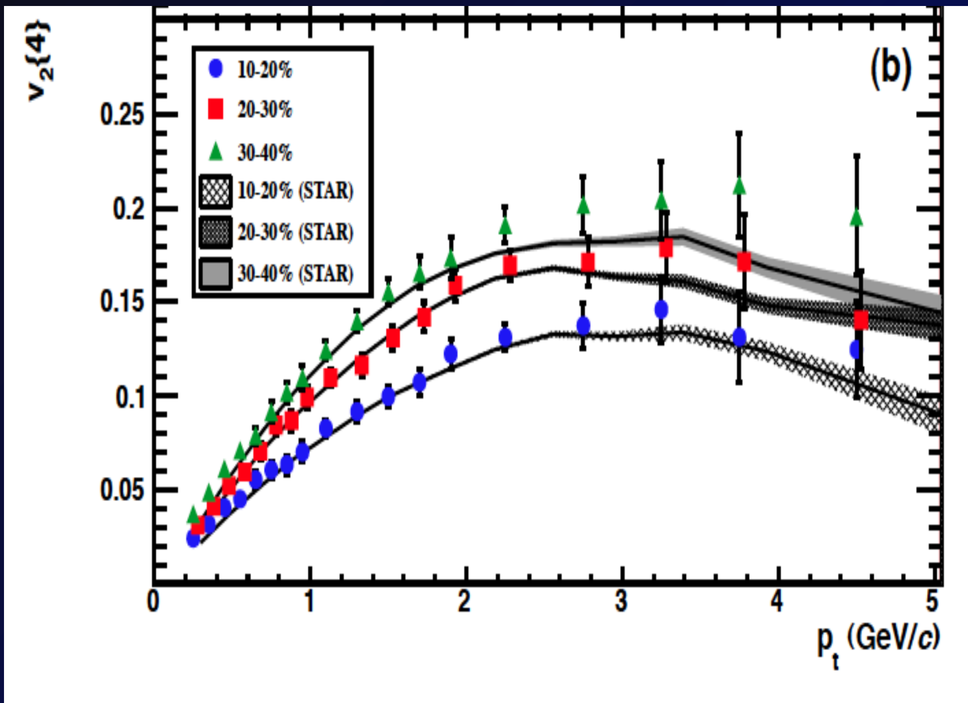


Still missing Chapmann-Enskog for massive + anisotropic cross section

S. Plumari et al., arXiV:1209.0589

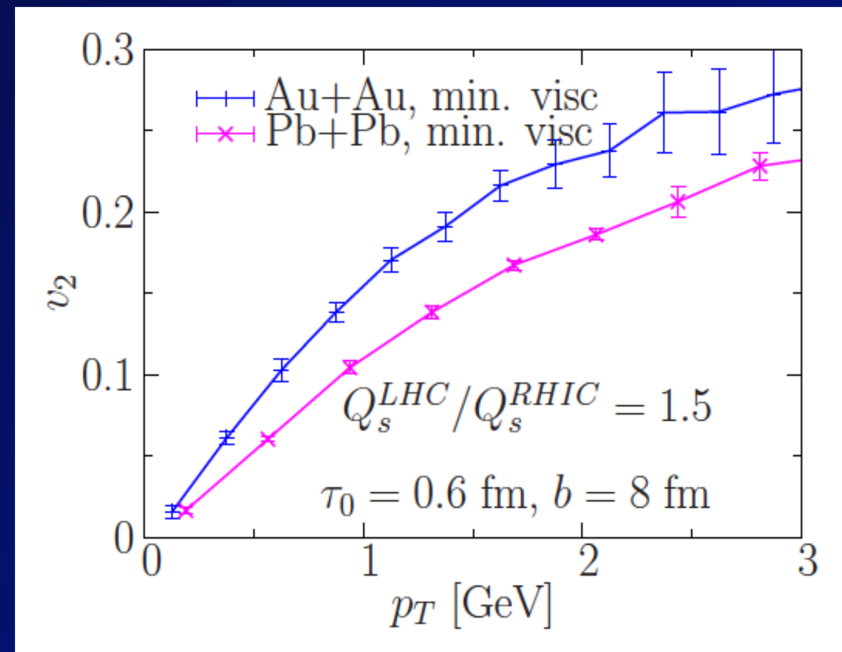
First Result on v_2: RHIC vs LHC

Going to LHC?



ALICE, PRL105 (2010)

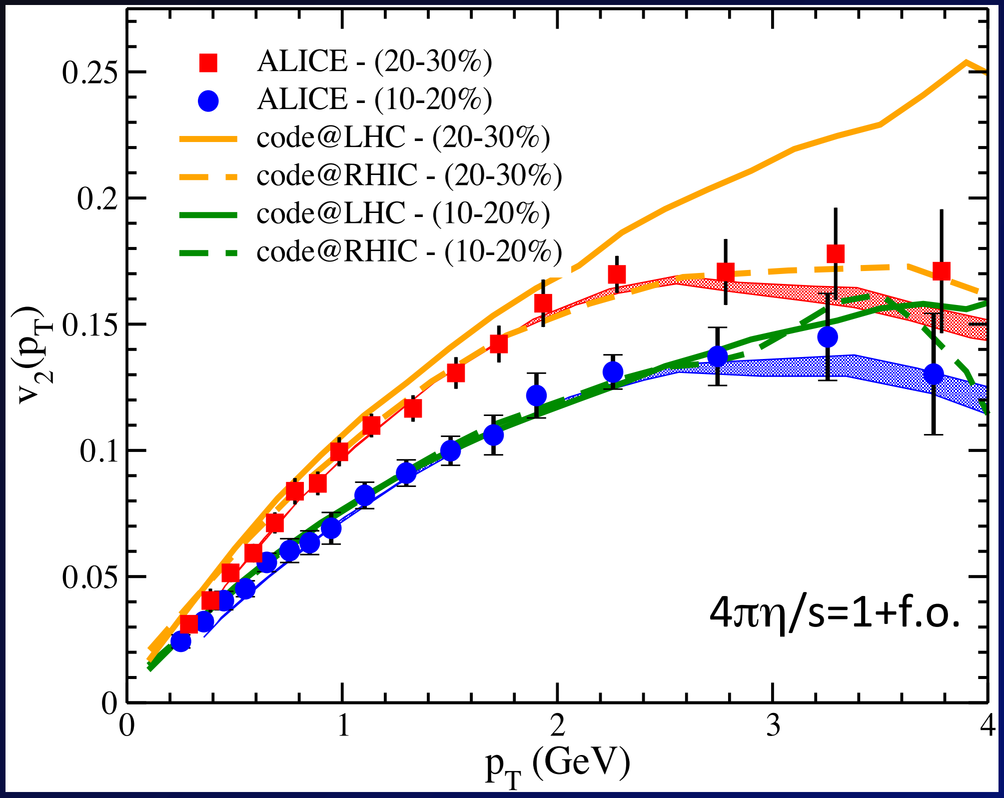
Cascade prediction



D. Molnar, LHC last call for prediction, JPG35

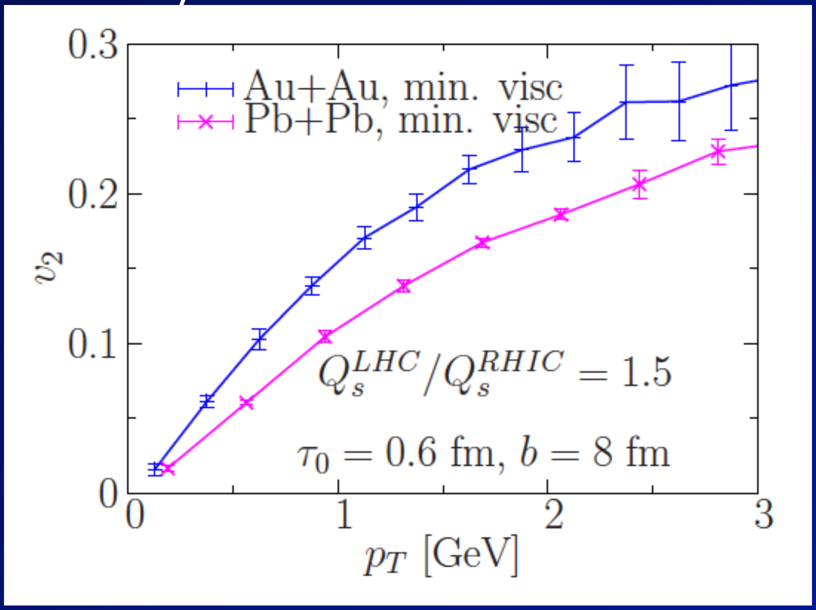
most pronounced for heavy particles. Models based on a parton cascade [19], including models that take into account quark recombination for particle production [20], predict a stronger decrease of the elliptic flow as function of transverse momentum compared to RHIC energies. Phenomenological extrapolations [21] and models

First application: v2 at RHIC & LHC



<u>At least 3 differences respect to MPC:</u>

- Initial conditions *(not only minijets)*
- way to fix η/s *(not on average+ with CE)*
- f.o. dynamics included



- ➢ Good agreement for 4πη/s=1 up to $p_T \approx 3$ GeV *(wider range than hydro)*
- ➢ <u>Same $v_2(p_T)$ at RHIC and LHC like in exp-data, up to p_T<2-3 GeV,</u> but for semi-central at LHC always over-predict larger v_2