# Fluctuations of conserved charges and freeze-out conditions in heavy ion collisions

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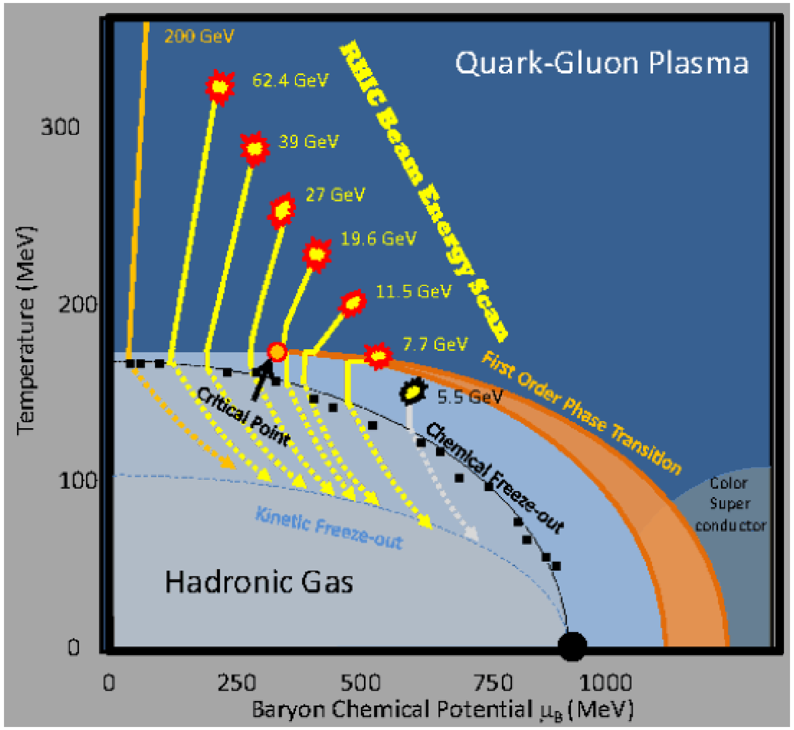
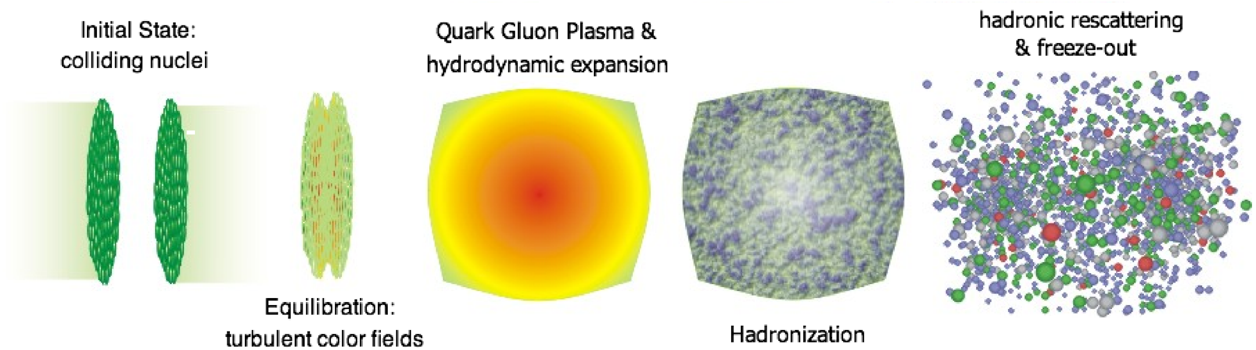
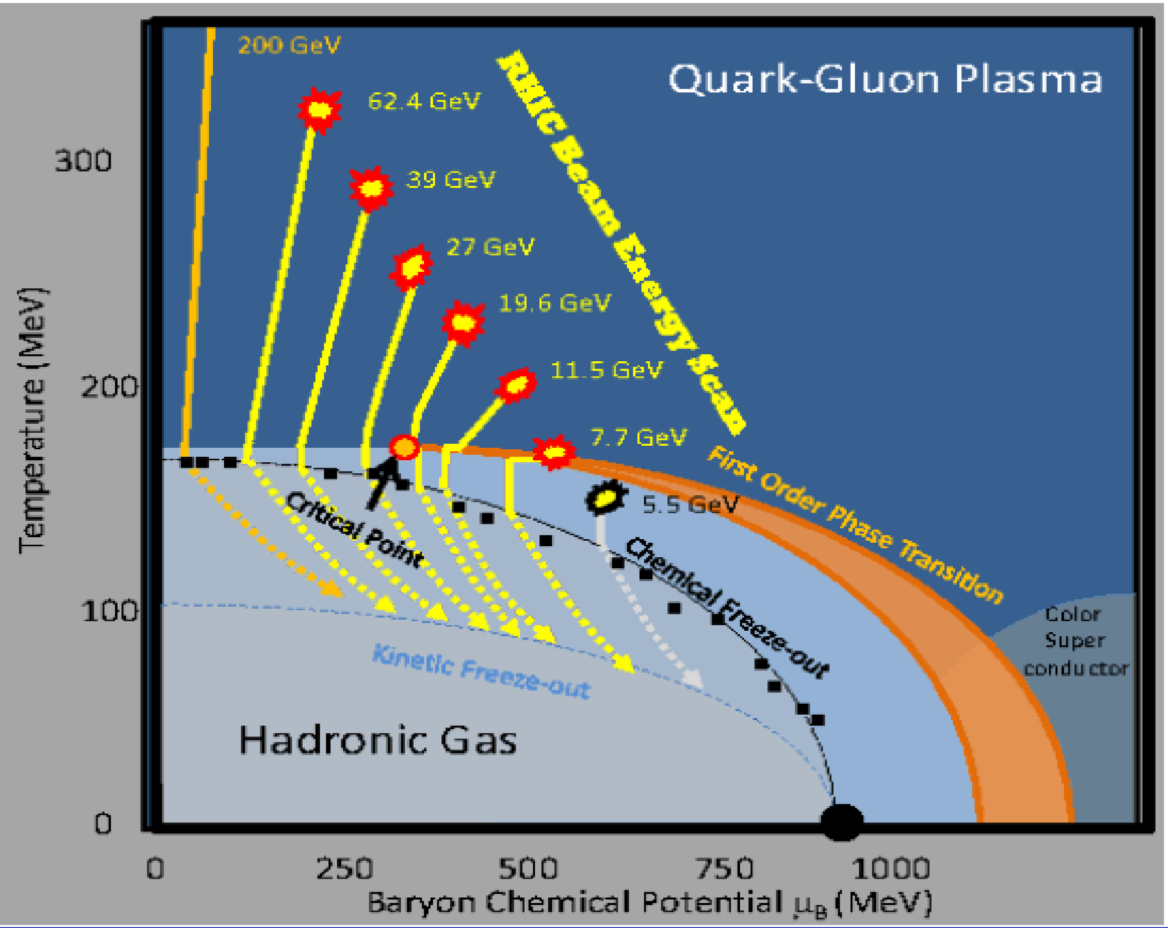



## OUTLINE

- The crossover transition close to $\mu_B = 0$
- Crossover transition and freeze-out
- conserved charge fluctuations and freeze-out

# Phase diagram and freeze-out in Heavy Ion Collisions

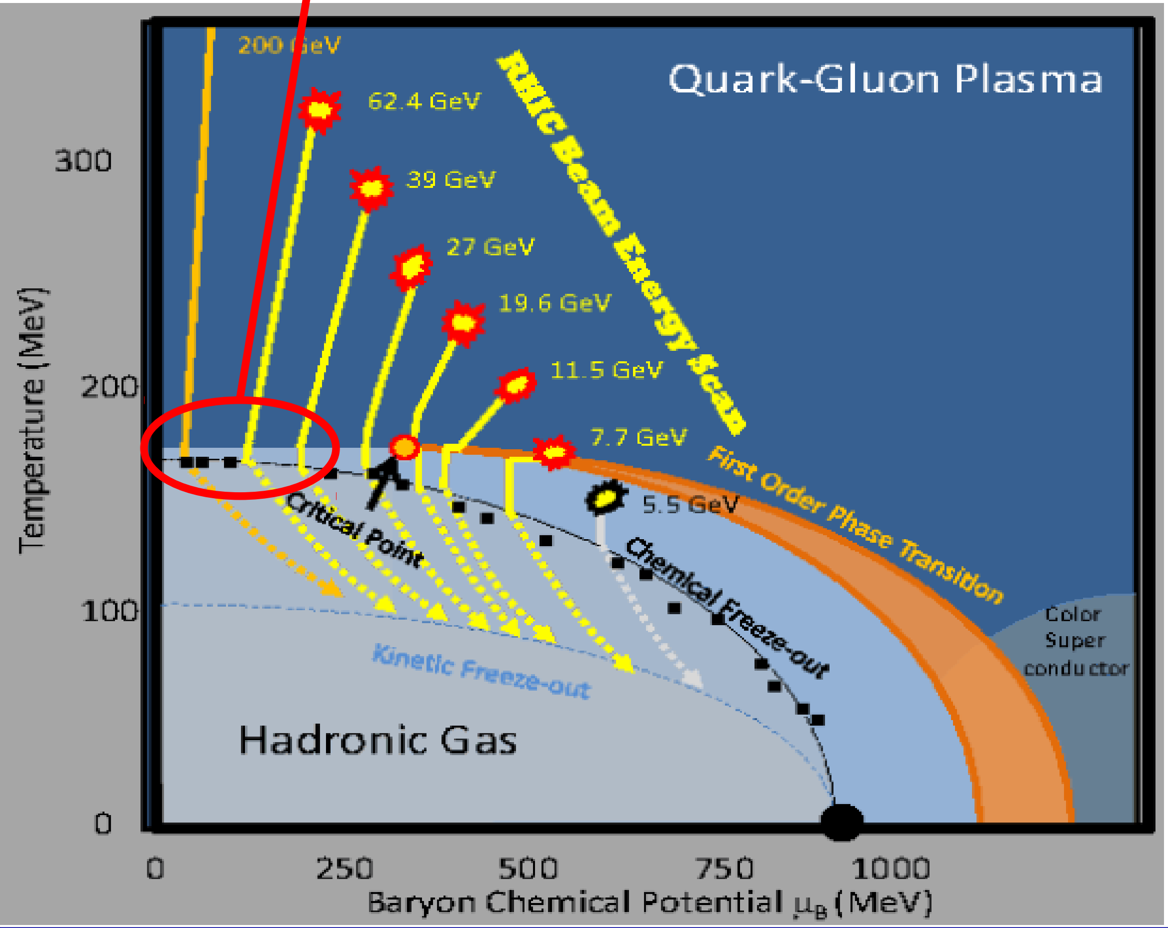



- lattice QCD predictions on the existence and/or location of a critical point are still ambiguous
- cumulants of conserved charges have been suggested to be sensitive probes for critical behavior
- the beam energy scan at RHIC provides first quantitative results for net proton and net electric charge fluctuations and observes deviations from hadron resonance gas model calculations

# Phase diagram and freeze-out in Heavy Ion Collisions

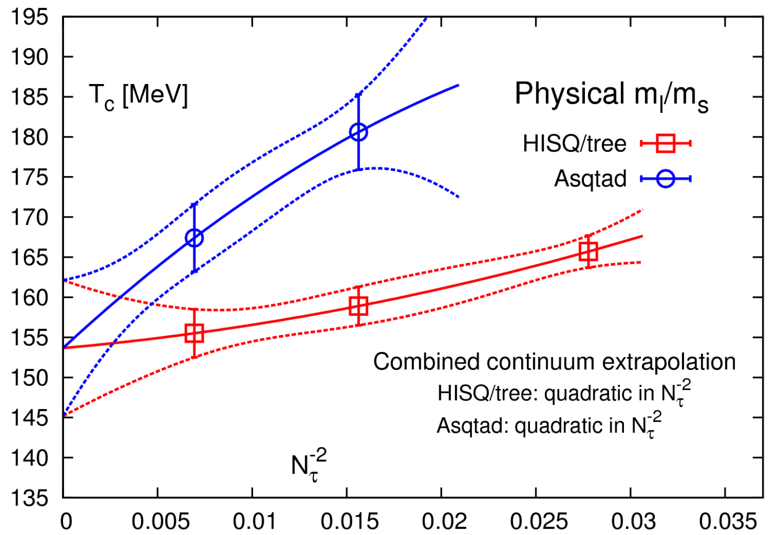
rather than hunting for the critical point right away, let's first establish basic features of cumulants of conserved charges at moderate values of the baryon chemical potential




- can cumulants be described by equilibrium thermodynamics with a unique set of temperature (T) and chemical potentials $\mu_B, \mu_S, \mu_Q$
- do cumulants characterize thermodynamics on the chemical freeze-out curve?
- are thermal (freeze-out) parameter obtained by comparing experimental data with HRG model calculations reproduced by QCD?

# Crossover temperature at and close to $\mu_B = 0$

## The transition temperature at vanishing chemical potential:




crossover identified by peak in the chiral susceptibility:

$$T_c = (154 \pm 9)\ \mathrm{MeV}$$

A. Bazavov et al (HotQCD Collaboration), Phys. Rev. D 85, 054503 (2012)

consistent with transition temperatures determined by the Budapest-Wuppertal collab.

Y. Aoki et al., JHEP 0906 (2009) 088

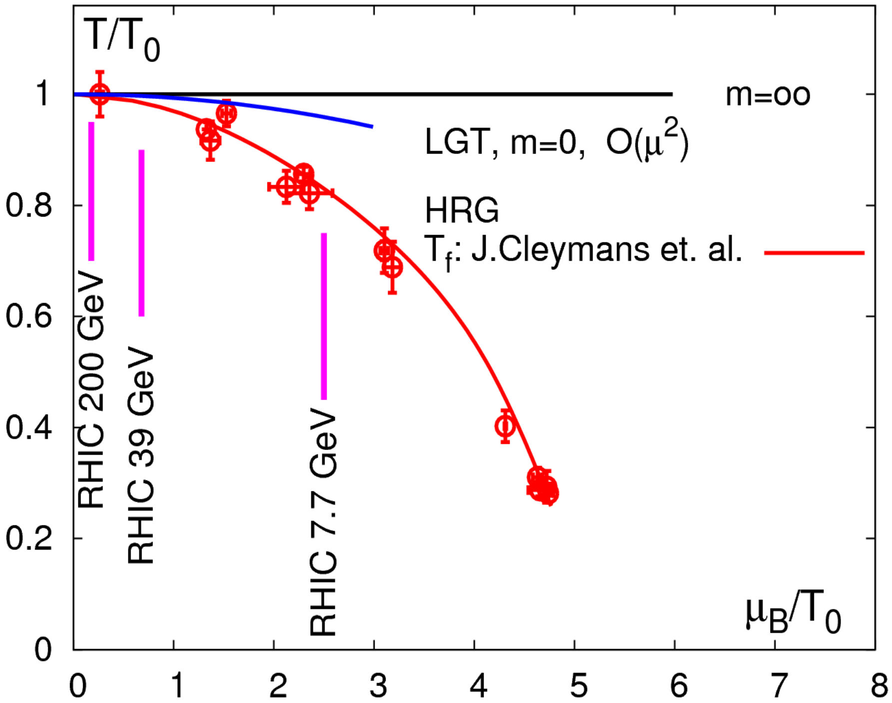
## Curvature of the transition line for small $\mu_B$ :

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - 0.0066(7)\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$

Bielefeld-BNL, Phys. Rev. D 83, 014504 (2011)

similar: G. Endrodi et al., JHEP 1104, 001 (2011)

# Chiral Transition and Freeze-out




(Crossover) transition from lattice QCD:

$$\frac{T(\mu_B)}{T_c} = 1 - 0.0066(7)\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$

freeze-out curve in HIC:
(using HRG as input)

$$\mu_B(\sqrt{s_{NN}}) = \frac{d}{1 + e\sqrt{s_{NN}}}$$

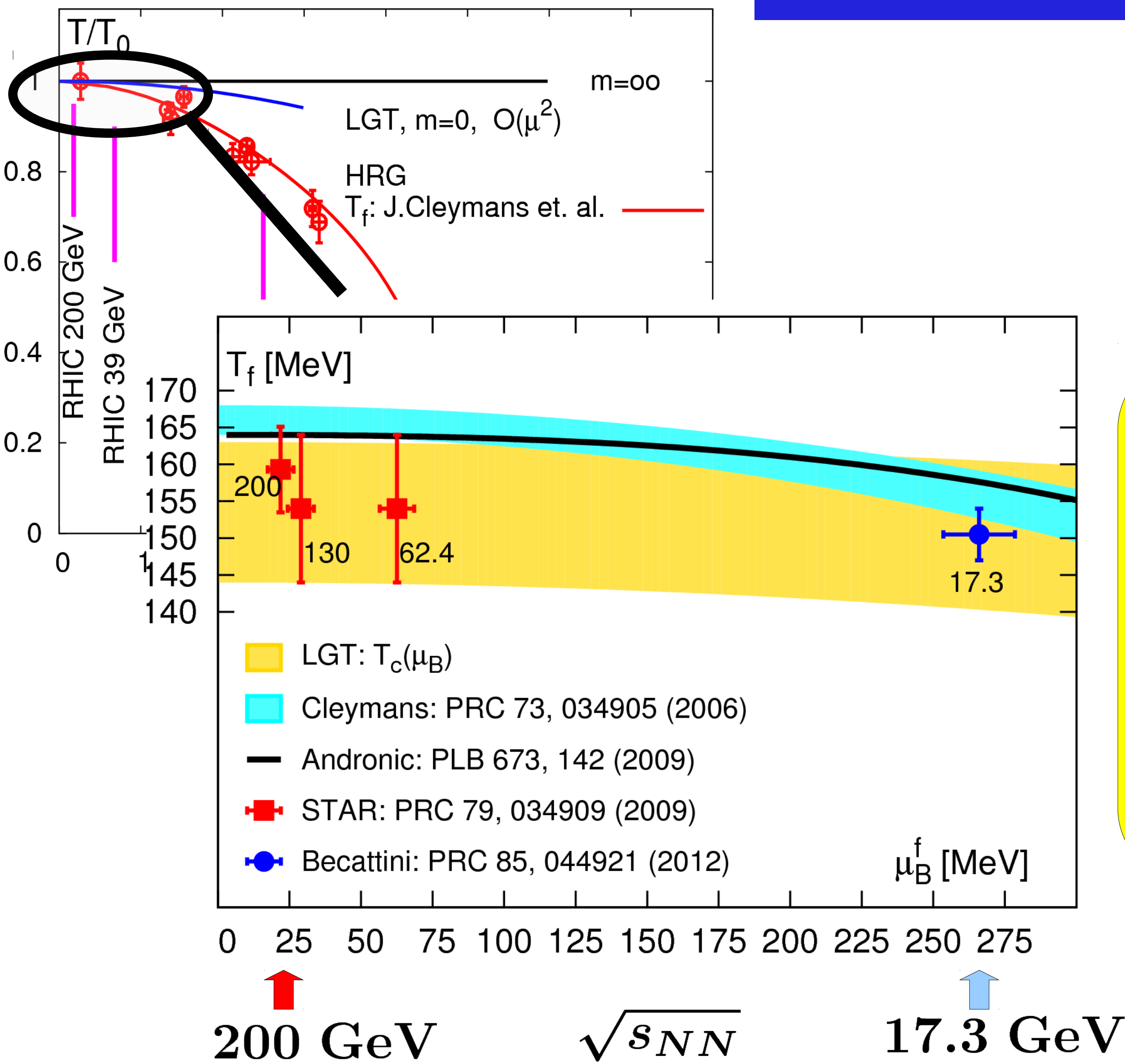
$$\frac{T(\mu_B)}{T_c} = 1 - 0.023\left(\frac{\mu_B}{T}\right)^2 - c\left(\frac{\mu_B}{T}\right)^4$$

J. Cleymans et al.,
Phys.Rev. C73, 034905 (2006)

$$T = \frac{164\mathrm{MeV}}{1 + \exp(2.60 - \ln(\sqrt{s_{NN}(\mathrm{GeV})})/0.45)}$$

A. Andronic et al.,
Phys. Lett. B673, 142 (2009)

# Chiral Transition and Freeze-out




phenomenological freeze-out curve, QCD transition line and experimental data (obtained by assuming the validity of the HRG model) are consistent for

$$\mu_B/T \lesssim 2$$

# QCD, HRG and Freeze-out in HIC

- in a wide range of beam energies covered by the beam energy scan at RHIC chemical freeze-out seems to happen close to the crossover temperature

Caveat / conceptual problem:

- freeze-out parameters have been determined by comparing experimental data to hadron resonance gas (HRG) model calculations
- in the transition region the HRG may not (does not?) provide a good description of the thermodynamics of strongly interacting matter

A. Andronic et al., arXiv:1201.0693

→ We need to determine basic thermal parameter (freeze-out ?) by comparing experimental data to QCD

Higher order cumulants of conserved charge fluctuations maybe good observables in this respect. They are (i) accessible to experiment, (ii) calculable in QCD and (iii) the HRG model

# Cumulants of Net Charge Fluctuations

pressure: $$\frac{p}{T^4} \equiv \frac{1}{VT^3} \ln Z(V,T,\mu_B,\mu_S,\mu_Q)$$
$$= \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

susceptibilities: $$\chi_{ijk,\mu}^{BQS} = \left.\frac{\partial^{i+j+k} p/T^4}{\partial(\mu_B/T)^i \partial(\mu_Q/T)^j \partial(\mu_S/T)^k}\right|_{\mu_B,\mu_Q,\mu_S}$$

moments of net-charge fluctuation

$$\mathcal{A}_n \sim \langle (\delta N_X)^n \rangle$$

cumulants: $$\chi_2^X \equiv \left.\frac{\partial^2 (p/T^4)}{\partial(\mu_X/T)^2}\right|_{\mu=0} = \frac{N_\tau}{N_\sigma^3} \mathcal{A}_2$$

X=B,Q, S

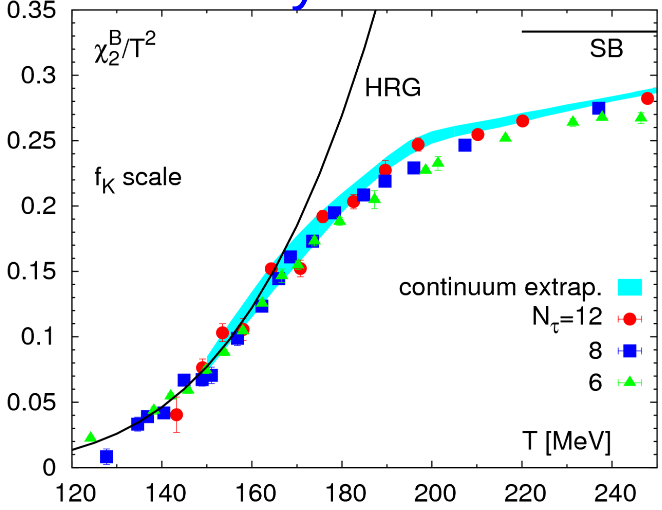
$$\chi_4^X \equiv \left.\frac{\partial^4 (p/T^4)}{\partial(\mu_X/T)^4}\right|_{\mu=0} = \frac{1}{N_\sigma^3 N_\tau} (\mathcal{A}_4 - 3\mathcal{A}_2^2)$$

$$\chi_6^X \equiv \left.\frac{\partial^6 (p/T^4)}{\partial(\mu_X/T)^6}\right|_{\mu=0} = \frac{1}{N_\sigma^3 N_\tau^3} (\mathcal{A}_6 - 15\mathcal{A}_4\mathcal{A}_2 + 30\mathcal{A}_2^3)$$

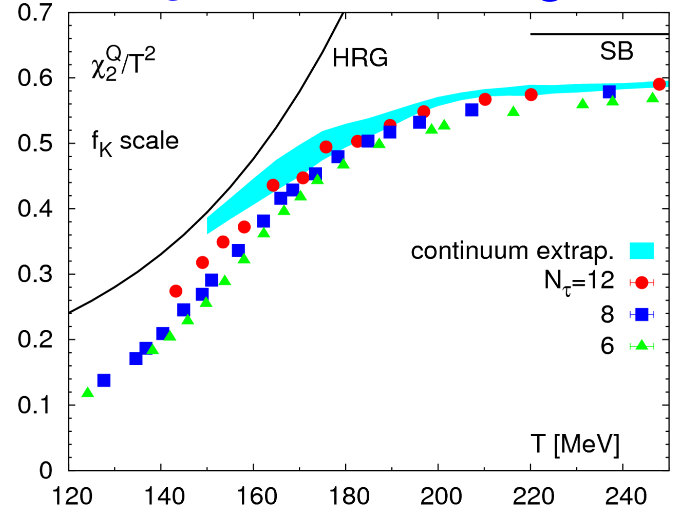
# Quadratic charge fluctuations: $\mu_B = 0$

continuum extrapolated results: A. Bazavov et al (hotQCD), arXiv:1203.0784

## B: baryon number



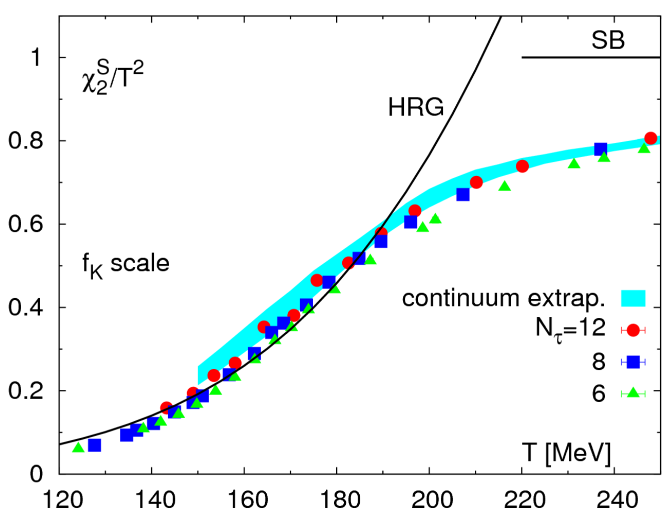

## Q: electric charge



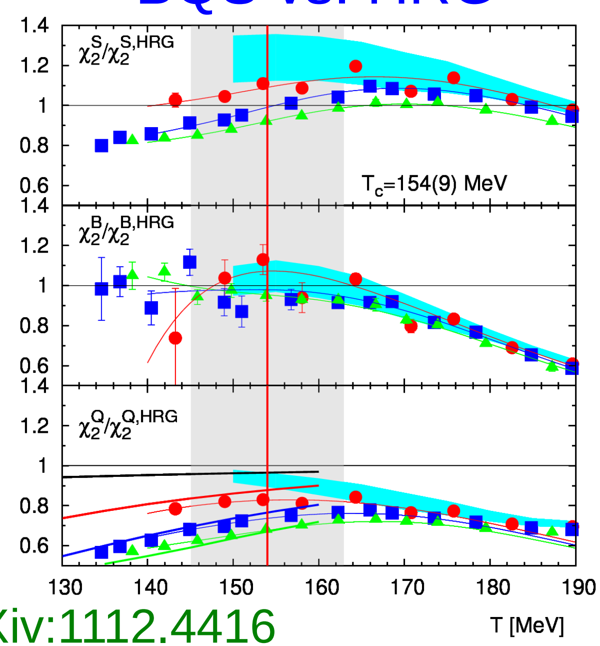

HISQ-action on $(4N_\tau)^3 \times N_\tau$ lattices;

statistics:
~3000 conf./T
~1500 source vec.

## S: strangeness




## BQS vs. HRG




(2+1)-flavor QCD physical strange quark sector;

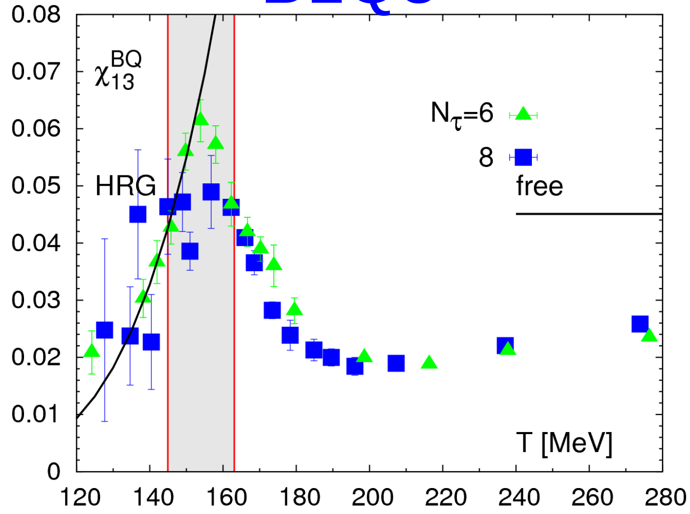
$m_l/m_s = 1/20$

$m_{ps} \simeq 160$ MeV

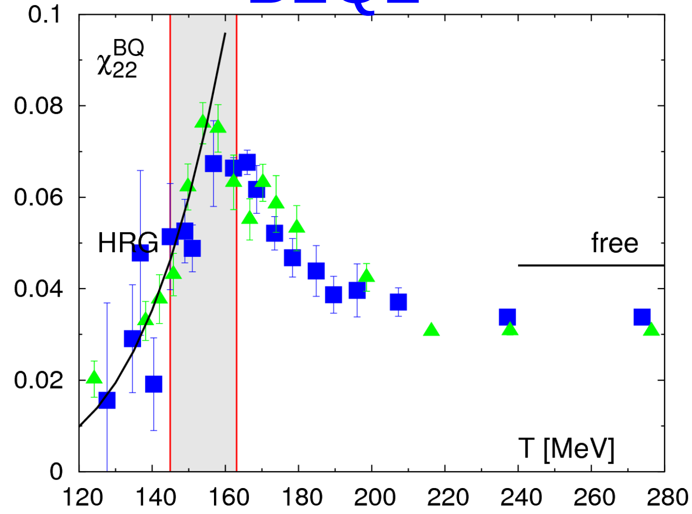
consistent with Wuppertal-Budapest, arXiv:1112.4416

# Some 4$^{th}$ order charge fluctuations: $\mu_B = 0$

## B1Q3



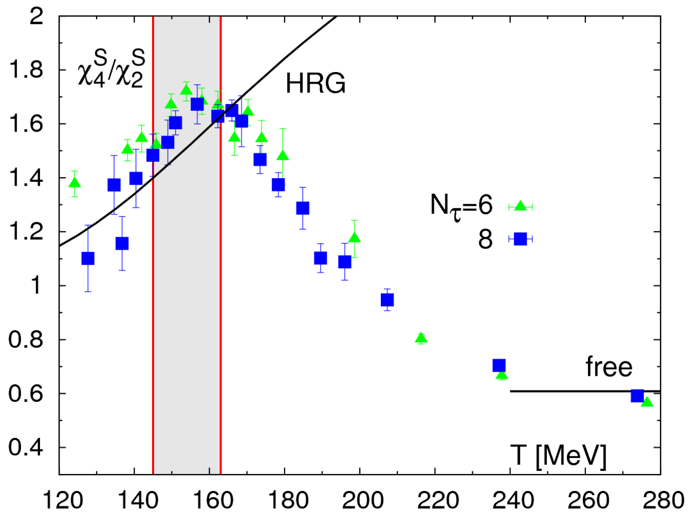

## B2Q2



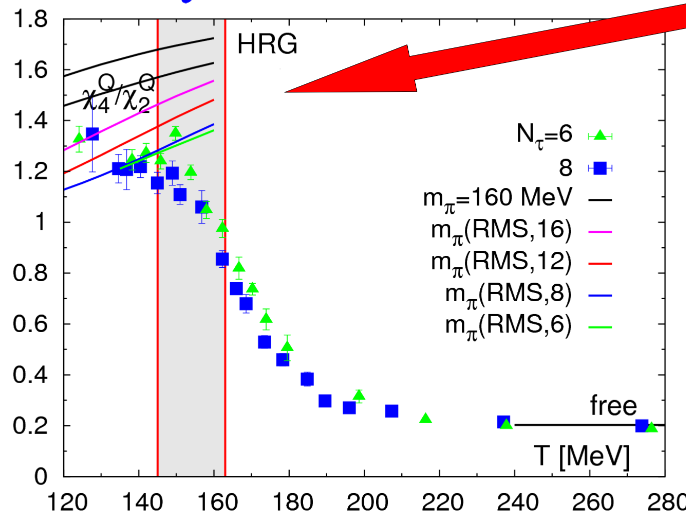

HISQ-action on $(4N_\tau)^3 \times N_\tau$ lattices;

statistics:
~3000 conf./T
~1500 source vec.

## $\kappa_S \sigma_S^2$ S4/S2




## $\kappa_Q \sigma_Q^2$ Q4/Q2



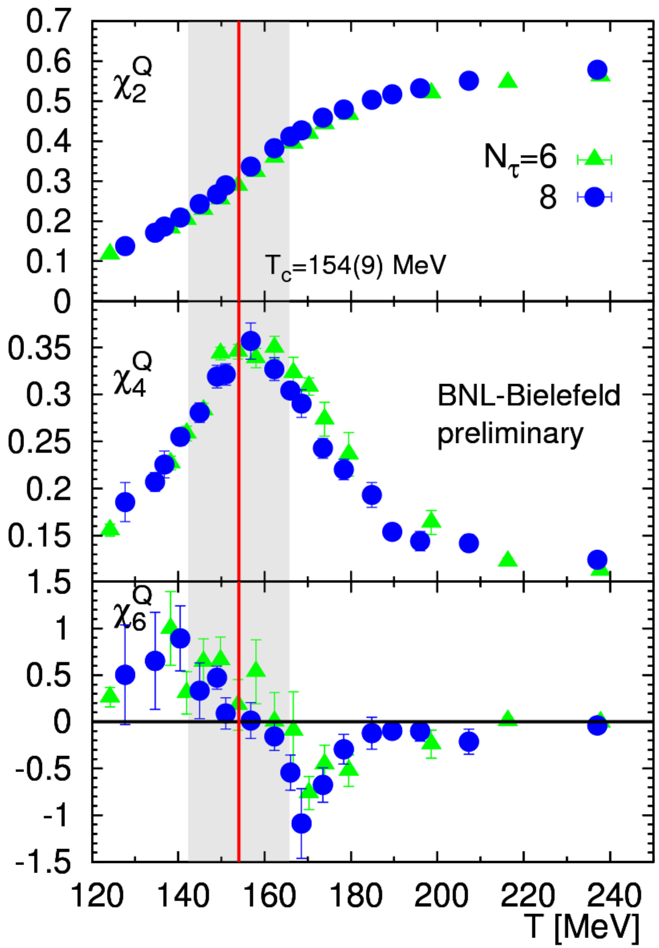

cut-off effects;
continuum limit for net electric charge fluctuations more difficult than for baryon number fluctuations

band:
$T_c = (154 \pm 9)\mathrm{MeV}$

Bielefeld-BNL, preliminary    similar: S. Borsanyi, Quark Matter 2012

# Electric charge fluctuations at the LHC

- Cumulants calculated at $\mu_B = 0$ can directly be compared to (eventually available) data taken at LHC

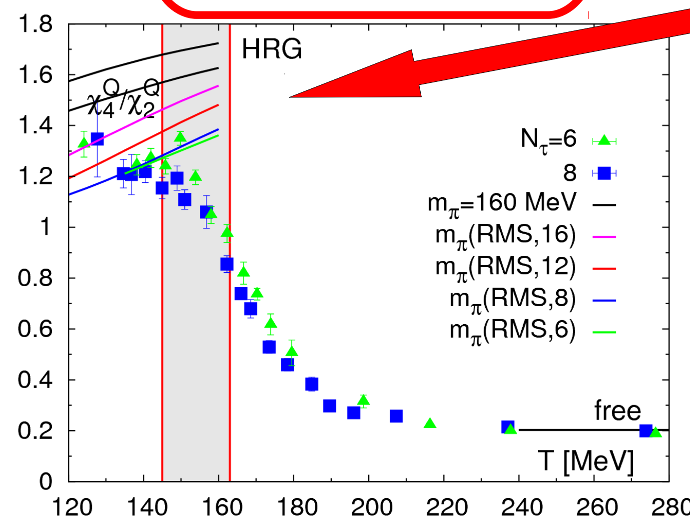



LHC: $\mu_B \simeq \mu_S \simeq \mu_Q \simeq 0$

T is the only free parameter

if $T_{freeze}^{cumulants} \simeq 160$ MeV

$$\chi_6^Q \lesssim 0$$
$$\chi_4^Q / \chi_2^Q \simeq 1$$




cut-off effects;
continuum limit for net electric charge fluctuations more difficult than for baryon number fluctuations

still need to take continuum limit!!

C. Schmidt (Bielefeld-BNL), Quark Matter 2012

# Taylor expansions of baryon number susceptibilities

$$\chi^B_{n,\mu} = \sum_{k=0}^{\infty} \frac{1}{k!} \chi^B_{k+n,0}(T) \left(\frac{\mu_B}{T}\right)^k$$

for simplicity: $\mu_s = \mu_Q = 0$

notation: $\chi^B_n \equiv \chi^B_{n,0}$

mean: $M_B = VT^3 \chi^B_{1,\mu} = VT^3 \left(\frac{\mu_B}{T}\chi^B_2 + ...\right)$

variance: $\sigma^2_B = VT^3 \chi^B_{2,\mu}$

$$= VT^3 \left(\chi^B_2 + \frac{1}{2}\left(\frac{\mu_B}{T}\right)^2 \chi^B_4 + ...\right)$$

skewness and kurtosis and volume independent ratios of susceptibilities

$$S_B \equiv \frac{\langle(\delta N_B)^3\rangle}{\sigma^3_B} \; , \quad \kappa_B \equiv \frac{\langle(\delta N_B)^4\rangle}{\sigma^4_B} - 3$$

$$\frac{\sigma^2_B}{M_B} = \frac{\chi^B_{2,\mu}}{\chi^B_{1,\mu}}, \qquad S_B\sigma_B = \frac{\chi^B_{3,\mu}}{\chi^B_{2,\mu}}, \qquad \kappa_B\sigma^2_B = \frac{\chi^B_{4,\mu}}{\chi^B_{2,\mu}}$$

# Cumulant ratios for fixed T and $\mu_B$

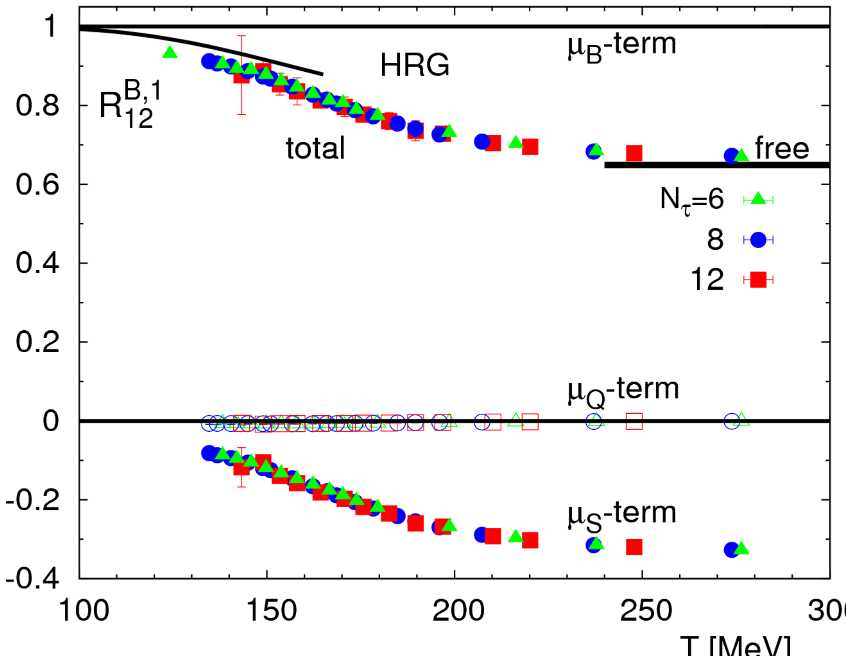
- How important is it to have $\mu_S \neq 0\ ,\ \mu_Q \neq 0$ ?
- Consider the simplest ratio

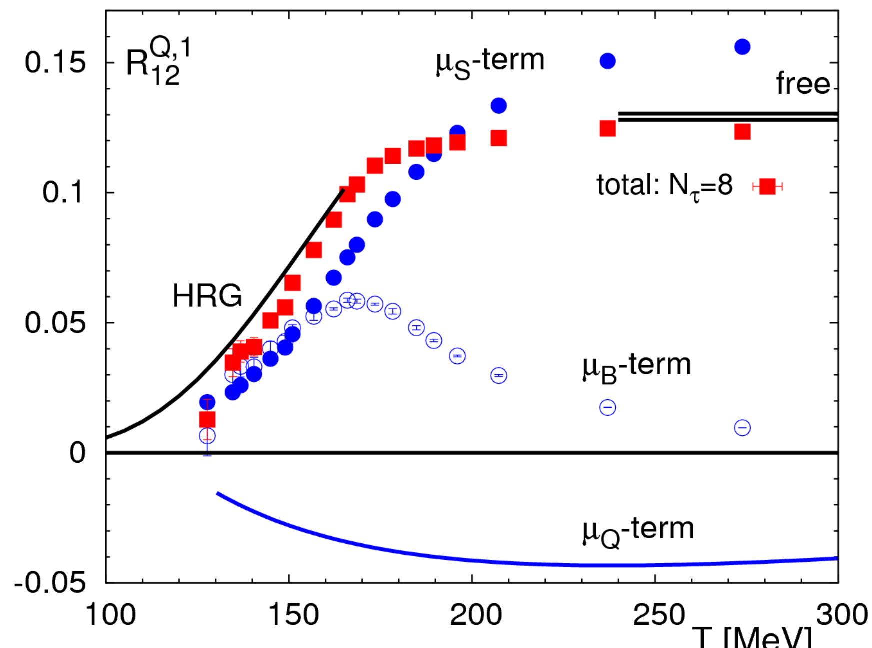
need (2+1)-f QCD

$$R_{12}^X \equiv \frac{M_X}{\sigma_X^2} = \frac{\mu_B}{T}\left(R_{12}^{X,1} + R_{12}^{X,3}\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)\right)$$

$$R_{12}^{B,1} = 1 + \frac{\mu_Q}{\mu_B}\frac{\chi_{11}^{BQ}}{\chi_2^B} + \frac{\mu_S}{\mu_B}\frac{\chi_{11}^{BS}}{\chi_2^B}$$

$$R_{12}^{Q,1} = \frac{\chi_{11}^{BQ}}{\chi_2^Q} + \frac{\mu_Q}{\mu_B} + \frac{\mu_S}{\mu_B}\frac{\chi_{11}^{QS}}{\chi_2^Q}$$




need $\mu_S \neq 0$

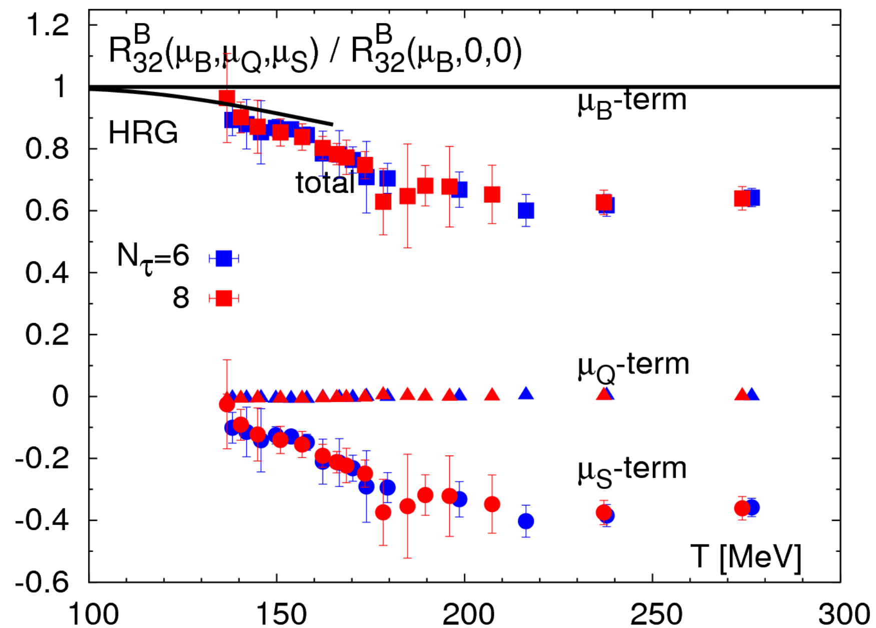
need $\mu_S \neq 0,\ \mu_Q \neq 0$

# Cumulant ratios of charge fluctuations

e.g. : $S_B\sigma_B = \frac{\chi^B_{3,\mu}}{\chi^B_{2,\mu}} \equiv R^B_{32} = \frac{\mu_B}{T}\left[\frac{\chi^B_4}{\chi^B_2} + \frac{\mu_S}{\mu_B}\frac{\chi^{BS}_{31}}{\chi^B_2} + \frac{\mu_Q}{\mu_B}\frac{\chi^{BQ}_{31}}{\chi^B_2} + \mathcal{O}(\mu^2)\right]$

– even-odd ratios are most sensitive to the baryon chemical potential
– even at leading order we need to control strangeness and electric charge chemical potentials

strangeness contribution is important when comparing LGT calculations with experiment




need to fix 4 parameter before LGT can make predictions

strategy: (I) fix $\mu_Q/\mu_B,\ \mu_S/\mu_B$
(II) get $\mu_B,\ T$ from data

# Thermal Conditions in Heavy Ion Collisions at RHIC

1) strangeness neutrality: $\langle N_S \rangle = 0$

2) isospin/charge asymmetry: $\langle N_Q \rangle = r \; \langle N_B \rangle$

HIC:
Au-Au; Pb-Pb
$r \simeq 0.4$

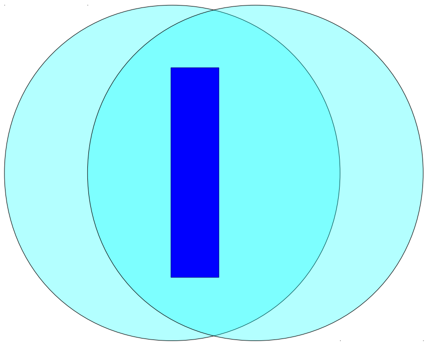
use (1) and (2) to determine strangeness and electric charge chemical potentials:

$$\mu_S \equiv \mu_S(\mu_B, T)$$
$$\mu_Q \equiv \mu_Q(\mu_B, T)$$

fulfill (1) and (2) order by order in a Taylor expansion:

$$\frac{\mu_S}{T} = s_1(T)\frac{\mu_B}{T} + s_3(T)\left(\frac{\mu_B}{T}\right)^3$$

$$\frac{\mu_Q}{T} = q_1(T)\frac{\mu_B}{T} + q_3(T)\left(\frac{\mu_B}{T}\right)^3$$



$$\frac{N_Q}{N_B} \equiv \frac{N_p}{N_p + N_n} = r$$

# Thermal Conditions in Heavy Ion Collisions

expand constraints (1) and (2) in Taylor series: $\hat{\mu}_B \equiv \mu_B/T$

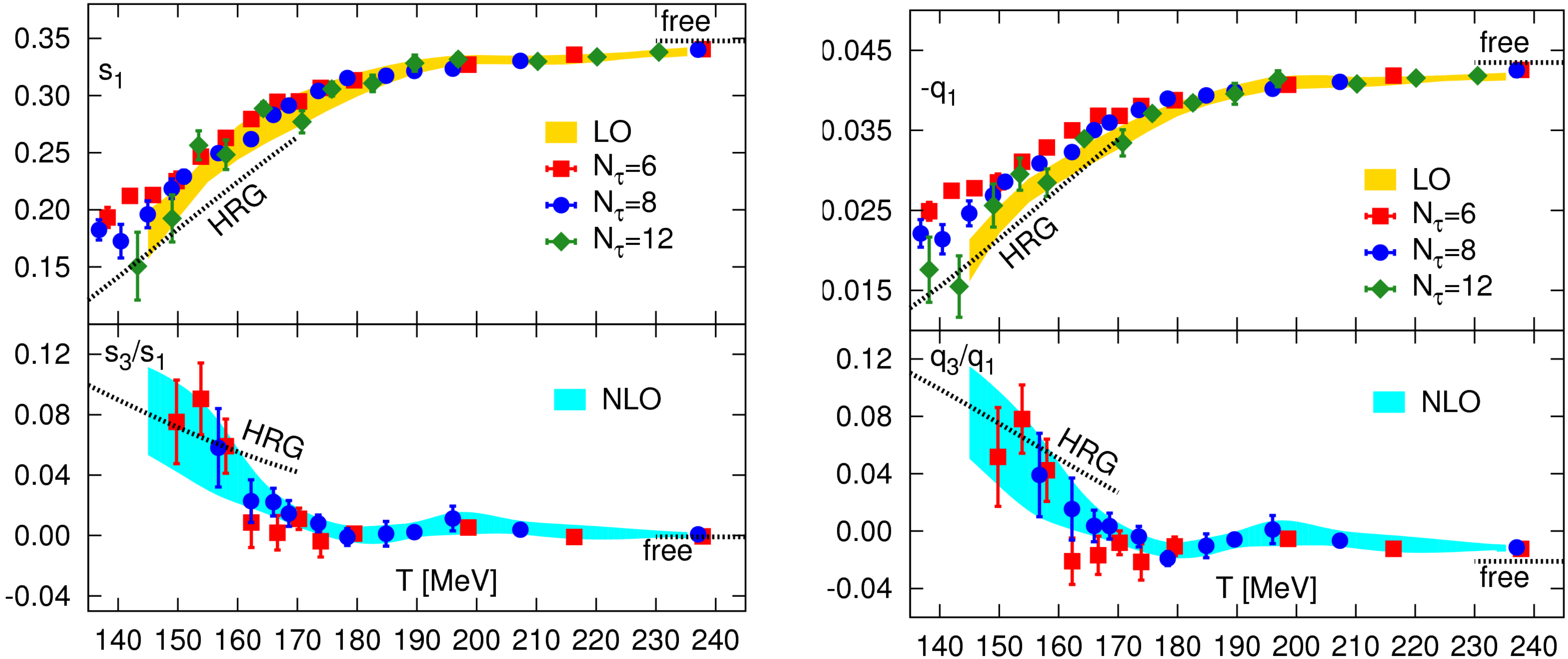
$$\langle N_S \rangle = \left( \chi_{11}^{BS} + q_1 \chi_{11}^{QS} + s_1 \chi_2^S \right) \hat{\mu}_B + \left( m_{S,3} + q_3 \chi_{11}^{QS} + s_3 \chi_2^S \right) \hat{\mu}_B^3$$

$$\begin{aligned} m_{S,3} = \; & \frac{1}{6}\chi_{31}^{BS} + \frac{1}{2}\chi_{211}^{BQS} q_1 + \frac{1}{2}\chi_{121}^{BQS} q_1^2 + \frac{1}{6}\chi_{31}^{QS} q_1^3 + \\ & \frac{1}{2}\chi_{22}^{BS} s_1 + \chi_{112}^{BQS} q_1 s_1 + \frac{1}{2}\chi_{22}^{QS} q_1^2 s_1 + \frac{1}{2}\chi_{13}^{BS} s_1^2 + \\ & \frac{1}{2}\chi_{13}^{QS} q_1 s_1^2 + \frac{1}{6}\chi_4^S s_1^3 \end{aligned}$$

similar for: $\langle N_B \rangle$ , $\langle N_Q \rangle$

solve for: $s_1$, $q_1$ , $s_3$, $q_3$ → $\mu_S(\mu_B, T)$, $\mu_Q(\mu_B, T)$

# Strangeness and Electric Charge Chemical Potentials




HRG at LO and NLO, respectively

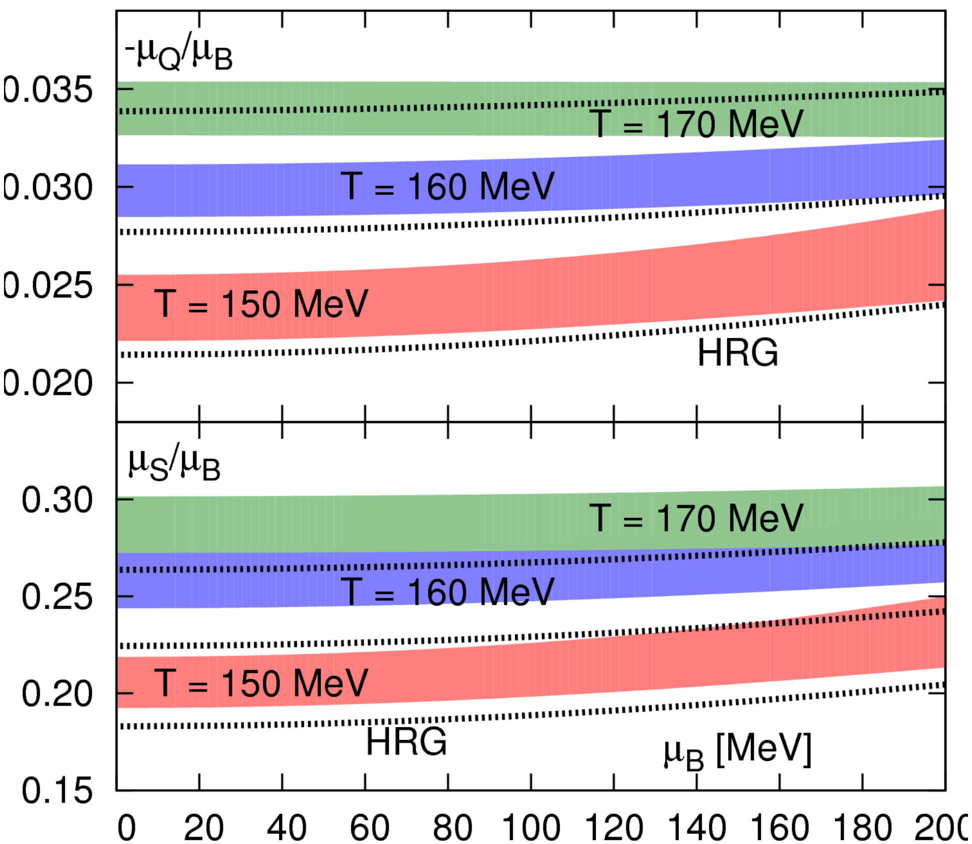
Bielefeld-BNL: arXiv:1208.1220

LO: continuum extrapolation
NLO: $N_\tau = 8$ interpolation

NLO correction is below 10% for all T > 140 MeV and $\mu_B/T \leq 1$

# Next to Leading Order (NLO) results at fixed T

for 150MeV < T < 170MeV QCD and HRG agree within ~10% on $\mu_S/\mu_B$ , $\mu_Q/\mu_B$




for orientation: $\mu_B = 1.3T \Leftrightarrow$

$\mu_B = 200$ MeV at $T = 160$ MeV

Bielefeld-BNL, arXiv:1208.1220

NLO Taylor expansions for electric charge and strangeness chemical potentials are well behaved for

$$\mu_B/T \lesssim 1.3$$

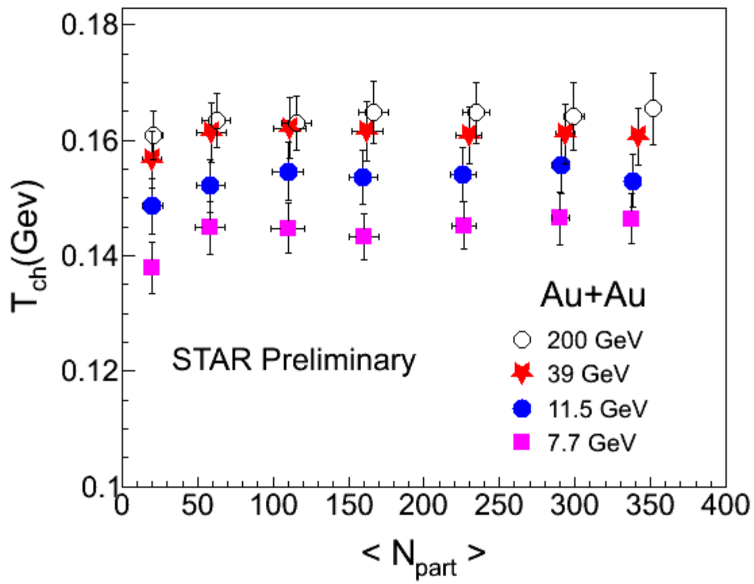
tempting to compare with STAR result (QM'12),

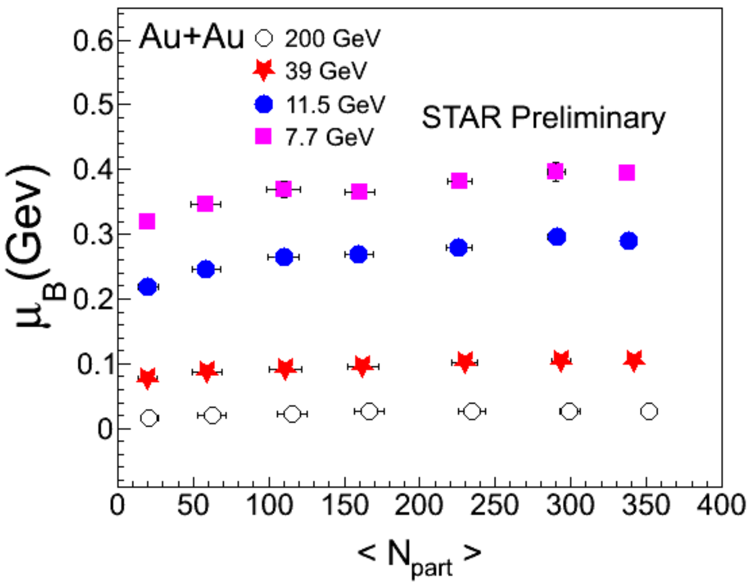
$$\frac{\mu_S}{\mu_B} \simeq (0.2 - 0.25)$$ However,..

this covers RHIC experiments down to

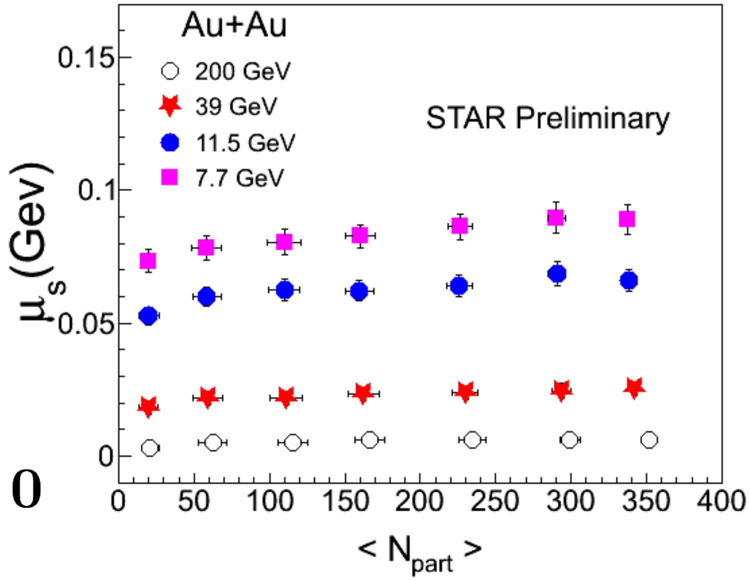
$$\sqrt{s_{NN}} \simeq 20 \text{ GeV}$$

# Chemical Freeze-out parameter




Sabita Das (STAR Collaboration),
Quark Matter 2012




$$\frac{\mu_S}{\mu_B} \simeq (0.2 - 0.25)$$

only weakly dependent on $\mu_B$

reflects strangeness neutrality: $\langle N_S \rangle = 0$

# Determination of T and $\mu_B$ from cumulant ratios

– in thermal equilibrium any two ratios of cumulants should allow to fix temperature and baryon chemical potential ♥

$$R^X_{n,m} = \frac{\chi^X_{n,\mu}}{\chi^X_{m,\mu}} \; , \quad X = B,\ Q,\ S$$

NLO Taylor expansion

– ratios with n+m even or odd show different sensitivity to T and $\mu_B$

$$R^X_{12} \equiv \frac{M_X}{\sigma^2_X} = \frac{\mu_B}{T}\left( R^{X,1}_{12} + R^{X,3}_{12}\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)\right) \; ,$$

$$R^X_{31} \equiv \frac{S_X \sigma^3_X}{M_X} = R^{X,0}_{31} + R^{X,2}_{31}\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4) \; ,$$

$M_X \equiv \chi^X_1$ : mean

$\sigma^2_X \equiv \chi^X_2$ : variance

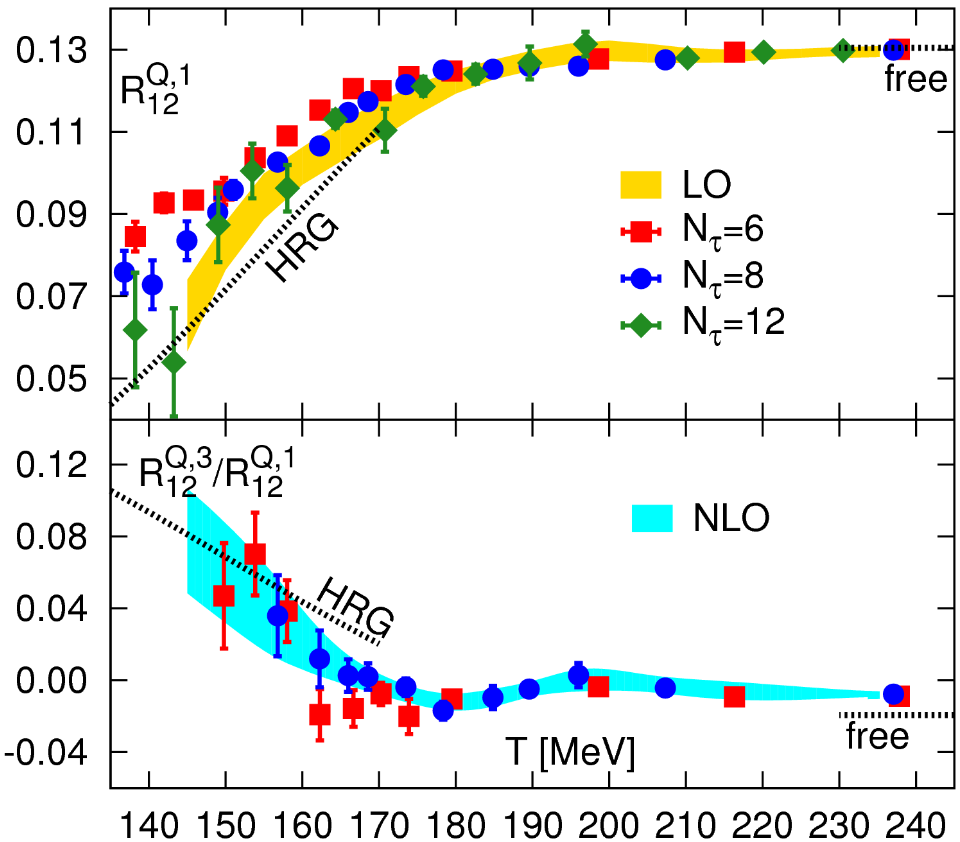
$S_X \equiv \chi^X_3/(\chi^X_2)^{3/2}$ :skewness

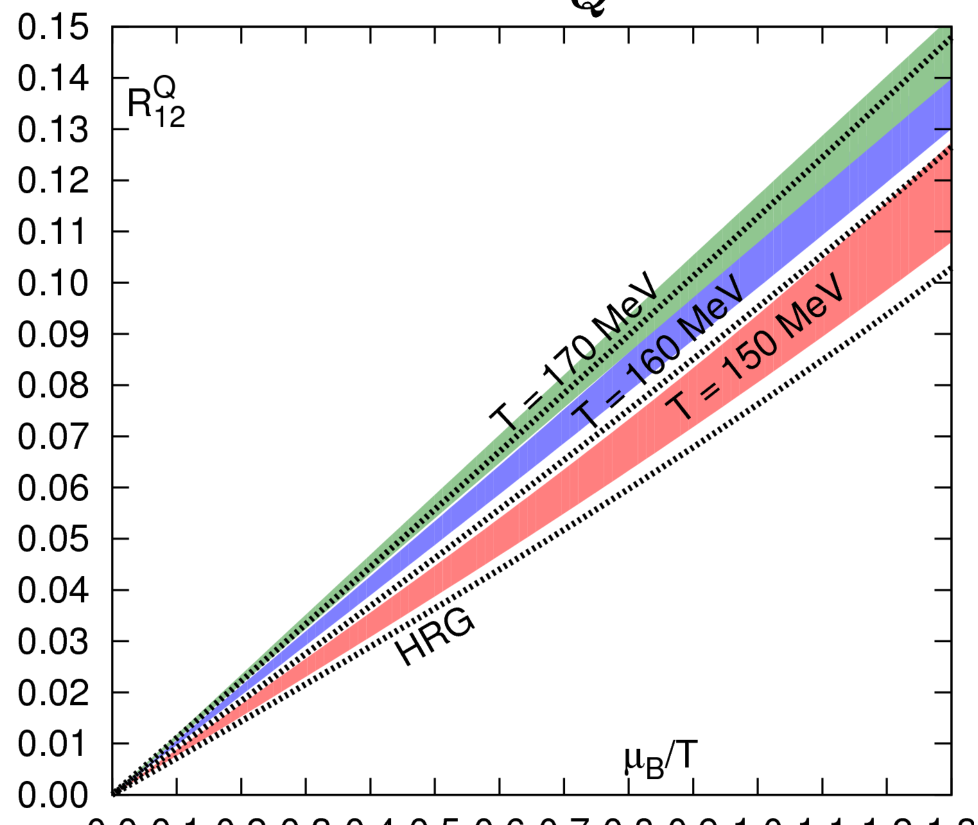
♥ if fluctuations are sensitive to equilibrium physics at a unique $(T, \mu_B)$ pair

# Baryo-meter: $R^{X}_{1,2}\ ,\ X = Q, B$

$R^{Q}_{12}$ provides stringent constraint on $\mu_B/T$

$$R^{Q}_{12} = \frac{M_Q}{\sigma^2_Q}$$




Bielefeld-BNL, arXiv:1208.1220

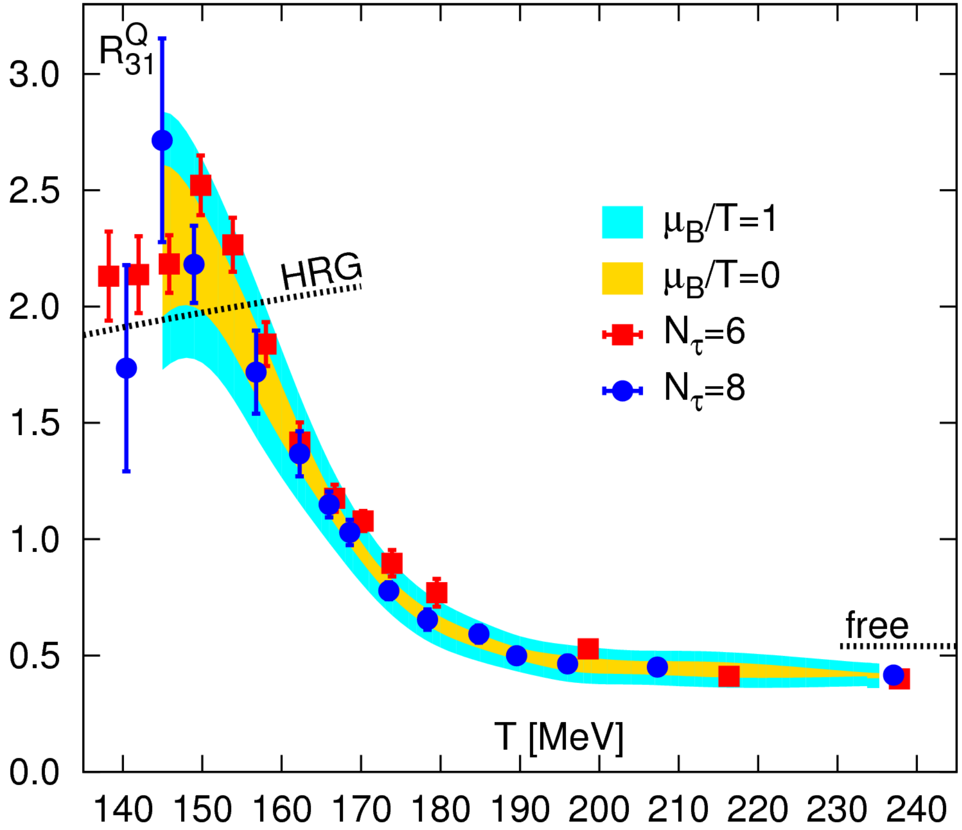
LO: continuum extrapolated
NLO: spline interpolation

NLO correction contributes less than 10% for T>140MeVand $\mu_B/T \leq 1$

# Thermo-meter: $R^X_{3,1}$ , $X = Q, B$

$$R^X_{31} \equiv \frac{S_X \sigma^3_X}{M_X} = R^{X,0}_{31} + R^{X,2}_{31}\ \hat{\mu}^2_B + \mathcal{O}(\hat{\mu}^4_B)$$

$$R^{Q,0}_{31} = \frac{\chi^{BQ}_{13} + q_1 \chi^Q_4 + s_1 \chi^{QS}_{31}}{q_1 \chi^Q_2 + \chi^{BQ}_{11} + s_1 \chi^{QS}_{11}}$$

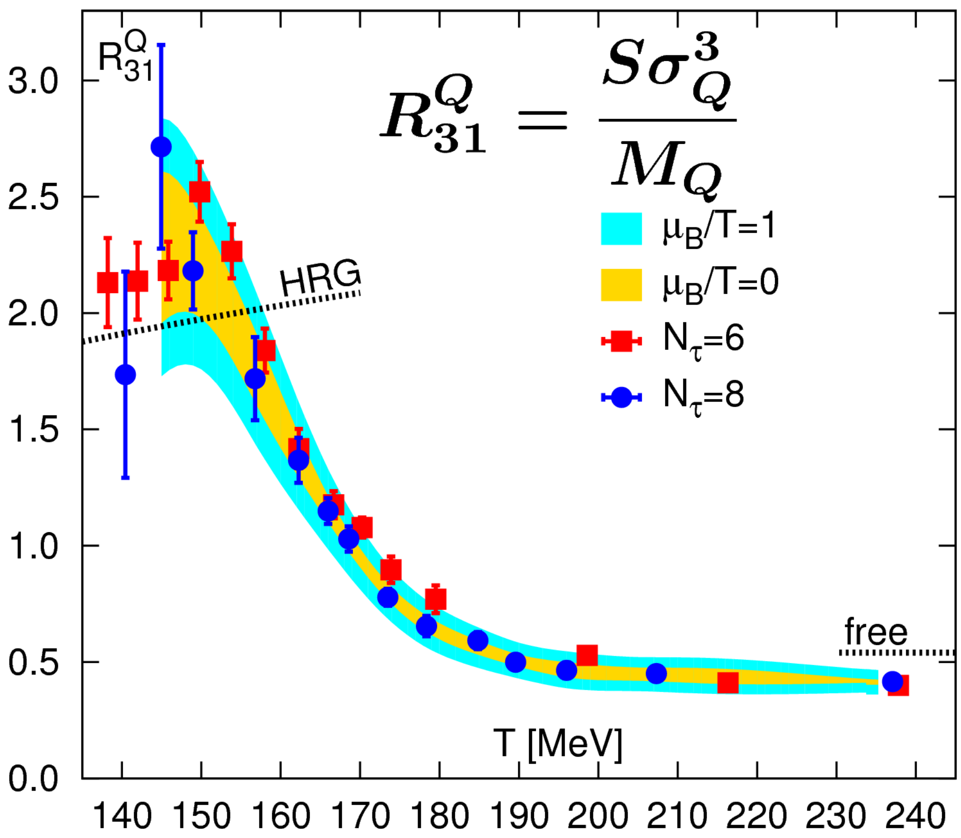


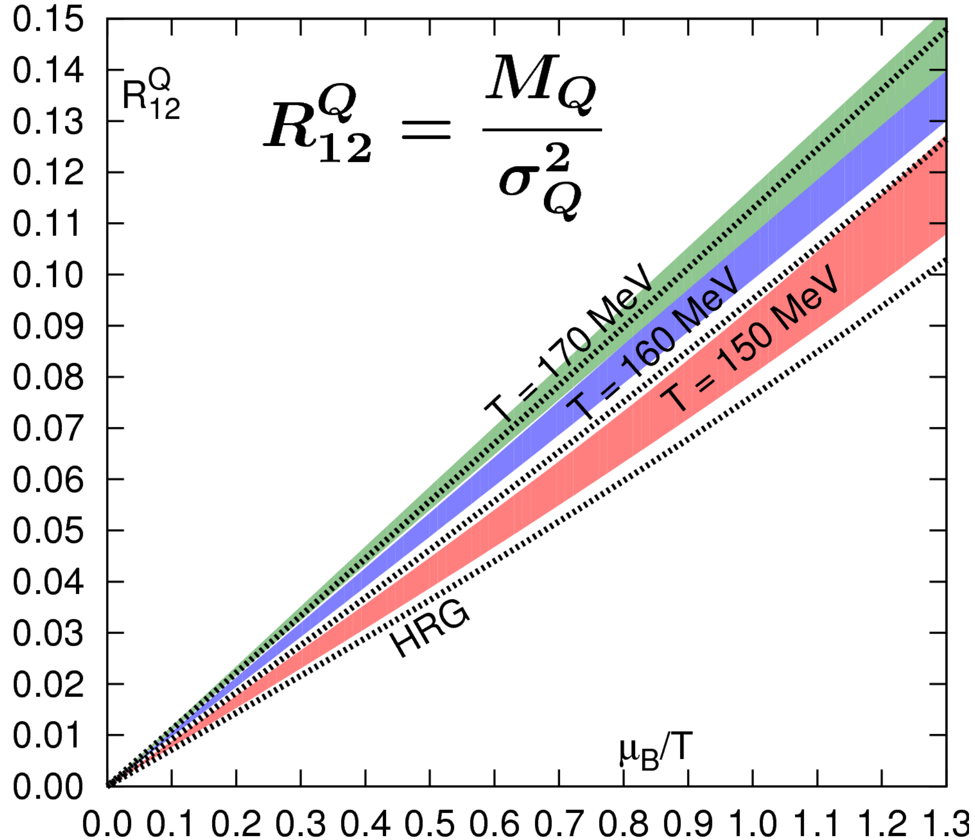

$R^{Q,2}_{31}$ requires 6th order coefficients (estimate of its magnitude) NLO corrections below 10%

$R^{Q,0}_{31}$ provides stringent constraint on T

large deviations from HRG for T>155 MeV

# Determination of T and $\mu_B$



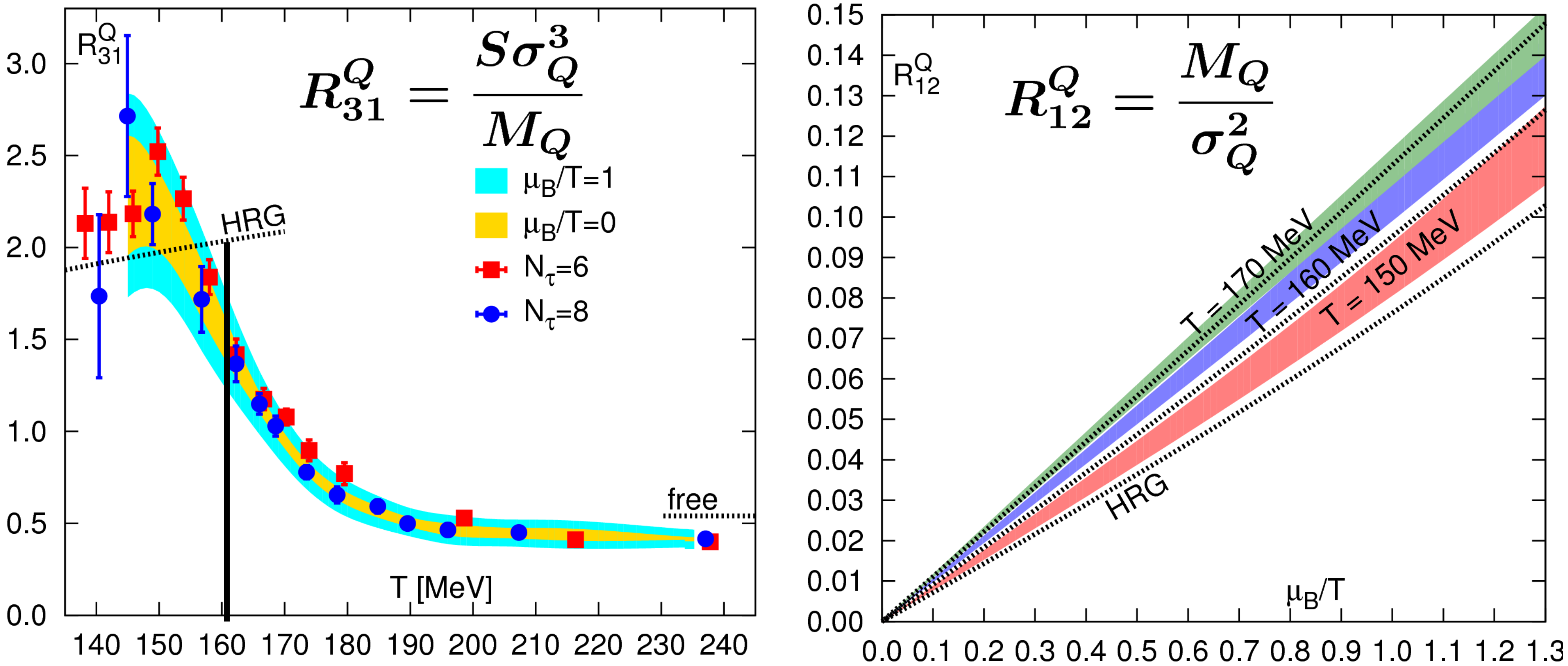

Bielefeld-BNL, arXiv:1208.1220

need data for these two observables to determine

$T,\ \mu_B$

from then on all other cumulant ratios probe thermodynamic consistency, i.e. our basic assumption of a unique freeze-out line, equilibrium thermodyn., etc

# Determination of T and $\mu_B$



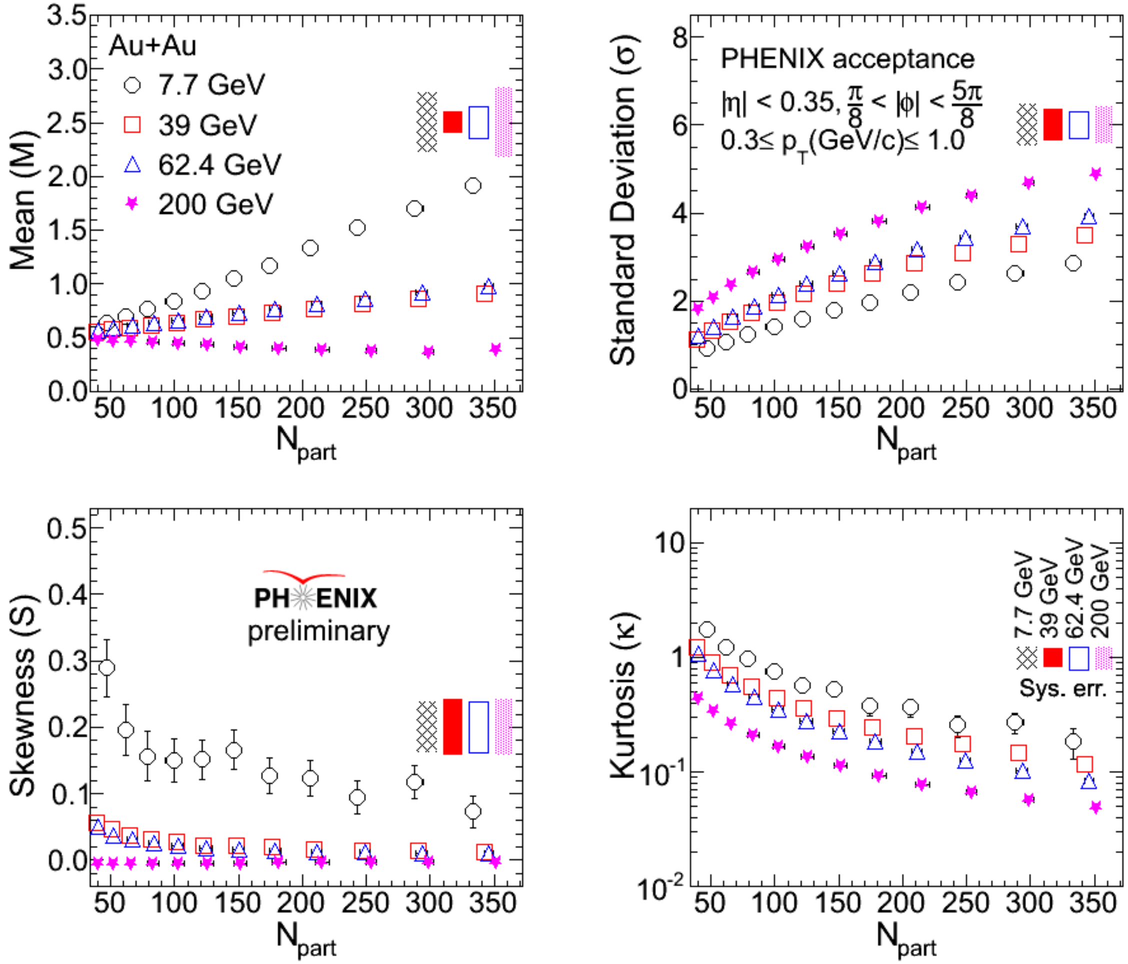

Bielefeld-BNL, arXiv:1208.1220

expect significant deviations for $R_{31}^Q$ from HRG, if
$T_{freeze} \geq 160$ MeV

from then on all other cumulant ratios probe thermodynamic consistency, i.e. our basic assumption of a unique freeze-out line, equilibrium thermodyn., etc

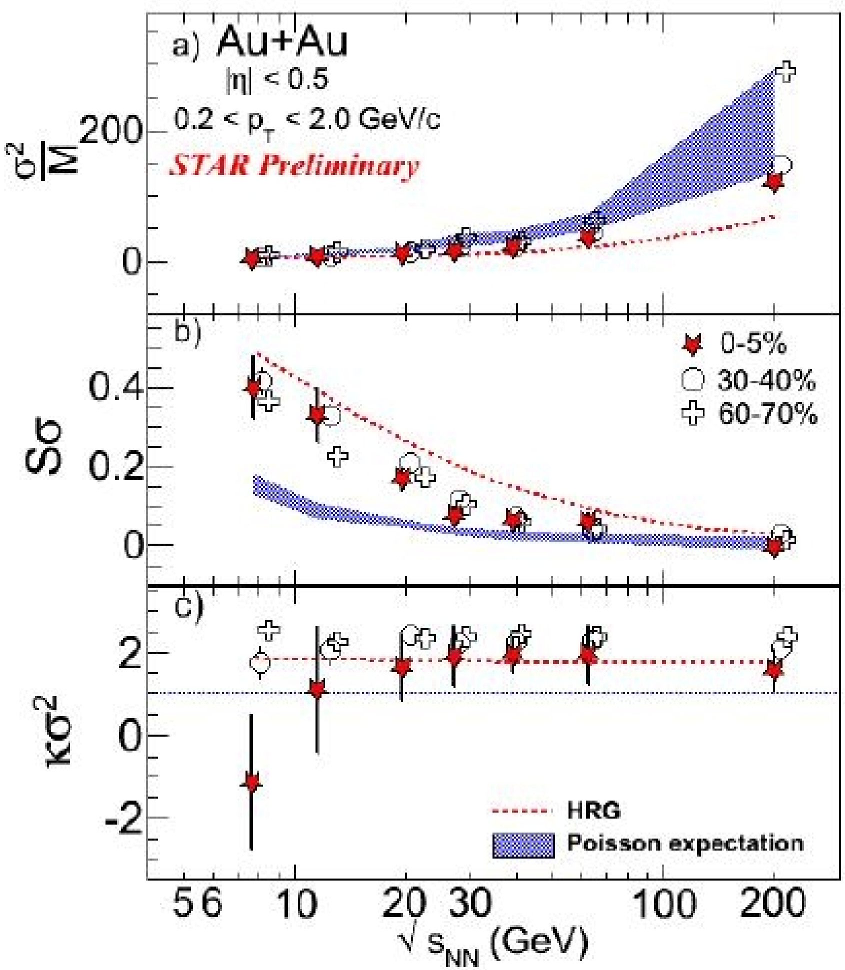
# PHENIX and STAR data on electric charge fluctuations

## PHENIX



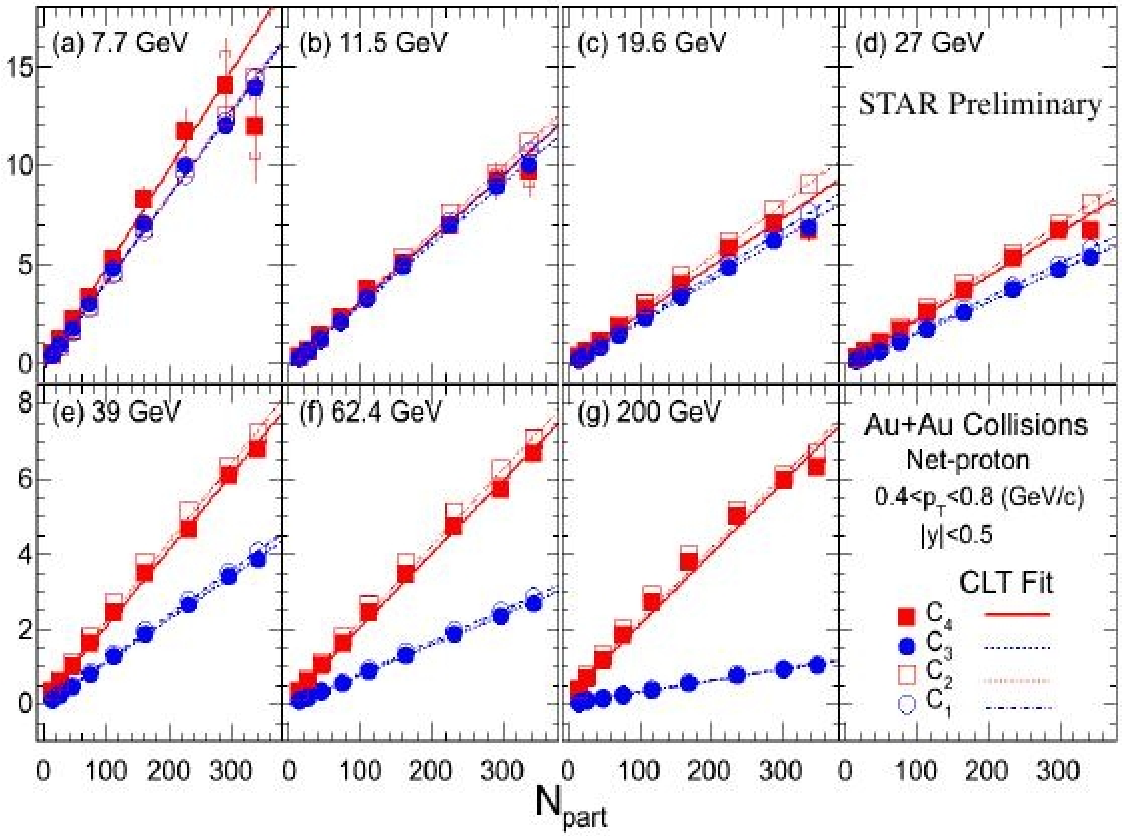

J. Mitchell, Quark Matter 2012

## STAR




L. Kumar, Quark Matter 2012

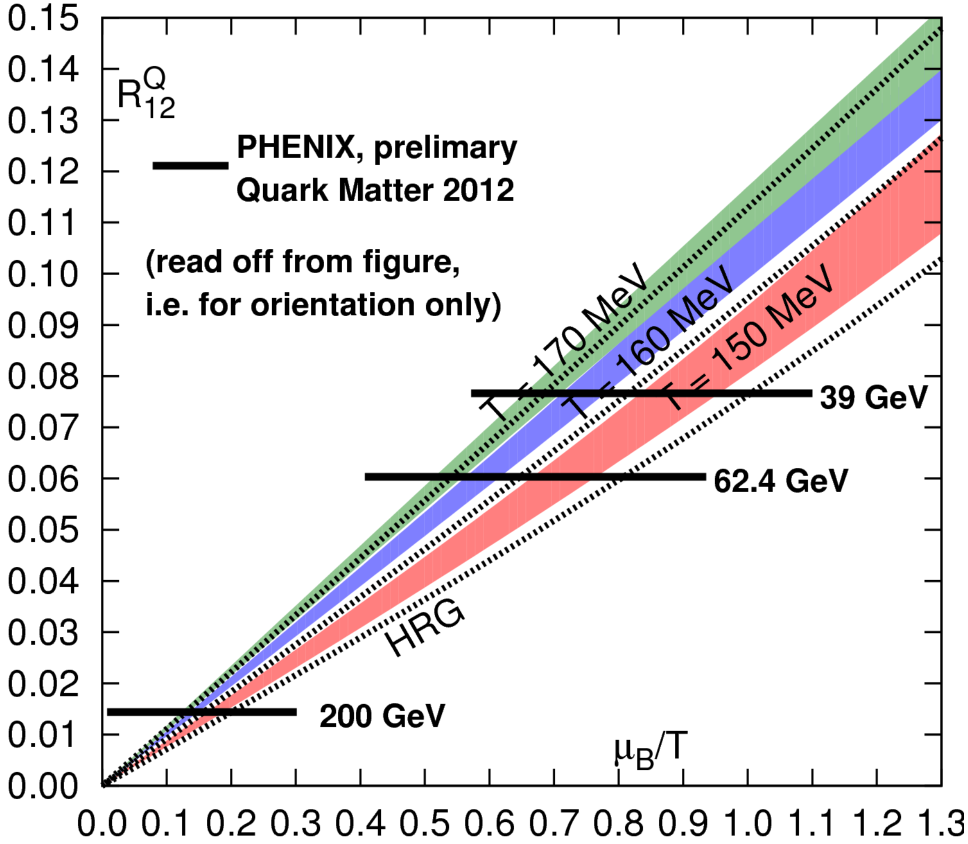
# STAR data on cumulants of net proton number fluctuations




X. Luo, Quark Matter 2012

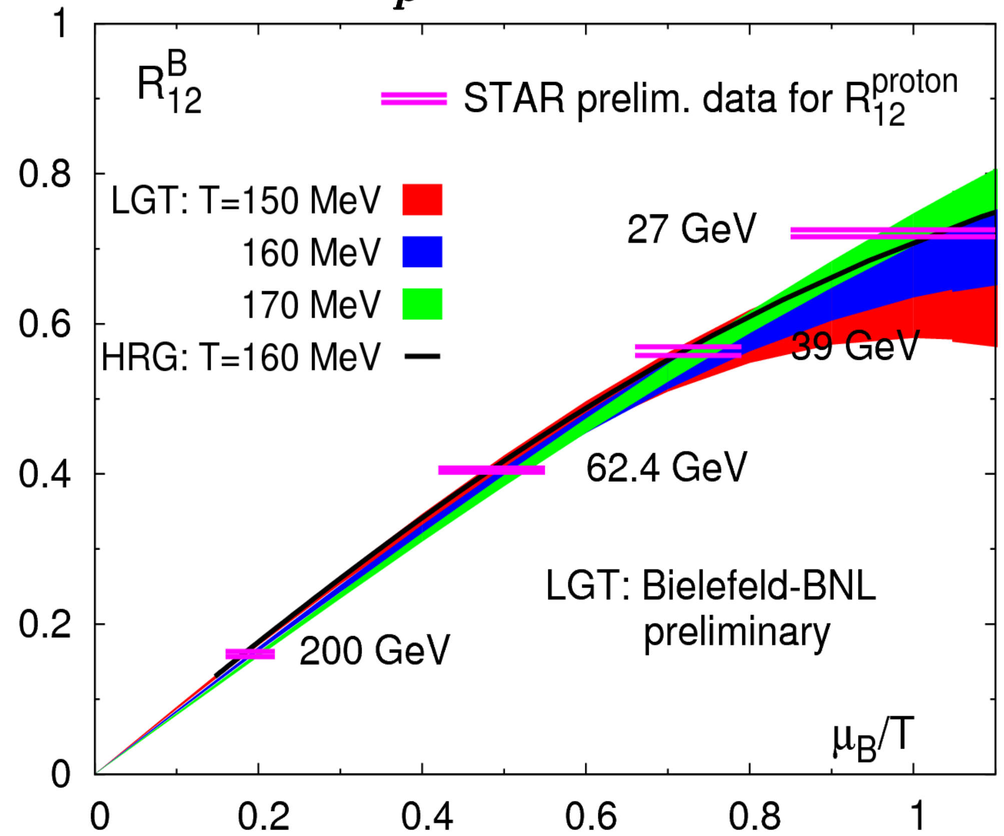
# Mean over variance: HIC vs. LGT

$$R_{12}^{Q} = \frac{M_Q}{\sigma_Q^2}$$




PHENIX data: J. Mitchell, Quark Matter 2012

$$R_{12}^{p} = \frac{M_p}{\sigma_p^2} \qquad R_{12}^{p} = R_{12}^{B}\ ??$$




STAR data: X. Luo, Quark Matter 2012

– need data for $R_{31}^{Q} = S_Q \sigma_Q^3 / M_Q$ to extract $T_{freeze}^{cumulants}$

# Mean over variance: HIC vs. LGT

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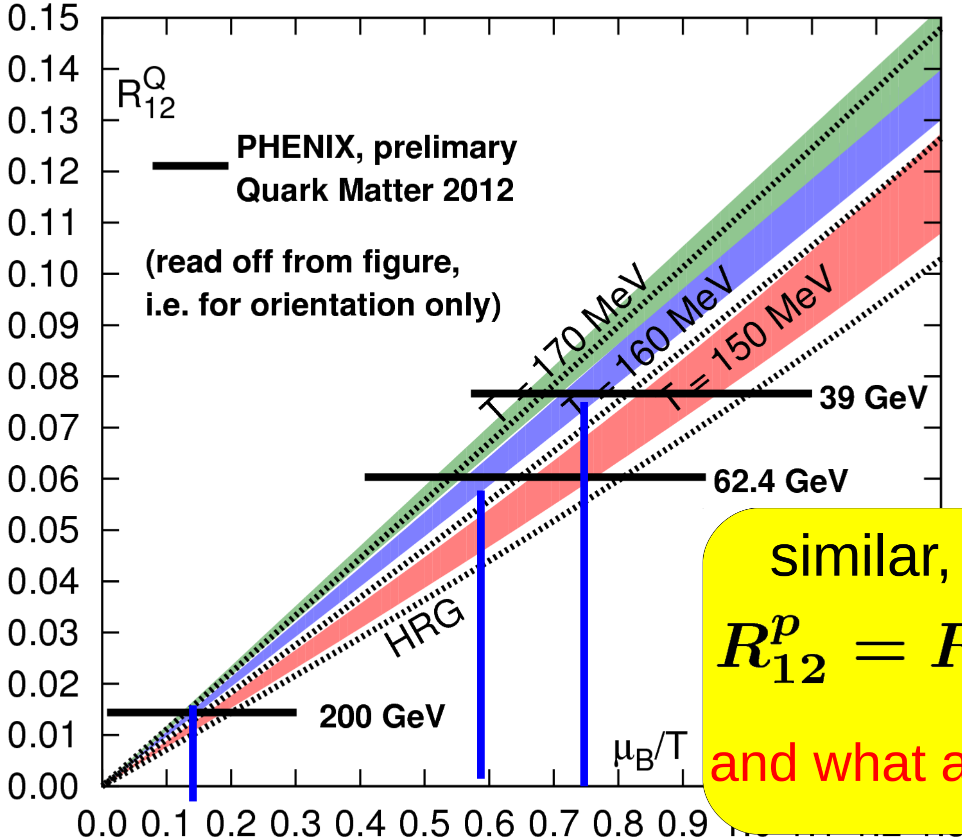


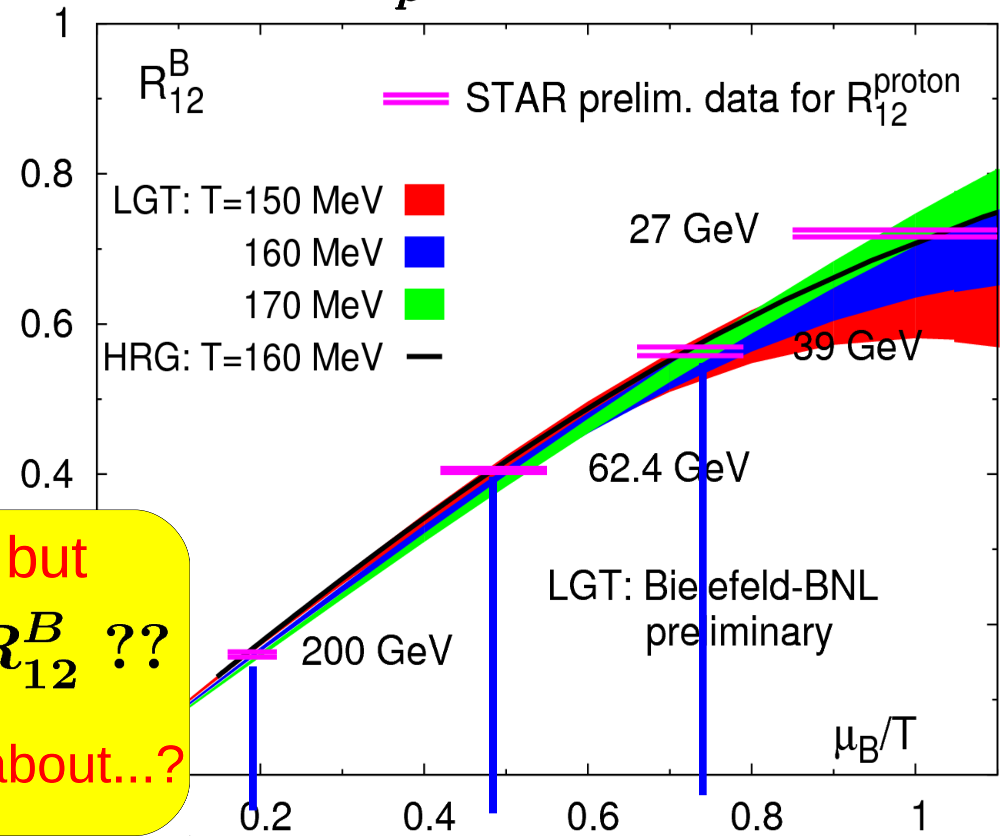


similar, but

$R_{12}^{p} = R_{12}^{B}$ ??

and what about...?

PHENIX data: J. Mitchell, Quark Matter 2012

STAR data: X. Luo, Quark Matter 2012

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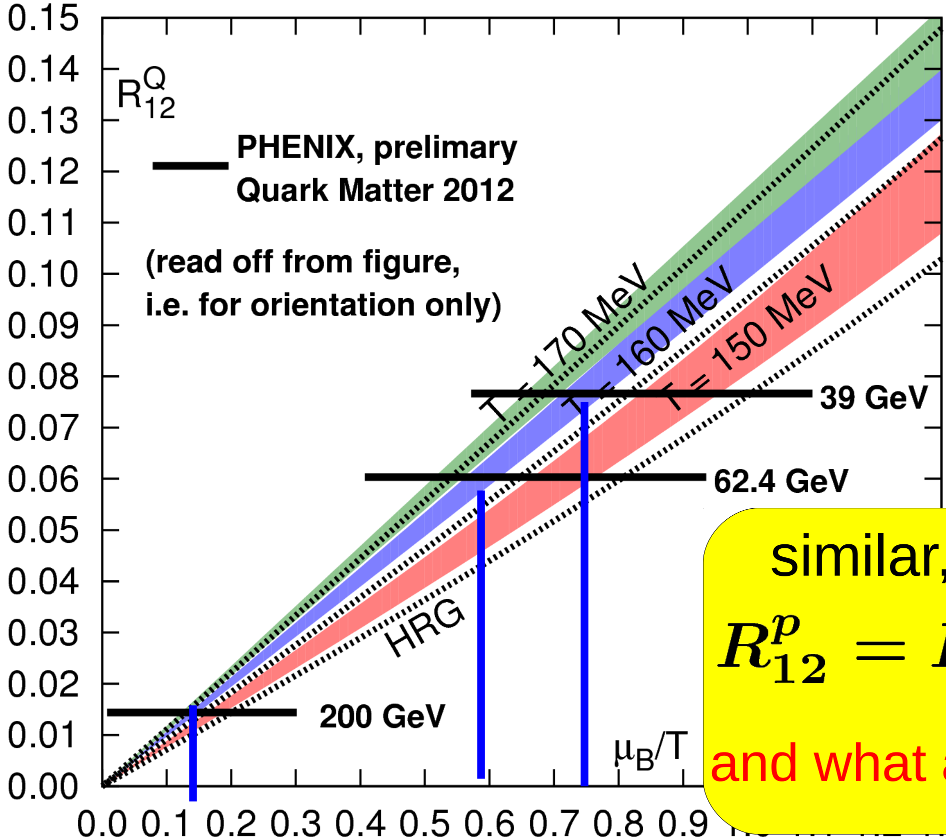
# Mean over variance: HIC vs. LGT

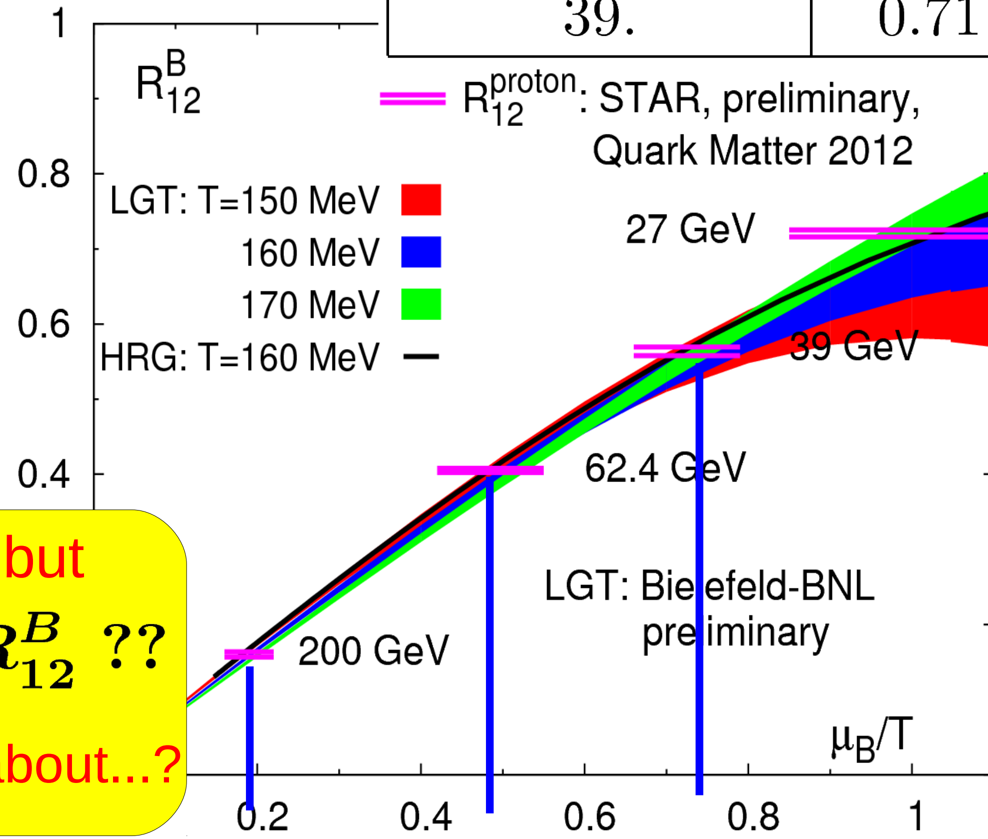
$$R_{12}^{Q} = \frac{M_Q}{\sigma_Q^2}$$

for comparison:
HRG values
(J. Cleymans et al.)

| $\sqrt{s_{NN}}$ [GeV] | $\mu_B/T$ |
|---|---|
| 200 | 0.15 |
| 62.4 | 0.45 |
| 39. | 0.71 |




PHENIX data: J. Mitchell, Quark Matter 2012




STAR data: X. Luo, Quark Matter 2012

similar, but
$R_{12}^{p} = R_{12}^{B}$ ??
and what about...?

– need data for $R_{31}^{Q} = S_Q \sigma_Q^3 / M_Q$ to extract $T_{freeze}^{cumulants}$

# Thermodynamic consistency

$$R_{12}^Q = \frac{M_Q}{\sigma_Q^2} \quad \text{and} \quad R_{12}^B = \frac{M_B}{\sigma_B^2}$$

should provide identical information on T and $\mu_B$

ratio of variances of electric charge and baryon number fluctuations

$$R_{QB} = \frac{R_{12}^Q}{R_{12}^B} = r \frac{\chi_2^B}{\chi_2^Q}$$

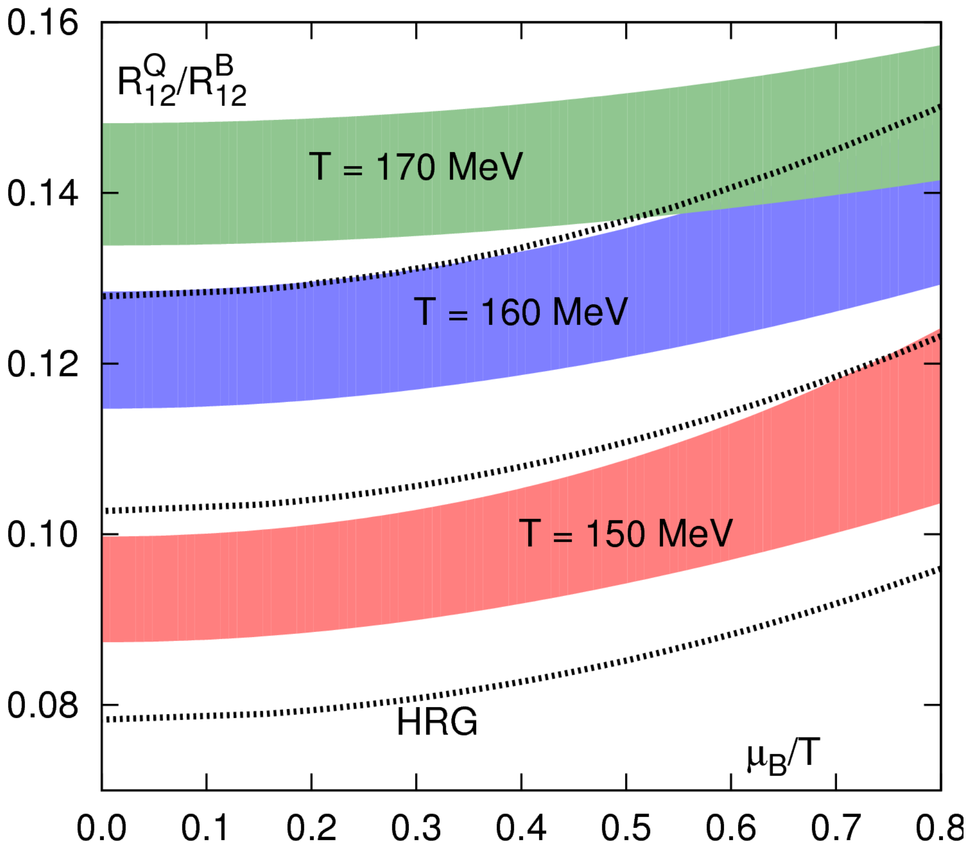
experimentally only net proton rather than net baryon number fluctuations are accessible

$$R_{nm}^B = R_{nm}^p \quad ???$$

STAR preliminary at 200 GeV:

$$\frac{R_{12}^Q}{R_{12}^{proton}} \simeq 0.06$$

a problem!!



# Conclusions

– the transition temperature and the freeze-out temperature agree within current statistical accuracy at zero and non-zero baryon chemical potential at least up to $\mu_B = 200\ \mathrm{MeV}$ which covers beam energies in heavy ion experiments down to about 20 GeV.

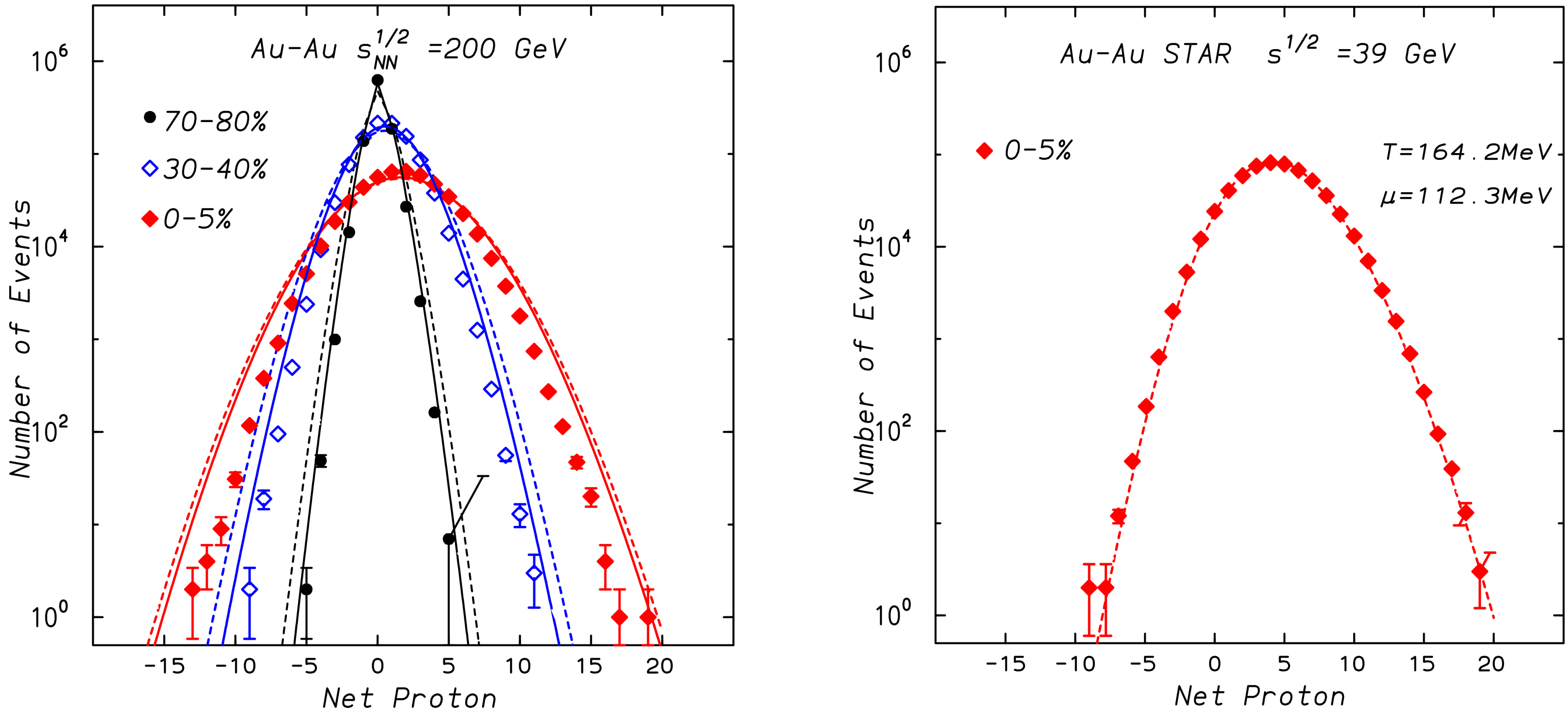
– higher order cumulants of net charge fluctuations are very promising observables to search for critical behavior and to make contact between (lattice) QCD and HIC experiments.

– through a comparison between equilibrium QCD calculations and HIC data on cumulants up to $6^{th}$ order it soon will become possible to test whether fluctuations of conserved charges can consistently be described by equilibrium thermodynamics with a unique set of freeze-out parameters.

– HRG and QCD calculations of freeze-out parameters seem to agree on the (10-20)% level – needs to be checked in more detail

# Back-up slides

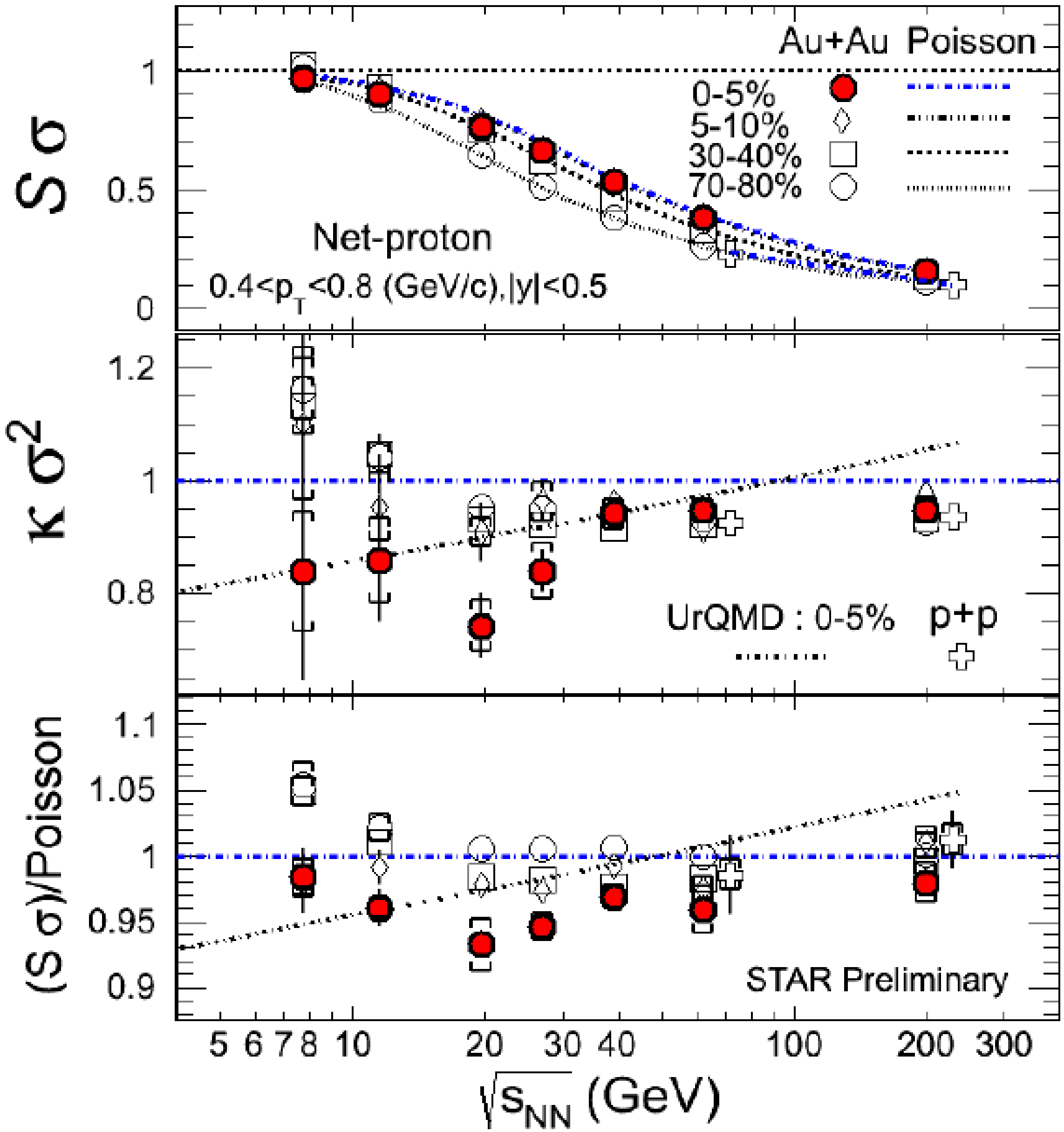
# Net-proton distributions




P. Braun-Munzinger, B. Friman, FK, K. Redlich, V. Skokov, Phys. Rev. C84, 064911 (2011)

curves: HRG model calculation using thermal parameter determined from freeze-out analysis of particle yields --- distribution is known as Skellam distribution

# STAR data on net proton number fluctuations




data deviate from HRG
(Poisson=HRG)

$$S\sigma \sim \mu_B/T$$

$$\kappa\sigma^2 < 1$$

where do HRG thermal parameters agree with QCD?

can the observed deviations be accomodated for in QCD equilibrium thermodynamics?

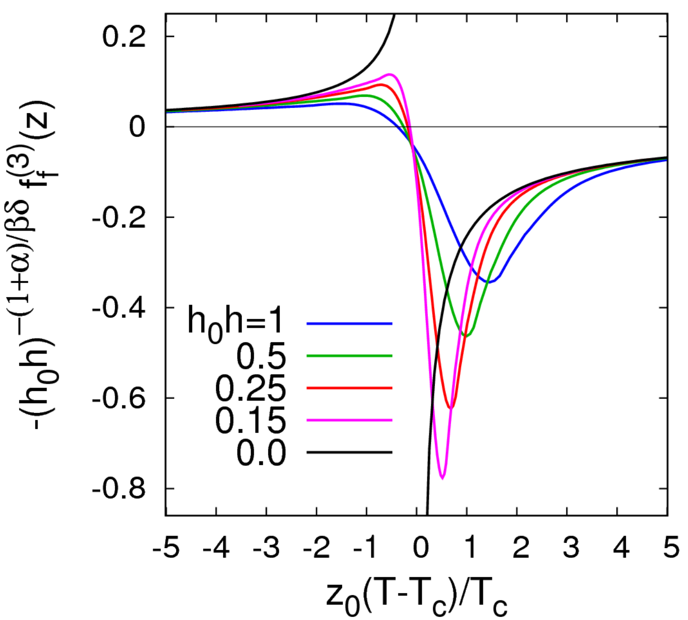
X. Luo, Quark Matter 2012

# Generalized Quark number susceptibilities

- quark number susceptibilities are sensitive to critical behavior described by (thermal) derivatives of the singular part of the free energy

$$\chi_{B,0}^{(n)} \sim h^{(2-\alpha-n/2)/\beta\delta} f_f^{(n/2)}(z) \ , \quad h \sim m_q/T \ , \ \alpha < 0$$

- for $\mu_B = 0, \ m_q = 0$ first divergent susceptibility for n=6



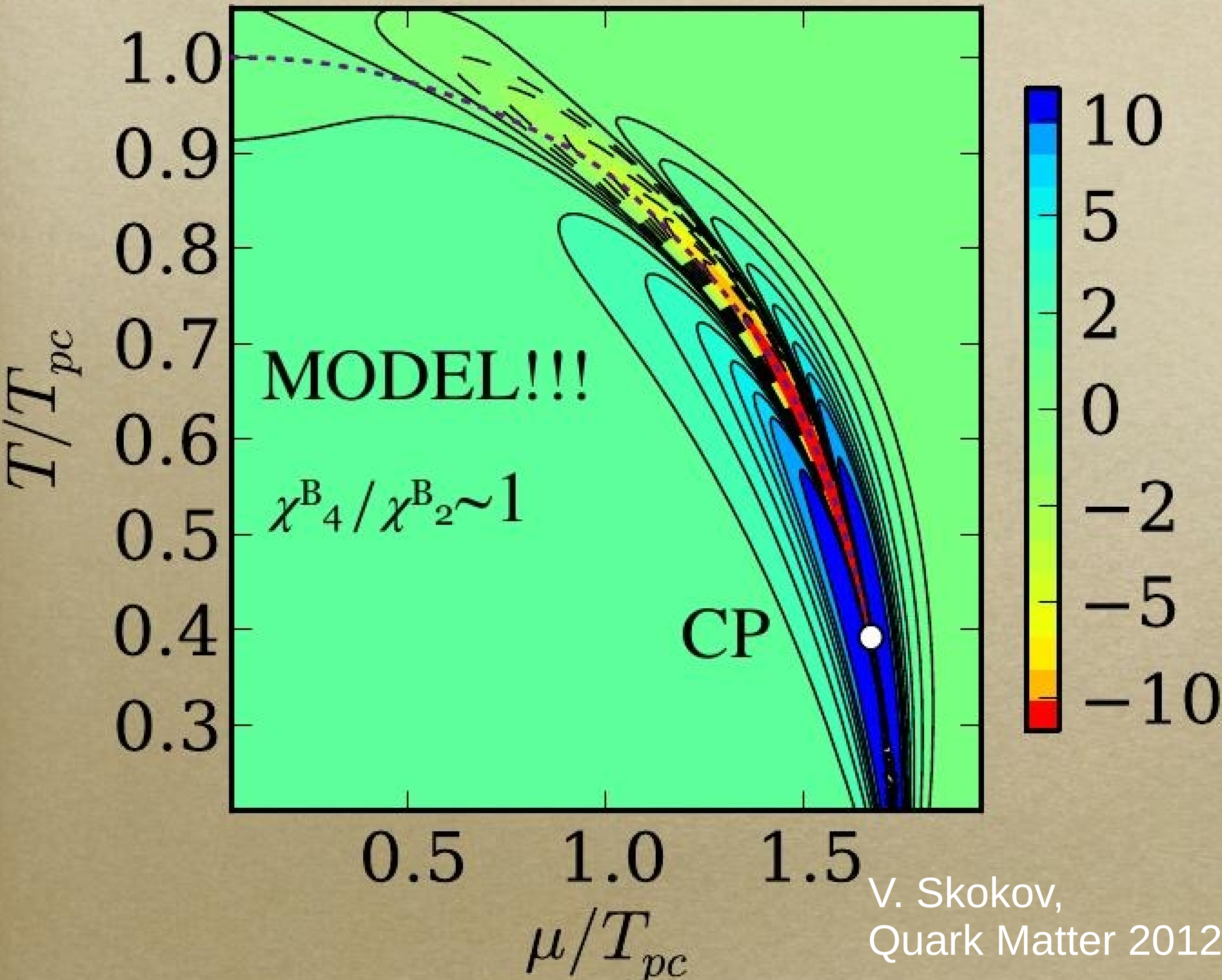

$$\chi_{B,0}^{(6)} \sim f_f'''(z)$$

third derivative of the singular part of the free energy

→ O(4) scaling function

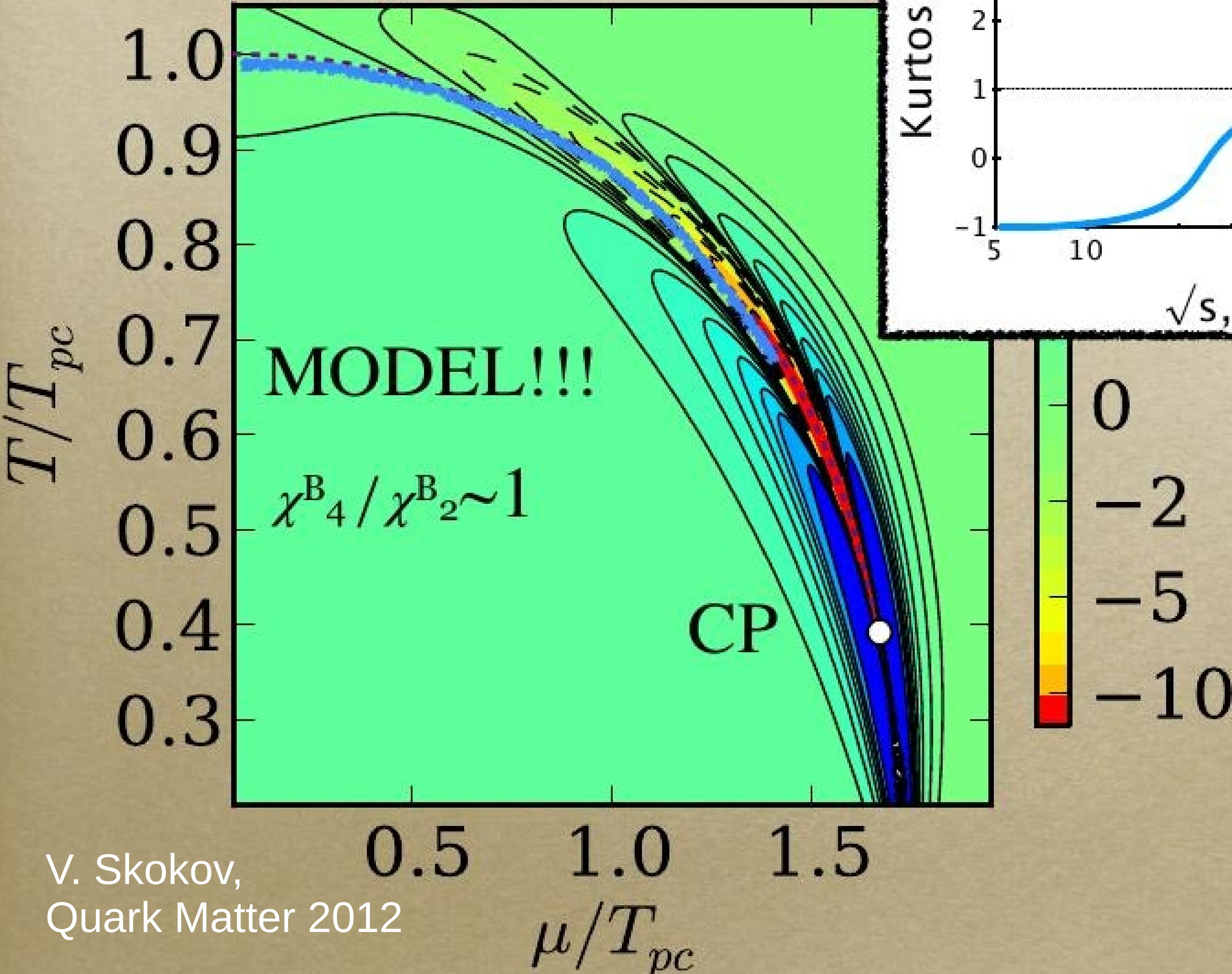
B. Friman, FK, K. Redlich, V. Skokov, arXiv:1103.3511

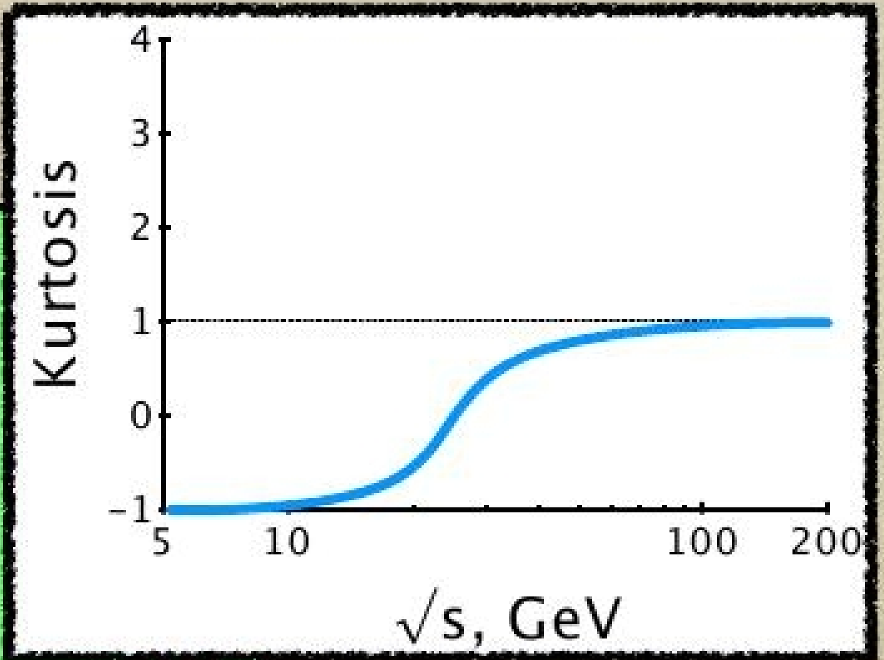
# Chiral model and negative $\chi^B_4/\chi^B_2$:




V. Skokov,
Quark Matter 2012

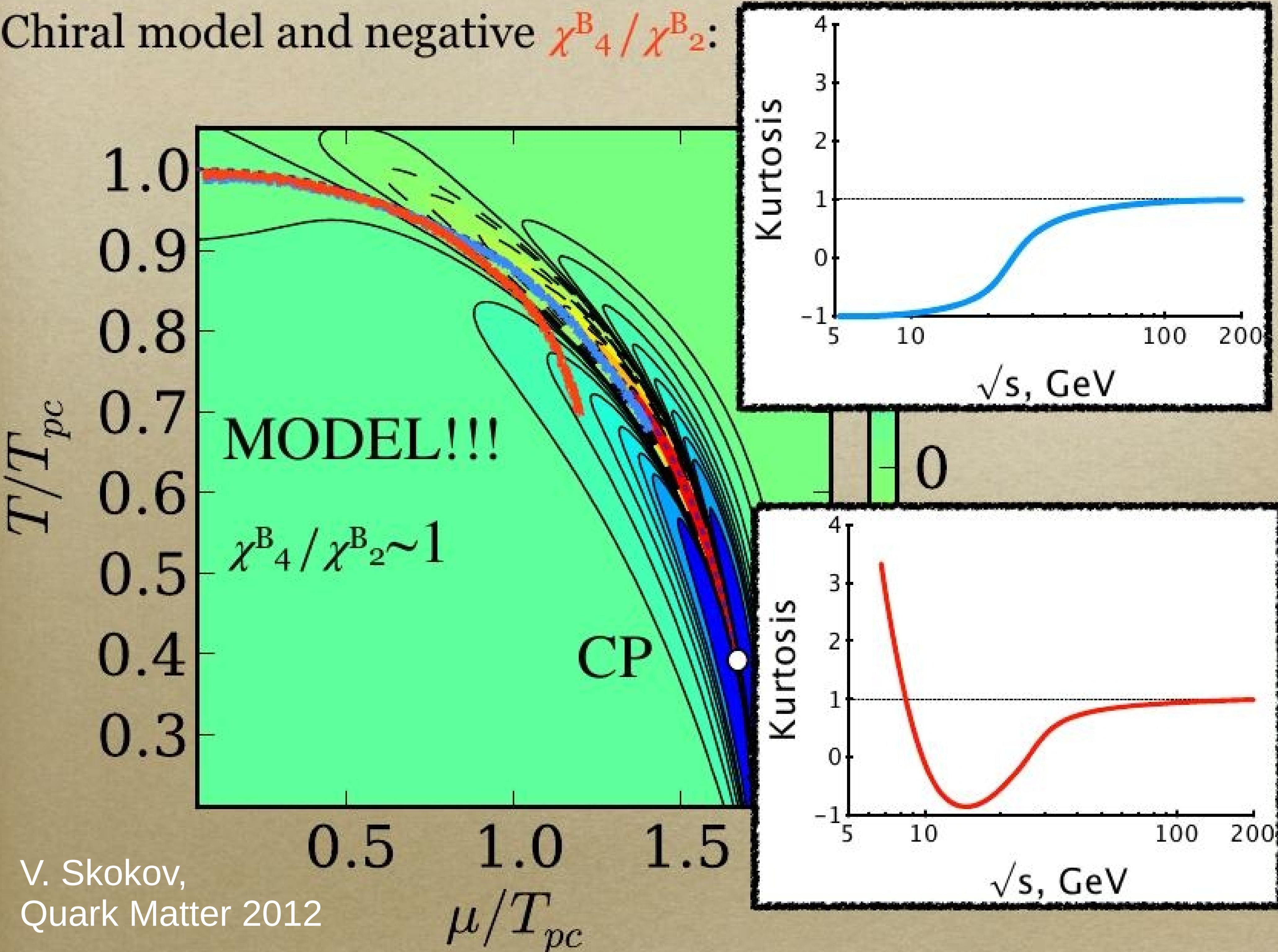
Chiral model and negative $\chi^B{}_4/\chi^B{}_2$:




V. Skokov,
Quark Matter 2012

## Chiral model and negative $\chi^B_4/\chi^B_2$:




V. Skokov,
Quark Matter 2012

# NLO expansion at high and low T

– Is it surprising that NLO expansions work that well for constraining the chemical potentials?

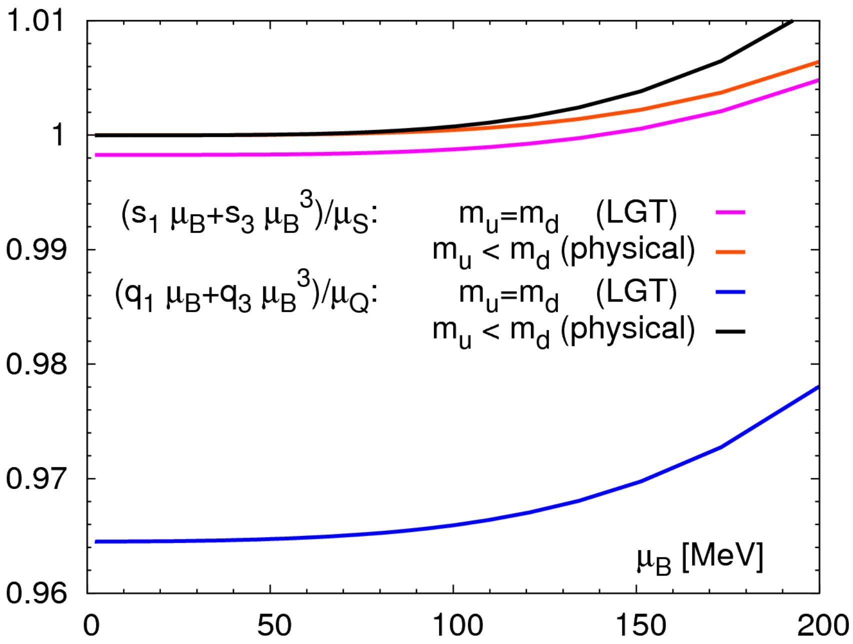
$T \to \infty$ : ideal quark gas, m=0

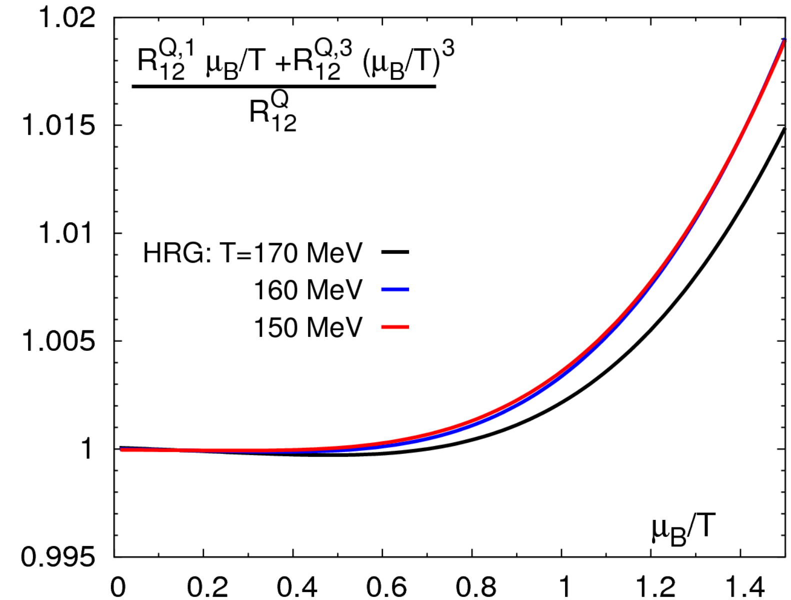
$$\left.\frac{p}{T^4}\right|_{\infty} = \frac{7 N_f \pi^2}{60} + \frac{N_f}{2}\left(\frac{\mu_q}{T}\right)^2 + \frac{N_f}{4\pi^2}\left(\frac{\mu_q}{T}\right)^4$$

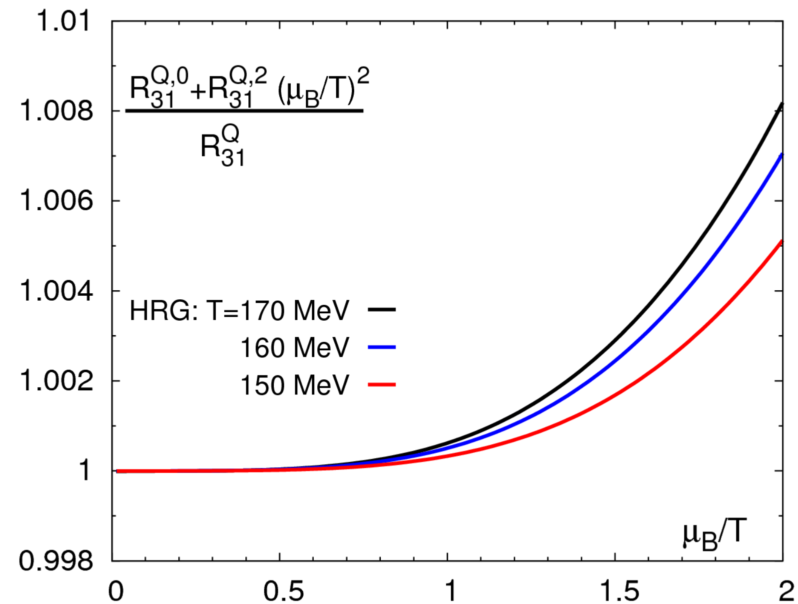
NLO is exact (also at $\mathcal{O}(g^2)$ )

$T < T_c$ : get guidance from HRG model

# NLO Taylor expansions in the HRG




NLO expansions work well in the HRG model (1% level) up to $\mu_B/T \simeq (1.3-2)$

this covers RHIC experiments down to $\sqrt{s_{NN}} \simeq 20\ \mathbf{GeV}$

# Our current working hypothesis on Critical point & conserved charge fluctuations

- at the critical point fluctuations (of conserved charges) become large
- if freeze-out happens close to the QCD crossover line (as it seems to be the case at muB=0) the existence of a critical point may show up in 'critical fluctuations'
- higher order cumulants of conserved charges become increasingly sensitive to critical behavior already in the crossover region or even at muB=0
- higher order cumulants calculated at muB=0 can provide insight into the existence of a critical point and allow to estimate its location

higher order cumulants of net charge fluctuations can be measured in HIC experiments AND can be calculated in QCD (as well as in the HRG model)