A No-Go Theorem for Critical Phenomena in QCD at finite temperature and density

Yoshimasa Hidaka
(RIKEN)

Based on Y.H. and N. Yamamoto, Phys. Rev. Lett. 108, 121601 (2012)

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What can we say about the QCD phase diagram from QCD inequalities?

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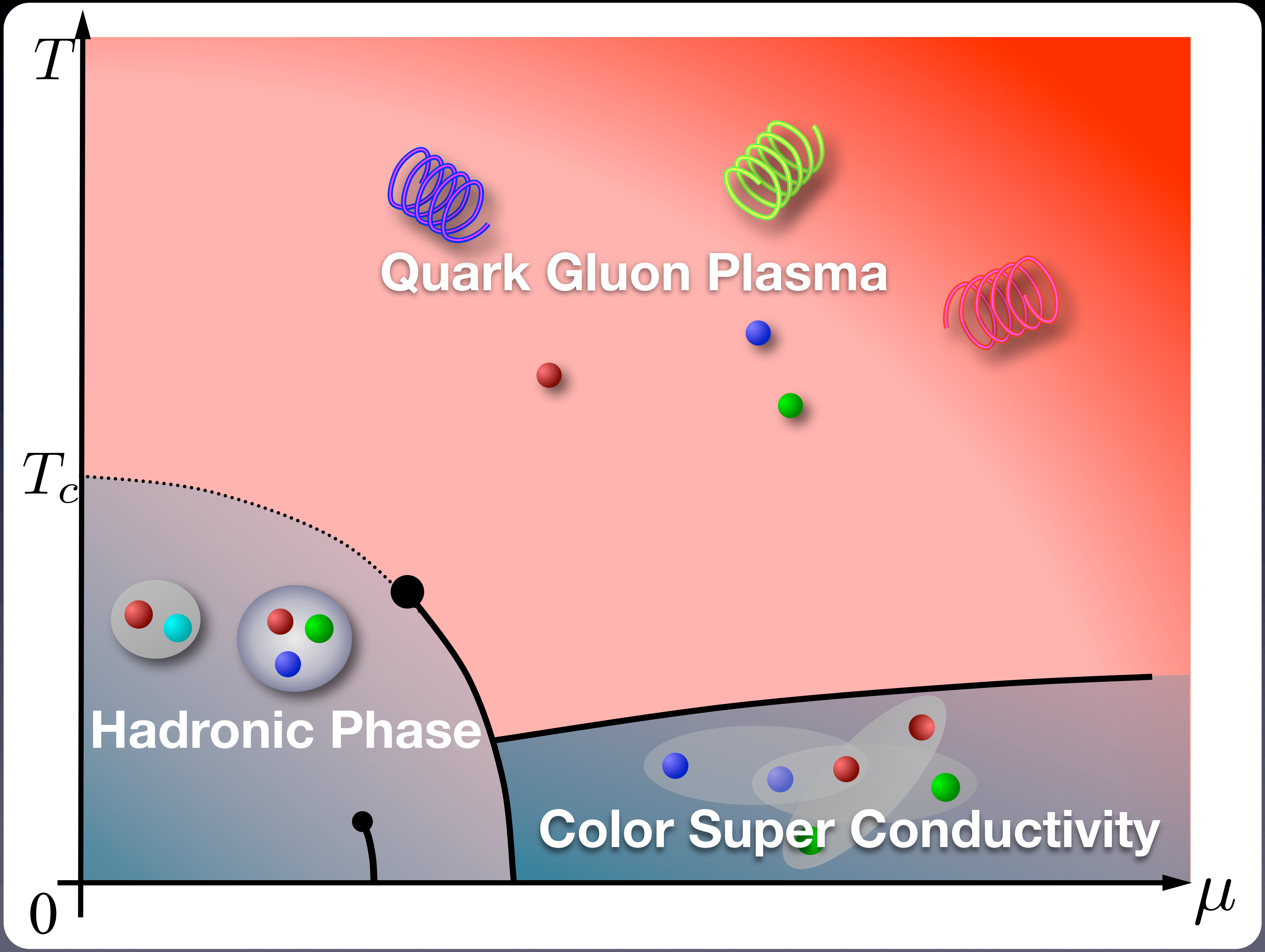
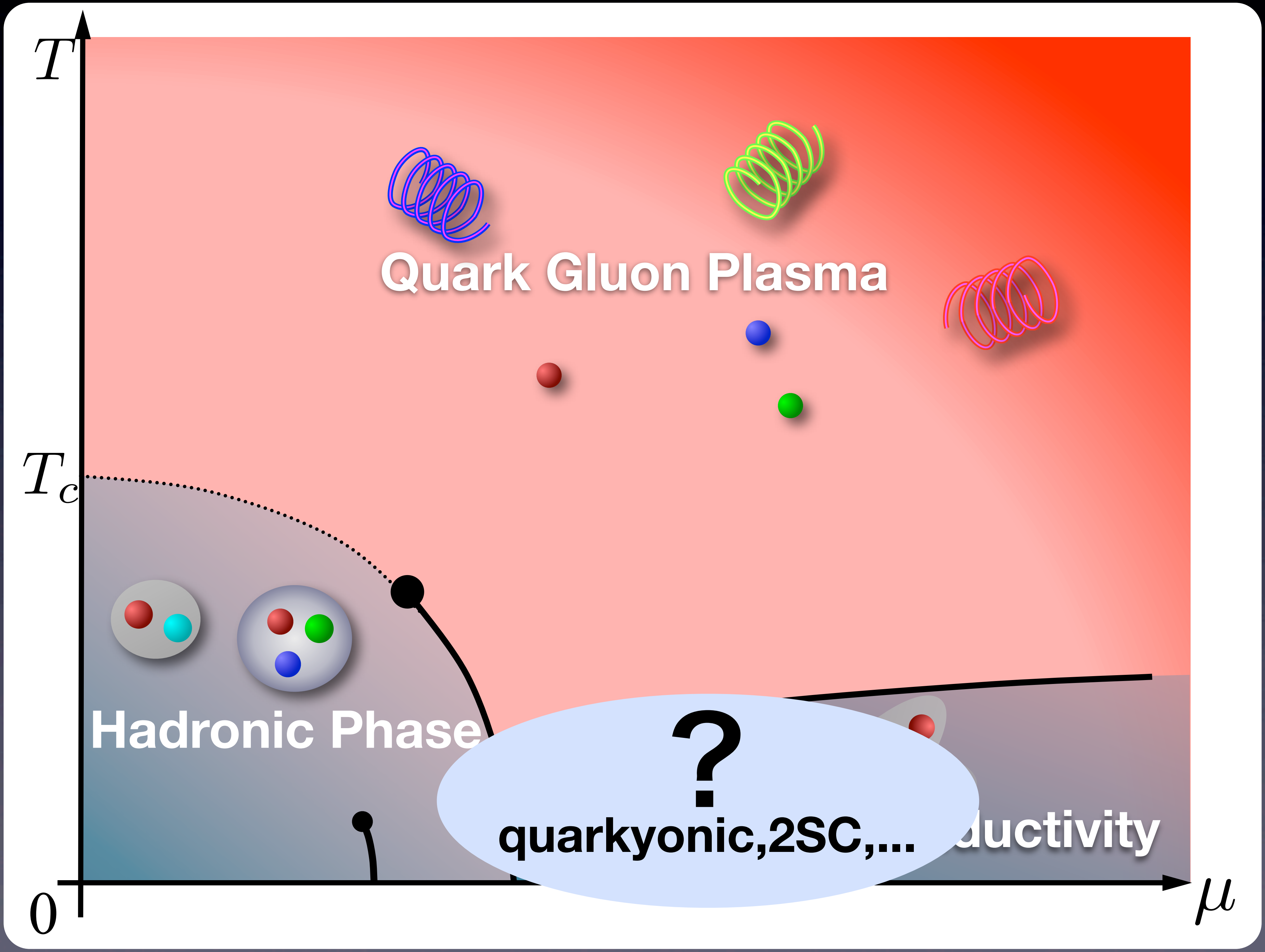
- QCD critical point at finite T and μ
- QCD at finite B and T

Where is the QCD critical point?

Phase diagram of QCD



Phase diagram of QCD



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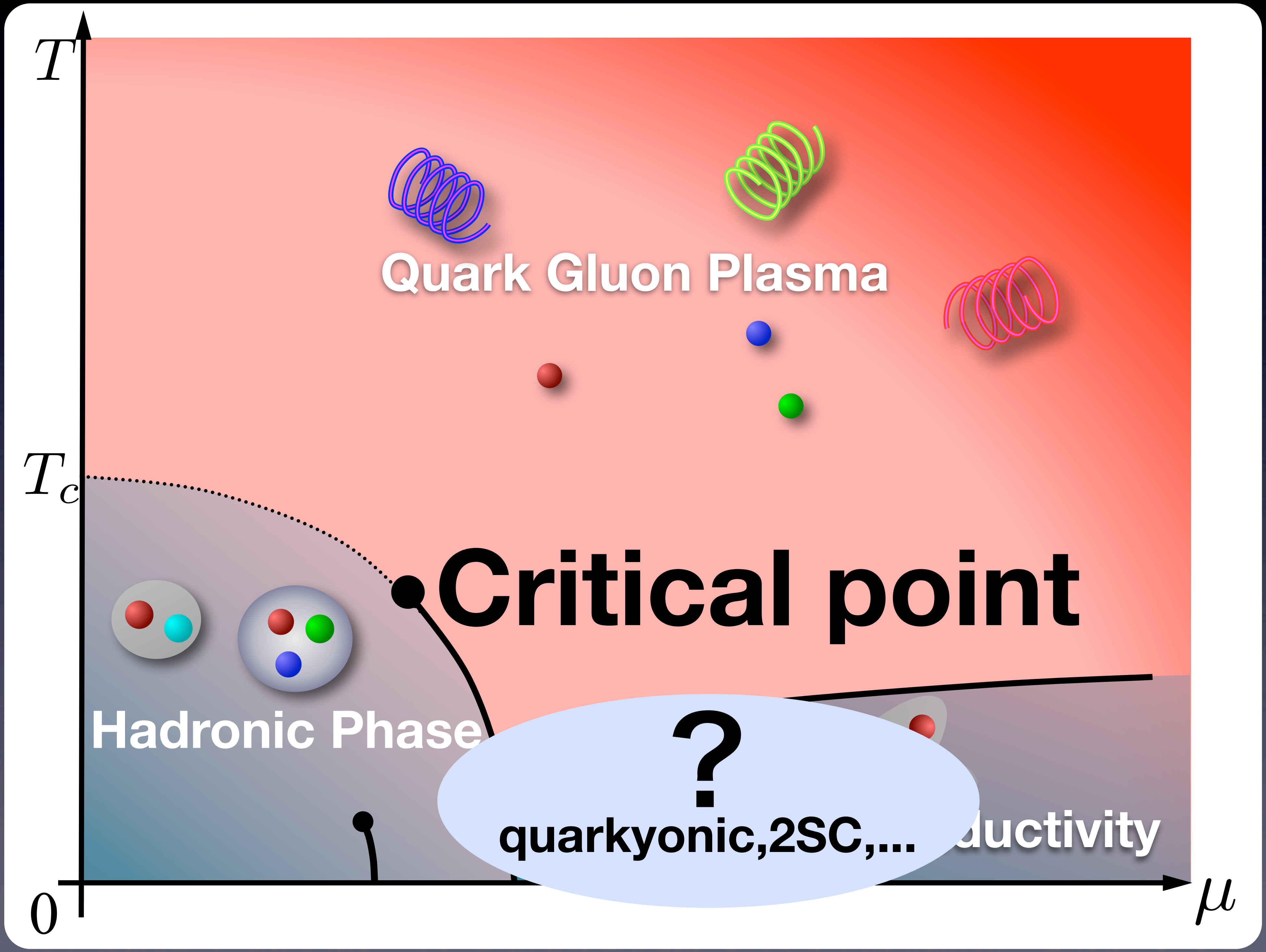

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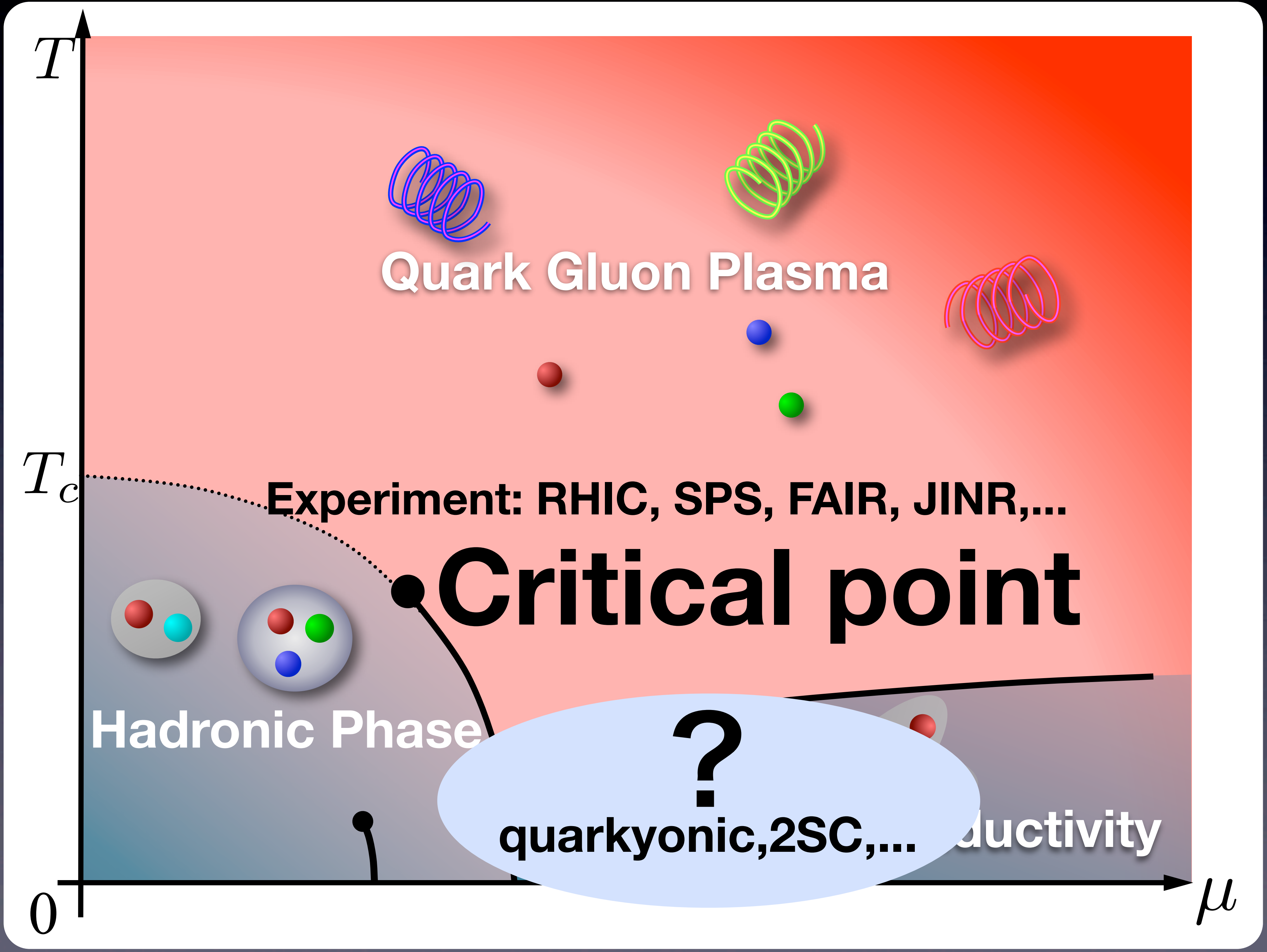
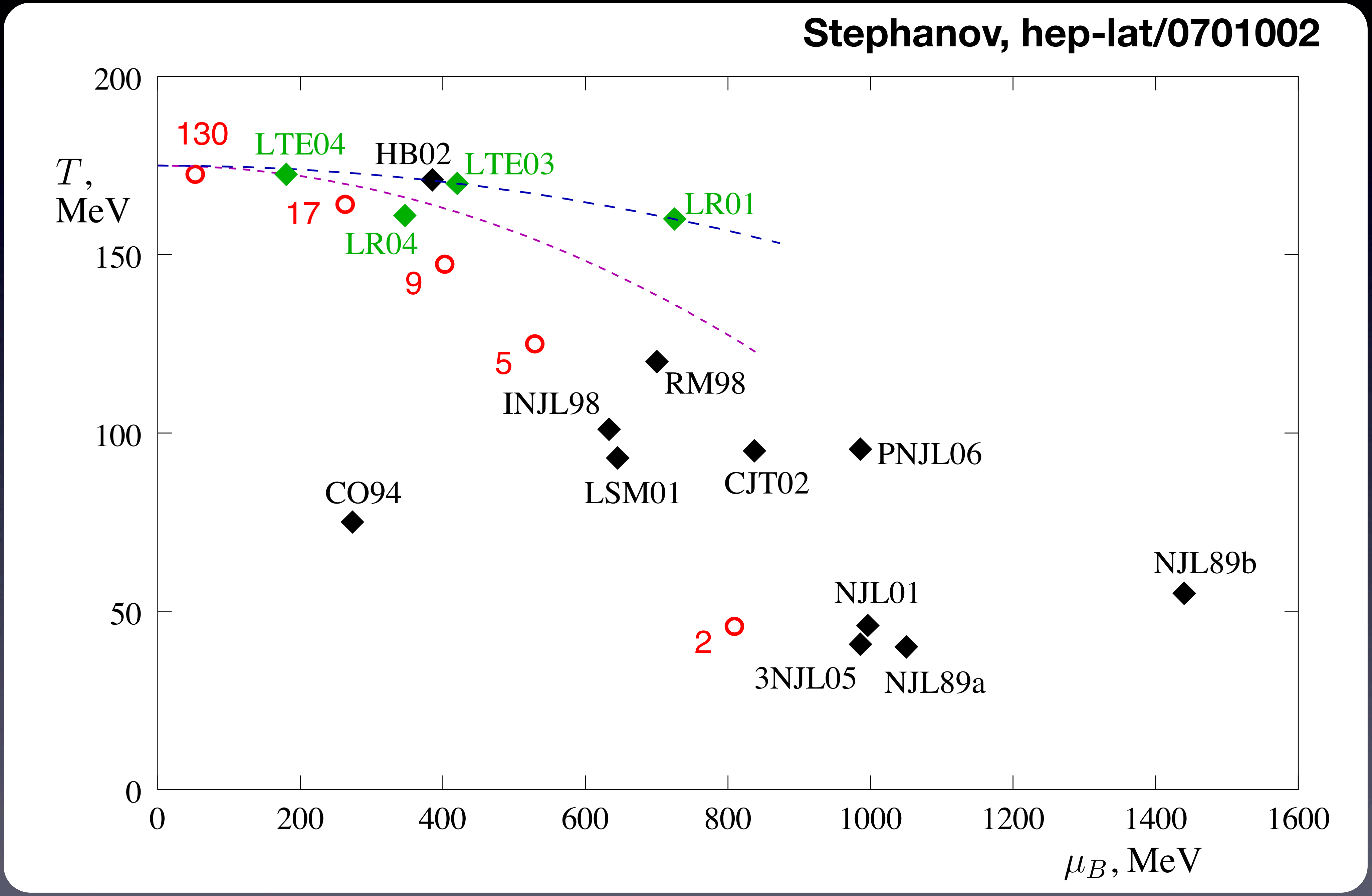
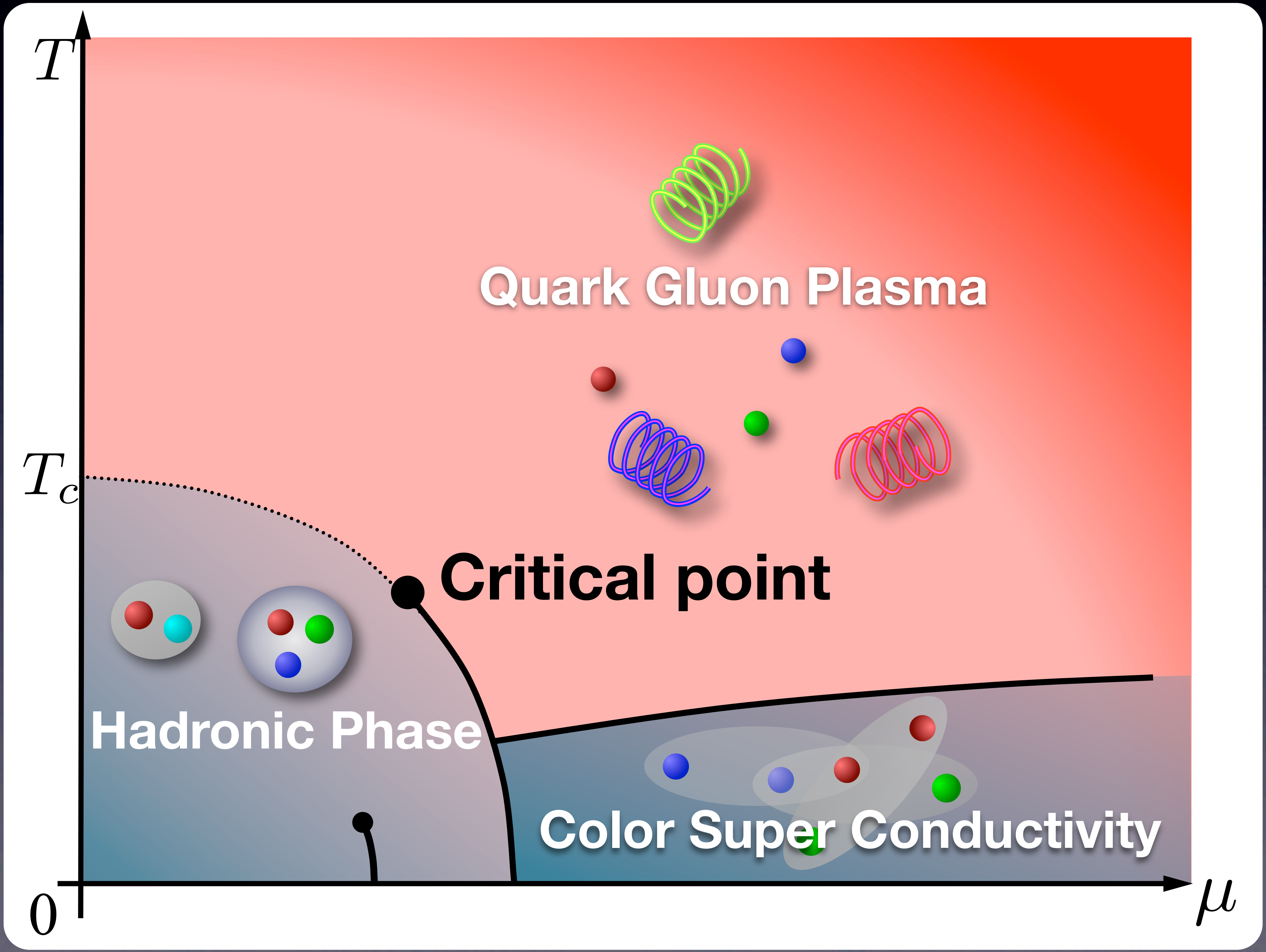


QCD critical point



We want to determine the allowed region.



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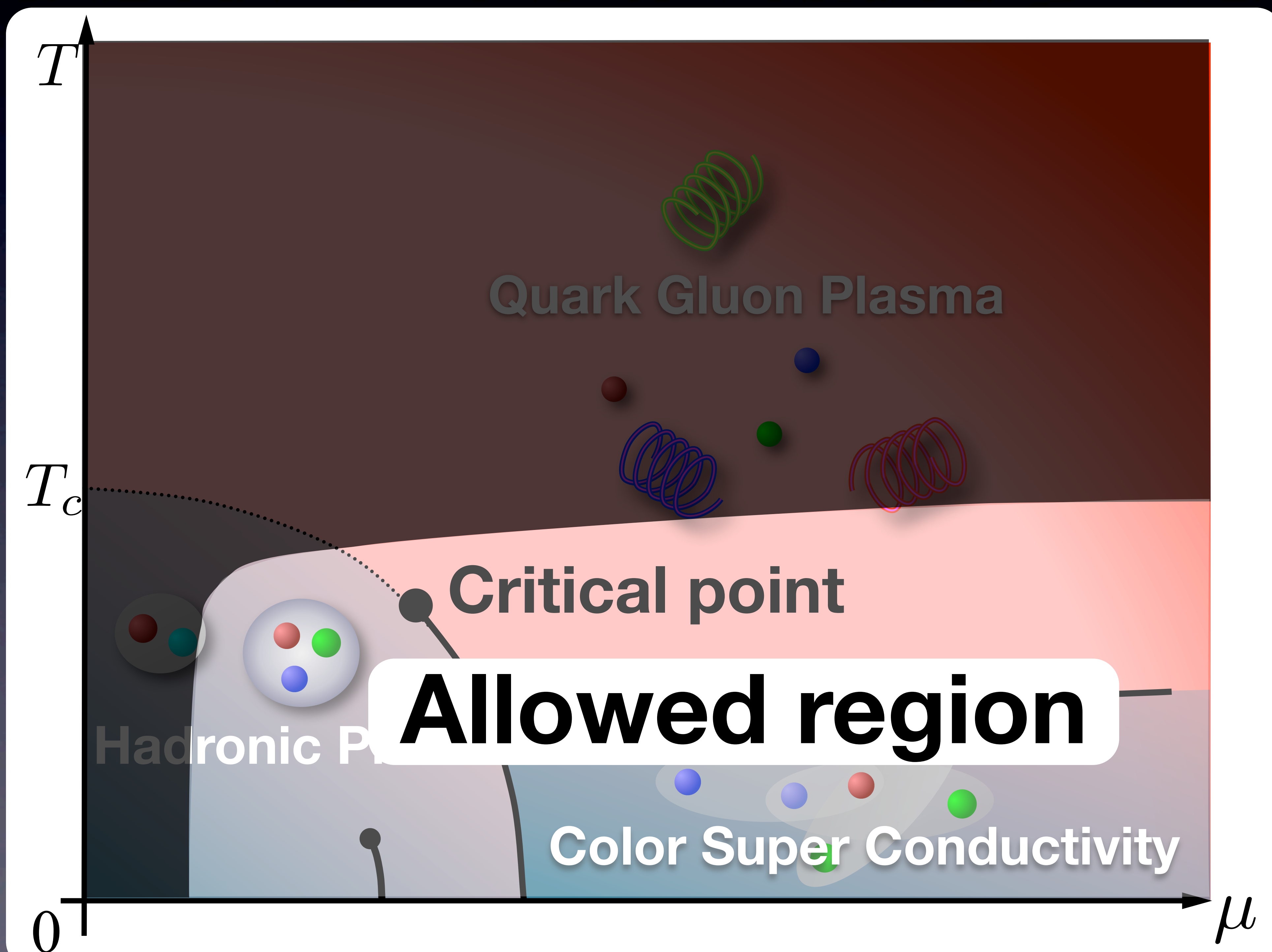

Fluctuation of the order parameter

$$\langle \delta\sigma(\boldsymbol{x})\delta\sigma(0)\rangle \sim \exp(-|\boldsymbol{x}|m_\sigma)$$

At the critical point

$$m_\sigma \to 0$$

QCD inequality

QCD inequality

+ some approximations

- Neglecting disconnected diagrams
- Neglecting quark loops mixing flavors

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Large-N_c QCD satisfies both approximations!

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Large-N_c QCD satisfies both approximations!

Several models such as NJL with mean field approximation, random matrix also satisfy.

QCD Inequality

Weingarten ('83), Witten ('83), Nussinov ('84),
Espriu, Gross, Wheater ('84)

At *T=0, μ=0*

For flavor nonsinglet channel

$$m_\Gamma \geq m_\pi$$

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No SSB of isospin and baryon symm.

Vafa-Witten ('84)

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Dirac operator $$D = \gamma_\mu(\partial_\mu + igA_\mu)$$

Anti-Hermite $$D^\dagger = -D$$

Chiral symmetry $$\gamma_5 D \gamma_5 = -D$$

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$$\mathcal{D} = D + m$$

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$$\Rightarrow \quad \mathcal{D} = D + m$$

$$\det \mathcal{D} \geq 0$$

For isospin chemical potential

$$\tau_1 \gamma_5 \mathcal{D} \gamma_5 \tau_1 = \mathcal{D}^\dagger \qquad \mathcal{D}(\mu_I) = D + \frac{\mu_I}{2}\gamma_0 \tau_3 + m$$

Alford, Kapustin and Wilczek ('99)

QCD Inequality

Weingarten ('83), Witten ('83), Nussinov ('84), Espriu, Gross, Wheater ('84)

Flavor nonsinglet operator: $M_\Gamma(x) = \bar{\psi}\Gamma\psi$

$$\langle M_\Gamma(x) M_\Gamma^\dagger(y)\rangle_{\psi,A} = -\langle \mathrm{tr}[S_A(x,y)\Gamma S_A(y,x)\bar{\Gamma}]\rangle_A$$

Cauchy–Schwarz inequality

$$\langle \mathcal{O}_1 \mathcal{O}_2 \rangle \leq \sqrt{\langle \mathcal{O}_1 \mathcal{O}_1^\dagger \rangle \langle \mathcal{O}_2 \mathcal{O}_2^\dagger \rangle}$$

QCD Inequality

Flavor nonsinglet operator

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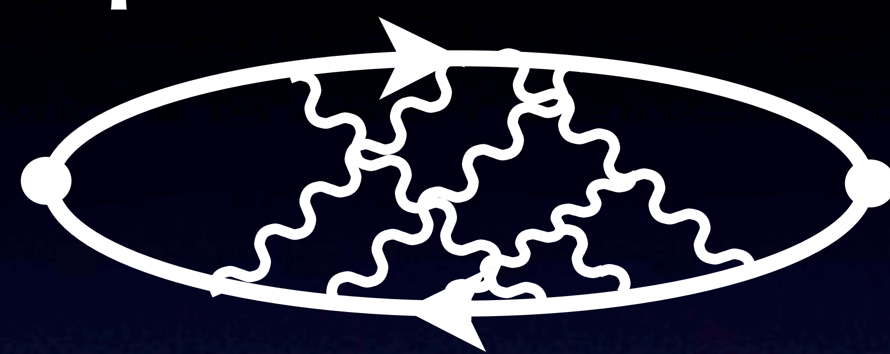


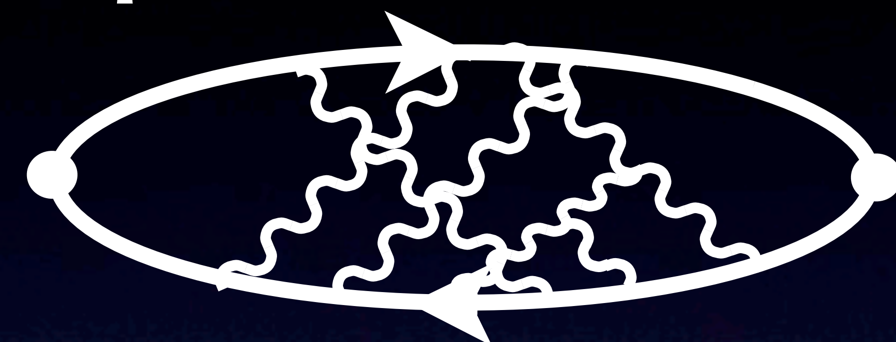
$$= -\langle \mathrm{tr}[S_A(x,y)\Gamma S_A(y,x)\bar{\Gamma}]\rangle_A$$

$$\leq \langle \mathrm{tr}[S_A(x,y) S_A^\dagger(y,x)]\rangle$$

$$= \langle M_\pi(x) M_\pi^\dagger(y)\rangle_{\psi,A}$$

QCD Inequality

Flavor nonsinglet operator

$$\langle M_\Gamma(x) M_\Gamma^\dagger(y)\rangle_{\psi,A} = $$ 

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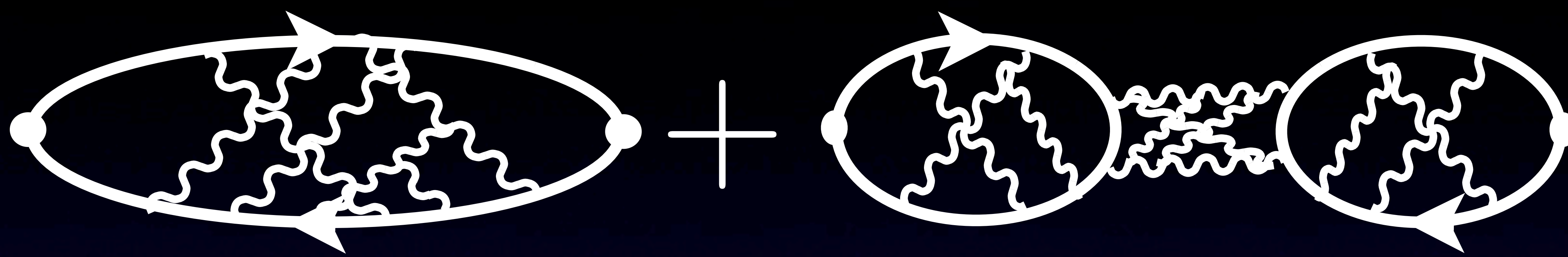
$$= \langle M_\pi(x) M_\pi^\dagger(y)\rangle_{\psi,A}$$

$$\Rightarrow \quad m_\Gamma \geq m_\pi$$

$$\langle M_\Gamma(x) M_\Gamma^\dagger(y)\rangle_{\psi,A} \sim \exp(-m_\Gamma |x-y|)$$

Flavor singlet operator



$$\langle M_{\Gamma}(x)M^{\dagger}_{\Gamma}(y)\rangle_{\psi,A} = -\langle \mathrm{tr}[S_A(x,y)\Gamma S_A(y,x)\bar{\Gamma}]\rangle_A$$

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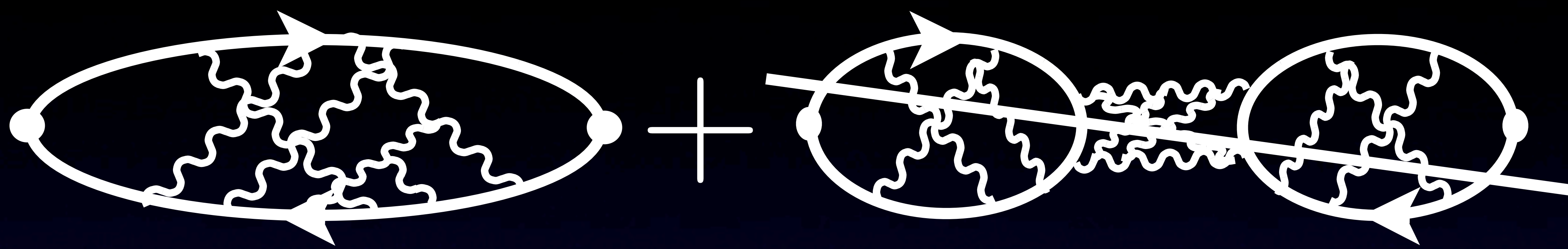

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If disconnected diagram is neglected, $m_\sigma \geq m_\pi$

No second order phase transition as long as $m_\pi \neq 0$

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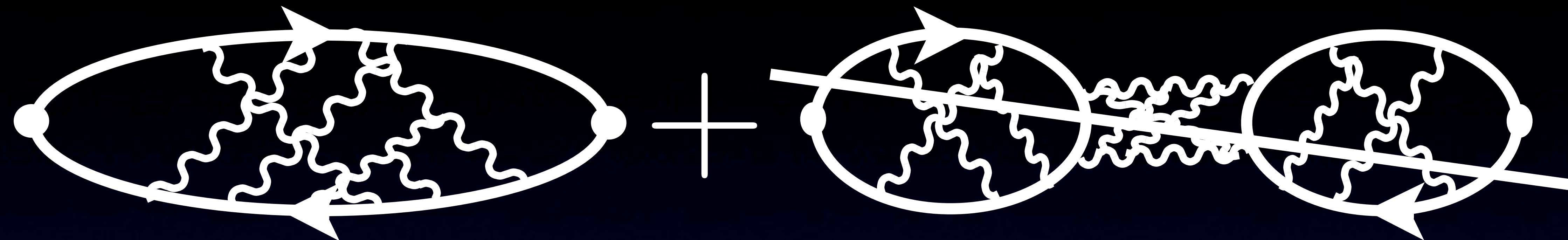

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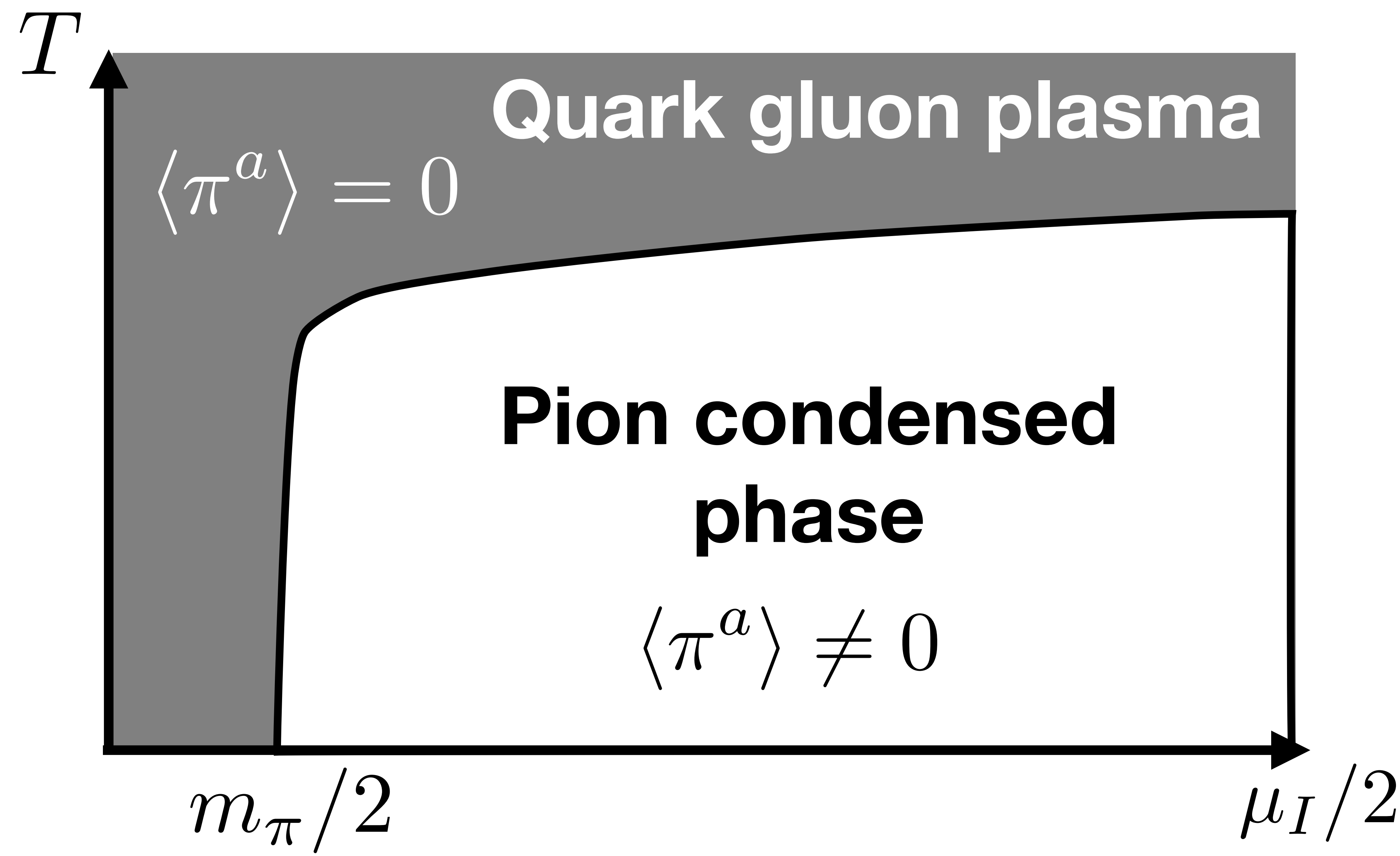
QCD inequality also works at finite *T* and μ_I.

Son and Stephanov ('01)

QCD phase diagram at μ_I

No critical point out side of the pion condensed phase

YH, Yamamoto('11)



QCD phase diagram at *μ*

QCD inequality does not work at *μ*...

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If some quark loops mixing flavors are negligible,

i.e., complex phase is negligible,

$$P = p(\mu_u^2, \mu_d^2) + \cancel{p_{\rm mix}(\mu_u\mu_d, \mu_u^2, \mu_d^2)}$$

OK, at large-N_c, outside of pion condensed phase

QCD phase diagram at *μ*

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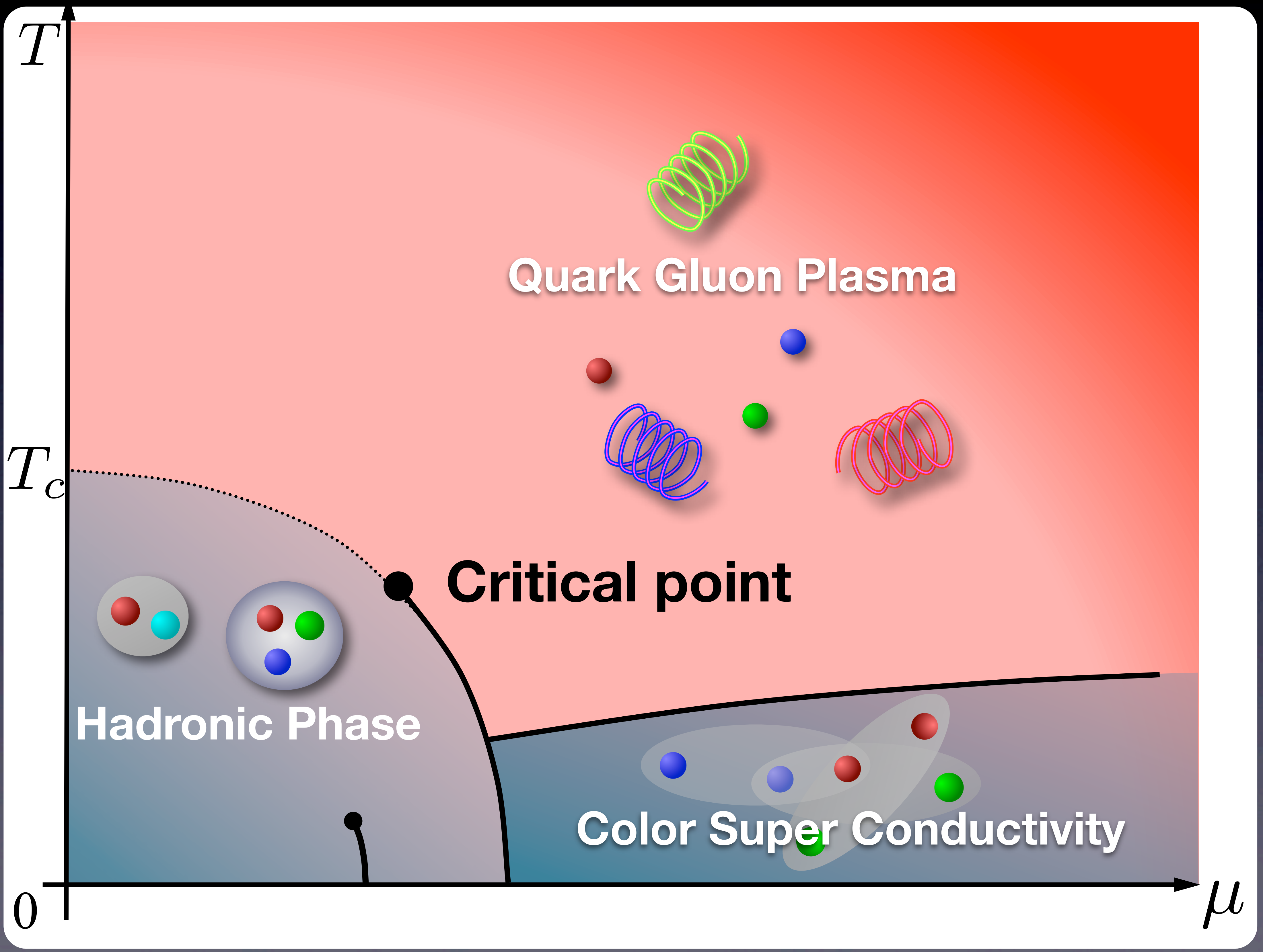
OK, at large-N_c, outside of pion condensed phase

The phase structure at finite *μ* $\approx$ **The phase structure at finite μ_I**

At large N_c, Hanada and Yamamoto ('11)

Phase diagram of QCD

YH, Yamamoto ('11)



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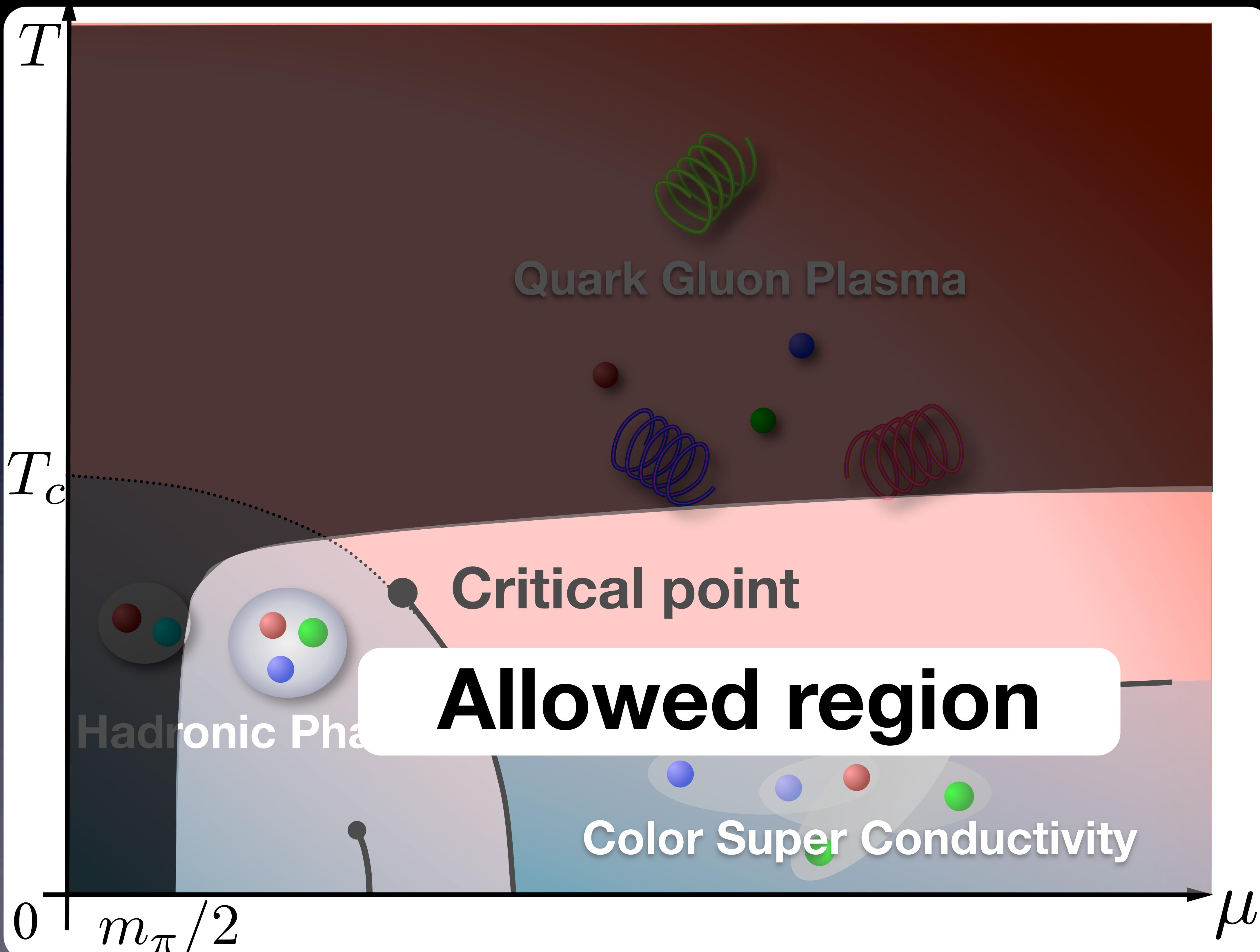
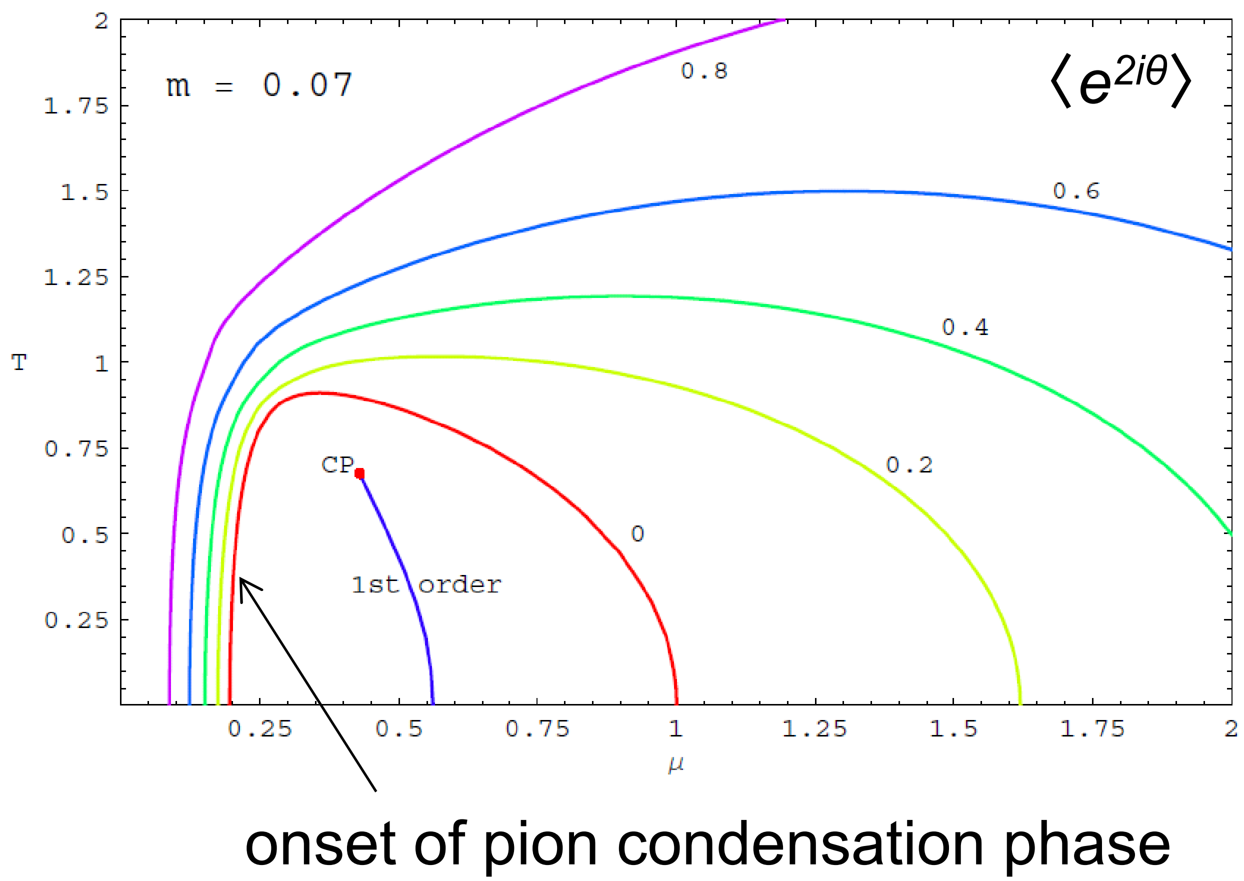


Model results

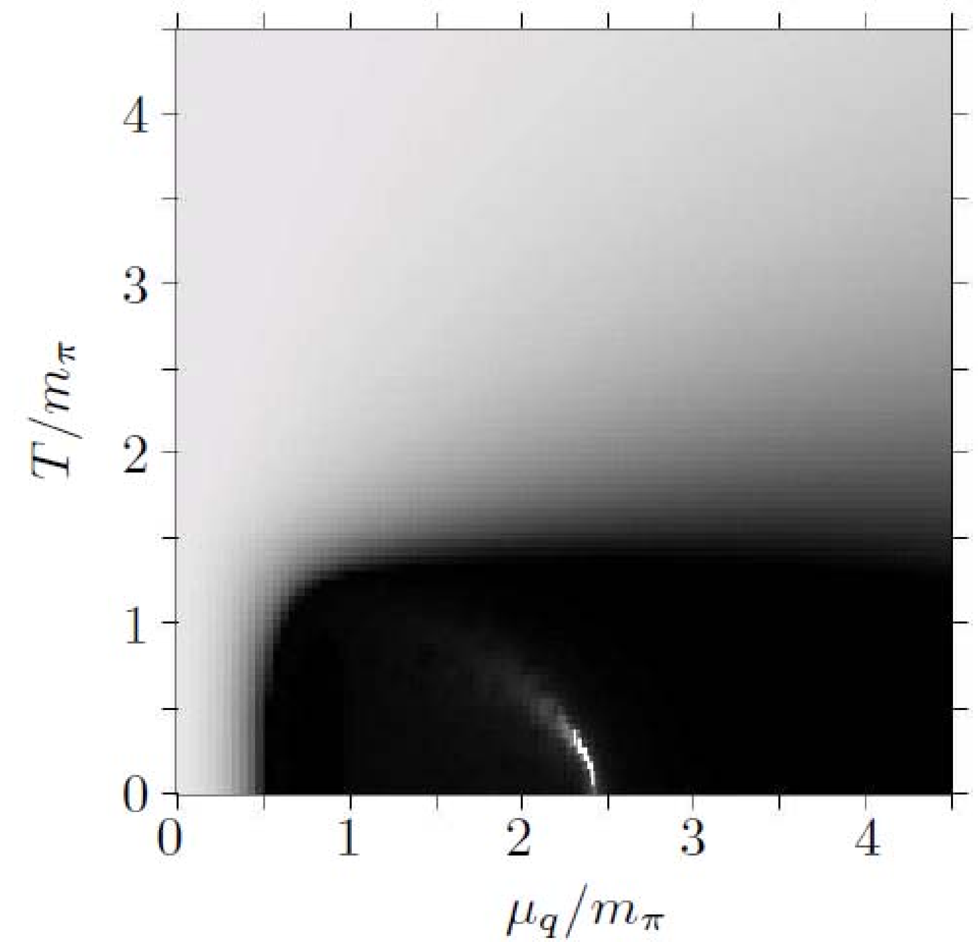
Random matrix model

Han, Stephanov (08)



NJL model

Andersen, Kyllingstad, Splittorff ('09)



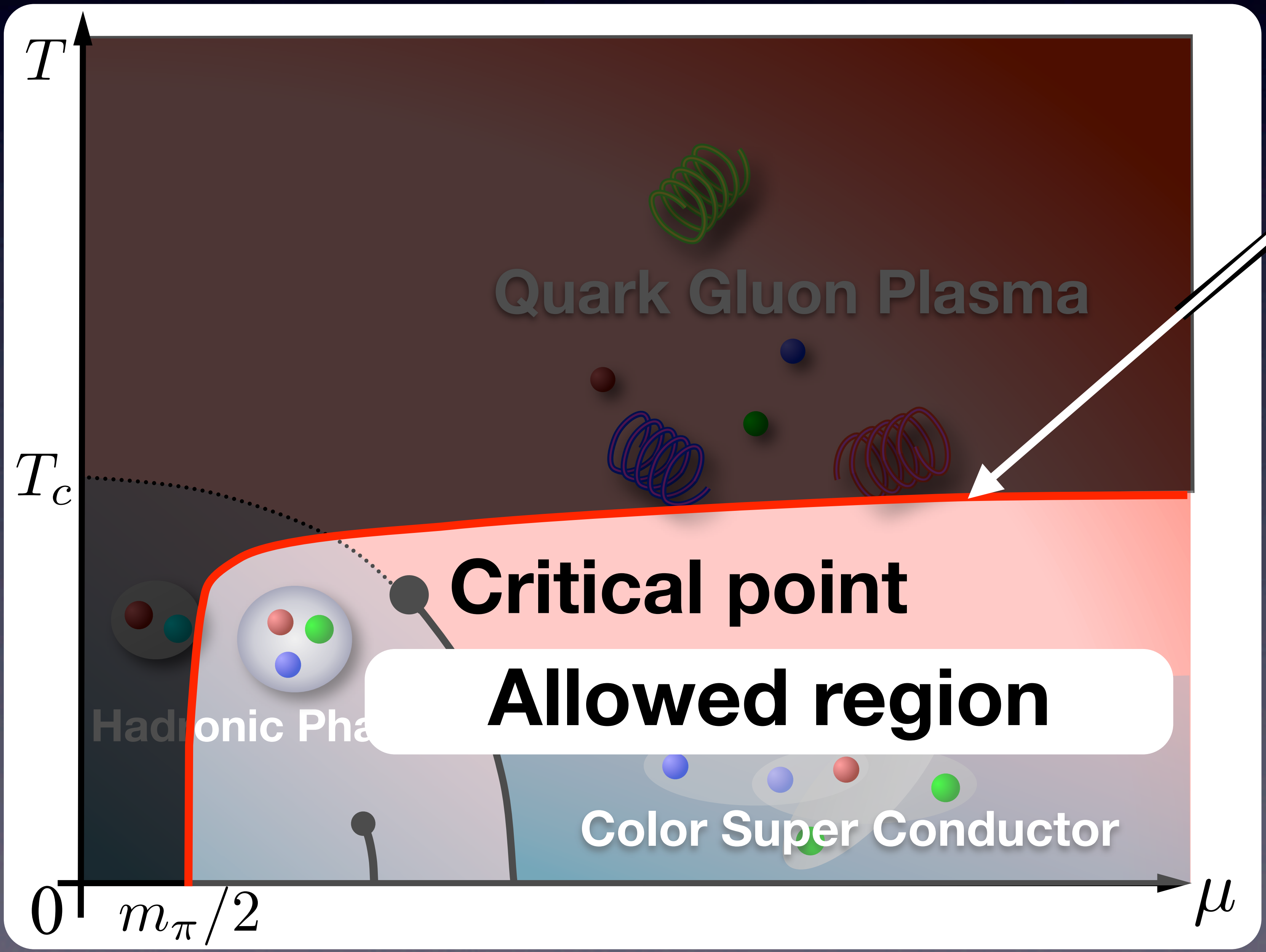
Similar result in PNJL model

Sakai, Sasaki, Kouno, Yahiro ('10)

Summary I

No critical point out side of the pion condensed phase.

(if quark loops and disconnected diagram are suppressed.)



Large N_c QCD, OK.

Lattice QCD can determine the boundary.

cf. Kogut, Sinclair ('04), ('06), ('07),
de Forcrand, Kratochvila ('06)
de Forcrand, Stephanov, Wenger ('07)
Detmold, Orginos, Shi ('12)

Lattice simulation in the pion condensed phase is a challenging problem.

We need to estimate contributions of disconnected diagrams.

Strong Magnetic field

Strong Magnetic field

Heavy ion collisions:

RHIC: $\sim m_\pi^2$

LHC: $\sim 10 m_\pi^2$

Magnetar:

$\sim 0.01 m_\pi^2$

The early universe:

$\sim m_W^2 \sim 10^5 m_\pi^2$

Interesting phenomena at finite *B*

Chiral magnetic effect:

Kharzeev, McLerran, Warringa ('07), Kharzeev, Fukushima, Warringa ('08),...

$$J_z = \frac{eBL^3}{2\pi}\mu_5$$

Magnetic catalysis:

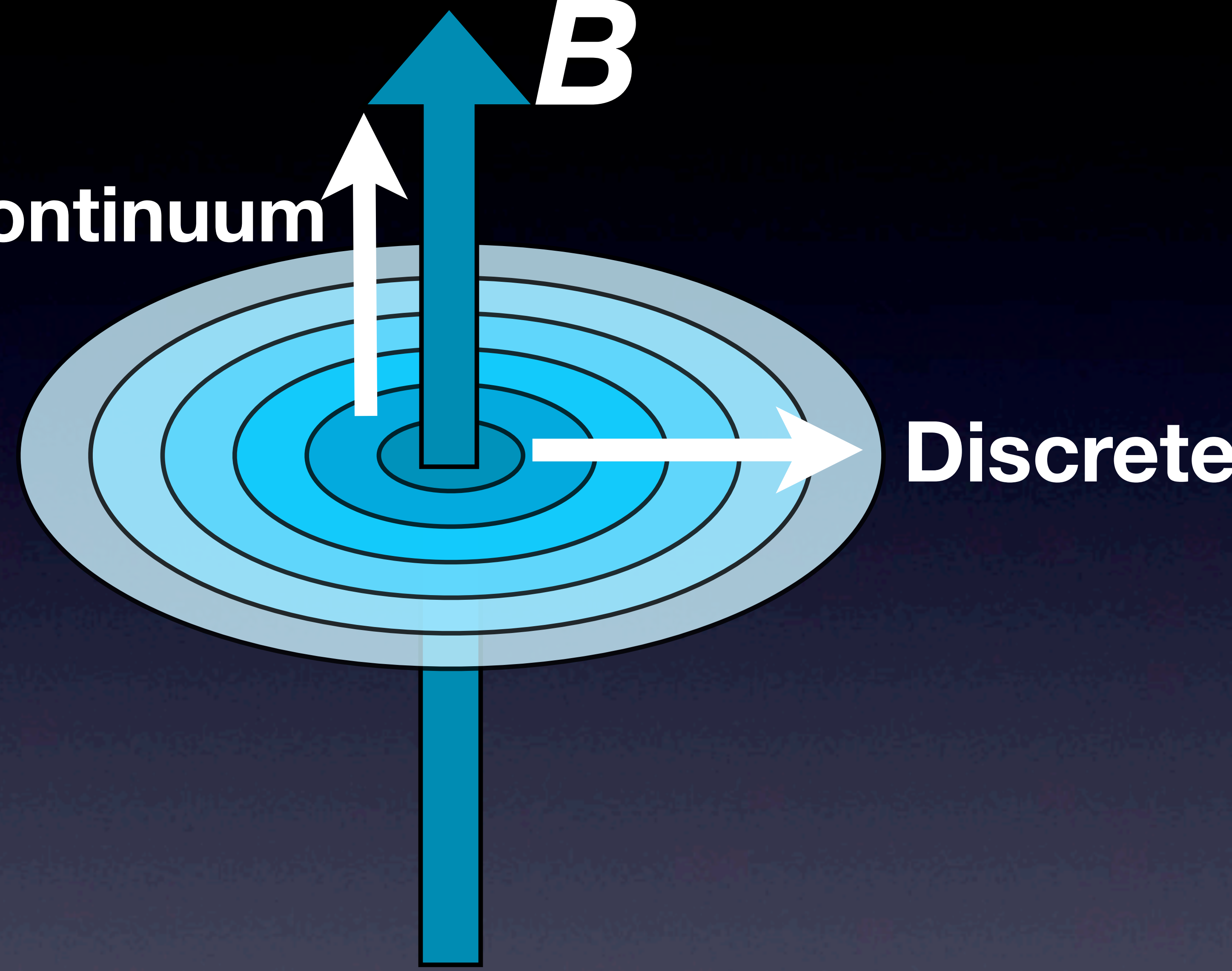
Suganuma, Tatsumi('91), Klimenko('92) Gusynin, Miransky, Shovkovy('94),Shushpanov, Smilga('97), ...

Chiral symmetry is always spontaneously broken at *T*=0

Synchrotron radiation, vacuum birefringence, ...

Tuchin ('10) ('12) Hattori, Itakura ('12)

Landau quantization



$$E^2 = p_z^2 + m^2 + (2n+1)qB - g s_z qB$$

Landau quantization **Zeeman splitting**

Vector meson

$$m_\rho^2(B) \approx m_\rho^2 - eB$$

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$$m_\rho^2(B = B_c) = 0$$

Vector meson condensation?

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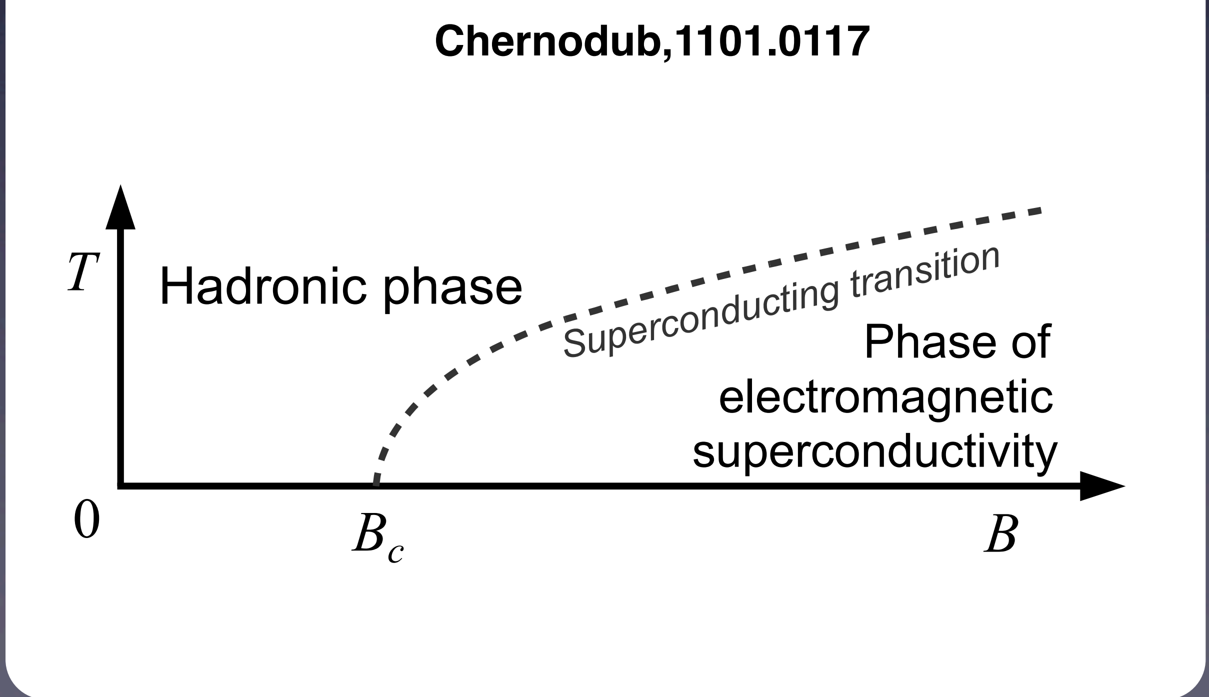
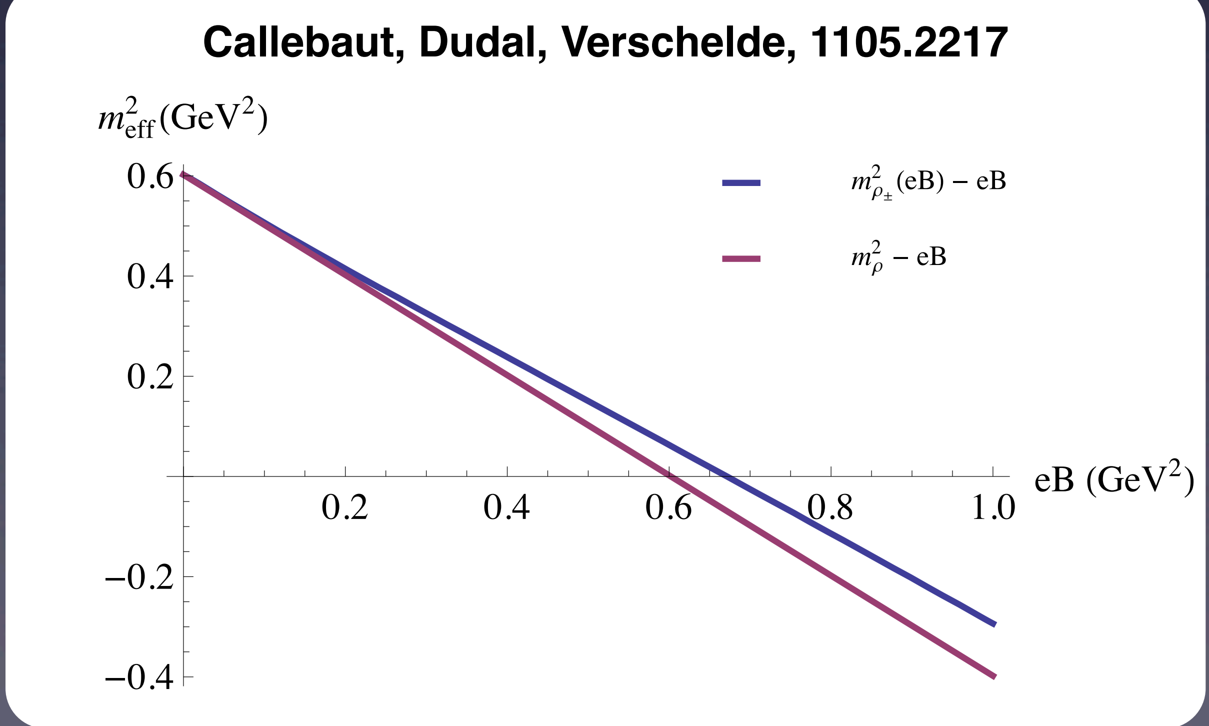
Model analysis:

Extended NJL model



AdS/CFT models



Does the vector meson condensation occur in QCD at finite *B*?

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The answer is No!

Important property

$$\not{D} = \gamma^\mu(\partial_\mu - igA_\mu + iqA_\mu^{\rm em})$$

$$\frac{1}{\not{D}+m}\frac{1}{\not{D}^\dagger+m} = \frac{1}{-\not{D}^2+m^2}$$
$$= \sum_\lambda \frac{1}{\lambda^2+m^2}|\lambda\rangle\langle\lambda|$$
$$\leq \sum_\lambda \frac{1}{m^2}|\lambda\rangle\langle\lambda| = \frac{1}{m^2}\ ,$$

bounded by quark mass

Vafa-Witten theorem

No isospin symmetry breaking occurs in vector like gauge theories.

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No isospin symmetry breaking occurs in vector like gauge theories.

Order parameter:

$$\phi \equiv \int d^4x \bar{\psi}(x) F \psi(x) \quad F = \gamma_+ \tau_+ f(x)$$

$$\mathcal{L} \to \mathcal{L} + \epsilon \bar{\psi} \Gamma \psi$$:Add an explicit breaking term

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$$|\langle \phi \rangle| = \epsilon \left| \langle \mathrm{Tr} \frac{1}{\not{D} + m} \Gamma \frac{1}{\not{D} + m} F \rangle_A \right| + \mathcal{O}(\epsilon^2)$$

$$\leq \epsilon \left\langle \sqrt{\mathrm{Tr} \frac{1}{\not{D}^\dagger + m} \frac{1}{\not{D} + m} \Gamma \Gamma^\dagger \mathrm{Tr} \frac{1}{\not{D}^\dagger + m} \frac{1}{\not{D} + m} F F^\dagger} \right\rangle_A + \mathcal{O}(\epsilon^2)$$

$$\leq \frac{\epsilon}{m^2} + \mathcal{O}(\epsilon^2) \to 0$$

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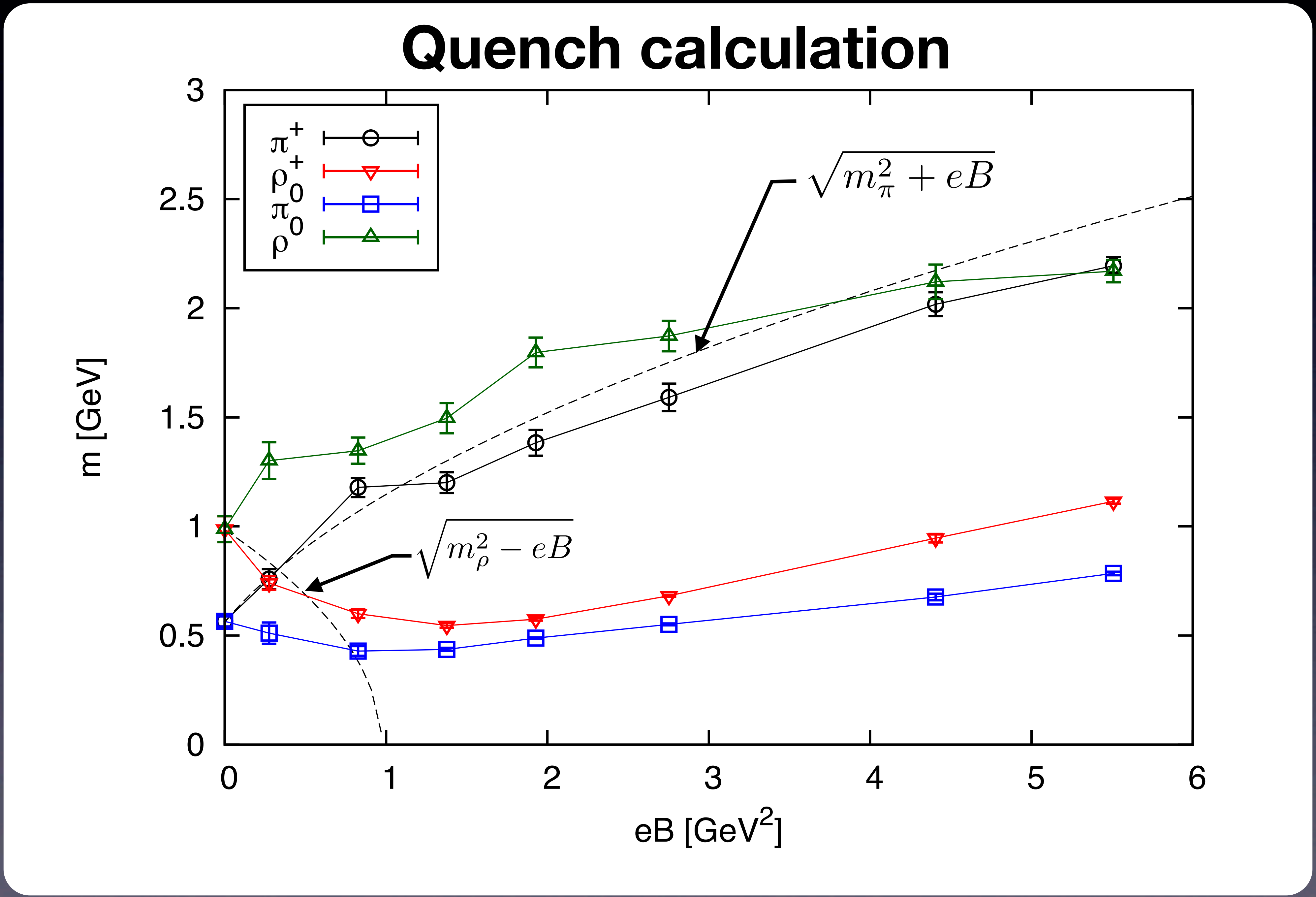
Vector meson cannot condense!

YH, A. Yamamoto, 1209.0007

Meson masses on the Lattice QCD

YH, A. Yamamoto, 1209.0007



Summary II

No vector meson condensation in QCD at finite *B* and *T*.

QCD inequality is useful tool to constrain effective models.

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No vector meson condensation in QCD at finite *B* and *T*.

QCD inequality is useful tool to constrain effective models.

Chernodub claims that

the electromagnetic superconductivity of vacuum in strong magnetic field background is consistent with the Vafa-Witten theorem because the charged vector meson condensates lock relevant internal global symmetries of QCD with the electromagnetic gauge group.

arXiv:1209.3587