A No-Go Theorem for Critical Phenomena in QCD at finite temperature and density

Yoshimasa Hidaka (RIKEN)

A No-Go Theorem for Critical Phenomena in QCD at finite temperature and density and magnetic field

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What can we say about the QCD phase diagram from QCD inequalities?
What can we say about the QCD phase diagram from QCD inequalities?

• QCD critical point at finite $T$ and $\mu$

• QCD at finite $B$ and $T$
Where is the QCD critical point?
Phase diagram of QCD

Quark Gluon Plasma

Hadronic Phase

Color Super Conductivity
Phase diagram of QCD
Phase diagram of QCD

Hadronic Phase

Quark Gluon Plasma

Critical point

$T_c$

$\mu$

Quarkyonic, 2SC, ...
Phase diagram of QCD

Quark Gluon Plasma

Critical point

Experiment: RHIC, SPS, FAIR, JINR,...

Hadronic Phase

? quarkyonic, 2SC, ...

Color Super Conductivity

T

T_c

0

µ
QCD critical point

Stephanov, hep-lat/0701002
We want to determine the allowed region.
We want to determine the allowed region.
Fluctuation of the order parameter

\[ \langle \delta \sigma(x) \delta \sigma(0) \rangle \sim \exp(-|x|m_\sigma) \]

At the critical point

\[ m_\sigma \rightarrow 0 \]
QCD inequality
QCD inequality

+ some approximations

- Neglecting disconnected diagrams
- Neglecting quark loops mixing flavors
QCD inequality

+ some approximations

- Neglecting disconnected diagrams
- Neglecting quark loops mixing flavors

Large-$N_c$ QCD satisfies both approximations!
QCD inequality

+ some approximations

- Neglecting disconnected diagrams
- Neglecting quark loops mixing flavors

Large-$N_c$ QCD satisfies both approximations!
Several models such as NJL with mean field approximation, random matrix also satisfy.
QCD Inequality

Weingarten ('83), Witten ('83), Nussinov ('84), Espriu, Gross, Wheater ('84)

At $T=0$, $\mu=0$

For flavor nonsinglet channel

\[ m_\Gamma \geq m_\pi \]
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At $T=0$, $\mu=0$
For flavor nonsinglet channel

$m_\Gamma \geq m_\pi$

No SSB of isospin and baryon symm.

Vafa-Witten ('84)
QCD Inequality

Weingarten ('83), Witten ('83), Nussinov ('84), Espriu, Gross, Wheater ('84)

Dirac operator

\[ D = \gamma_\mu (\partial_\mu + igA_\mu) \]

Anti-Hermite

\[ D^\dagger = -D \]

Chiral symmetry

\[ \gamma_5 D \gamma_5 = -D \]
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\[ \mathcal{D} = D + m \]

\[ \det \mathcal{D} \geq 0 \]
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\[ \mathcal{D} = D + m \]

\[ \det \mathcal{D} \geq 0 \]

For isospin chemical potential

\[ \tau_1 \gamma_5 \mathcal{D} \gamma_5 \tau_1 = \mathcal{D}^\dagger \]

\[ \mathcal{D}(\mu_I) = D + \frac{\mu_I}{2} \gamma_0 \tau_3 + m \]

Alford, Kapustin and Wilczek ('99)
QCD Inequality

Weingarten ('83), Witten ('83), Nussinov ('84), Espriu, Gross, Wheater ('84)

Flavor nonsinglet operator: $M_\Gamma(x) = \bar{\psi} \Gamma \psi$

$\langle M_\Gamma(x) M_\Gamma^\dagger(y) \rangle_{\psi,A} = - \langle \text{tr}[S_A(x,y) \Gamma S_A(y,x) \bar{\Gamma}] \rangle_A$

Cauchy–Schwarz inequality

$\langle O_1 O_2 \rangle \leq \sqrt{\langle O_1 O_1^\dagger \rangle \langle O_2 O_2^\dagger \rangle}$
QCD Inequality

Flavor nonsinglet operator

\[
\langle M_\Gamma(x)M_\Gamma^\dagger(y)\rangle_{\psi,A} = -\langle \text{tr}[S_A(x,y)\Gamma S_A(y,x)\bar{\Gamma}]\rangle_A \\
\leq \langle \text{tr}[S_A(x,y)S_A^\dagger(y,x)]\rangle \\
= \langle M_\pi(x)M_\pi^\dagger(y)\rangle_{\psi,A}
\]
QCD Inequality

Flavor nonsinglet operator

\[ \langle M_\Gamma(x) M_\Gamma^\dagger(y) \rangle_{\psi,A} = - \langle \text{tr} [S_A(x,y) \Gamma S_A(y,x) \bar{\Gamma}] \rangle_A \]

\[ \leq \langle \text{tr} [S_A(x,y) S_A^\dagger(y,x)] \rangle \]

\[ = \langle M_\pi(x) M_\pi^\dagger(y) \rangle_{\psi,A} \]

\[ m_\Gamma \geq m_\pi \]

\[ \langle M_\Gamma(x) M_\Gamma^\dagger(y) \rangle_{\psi,A} \sim \exp(-m_\Gamma |x - y|) \]
Flavor singlet operator

\[
\langle M_\Gamma(x)M^{\dagger}_\Gamma(y) \rangle_{\psi,A} = -\langle \text{tr}[S_A(x,y)\Gamma S_A(y,x)\bar{\Gamma}] \rangle_A \\
+ \langle \text{tr}[S_A(x,x)\Gamma] \text{tr}[S_A(y,x)\bar{\Gamma}] \rangle_A
\]
Flavor singlet operator

\[
\langle M_\Gamma(x) M_\Gamma^\dagger(y) \rangle_{\psi,A} = -\langle \text{tr}[S_A(x,y) \Gamma S_A(y,x) \bar{\Gamma}] \rangle_A \\
+ \langle \text{tr}[S_A(x,x) \Gamma] \text{tr}[S_A(y,x) \bar{\Gamma}] \rangle_A
\]

If disconnected diagram is neglected, \( m_\sigma \geq m_\pi \)

No second order phase transition as long as \( m_\pi \neq 0 \)
Flavor singlet operator

\[ \langle M_{\Gamma}(x)M_{\Gamma}^\dagger(y) \rangle_{\psi,A} = -\langle \text{tr}[S_A(x,y)\Gamma S_A(y,x)\bar{\Gamma}] \rangle_A \]

\[ +\langle \text{tr}[S_A(x,x)\Gamma]\text{tr}[S_A(y,x)\bar{\Gamma}] \rangle_A \]

If disconnected diagram is neglected, \( m_\sigma \geq m_\pi \)

No second order phase transition as long as \( m_\pi \neq 0 \)

QCD inequality also works at finite \( T \) and \( \mu_1 \).

Son and Stephanov (’01)
QCD phase diagram at $\mu_I$

No critical point outside of the pion condensed phase

YH, Yamamoto (’11)
QCD phase diagram at $\mu$

QCD inequality does not work at $\mu$...
QCD phase diagram at $\mu$

QCD inequality does not work at $\mu$...

If some quark loops mixing flavors are negligible, i.e., complex phase is negligible,

$$P = p(\mu_u^2, \mu_d^2) + p_{\text{mix}}(\mu_u \mu_d, \mu_u^2, \mu_d^2)$$

OK, at large-$N_c$, outside of pion condensed phase
QCD phase diagram at $\mu$

QCD inequality does not work at $\mu$...

If some quark loops mixing flavors are negligible, i.e., complex phase is negligible,

$$P = p(\mu_u^2, \mu_d^2) + p_{\text{mix}}(\mu_u\mu_d, \mu_u^2, \mu_d^2)$$

OK, at large-$N_c$, outside of pion condensed phase

The phase structure at finite $\mu$ \(\approx\) The phase structure at finite $\mu$!

At large $N_c$, Hanada and Yamamoto (’11)
Phase diagram of QCD

YH, Yamamoto ('11)

Quark Gluon Plasma

Critical point

Hadronic Phase

Color Super Conductivity

Yamamoto ('11)
Phase diagram of QCD

YH, Yamamoto ('11)

Critical point

Allowed region

Hadronic Phase

Quark Gluon Plasma

Color Super Conductivity

m_\pi / 2

\mu

0
Model results

Random matrix model
Han, Stephanov (08)

NJL model
Andersen, Kyllingstad, Splittorff (’09)

onset of pion condensation phase

Similar result in PNJL model
Sakai, Sasaki, Kouno, Yahiro (’10)
Summary I
No critical point out side of the pion condensed phase.

( if quark loops and disconnected diagram are suppressed.)

Large $N_c$ QCD, OK.

Lattice QCD can determine the boundary.

cf. Kogut, Sinclair ('04), ('06), ('07), de Forcrand, Kratochvila ('06)
de Forcrand, Stephanov, Wenger ('07)
Detmold, Orginos, Shi ('12)

Lattice simulation in the pion condensed phase is a challenging problem.

We need to estimate contributions of disconnected diagrams.
Strong Magnetic field
Strong Magnetic field

Heavy ion collisions:

RHIC: \( \sim m^2_\pi \)

LHC: \( \sim 10m^2_\pi \)

Magnetar:

\( \sim 0.01m^2_\pi \)

The early universe:

\( \sim m^2_W \sim 10^5 m^2_\pi \)
Interesting phenomena at finite $B$

**Chiral magnetic effect:**
Kharzeev, McLerran, Warringa ('07), Kharzeev, Fukushima, Warringa ('08), ...

$$J_z = \frac{eBL^3}{2\pi} \mu_5$$

**Magnetic catalysis:**
Suganuma, Tatsumi ('91), Klimenko ('92) Gusynin, Miransky, Shovkovy ('94), Shushpanov, Smilga ('97), ...

Chiral symmetry is always spontaneously broken at $T=0$

**Synchrotron radiation, vacuum birefringence,** ...

Tuchin ('10) ('12) Hattori, Itakura ('12)
Landau quantization

\[ E^2 = p_z^2 + m^2 + (2n + 1)qB - gs_zqB \]

Landau quantization

Zeeman splitting
Vector meson

\[ m^2_\rho (B) \approx m^2_\rho - eB \]
Vector meson

\[ m_{\rho}^2(B) \approx m_{\rho}^2 - eB \]

\[ m_{\rho}^2(B = B_c) = 0 \]

Vector meson condensation?
Vector meson

\[ m_{\rho}^2(B) \approx m_{\rho}^2 - eB \]

\[ m_{\rho}^2(B = B_c) = 0 \]

Vector meson condensation?

Model analysis:

Extended NJL model

AdS/CFT models
Does the vector meson condensation occur in QCD at finite $B$?
Does the vector meson condensation occur in QCD at finite $B$?

The answer is No!
Important property

\[ \mathcal{D} = \gamma^\mu (\partial_\mu - igA_\mu + iqA^{em}_\mu) \]

\[ \frac{1}{\mathcal{D} + m} \frac{1}{\mathcal{D}^\dagger + m} = \frac{1}{\mathcal{D}^2 + m^2} \]

\[ = \sum_{\lambda} \frac{1}{\lambda^2 + m^2} |\lambda\rangle \langle \lambda| \]

\[ \leq \sum_{\lambda} \frac{1}{m^2} |\lambda\rangle \langle \lambda| = \frac{1}{m^2} , \]

bounded by quark mass
Vafa-Witten theorem
No isospin symmetry breaking occurs in vector like gauge theories.
Vafa-Witten theorem

No isospin symmetry breaking occurs in vector like gauge theories.

Order parameter:

\[ \phi \equiv \int d^4x \bar{\psi}(x) F \psi(x) \quad F = \gamma_+ \tau_+ f(x) \]

\[ \mathcal{L} \rightarrow \mathcal{L} + \epsilon \bar{\psi} \Gamma \psi \text{ :Add an explicit breaking term} \]
Vafa-Witten theorem

No isospin symmetry breaking occurs in vector like gauge theories.

Order parameter:

$$\phi \equiv \int d^4 x \bar{\psi}(x) F \psi(x) \quad F = \gamma_+ \tau_+ f(x)$$

$$\mathcal{L} \rightarrow \mathcal{L} + \epsilon \bar{\psi} \Gamma \psi : \text{Add an explicit breaking term}$$

$$|\langle \phi \rangle| = \epsilon \left| \langle \text{Tr} \frac{1}{\slashed{D} + m} \Gamma \frac{1}{\slashed{D} + m} F \rangle_A \right| + \mathcal{O}(\epsilon^2)$$

$$\leq \epsilon \left\langle \sqrt{\text{Tr} \frac{1}{\slashed{D} + m} \frac{1}{\slashed{D} + m} \Gamma \Gamma^\dagger \text{Tr} \frac{1}{\slashed{D} + m} \frac{1}{\slashed{D} + m} F F^\dagger} \right\rangle_A + \mathcal{O}(\epsilon^2)$$

$$\leq \frac{\epsilon}{m^2} + \mathcal{O}(\epsilon^2) \rightarrow 0$$
Vafa-Witten theorem

No isospin symmetry breaking occurs in vector like gauge theories.

Order parameter:

\[
\phi \equiv \int d^4 x \bar{\psi}(x) F \psi(x) \quad F = \gamma_+ \tau_+ f(x)
\]

\[
\mathcal{L} \to \mathcal{L} + \epsilon \bar{\psi} i \Gamma \psi : \text{Add an explicit breaking term}
\]

\[
|\langle \phi \rangle| = \epsilon \left| \left< \text{Tr} \frac{1}{\overline{\Phi} + m} \Gamma \frac{1}{\overline{\Phi} + m} F \right> \right|_A + \mathcal{O}(\epsilon^2)
\]

\[
\leq \epsilon \left< \left< \sqrt{\text{Tr} \frac{1}{\overline{\Phi} + m} \frac{1}{\overline{\Phi} + m} \Gamma \Gamma^\dagger \text{Tr} \frac{1}{\overline{\Phi} + m} \frac{1}{\overline{\Phi} + m} FF^\dagger} \right> \right>_A + \mathcal{O}(\epsilon^2)
\]

\[
\leq \frac{\epsilon}{m^2} + \mathcal{O}(\epsilon^2) \to 0
\]

Vector meson cannot condense!

YH, A. Yamamoto, 1209.0007
Meson masses on the Lattice QCD

YH, A. Yamamoto, 1209.0007

Quench calculation

\[ m \approx \sqrt{m^2 - eB} \]

\[ m \approx \sqrt{m^2 + eB} \]
Summary II

No vector meson condensation in QCD at finite $B$ and $T$.

QCD inequality is useful tool to constrain effective models.
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No vector meson condensation in QCD at finite $B$ and $T$.

QCD inequality is useful tool to constrain effective models.

Chernodub claims that
the electromagnetic superconductivity of vacuum in strong magnetic field background is consistent with the Vafa-Witten theorem because the charged vector meson condensates lock relevant internal global symmetries of QCD with the electromagnetic gauge group.

arXiv:1209.3587