The Ultra-High-Energy

Cosmic-Neutrino-Nucleon Cross Section

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1. Introduction

For present and future search and investigation of ultra-high-energy cosmic neutrinos a prediction of the neutrino-nucleon cross section is indispensable.

Large extension of the kinematic range where experimental data available is necessary.

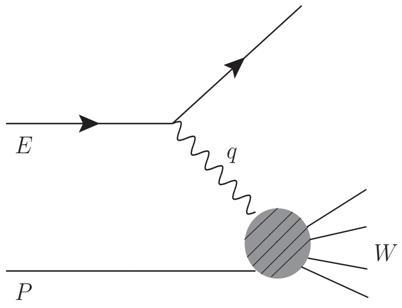
2. Neutrino-Nucleon Cross Section

$$\sigma_{\nu N}(E) = \frac{G_F^2}{2\pi} \int_{Q^2_{min.}}^{s-M_p^2} dQ^2 \left(\frac{M_W^2}{Q^2 + M_W^2} \right)^2 \int_{M_p^2}^{s-Q^2} \frac{dW^2}{W^2} \sigma_r(x, Q^2).$$

e.g. Goncalves and Hepp (2011)

$$s = 2M_pE + M_p^2 \cong 2M_pE,$$

$$x = \frac{Q^2}{2qP} = \frac{Q^2}{W^2 + Q^2 - M_p^2} \cong \frac{Q^2}{W^2},$$



$$\sigma_r(x, Q^2) = \frac{1 + (1-y)^2}{2} F_2^\nu(x, Q^2) - \frac{y^2}{2} F_L^\nu(x, Q^2) + y(1 - \frac{y}{2}) x F_3^\nu(x, Q^2).$$

$$y = \frac{Q^2}{2M_pEx} \cong \frac{W^2}{s}.$$

For $s \gg M_W^2 \approx 10^4 \mathrm{GeV}^2$,

dominant contribution from $Q^2 \cong M_W^2$,

$$x \cong \frac{M_W^2}{s} \ll 0.1,$$

3. Connection to ep deep inelastic scattering (DIS)

HERA (1990 to 2007): DIS at low values of

$$x \equiv x_{bj} \simeq \frac{Q^2}{W^2}, \text{ where}$$

$$5 \cdot 10^{-4} \le x \le 10^{-1}$$

$$0 \le Q^2 \le 100\text{GeV}^2$$

For n_f actively contributing quark flavors:

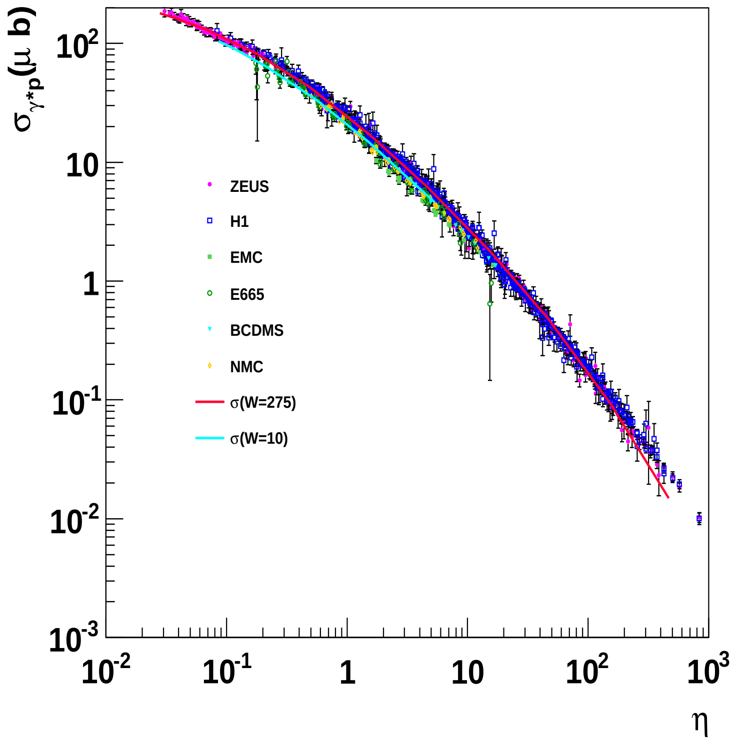
$$\frac{1}{n_f} F_2^{\nu N}(x, Q^2) = \frac{1}{\sum_q Q_q^2} F_2^{eN}(x, Q^2);$$

$$F_{2,L}^{\nu N}(x, Q^2) = \frac{n_f}{\sum_q^{n_f} Q_q^2} F_{2,L}^{eN}(x, Q^2), \quad \text{with } \frac{n_f}{\sum_q^{n_f} Q_q^2} = \frac{5}{18} \quad (\text{for } n_f = 4).$$

$$F_2^{ep}(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha} \sigma_{\gamma^* p}(W^2, Q^2).$$

4. DIS – Empirical Results

Low-x Scaling



Schildknecht, Surrow, Tentyukov (2000)

Empirically : $\eta(W^2, Q^2) \equiv \dfrac{Q^2 + m_0^2}{\Lambda_{sat}^2(W^2)},$

$$\Lambda_{sat}^2(W^2) \sim (W^2)^{C_2}$$

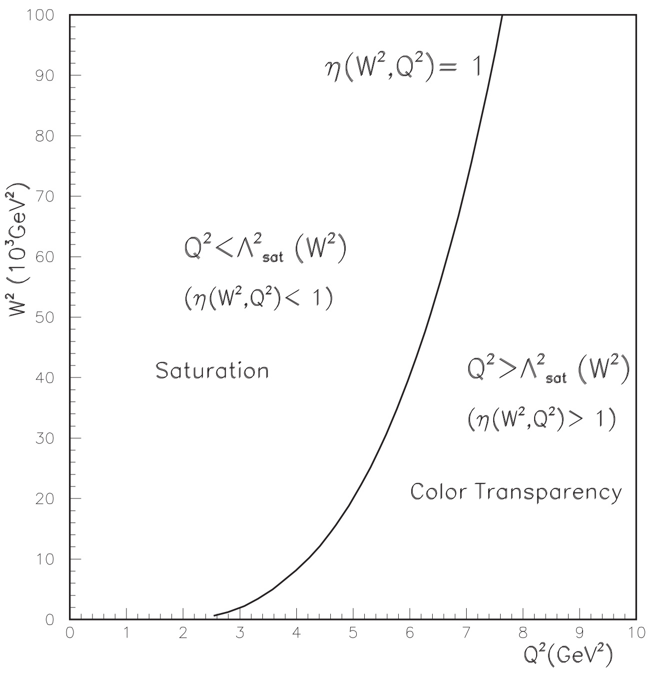
$$\begin{aligned} \sigma_{\gamma^* p}(W^2, Q^2) &= \sigma_{\gamma^* p}(\eta(W^2, Q^2)) \\ &\sim \sigma^{(\infty)} \begin{cases} ln\frac{1}{\eta(W^2,Q^2)} & , \quad \text{for } \eta(W^2, Q^2) \ll 1 \\ \frac{1}{\eta(W^2,Q^2)} & , \quad \text{for } \eta(W^2, Q^2) \gg 1 \end{cases} \end{aligned}$$

The limit of $\eta(W^2, Q^2) \to 0$, or $W^2 \to \infty$ at Q^2 fixed

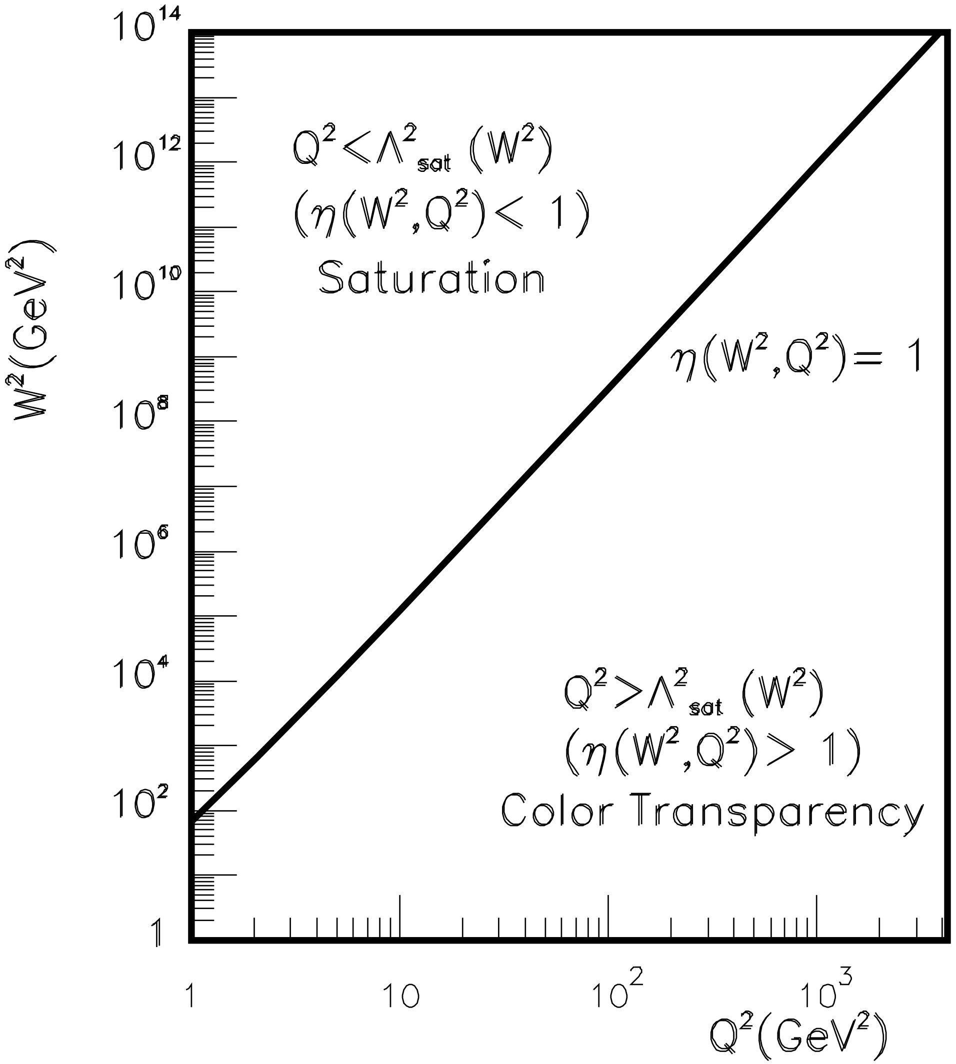
$$\lim_{\substack{W^2\to\infty \\ Q^2 \text{ fixed}}} \frac{\sigma_{\gamma^* p}(\eta(W^2,Q^2))}{\sigma_{\gamma^* p}(\eta(W^2,Q^2=0))} = \lim_{\substack{W^2\to\infty \\ Q^2 \text{ fixed}}} \frac{\ln\left(\frac{\Lambda^2_{sat}(W^2)}{m_0^2}\frac{m_0^2}{(Q^2+m_0^2)}\right)}{\ln\frac{\Lambda^2_{sat}(W^2)}{m_0^2}} = 1 + \lim_{\substack{W^2\to\infty \\ Q^2 \text{ fixed}}} \frac{\ln\frac{m_0^2}{Q^2+m_0^2}}{\ln\frac{\Lambda^2_{sat}(W^2)}{m_0^2}} = 1.$$

$$\sigma_{\gamma^* p}\left(\eta(W^2,Q^2=0)\right) = \sigma_{\gamma p}(W^2)$$

D. Schildknecht, DIS 2001 (Bologna)



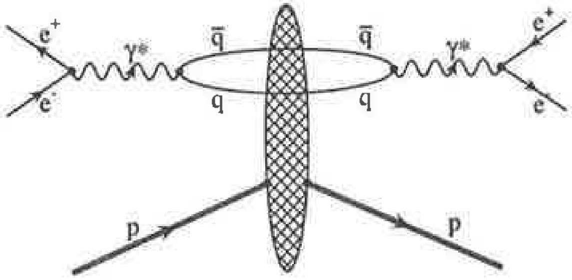
$Q^2[GeV^2]$	$W^2[GeV^2]$	$\frac{\sigma_{\gamma^* p}(\eta(W^2,Q^2))}{\sigma_{\gamma p}(W^2)}$
1.5	2.5×10^7	**0.5**
	1.26×10^{11}	**0.63**



$W^2(GeV^2)$
$Q^2(GeV^2)$
$Q^2 < \Lambda^2_{sat}(W^2)$
$(\eta(W^2,Q^2) < 1)$
Saturation
$\eta(W^2,Q^2) = 1$
$Q^2 > \Lambda^2_{sat}(W^2)$
$(\eta(W^2,Q^2) > 1)$
Color Transparency

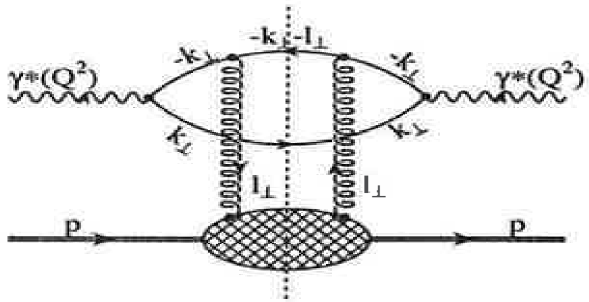
5. The Color Dipole Picture (CDP).

The longitudinal and the transverse photoabsorption cross section

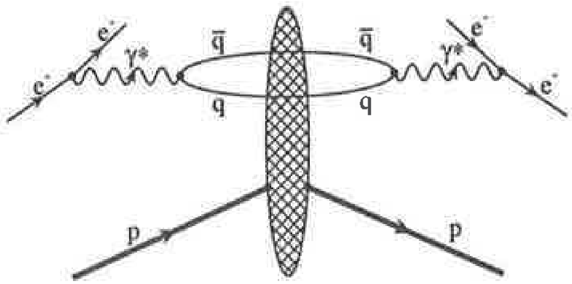


(a)

channel 1:

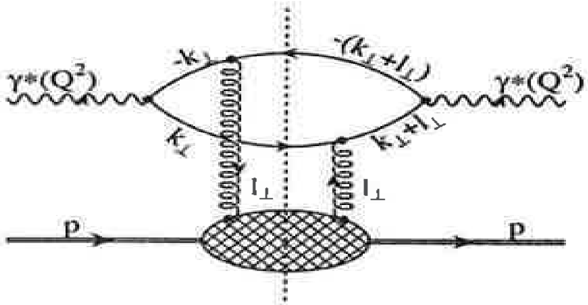


(a)

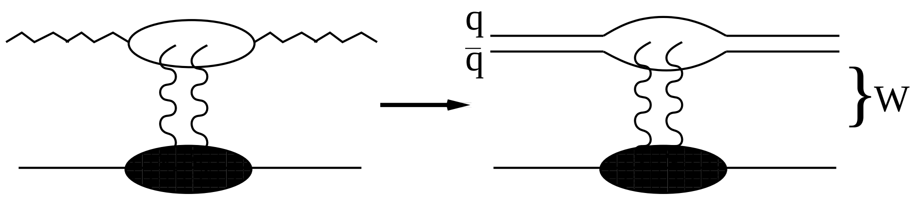


(b)

channel 2:



(b)



$\gamma^* p \rightarrow \gamma^* p \longrightarrow (q\bar{q})p \rightarrow (q\bar{q})p$

Sakurai, Schildknecht (1972)

Low (1975)

A) $$\sigma_{\gamma^*_{L,T}}(W^2, Q^2) = \int dz \int d^2\vec{r}_\perp |\psi_{L,T}(\vec{r}_\perp, z(1-z), Q^2)|^2 \ \ \sigma_{(q\bar{q})p}(\vec{r}_\perp, z(1-z), W^2)$$

Remarks: i) $|\psi_{L,T}(\vec{r}_\perp, z(1-z), Q^2)|$: Probability for $\gamma^*_{L,T} \to q\bar{q}$ fluctuation

ii) $\sigma_{(q\bar{q})p}(\vec{r}_\perp, z(1-z), W^2)$: $(q\bar{q})p$ cross section dependent on W^2 (not on $x \equiv \frac{Q^2}{W^2}$)

B) Gauge-invariant two-gluon coupling:

$$\sigma_{(q\bar{q})p}(\vec{r}_\perp, z(1-z), W^2) = \int d^2\vec{l}_\perp \tilde{\sigma}(\vec{l}_\perp^{\,2}, z(1-z), W^2) \left(1 - e^{-i\ \vec{l}_\perp \cdot \vec{r}_\perp}\right)$$

Nikolaev, Zakharov (1991)

Cvetic, Schildknecht, Shoshi(2000)

Schildknecht, Surrow, Tentyukov (2001)

Equivalently, in terms of the variables:

$$\vec{r}_\perp{}' = \sqrt{z(1-z)}\,\vec{r}_\perp,$$

$$\vec{l}_\perp{}' = \frac{\vec{l}_\perp}{\sqrt{z(1-z)}},$$

Photon wave function (e.g. L):

$$K_0(r'_\perp Q) = \frac{1}{2\pi}\int d^2\vec{k}_\perp{}' \frac{1}{Q^2+\vec{k}_\perp{}'^2} e^{-i\vec{r}_\perp{}'\cdot\vec{k}_\perp{}'}$$

$$\gamma^* q\bar{q} \text{ coupling :} \qquad \sum_{\lambda=-\lambda=\pm 1} |j_L^{\lambda,\lambda'}|^2 = 4M_{q\bar{q}}^2 \left(d^1_{10}(z)\right)^2,$$

$$\sum_{\lambda=-\lambda'=\pm 1} |j_T^{\lambda,\lambda'}(+)|^2 = \sum_{\lambda=-\lambda=\pm 1} |j_T^{\lambda,\lambda'}(-)|^2 = 4M_{q\bar{q}}^2 \frac{1}{2}\left((d^1_{1-1}(z))^2 + (d^1_{11}(z))^2\right).$$

Upon introducing the cross section $\sigma_{(q\bar{q})^{J=1}_{L,T}p}(r'_\perp, W^2)$, for $(q\bar{q})^{J=1}_{L,T}p$ scattering

A) $\sigma_{\gamma^*_{L,T}p}(W^2, Q^2) = \frac{\alpha}{\pi} \sum_q Q_q^2 Q^2 \int dr'^2_\perp K^2_{0,1}(r'_\perp Q) \sigma_{(q\bar{q})^{J=1}_{L,T}p}(r'_\perp, W^2).$

Kuroda, Schildknecht (2011)

and

B)
$$\begin{aligned}
\sigma_{(q\bar{q})^{J=1}_{L,T}p}(\vec{r}_\perp{}', W^2) &= \int d^2\vec{l}_\perp{}' \bar{\sigma}_{(q\bar{q})^{J=1}_{L,T}p}(\vec{l}_\perp{}'^2, W^2)(1 - e^{-i\vec{l}_\perp{}' \cdot \vec{r}_\perp{}'}) \\
&= \pi \int d\vec{l}_\perp{}'^2 \bar{\sigma}_{(q\bar{q})^{J=1}_{L,T}p}(\vec{l}_\perp{}'^2, W^2) \cdot \left(1 - \frac{\int d\vec{l}_\perp{}'^2 \bar{\sigma}_{(q\bar{q})^{J=1}_{L,T}p}(\vec{l}_\perp{}'^2, W^2) J_0(l'_\perp r'_\perp)}{\int d\vec{l}_\perp{}'^2 \bar{\sigma}_{(q\bar{q})^{J=1}_{L,T}p}(\vec{l}_\perp{}'^2, W^2)} \right)
\end{aligned}$$

For fixed dipole size, $r'_\perp$, dominant contribution to dipole cross section

$$\vec{l}_\perp{}'^2 \leq \vec{l}_{\perp \mathrm{Max}}'^2(W^2).$$

Saturation $(r_\perp'^2 \Lambda^2_{\rm sat}(W^2) \gg 1)$:

$$\sigma_{(q\bar{q})^{J=1}p}(r_\perp'^2, W^2) \sim \int d\vec{l}_\perp'^2 \bar{\sigma}_{(q\bar{q})p}(\vec{l}_\perp'^2, W^2) \sim \sigma^{(\infty)}(W^2)$$

Color Transparency $(r_\perp'^2 \Lambda^2_{\rm sat}(W^2) \ll 1)$:

$$\begin{aligned} \sigma_{(q\bar{q})^{J=1}p}(r_\perp'^2, W^2) &\sim \vec{r}_\perp'^2 \int d\vec{l}_\perp'^2 \vec{l}_\perp'^2 \bar{\sigma}_{(q\bar{q})^{J=1}p}(\vec{l}_\perp'^2, W^2) \\ &= \vec{r}_\perp'^2 \Lambda^2_{\rm sat}(W^2) \sigma^{(\infty)}(W^2) \end{aligned}$$

Photoabsorption cross section:

$$\sigma_{\gamma^* p}(W^2, Q^2) = \sigma_{\gamma^* p}(\eta(W^2, Q^2)) \sim \sigma^{(\infty)} \begin{cases} ln \frac{1}{\eta(W^2,Q^2)}, & \text{for } \eta(W^2, Q^2) \ll 1, \text{ saturation} \\ \frac{1}{2\eta(W^2,Q^2)}, & \text{for } \eta(W^2, Q^2) \gg 1, \text{ color transparency} \end{cases}$$

6. The Neutrino-Nucleon Cross Section in the CDP

$$\sigma_{\nu N}(E) = \frac{G_F^2 M_W^4}{8\pi^3\alpha} \frac{n_f}{\sum_q Q_q^2} \int_{Q^2_{Min}}^{s-M_p^2} dQ^2 \frac{Q^2}{(Q^2+M_W^2)^2} \times \int_{M_p^2}^{s-Q^2} \frac{dW^2}{W^2} \frac{1}{2}(1+(1-y)^2)\sigma_{\gamma^* p}(\eta(W^2,Q^2)).$$

Kuroda, Schildknecht, arXiv:1305.0440v3, Phys. Rev. in print

Contribution from saturation region

$$r(E) = \frac{\sigma_{\nu N}(E)_{\eta(W^2,Q^2)<1}}{\sigma_{\nu N}(E)}.$$

$$r(E) < \bar{r}(E),$$

$$\bar{r}(E) = \frac{2\int_{Q^2_{Min}}^{Q^2_{Max}(s)} dQ^2 \frac{Q^2}{(Q^2+M_W^2)^2} \int_{W^2(Q^2)_{Min}}^{s-Q^2} \frac{dW^2}{W^2} \ln \frac{1}{\eta(W^2,Q^2)}}{\int_{Q^2_{Min}}^{s-M_p^2} dQ^2 \frac{Q^2}{(Q^2+M_W^2)^2} \int_{M_p^2}^{s-Q^2} \frac{dW^2}{W^2} \frac{1}{2\eta(W^2,Q^2)}}.$$

$$r(E) < \bar{r}(E) = \frac{1}{2}\frac{\Lambda^2_{sat}(s)}{M_W^2} = \begin{cases} 1.74 \times 10^{-3} & for\ E = 10^6 \text{GeV} \\ 2.51 \times 10^{-2} & for\ E = 10^{10} \text{GeV} \\ 3.63 \times 10^{-1} & for\ E = 10^{14} \text{GeV} \end{cases}.$$

Explicit expression for $\sigma_{\gamma^* p}(W^2 Q^2)$ in the CDP:

Ansatz: $\sigma_{(q\bar{q})^{J=1}_{L,T} p}(r'_\perp \Lambda_{\rm sat}(W^2)) = \sigma^{(\infty)}(W^2)(1 - J_0(r'_\perp \Lambda_{\rm sat}(W^2)))$

$$\begin{aligned}\sigma_{\gamma^* p}(W^2, Q^2) &= \sigma_{\gamma^* p}(\eta(W^2, Q^2)) + O\left(\frac{m_0^2}{\Lambda^2_{\rm sat}(W^2)}\right) = \\ &= \frac{\alpha R_{e^+e^-}}{3\pi}\sigma^{(\infty)}(W^2) I_0(\eta(W^2, Q^2)) + O\left(\frac{m_0^2}{\Lambda^2_{\rm sat}(W^2)}\right), \quad \Lambda^2_{\rm sat}(W^2) = C_1 \left(\frac{W^2}{1{\rm GeV}^2}\right)^{C_2}.\end{aligned}$$

$$I_0(\eta(W^2, Q^2)) = \frac{1}{\sqrt{1 + 4\eta(W^2, Q^2)}} \ln \frac{\sqrt{1 + 4\eta(W^2, Q^2)} + 1}{\sqrt{1 + 4\eta(W^2, Q^2)} - 1} \cong$$

$$\cong \begin{cases} \ln \frac{1}{\eta(W^2, Q^2)} + O(\eta \ln \eta), & \text{for } \eta(W^2, Q^2) \to \frac{m_0^2}{\Lambda^2_{\rm sat}(W^2)}, \\ \frac{1}{2\eta(W^2, Q^2)} + O\left(\frac{1}{\eta^2}\right), & \text{for } \eta(W^2, Q^2) \to \infty, \end{cases}$$

$$R_{e^+e^-} = 3 \sum_q Q_q^2.$$

Photoproduction limit:

$$\sigma^{(\infty)}(W^2) = \frac{3\pi}{R_{e^+e^-}\alpha} \frac{1}{\ln \frac{\Lambda^2_{sat}(W^2)}{m_0^2}} \begin{cases} \sigma^{Regge}_{\gamma p}(W^2), \\ \sigma^{PDG}_{\gamma p}(W^2). \end{cases} \qquad \text{where} \begin{cases} \sigma^{Regge}_{\gamma p}(W^2) & \sim (W^2)^{0.0808} \\ \sigma^{PDG}_{\gamma p}(W^2) & \sim (\ln W^2)^2 \end{cases}$$

For $\eta(W^2, Q^2) \gg 100$, exclusion of high-mass $q\bar{q}$ fluctuations:

$$m_1^2(W^2) = \xi \Lambda^2_{\rm sat}(W^2), \text{ where empirically } \xi = 130.$$

$$\begin{aligned}
\sigma_{\gamma^* p}(W^2, Q^2) &= \frac{\alpha R_{e^+e^-}}{3\pi} \sigma^{(\infty)}(W^2) I_0(\eta(W^2, Q^2)) \\
&\times \frac{1}{3}\left(G_L\left(\frac{\xi}{\eta(W^2,Q^2)}\right) + 2 G_T\left(\frac{\xi}{\eta(W^2,Q^2)}\right)\right) \\
&+ O\left(\frac{m_0^2}{\Lambda^2_{\rm sat}(W^2)}\right)
\end{aligned}$$

where

$$\frac{1}{3}\left(G_L\left(\frac{\xi}{\eta(W^2,Q^2)}\right) + 2 G_T\left(\frac{\xi}{\eta(W^2,Q^2)}\right)\right) \cong \begin{cases} 1, & for\ \eta(W^2,Q^2) \ll \xi = 130 \\ \frac{\xi}{\eta(W^2,Q^2)}, & for\ \eta(W^2,Q^2) \gg \xi = 130 \end{cases}$$

Parameters:

$$\eta(W^2, Q^2) = \frac{Q^2 + m_0^2}{\Lambda_{sat}^2(W^2)};$$

$m_0^2 = 0.15 \text{ GeV}^2 \quad (m_0^2 < m_\rho^2)$

$$\Lambda_{sat}^2(W^2) = C_1 \left(\frac{W^2}{1 \text{ GeV}^2}\right)^{C_2},$$

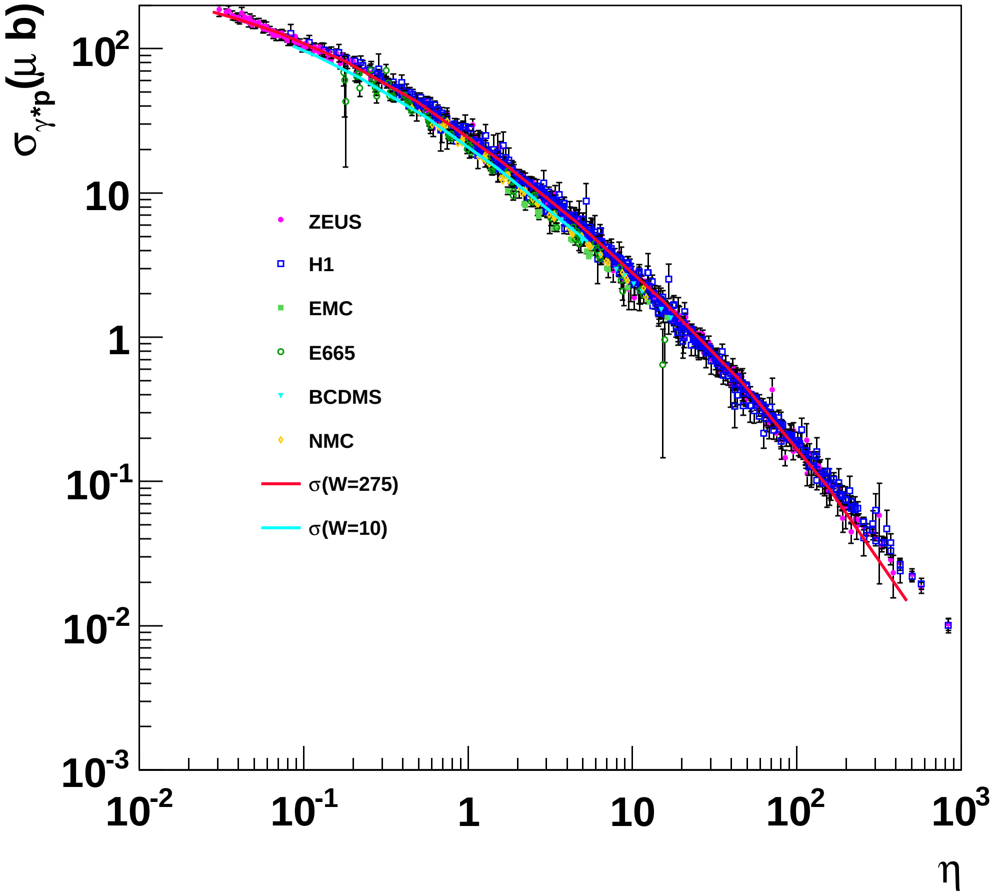
$C_1 = 0.34 \text{ GeV}^2$ (fit parameter)

$C_2 = 0.29$ (consistency with DGLAP evolution)

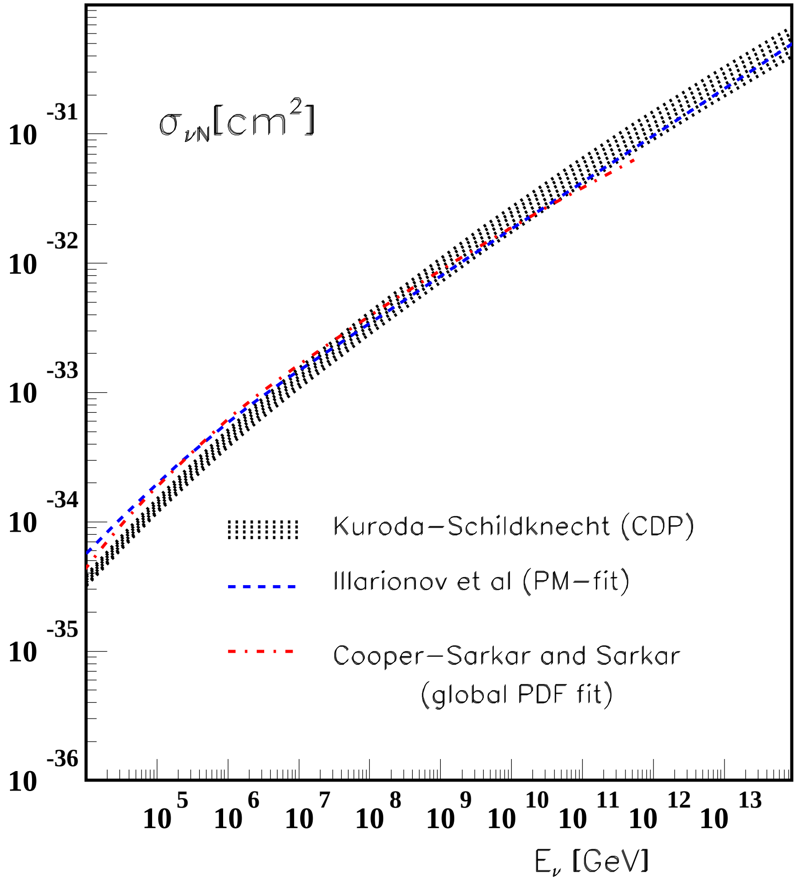
$$M_{q\bar{q}}^2 \le m_1^2(W^2) = \xi \Lambda_{sat}^2(W^2);$$

$\xi = 130$ (fit parameter)

The photoabsorption cross section $\sigma_{\gamma^* p}(\eta(W^2, Q^2))$



The (charged-current) neutrino-nucleon cross section, $\sigma_{\gamma N}(E)$



7. Comparison with “Froissart-inspired” ansatz

Heisenberg (1953):

Lorentz-contracted sphere with experimentally decreasing edge
Minimum “blackness” necessary for particle production.
Collision radius then given by radius of “suffiently black” region.
$\sigma_{hN}(W^2) \sim (\ln W^2)^2$

Froissart (1961):

From unitarity and analyticity, upper bound,
$\sigma_{hN}(W^2) < (\ln W^2)^2$.

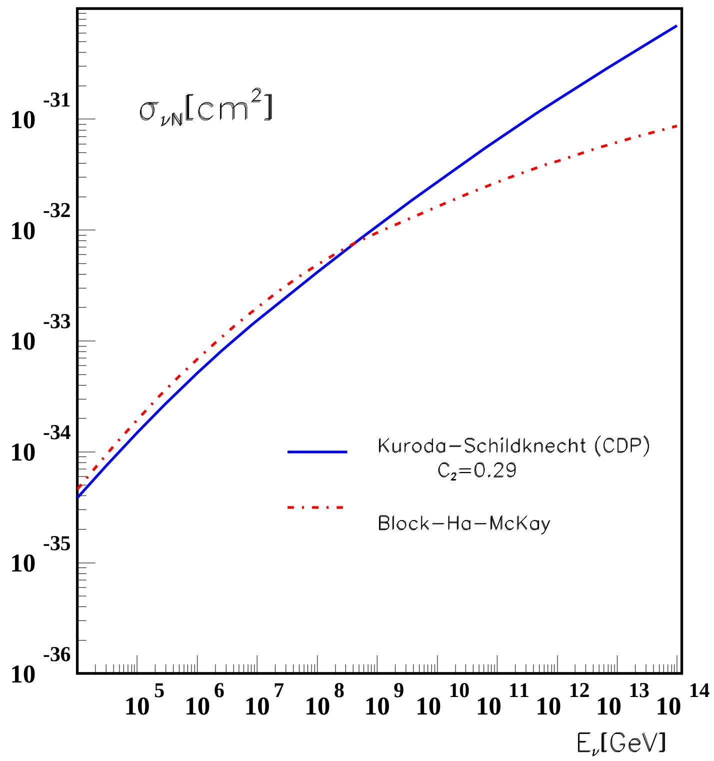
“Froissart-inspired ansatz”:

$$F_2^{ep}(x_1, Q^2) \sim \sum_{n,m=0,1,2} a_{nm} (\ln Q^2)^n (\ln(1/x))^m$$

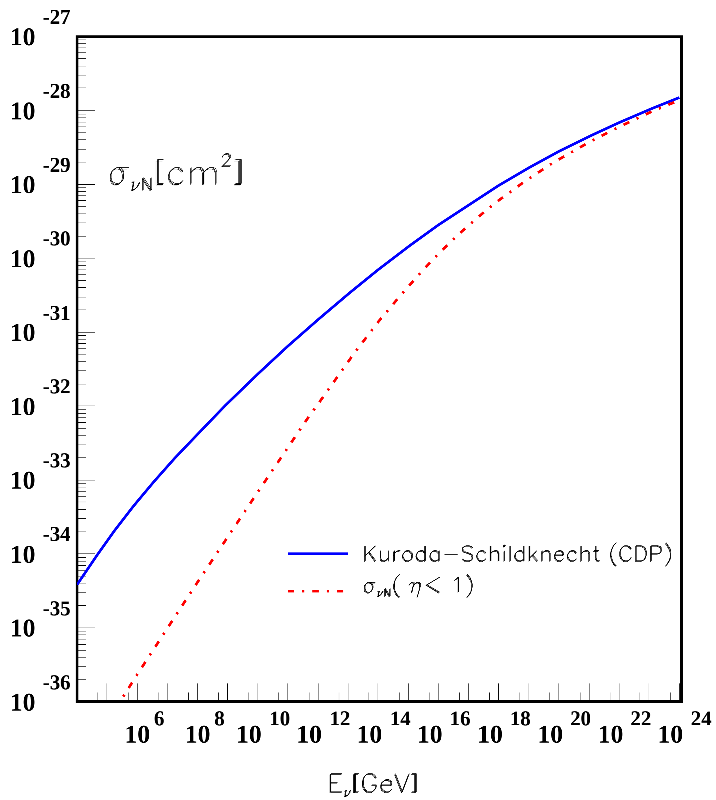
Block et al. (2006 to 2013)

Fit to HERA data with seven fit parameters.

Comparison of $\sigma_{\nu N}(E)$ from the CDP with the results from the "Froissart-inspired" ansatz



Prediction of $\sigma_{\nu N}(E)$ from the CDP for ultra-ultra-high energies, $E \leq 10^{24}$GeV. Reduced growth of cross section for $E \geq 10^{14}$ GeV



$$\sigma_{\nu N}^{(c)}(E) = \sigma_{\nu N}^{(c)}(E)_{\eta(W^2,Q^2)<1} + \sigma_{\nu N}(E)^{(c)}_{\eta(W^2,Q^2)>1}$$

8. Conclusions

- **Predictions for the charged-current neutrino-nucleon cross section based on the CDP are consistent with results obtained from pQCD fits ($E \leq 10^{14}$ GeV).**

- **The results based on the CDP disagree with results from the "Froissart-inspired" ansatz that predicts a strong suppression of the neutrino-nucleon cross section for energies $E \geq 10^{9}$ GeV**

- **A suppression of the cross section, i.e., a weaker growth with increasing energy, is predicted, however, to occur for ultra-ultra-high energies of $E \geq 10^{14}$ GeV**