Problems with single pion production

International School of Nuclear Physics
35th Course
Neutrino Physics: Present and Future
Erice-Sicily

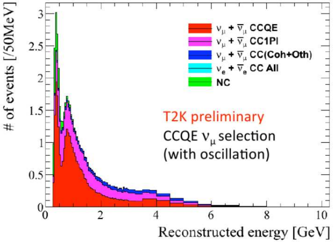
Jakub Żmuda

Institute of Theoretical Physics, University of Wrocław

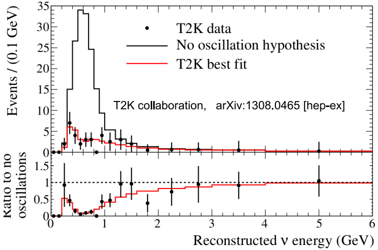
September 23, 2013

Motivation

- Motivation: from ν oscillation experiments. Example: the T2K experiment.



- Single pion production (SPP): second to quasielastic.

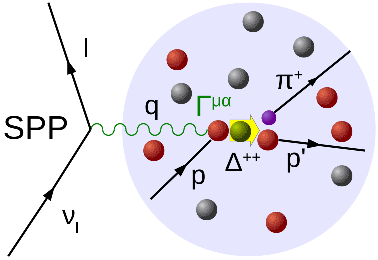


- T2K and other: huge no oscillation/ best fit difference $\rightarrow \Theta_{ij}$, Δm^2_{ij}, ΔCP?, ...?

Predictions of # events with/without oscillation

Based on Monte Carlo. What you put is what you get $\rightarrow$ dependency on theoretical lepton-nucleus interaction modeling.

Single pion production



- At T2K mean energy $\nu_\mu p \to \mu^- p \pi^+$ channel: intermediate $\Delta(1232)$ baryon.

$$\Gamma^{\mu\alpha} = \left[\frac{C_3^V}{M}(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu) + \frac{C_4^V}{M^2}(g^{\alpha\mu}q\cdot p_\Delta - q^\alpha p_\Delta^\mu) + \frac{C_5^V}{M^2}(g^{\alpha\mu}q\cdot p - q^\alpha p^\mu) + g^{\alpha\mu}C_6^V\right]\gamma^5 +$$

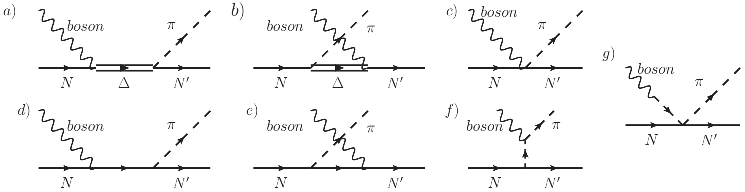
$$+ \left[\frac{C_3^A}{M}(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu) + \frac{C_4^A}{M^2}(g^{\alpha\mu}q\cdot p_\Delta - q^\alpha p_\Delta^\mu) + C_5^A g^{\alpha\mu} + \frac{C_6^A}{M^2}q^\alpha q^\mu\right]$$

- Vector part: (rather) well-known from pion photo- and electroproduction data.
- Axial part: dominated by $C_5^A(Q^2) = \frac{C_5^A(0)}{(1+Q^2/M_{A\Delta}^2)^2}$.
  $C_{3/4}^A$- Adler's relations, C_6^A-PCAC. $C_5^A(0) \approx 1.2 \propto f_{\pi N\Delta}$ (Goldberger-Treiman). $M_{A\Delta}$: fits to ANL/BNL data.

Uniwersytet Wrocławski

The HNV model

- Other isospin channels: large nonresonant background (but in $\nu_\mu p \to \mu^- p \pi^+$ also non-negligible).
- Hernandez, Nieves, Valverde (HNV) model from Phys. Rev. D 76, 033005 (2007):



- Alltogether 7 currents: 2 from Δ resonance (a) and b)), rest from chiral effective field theory.
- Fit to ANL/BNL data with background: E. Hernandez et al. Phys. Rev. D 81, 085046 (2010): $C_5^A(0) = 1.0 \pm 0.11$, $M_{A\Delta} = 0.93 \pm 0.07$ GeV.

Uniwersytet Wrocławski

The HNV model

- Amplitudes in HNV $\mathcal{J}^{\mu}_{hadr.} = \langle N'\pi \,| s^{\mu} |\, N\rangle$. Example:

$$
\begin{aligned}
s^{\mu}_{\Delta P} &= iC_{\Delta P}\frac{f^*}{m_\pi}k^\alpha G_{\alpha\beta}(p+q)\Gamma^{\beta\mu}(p,q) \\
s^{\mu}_{NP} &= -iC_{NP}\frac{g_A}{\sqrt{2}f_\pi}\not{k}\gamma^5\frac{(\not{p}+\not{q}+M)}{(p+q)^2-M^2+i\epsilon}\Gamma^{\mu}(q)F_\pi(k-q)
\end{aligned}
$$

- $\Gamma^{\beta\mu}$: Δ excitation vertex, Γ^{μ}-nucleon weak vertex, $G_{\alpha\beta}$: Rarita-Schwinger propagator, $\Gamma_\Delta(p^2_\Delta)$- $\Delta \to \pi N$ decay width.

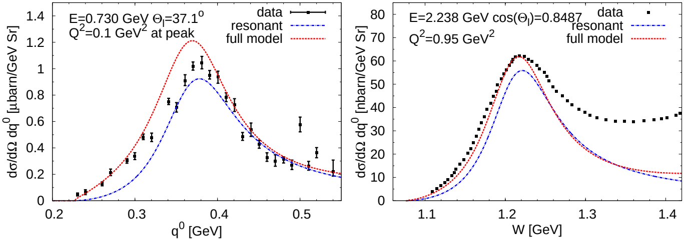
$$
G_{\alpha\beta}(p_\Delta) = \frac{-(\not{p}_\Delta + M_\Delta)}{p^2_\Delta - M^2_\Delta + iM_\Delta\Gamma_\Delta(p^2_\Delta)}\left(g_{\alpha\beta} - \frac{1}{3}\gamma^\alpha\gamma^\beta - \frac{2}{3}\frac{p^\alpha_\Delta p^\beta_\Delta}{M^2_\Delta} + \frac{1}{3}\frac{p^\alpha_\Delta\gamma^\beta - p^\beta_\Delta\gamma^\alpha}{M_\Delta}\right)
$$

- High complexity- Dirac and Lorentz algebra in numerical C++ code.

Uniwersytet Wrocławski

Tests: inclusive electron data

- O. Lalakulich, E. A. Paschos, and G. Piranishvili, Phys. Rev. D , 014009 (2006) electromagnetic Delta form factors.
- Test: inclusive electron-proton scattering data (J. S. O'Connell *et al.*, Phys. Rev. Lett. 53, 1627 (1984) (left), M. Christy and P. E. Bosted, Phys. Rev. C 81, 055213 (2010) (right)).



- Background contribution in interesting energy range (730 MeV)-large.
- Some imperfections for low energies. Above $W \approx 1.25$ GeV- 2π channel, then heavy resonances and DIS

Uniwersytet Wrocławski

Lepton-nucleus cross section

- Double-differential cross section w.r.t. final lepton variables (angle Ω' amd energy E'):

$$\frac{d^3\sigma}{d\Omega' dE'} = \frac{G_F^2 \cos^2\theta_C}{4\pi^2} \frac{|\vec{l'}|}{\sqrt{(l\cdot P_i)^2}} \left(-\frac{1}{\pi}\Im L_{\mu\nu}\Pi^{\mu\nu}\right)$$

$$L_{\mu\nu} = \frac{1}{8}\mathrm{Tr}[(\not{l}' + m_x)\gamma_\mu(1\mp\gamma^5)(\not{l} + m_{\nu_x})\gamma_\nu(1\mp\gamma^5)]$$

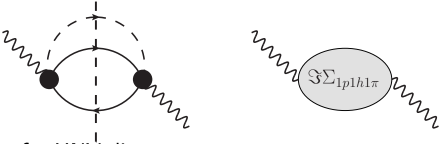
- Information about nuclear system excitations in polarization tensor, $\Pi^{\mu\nu}$:

$$\Pi^{\mu\nu}(q) \equiv i\Omega \int d^4x e^{iqx} \overline{\sum_i} \left\langle i \middle| T\left\{\hat{J}_I^{\nu\dagger}(x)\hat{J}_I^{\mu}(0)\exp\left(-i\int d^4y \hat{H}_{int.}(y)\right)\right\}\middle| i\right\rangle E_i$$

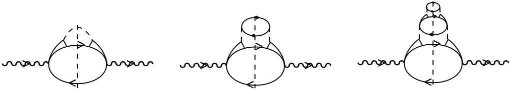
- $\hat{J}^\mu$- nuclear current operator, $\hat{H}_{int.}$- interaction Hamiltonian (here-from chiral field theory).
- $\Pi^{\mu\nu}$: gauge boson self-energy in nuclear matter.

Uniwersytet Wrocławski

- Single pion production contribution to polarization tensor (following IFIC group Phys. Rev. C 83, 045501 (2011)):



- Black circle- sum of 7 HNV diagrams
- Nuclear effects in local density approximation:
  - Fermi motion: local Fermi gas
  - Pauli blocking
  - Δ self-energy in nuclear matter Σ_Δ ($npnh$ excitations, part of MEC) $\Gamma_\Delta \rightarrow \Gamma_\Delta^{PB} - 2\Im\Sigma_\Delta$, parameterization from E. Oset Nucl. Phys. A 468, 631 (1987).



Uniwersytet Wrocławski

SPP on atomic nuclei

- Basic formula with sum over isospins N, N', t_π, integration over nuclear volume d^3r, nucleon d^3p and pion d^3k momenta:

$$\frac{d^3\sigma}{d\Omega' dE'} = \frac{G_F^2 \cos^2\theta_C}{4\pi^2}\frac{|\vec{l}'|}{E}\sum_{N,N',t_\pi}\int d^3r\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\frac{1}{8E_\pi(k)E(p)E(p')}$$

$$\left[n_N(p)(1-n_{N'}(p'))+n_{N'}(p')(1-n_N(p))\right]$$

$$\delta(E(p')-\tilde{q}^0+E_\pi-E(p))\mathrm{Tr}\left[A^{\mu\nu}_{1p1h1\pi}(p,q,k)\right]L_{\mu\nu}.$$

- $A^{\mu\nu}_{1p1h1\pi} \equiv \mathrm{Tr}\left[\mathcal{J}^{\nu}_{hadr.}\mathcal{J}^{\mu *}_{hadr.}\right]$, $n_N(p) = \Theta(k_F^N(r)-p)$.
- Separate piece for Δ-in-medium excitations:

$$\frac{d^3\sigma}{dE' d\Omega'} \approx \frac{G_F^2\cos^2(\Theta_C)|\boldsymbol{l}'|}{16\pi^5|\boldsymbol{l}|}\int dr r^2\sum_{iso}C_{iso}\int d^3p\frac{n_N(p)(\frac{1}{2}\tilde{\Gamma}-\Im\Sigma_\Delta)}{E(p)(M_\Delta+W)}\cdot$$

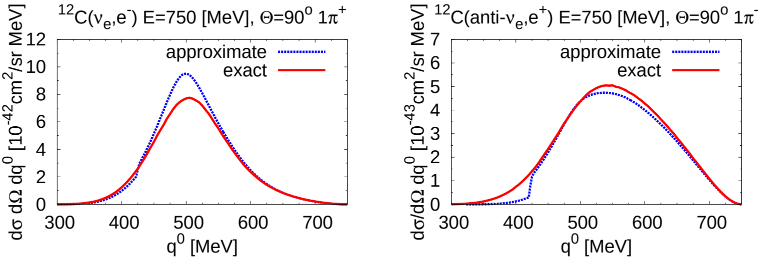
$$\cdot\frac{\mathrm{Tr}\left[\gamma^0\Gamma^{\alpha\mu^\dagger}\gamma^0 P^{3/2}_{\alpha\beta}(p_\Delta)\Gamma^{\beta\nu}(\not{p}+M)\right]L_{\mu\nu}}{(W-(M_\Delta+\Re\Sigma_\Delta))^2+(\frac{1}{2}\tilde{\Gamma}-\Im\Sigma_\Delta)^2}. \quad (1)$$

- Problem: total σ Δ+background integration in 8 dimensions.
  1. Set $<|\vec{p}|>$ in $A^{\mu\nu}$ $\rightarrow$ integral factorization (literal Phys. Rev. C 83, 045501 (2011) solution)
  2. Monte Carlo integration with Vegas algorithm from GSL $\rightarrow$ "exact" solution (J. Sobczyk and J. Żmuda PRC87, 065503 (2013)).

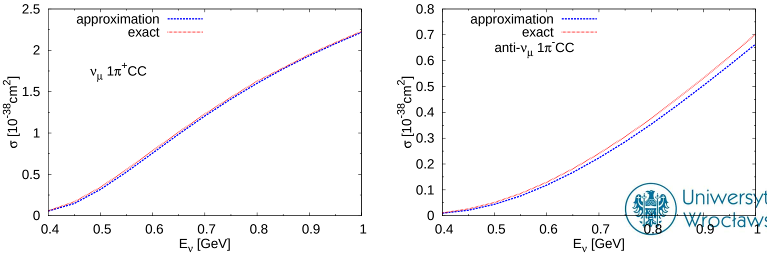
Uniwersytet Wrocławski

"Approximate" and "exact" integration

- Difference for double-differential cross sections (rather large):



- Total cross-sections: good approximation



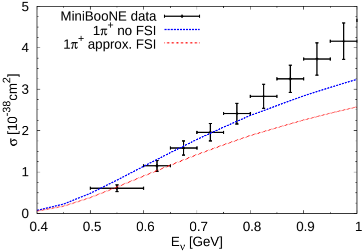
Uniwersytet Wrocławski

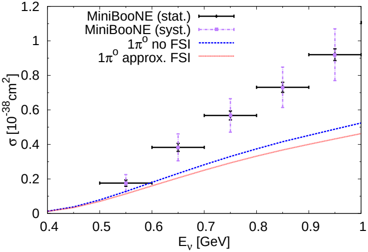
Comparison with MiniBooNE

- MiniBooNE: Phys. Rev. D83, 052007 and 052009 (2011). All 1π final state included!
- Strong final state interactions, no MC simulation yet. Approximate ^{16}O at 1 GeV (T. Golan et al., Acta Phys. Polon. B 40, 2519 (2009)):

$$P(\pi^0 \to \pi^0) = 67\%, \quad P(\pi^0 \to \pi^+) = 5\% \qquad (2)$$
$$P(\pi^+ \to \pi^+) = 69\%, \quad P(\pi^+ \to \pi^0) = 5\%.$$

- In use: $C_5^A(0) = 1.0$ and $M_{A\Delta} = 0.93$ GeV.



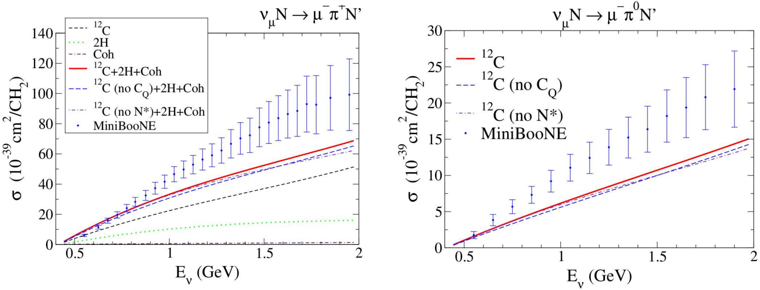


- Data underestimated, but:
- Lack of coherent process, no 2nd resonance region or DIS (multiple mesons) + absorption (final state interactions) $\to 1\pi$ final state.

Uniwersytet Wrocławski

Comparison with MiniBooNE

- (Almost) the same model with same $C_5^A(Q^2)$ E. Hernández, J. Nieves, M. J. Vicente Vacas, Phys. Rev. D 87, 113009 (2013).
- Additional coherent process, D_{13} resonance. FSI in cascade model.

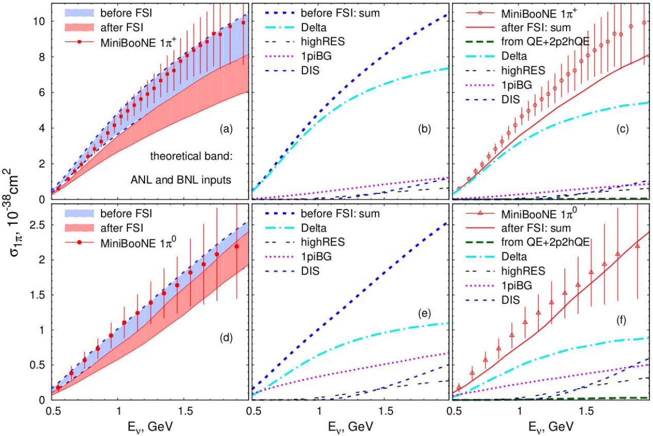


- Two extra mechanisms for SPP, actual FSI and still not enough.

Uniwersytet Wrocławski

Comparison with MiniBooNE

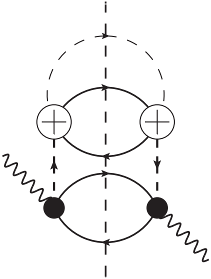
- Different model O. Lalakulich, U. Mosel Phys.Rev. C87 (2013) 014602.
- Different background model, more resonances, migration from other channels through FSI (GiBUU transport model)



- Error band: C_5^A from ANL (lower σ) and BNL (higher σ) experiments. Again, theory below data.

Uniwersytet Wrocławski

Possible explanations



- Atomic nucleus: more mechanisms possible
- Example: SPP on top of meson exchange current (MEC $2p2h1\pi$-type).
- Requirements: good understanding of SPP on free targets and MEC itself.

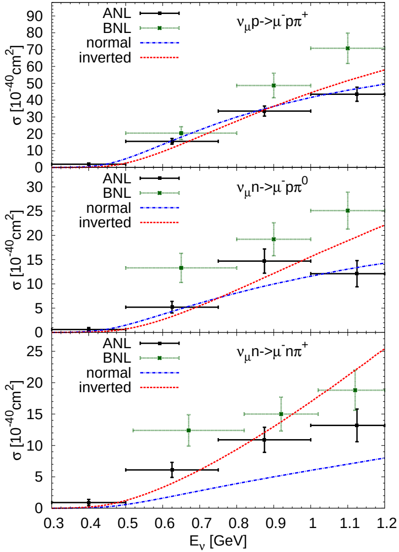
- Other possibilities: need for consistent treatment of Δ off-shell propagator. In use: on-shell propagator (solution to free Rarita-schwinger equation), width put "by hand":

$$G_{\alpha\beta}(p_\Delta) = \frac{-(\not{p}_\Delta + M_\Delta)}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta(p_\Delta^2)}\left(g_{\alpha\beta} - \frac{1}{3}\gamma^\alpha\gamma^\beta - \frac{2}{3}\frac{p_\Delta^\alpha p_\Delta^\beta}{M_\Delta^2} + \frac{1}{3}\frac{p_\Delta^\alpha\gamma^\beta - p_\Delta^\beta\gamma^\alpha}{M_\Delta}\right)$$

- Another possible statement: *MiniBooNE (carbon) "prefers" higher C_5^A, than ANL/BNL (deuteron)"*: medium modification of Δ excitation vertex?

Uniwersytet Wrocławski

Possible explanations



- Full model $C_5^A(0) = 1.0$, $M_{A\Delta} = 0.93$ GeV, no cut in invariant mass W.
- Deuteron effects $\sigma^{^2H} = \int d^3p \frac{f(p)}{v_{rel.}} \sigma^{free}(p)$.
- Norm of deuteron wave function $f(p)$, binding energy $B = 2\sqrt{p^2 + M^2} - M_{^2H}$.
- Toying around with Delta-background interference sign.
- Interference phase- possibly large effect in neutron channels.
- Actual solution: unitarization with Watson's theorem (J. Nieves, L. Alvarez-Ruso)→ dynamical Delta-background complex phase (not a sign flip!).

Uniwersytet Wrocławski

Conclusions and outlook

1. SPP problem leaves us with more questions, than definite answers. Still no completely satisfying SPP model.
2. MiniBooNE CH_2 data: underestimated, problem with simultaneous ANL/BNL data reproduction in all isospin channels.
   - Proper off-shell Δ treatment; possible in-medium excitation vertex modifications?
   - Proper weak Delta-background phases?
   - Extra channels coming from MEC $2p2h1\pi$ and multi-pion+FSI$\rightarrow$ 1π?
3. ANL-BNL-MiniBooNE data: seemingly different, large normalization errors. Need for more precise experiments.

Uniwersytet Wrocławski

Thank you

- Thank you for your attention!
- This project has been sponsored by Polish National Science Centre grant UMO-2011/01/N/ST2/03224.

Uniwersytet Wrocławski