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$2\nu\beta\beta$ decay within SO(5) and SO(8) models and energy - weighted sum rule involving $\Delta Z=2$ nuclei

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Outline

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1.INTRODUCTION

Motivation for study

a) In the limit $2E_n - E_i - E_f \rightarrow 0$ we get $M^{2\nu} \rightarrow 0$, we shall adress the issue what is the origin of this important quantity?

b) What is beyond the single state dominance hypothesis in the $2\nu\beta\beta$? Problem will be studied in the SO(5) and SO(8) models by taking advantage of perturbation theory.

The half life for 2νββ decay can be write in form

$$\left[T^{2\nu\beta\beta}\right]^{-1} = \frac{\frac{1}{4}\int d\Gamma}{\ln 2} = \frac{m_e}{8\pi^7 \ln 2}(G_\beta m_e^2)^4 I^{2\nu}\left(0^+\right)$$

with

$$I^{2\nu}\left(0^+\right) = \frac{1}{m_e^9}\int_{m_e}^{Q-m_e} F_0(Z_f, p_{10}) p_1 p_{10} dp_{10}$$

M.E.

$$\times \int_{m_e}^{Q-p_{10}} F_0(Z_f, p_{20}) p_2 p_{20} dp_{20} \int_0^{Q-p_{10}-p_{20}} k_{10}^2 k_{20}^2 |M^{2\nu}|^2 dk_{10},$$

The nuclear matrix element is defined as

$$|M^{2\nu}|^2 = g_V^4\left[|M_K^F|^2 - Re\left\{M_K^{F*}M_L^F\right\} + |M_L^F|^2\right] -$$
$$g_V^2 g_A^2 Re\left\{M_K^{F*}M_L^{GT} + M_K^{GT*}M_L^F\right\} + g_A^4\left[\frac{|M_K^{GT}+M_L^{GT}|^2}{4} + \frac{1}{12}|M_K^{GT} - M_L^{GT}|^2\right].$$

Where the Fermi and Gamow-Teller matrix elements are defined as

$$M_\Omega^F = \frac{1}{2}\sum_n \left\langle 0_f^+ \left| \sum_m \tau_m^+ \right| 0_n^+ \right\rangle \left\langle 0_n^+ \left| \sum_m \tau_m^+ \right| 0_i^+ \right\rangle \Omega_n \ ,$$

$$M_\Omega^{GT} = \frac{1}{2}\sum_n \left\langle 0_f^+ \left| \sum_m \tau_m^+ (\sigma_m)_k \right| 1_n^+ \right\rangle \left\langle 1_n^+ \left| \sum_m \tau_m^+ (\sigma_m)_k \right| 0_i^+ \right\rangle \Omega_n \ ,$$

with Ω_n=K_n, L_n

and

$$K_n = \frac{1}{(2E_n - E_i - E_f)/2 + \epsilon_K} + \frac{1}{(2E_n - E_i - E_f)/2 - \epsilon_K}$$

$$L_n = \frac{1}{(2E_n - E_i - E_f)/2 + \epsilon_L} + \frac{1}{(2E_n - E_i - E_f)/2 - \epsilon_L}$$

with lepton energies

$$\epsilon_K = (p_{20} + k_{20} - p_{10} - k_{10})/2 \qquad \epsilon_L = (p_{10} + k_{20} - p_{20} - k_{10})/2$$

From above expressions we can see that in the limit, $2E_n - E_i - E_f = 0$ **(n=1,2...) the Fermi and Gamow-Teller matrix elements vanish.**

If we execute approximation by replacing the sum of outgoing lepton energies in the denominators of the 2νββ matrix elements of with its Q value

$$p_{10} + k_{10} \approx p_{20} + k_{20} \approx (E_i - E_f)/2 \equiv Q/2$$

$$p_{10} + k_{20} \approx p_{20} + k_{10} \approx Q/2$$

we can factorize the lepton and nuclear parts in the calculation of the 2νββ decay rate.

$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} = G_{2\nu}(Q,Z)\left|M^{2\nu}\right|^2,$$

Than the nuclear matrix element is independent on the lepton energies

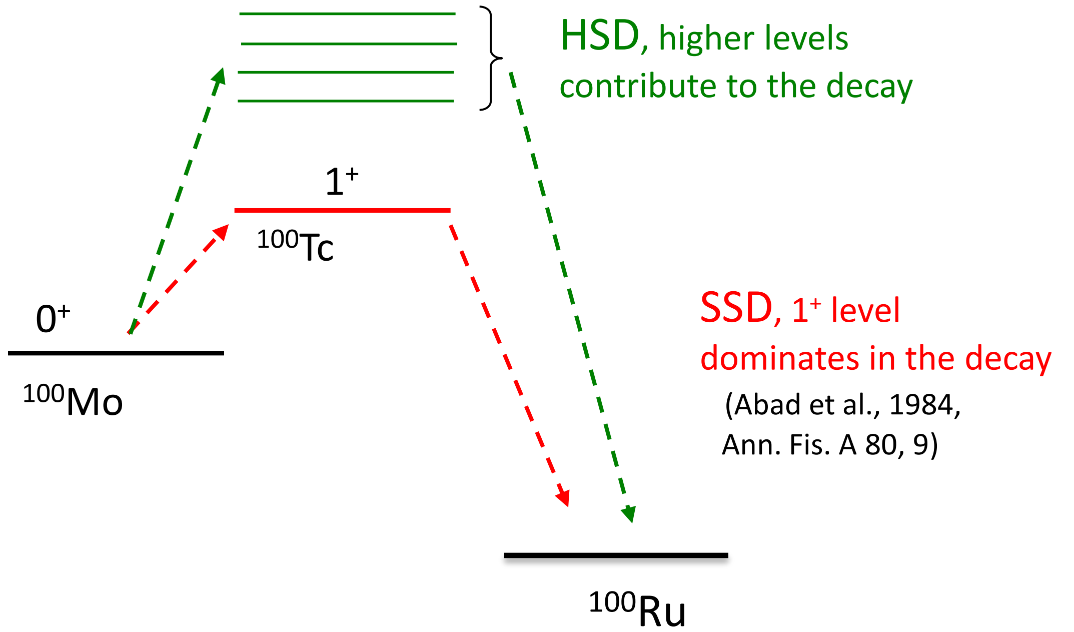
$$|M^{2\nu}|^2 = \left|g_V^2 M^F - g_A^2 M^{GT}\right|^2$$

with Fermi and Gamow-Teller matrix elements defined as

$$M^F = \sum_n \frac{\left\langle 0_f^+ \left| \sum_m \tau_m^+ \right| 0_n^+ \right\rangle \left\langle 0_n^+ \left| \sum_m \tau_m^+ \right| 0_i^+ \right\rangle}{(2E_n - E_i - E_f)/2}$$

$$M^{GT} = \sum_n \frac{\left\langle 0_f^+ \left| \sum_m \tau_m^+ (\sigma_m)_k \right| 1_n^+ \right\rangle \left\langle 1_n^+ \left| \sum_m \tau_m^+ (\sigma_m)_k \right| 0_i^+ \right\rangle}{(2E_n - E_i - E_f)/2}$$

Single State Dominance (^{100}Mo, ^{106}Cd, ^{116}Cd, ^{128}Te …)



In SSD only one term with n=1 is dominant

$$M^{GT} = \sum_n \frac{\left\langle 0_f^+ \left| \sum_m \tau_m^+ (\sigma_m)_k \right| 1_n^+ \right\rangle \left\langle 1_n^+ \left| \sum_m \tau_m^+ (\sigma_m)_k \right| 0_i^+ \right\rangle}{(2E_n - E_i - E_f)/2}$$

$$\approx \frac{\left\langle 0_f^+ \left| \sum_m \tau_m^+ (\sigma_m)_k \right| 1_1^+ \right\rangle \left\langle 1_1^+ \left| \sum_m \tau_m^+ (\sigma_m)_k \right| 0_i^+ \right\rangle}{(2E_n - E_i - E_f)/2}$$

2.Study of the 2νββ Fermi nuclear matrix elements within SO(5) model

We shall study an exactly solvable model for the description 2νββ processes of the Fermi type in order to understand the role of diferent components of H. The schematic Hamiltonian H includes a single-particle term, a pairing term for protons and neutrons, and a schematic charge-dependent residual interaction with both particle-hole and particle-particle channels in a single-one-shell limit and for monopole (J=0) excitations.

$$H = e_p N_p + e_n N_n - G_p S_p^\dagger S_p - G_n S_n^\dagger S_n + 2\chi\beta^- \beta^+ - 2\kappa P^- P^+$$

Where operators are defined as

$$N_i = \sum_m a^\dagger_{m,T_i} a_{m,T_i}, \quad \beta^- = \sum_m a^\dagger_{m,-\frac{1}{2}} a_{m,\frac{1}{2}}, \quad P^- = \sum_m (-1)^{j-m} a^\dagger_{m,-\frac{1}{2}} a^\dagger_{-m,\frac{1}{2}}$$

$$S_i^\dagger = \sum_m \frac{(-1)^{j-m}}{2} a^\dagger_{m,T_i} a^\dagger_{-m,T_i}, \text{ where } i = p, n, \quad T_p = -\frac{1}{2}, \qquad T_n = \frac{1}{2}$$

J. Hirsch, P. Hess, and O. Civitarese, Phys. Rev. C **56,** 199 (1997).

The schematic Hamiltonian can be expressed in terms of following operators (and their hermitian conjugated)

$$A^{\dagger}\left(M_T\right) = \frac{1}{\sqrt{2}}\left[a^{\dagger} \otimes a^{\dagger}\right]^{1}_{M_T} \equiv \sum_{m} C^{00}_{jmj-m} \sum_{m_T, m'_T} C^{1M_T}_{\frac{1}{2}m_T\frac{1}{2}m'_T} a^{\dagger}_{m,m_T} a^{\dagger}_{-m,m'_T}$$

$$T^{-} = -\sqrt{2\Omega}\sum_{m} a^{\dagger}_{m,-\frac{1}{2}} a_{m,\frac{1}{2}}, \quad N = N_p + N_n, \quad T_z = \frac{N_p - N_n}{2}$$

This ten operators represent ten generators of the SO(5) group. In the case of the zero seniority *v=0* (all particles are coupled in J=0) irreducible representations of the SO(5) algebra are well known and the schematic Hamiltonian can be solved exactly.

J. Parikh, Nucl. Phys. **63, 214 (1965).**

For the seniority v=0 or 1, it turns out that the quantum numbers Ω, N, T and T_z, are sufficient to label the wave functions.

$$\Omega=j+1/2$$

$$|N, T, T_z>$$

Possible values of T with given N for seniority s=0.

N	T					
4Ω	0					
4Ω-1		1				
4Ω-2	0		2			
4Ω-3		1		3		
4Ω-4	0		2		4	
|						
2Ω						Ω
|						
8	0		2		4	
6		1		3		
4	0		2			
2		1				
0	0					

For given N, two neighbors states differs in isospin by two units, T±2.

The Hamiltonian can be divided into isoscalar, isovector and isotensor parts

$$H = \left[e_n + e_p - \frac{1}{3}\left(3 + 2\Omega - \frac{N}{2}\right)\left(\frac{G_p + G_n}{2} + 2\kappa\right)\right]\frac{N}{2} + \left[\epsilon_n - \epsilon_p - 2\chi(T_z + 1)\right] T_z$$
$$+ \left[2\chi + \frac{1}{3}\left(\frac{G_p + G_n}{2} + 2\kappa\right)\right] T(T+1)$$
$$+ \frac{\Omega}{\sqrt{2}}\left(\frac{G_p - G_n}{2}\right)[A^\dagger \tilde{A}]^1_0 + \sqrt{\frac{2}{3}}\Omega\left(4\kappa - \frac{G_p + G_n}{2}\right)[A^\dagger \tilde{A}]^2_0.$$

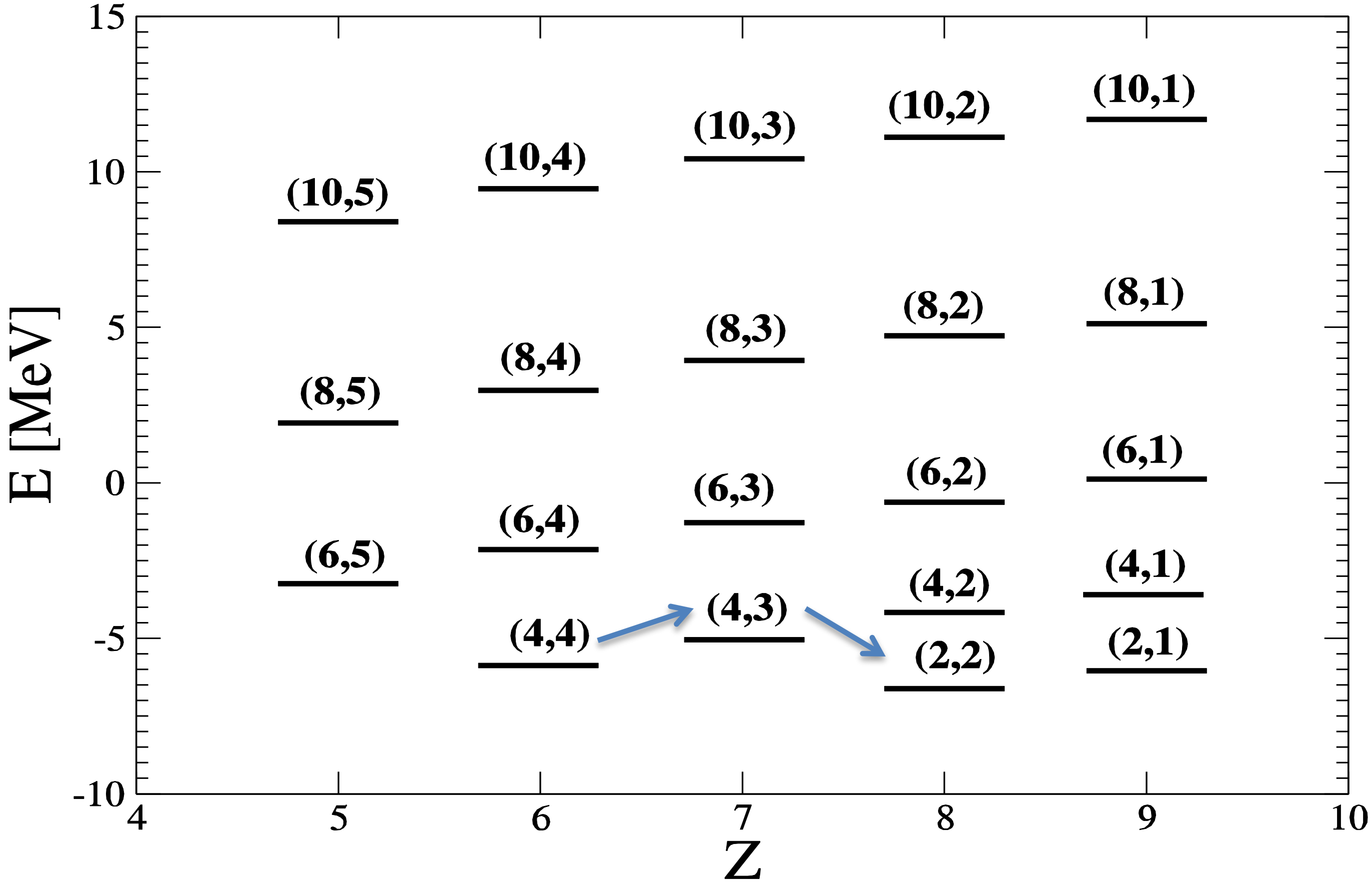
This isotensor term is responsible for SU(2) isospin breaking

In general, Hamiltonian is not diagonal in the $|N, T, T_z\rangle$ basis and diagonalization requires calculation of these matrix elements $\langle N, T, T_z| H|N, T, T_z\rangle$ and $\langle N, T\pm2, T_z| H|N, T, T_z\rangle$

We assume next sets of parameters, for which the Hamiltonian has only isoscalar terms

$$G/2 \equiv G_p = G_n, \quad \kappa = G/4, \quad \Omega = 10, \quad N = 20, \quad e_n = 0.1\text{MeV},$$

$$e_p = 0.3\text{MeV}, \quad G = 0.165\text{MeV}, \quad \chi = 0.044\text{MeV}$$



We will investigate double beta decay

$$(4,4) \longrightarrow (2,2) + 2e^- + 2\bar{\nu}_e$$

We see that Fermi matrix element is zero in the limit of isospin SU(2) conservation

$$M_F = \sum_{T=4,6,8,10} \frac{\left\langle 2,2 \left| T^- \right| T,3 \right\rangle \left\langle T,3 \left| T^- \right| 4,4 \right\rangle}{\left(2E_{T,3} - E_{4,4} - E_{2,2}\right)/2}$$

We switch on the isotensor part of Hamiltonian, which we will consider as a small perturbation to the isoscalar part of Hamiltonian. It means, that we assume (4κ-G)≈0.

Perturbation theory allows us to obtain expression for 2νββ matrix elements close to a point of the restoration of isospin symmetry in terms of the parameters of the nuclear Hamiltonian. It will allow us to understand better the role of the single particle mean field, pairing and residual interactions in the evaluation of 2νββ matrix transition elements.

Now the eigenstates of the Hamiltonian are a superposition of eigenstates of isospin operator T^2 and the Fermi matrix element is non zero.

$$|E, T_z> = \sum_T C_T^E |T, T_z>$$

Analysis of Fermi double matrix element within the perturbation theory reveals, that only the transition trough the lowest 0^+ state of intermediate nucleus, which main component has the same isospin as the intial nucleus, is important.

$$M^F = \frac{\left\langle E_f, 2|T^-|E_{n_1}, 3\right\rangle \left\langle E_{n_1}, 3|T^-|E_i, 4\right\rangle}{(2E_{n_1} - E_i - E_f)/2}$$

Where the numerator can be expressed - up to the second order of perturbation theory - as

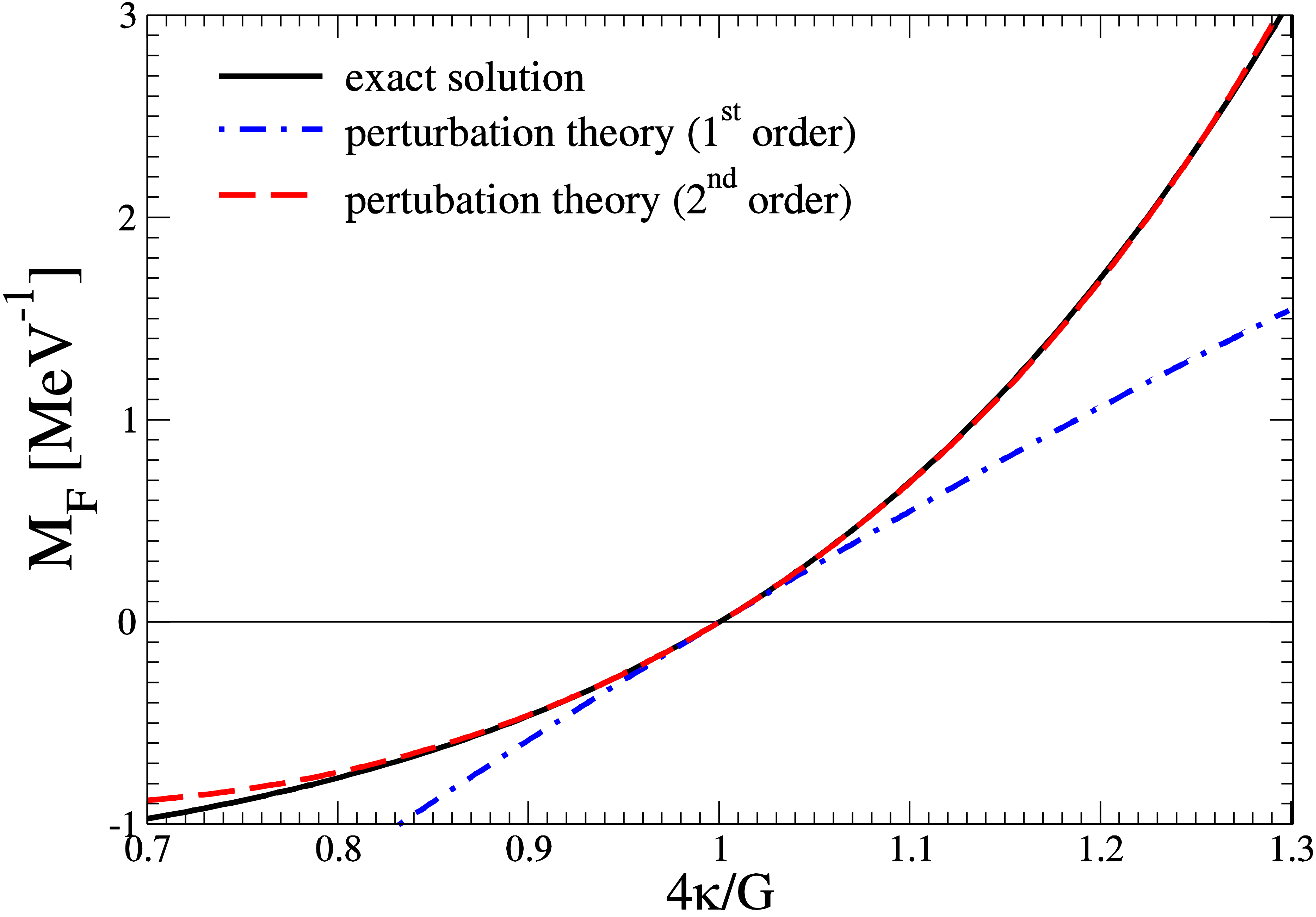
$$\left\langle 42\left|T^-\right|43\right\rangle\left\langle 43\left|T^-\right|44\right\rangle\left[\sqrt{\frac{2}{3}}\Omega(G-4\kappa)\frac{\left\langle 42\left|[A^\dagger\tilde{A}]^2_0\right|22\right\rangle}{(28\chi+14/3(G+2\kappa))}+\frac{2}{3}\frac{\Omega^2(G-4\kappa)^2}{(28\chi+14/3\,(G+2\kappa))^2}\times\right.$$
$$\left.\times\left[\left(\left\langle 42\left|[A^\dagger\tilde{A}]^2_0\right|42\right\rangle\left\langle 42\left|[A^\dagger\tilde{A}]^2_0\right|22\right\rangle-\left\langle 22\left|[A^\dagger\tilde{A}]^2_0\right|22\right\rangle\left\langle 42\left|[A^\dagger\tilde{A}]^2_0\right|22\right\rangle\right)\right]\right]$$

And energy denominator as

$$\left(2E_{n=1}-E_i-E_f\right)/2=16\chi+\frac{7}{3}(G+2\kappa)$$
$$+\sqrt{\frac{1}{6}}\Omega\,(4\kappa-G)\left[2\left\langle 43\left|[A^\dagger\tilde{A}]^2_0\right|43\right\rangle-\left\langle 44\left|[A^\dagger\tilde{A}]^2_0\right|44\right\rangle-\left\langle 22\left|[A^\dagger\tilde{A}]^2_0\right|22\right\rangle\right]+$$
$$+\frac{1}{3}\Omega^2\,(4\kappa-G)^2\left[\frac{\left\langle 64\left|[A^\dagger\tilde{A}]^2_0\right|44\right\rangle^2}{44\chi+22/3\,(G+2\kappa)}+\frac{\left\langle 42\left|[A^\dagger\tilde{A}]^2_0\right|22\right\rangle^2}{28\chi+14/3\,(G+2\kappa)}-2\frac{\left\langle 63\left|[A^\dagger\tilde{A}]^2_0\right|43\right\rangle^2}{44\chi+22/3\,(G+2\kappa)}\right].$$

There is no explicit dependence on the mean field parameters e_p, e_n.

D. Štefánik, F. Šimkovic, K. Muto and A. Faessler, Phys. Rev. C **88**, 025503 (2013)



exact solution
perturbation theory (1st order)
pertubation theory (2nd order)
M_F [MeV^{-1}]
$4\kappa/G$
3
2
1
0
-1
0.7
0.8
0.9
1
1.1
1.2
1.3

3. Study of the 2νββ Gamow-Teller nuclear matrix elements within the SO(8) model

The Hamiltonian will contain $T=0,S=1,L=0$ and $T=1,S=0,L=0$ two body particle-particle interactions and $T=1,S=1,L=0$ particle-hole interactions and will be expressed in terms of the generators of an SO(8) algebra. We denote it as the SO(8) model.

$$H = -g_{pair}\sum_{M_T} A^{\dagger}_{0,1}(0,M_T)A_{0,1}(0,M_T) - g_{pp}^{T=0}\sum_{M_S} A^{\dagger}_{1,0}(M_S,0)A_{1,0}(M_S,0) + g_{ph}\sum_{\mu,\nu} F^{\mu\dagger}_{\nu}F^{\mu}_{\nu}$$

$$A^{\dagger}_{S,T}(M_S,M_T) = \sum_{m_S,m_T,m}\sqrt{\frac{\Omega}{2}}C^{00}_{lmlm'}C^{TM_T}_{\frac{1}{2}m_t\frac{1}{2}m'_t}C^{SM_S}_{\frac{1}{2}m_s\frac{1}{2}m'_s}a^{\dagger}_{lmm_sm_t}a^{\dagger}_{lm'm'_sm'_t}$$

$$F^{b}_{a} = \sum_{m,m_s,m_t}\langle (m_s+a)(m_t+b)\,|\sigma_a\tau_b|\,m_sm_t\rangle\, a^{\dagger}_{nlm(m_s+a)(m_t+b)}a_{nlmm_sm_t}$$

The 12 operators of type A and 9 operators of type F, together with 7 operators **S**, **T** and N, represents 28 generators of group SO(8).

Engel J. et al., Phys. Rev. C **55,** 1781(1997)

In the case of zero seniority, the wave functions for given *l* shell are labeled as

$$|N, S, S_z, T, T_z, n>$$

Matrix elements of the SO(8) generators are known in this basis.

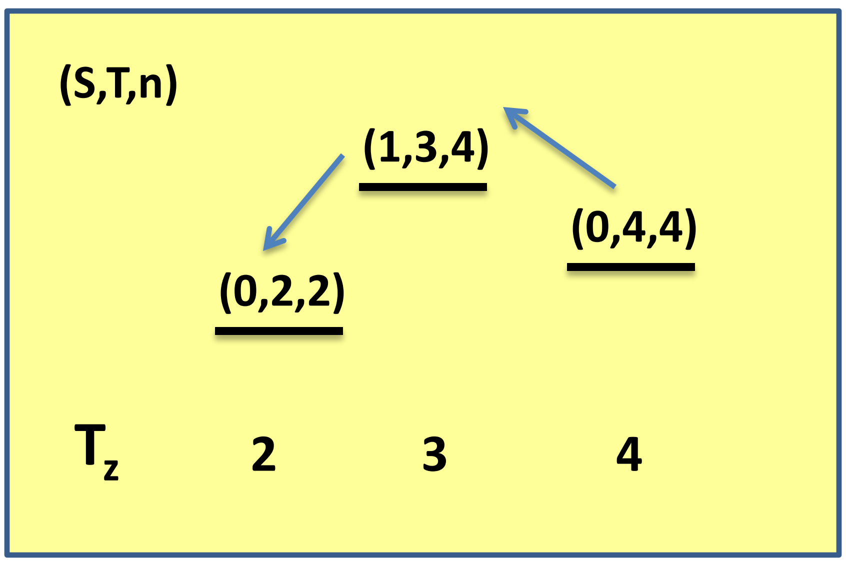
When the $g_{pp}^{T=0} = g_{pair}$, the Hamiltonian is diagonal in this basis, on the other hand it mixes the quantum numbers $n \pm 2$.

For given (S,T), the values of SU(4) quantum numbers are $n = S+T,\ S+T+2, \ldots, N/2$

S. Pang, Nucl. Phys. **A128,** 497 (1969)
K. Hecht and S. C. Pang, J. Math. Phys. **10,** 1571 (1969)

We separate the schematic Hamiltonian in the SU(4) spin-isospin symmetry conservation part and in the SU(4) breaking part.

$$H = \underbrace{-g_{pair}\left(\sum_{M_T} A^{\dagger}_{0,1}(0,M_T)A_{0,1}(0,M_T) + \sum_{M_S} A^{\dagger}_{1,0}(M_S,0)A_{1,0}(M_S,0)\right) + g_{ph}\sum_{\mu,\nu} F^{\mu\dagger}_{\nu}F^{\mu}_{\nu}}_{H_0}$$
$$\underbrace{+\,(g_{pair} - g^{T=0}_{pp})\sum_{M_S} A^{\dagger}_{1,0}(M_S,0)A_{1,0}(M_S,0)}_{H_I}$$



Because the Gamow-Teller operators connect only states with the same SU(4) quantum numbers, the Gamow-Teller nuclear matrix element vanishes in the case of restoration of the SU(4) symmetry.

Using the perturbation theory up to the second order of the parameter (g_{pair}-$g_{pp}^{T=0}$), we can find that only the transition through ground state of the intermediate nucleus give dominant contribution to the G-T matrix element.

$$M_{GT}^{2\nu} \simeq \frac{\left\langle 0 \left| \vec{\sigma}\tau^{-} \right| 1_{n=1} \right\rangle \left\langle 1_{n=1} \left| \vec{\sigma}\tau^{-} \right| 0 \right\rangle}{(2E_{n=1} - E_f - E_i)/2}$$

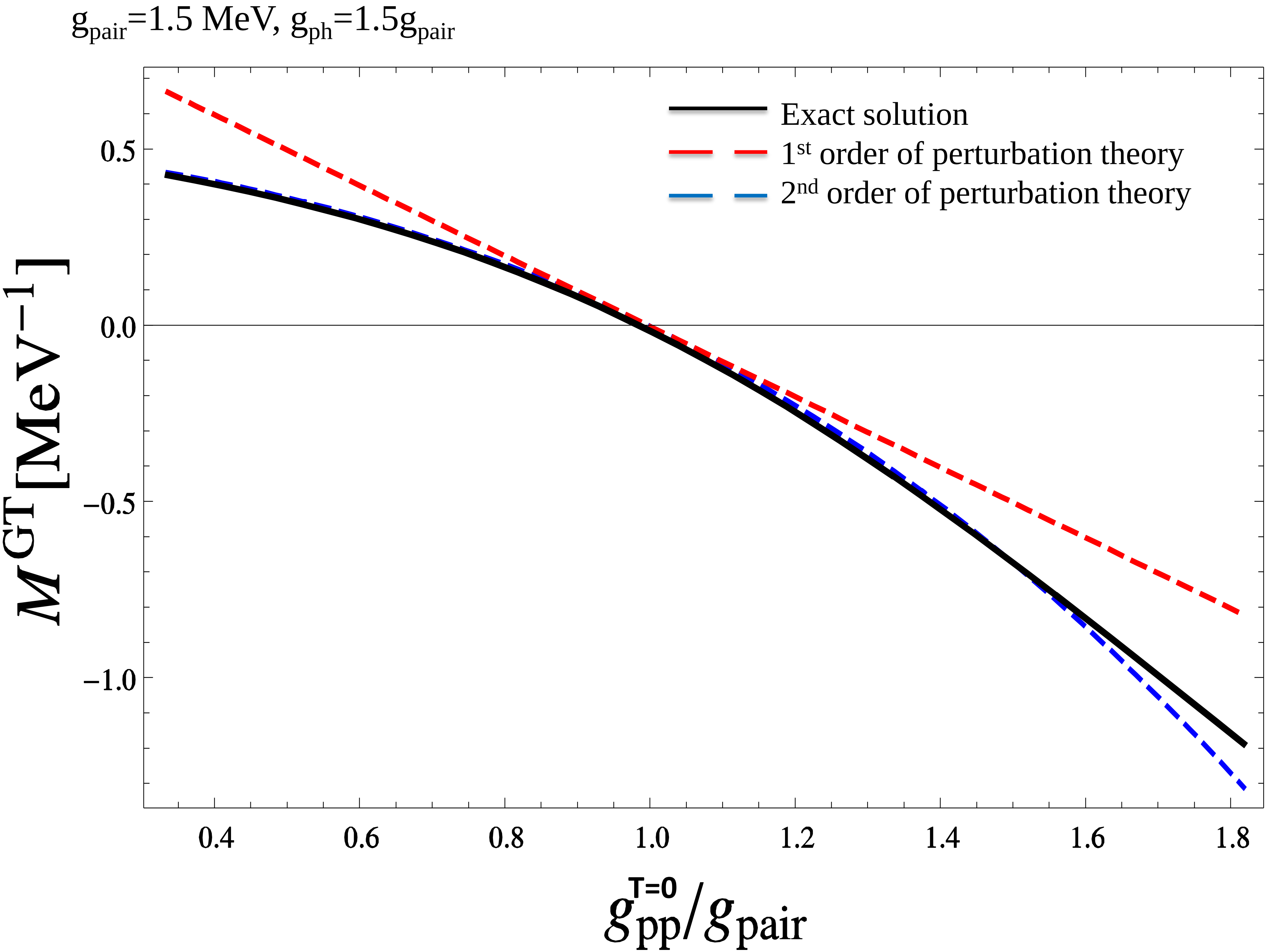
where the numerator has the form

Ω=12, N=20,

$$144\sqrt{\frac{231}{35}}\left(\frac{(g_{pair} - g_{pp}^{T=0})}{10g_{pair} + 20g_{ph}} - \frac{267(g_{pair} - g_{pp}^{T=0})^2}{35(-10g_{pair} - 20g_{ph})^2}\right)$$

and the energy denominator

$$(2E_{n=1}-E_i-E_f)/2 = 5g_{pair}+9g_{ph}+(g_{pair}-g_{pp}^{T=0})\frac{39}{5}+\frac{(g_{pair} - g_{pp}^{T=0})^2}{g_{pair} + 2g_{ph}}\left(\frac{1249263}{171500}\right)$$



g_pair=1.5 MeV, g_ph=1.5g_pair
Exact solution
1st order of perturbation theory
2nd order of perturbation theory
0.5
0.0
−0.5
−1.0
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
M^GT [MeV^−1]
g_pp^T=0/g_pair

Šimkovic, Rodin, Faessler, Vogel Phys. Rev. C **87,** 045501 (2013).

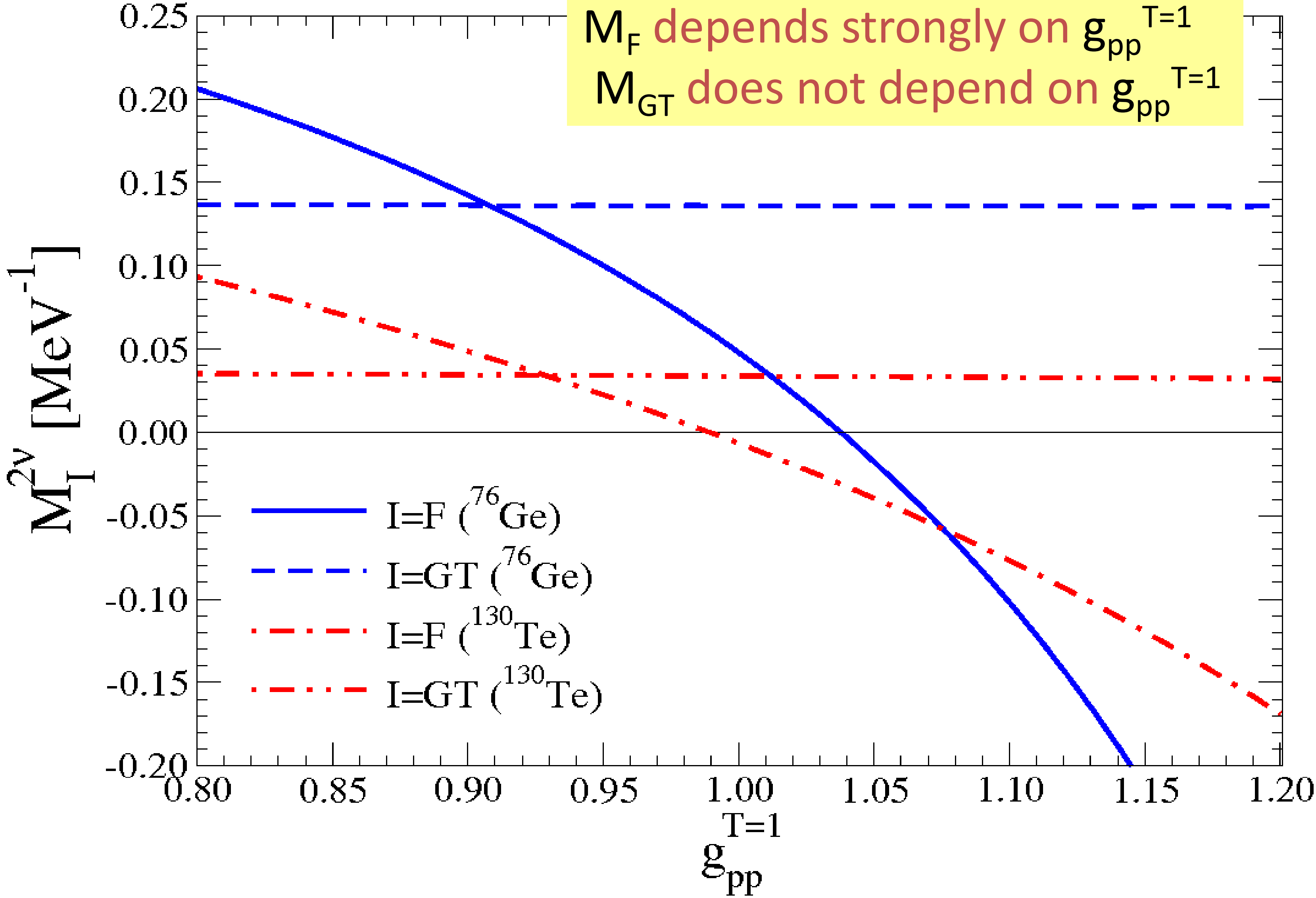
M_F

M_{GT}

$$\mathbf{H = H_0 + g_{ph}\, H_{ph} + g_{pp}^{T=0}\, H_{pp}^{T=0} + g_{pp}^{T=1}\, H_{pp}^{T=1}}$$

M_F depends strongly on $g_{pp}^{T=1}$

M_{GT} does not depend on $g_{pp}^{T=1}$



Adding term, which breaks also SU(2) symmetry of Hamiltonian.

$$H_N = H - (g_{pp}^{T=1} - g_{pair}) A^{\dagger}_{0,1}(0,0) A_{0,1}(0,0)$$

We investigate the dependence of Fermi a Gamow-Teller NME on parameters $g_{pp}^{T=1}$ and $g_{pp}^{T=0}$.

Up to the first order of perturbation theory we get

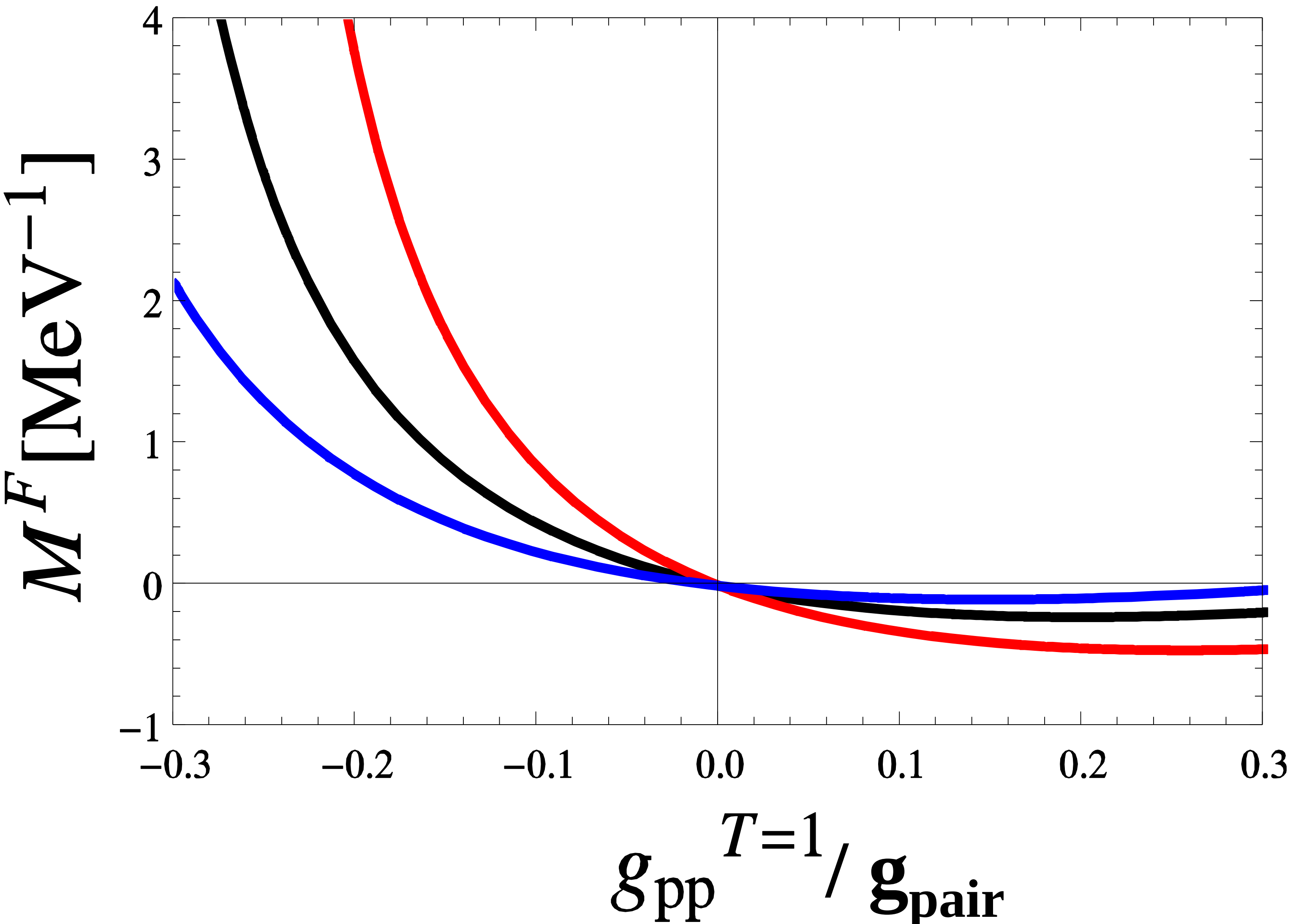
Ω=12, N=20

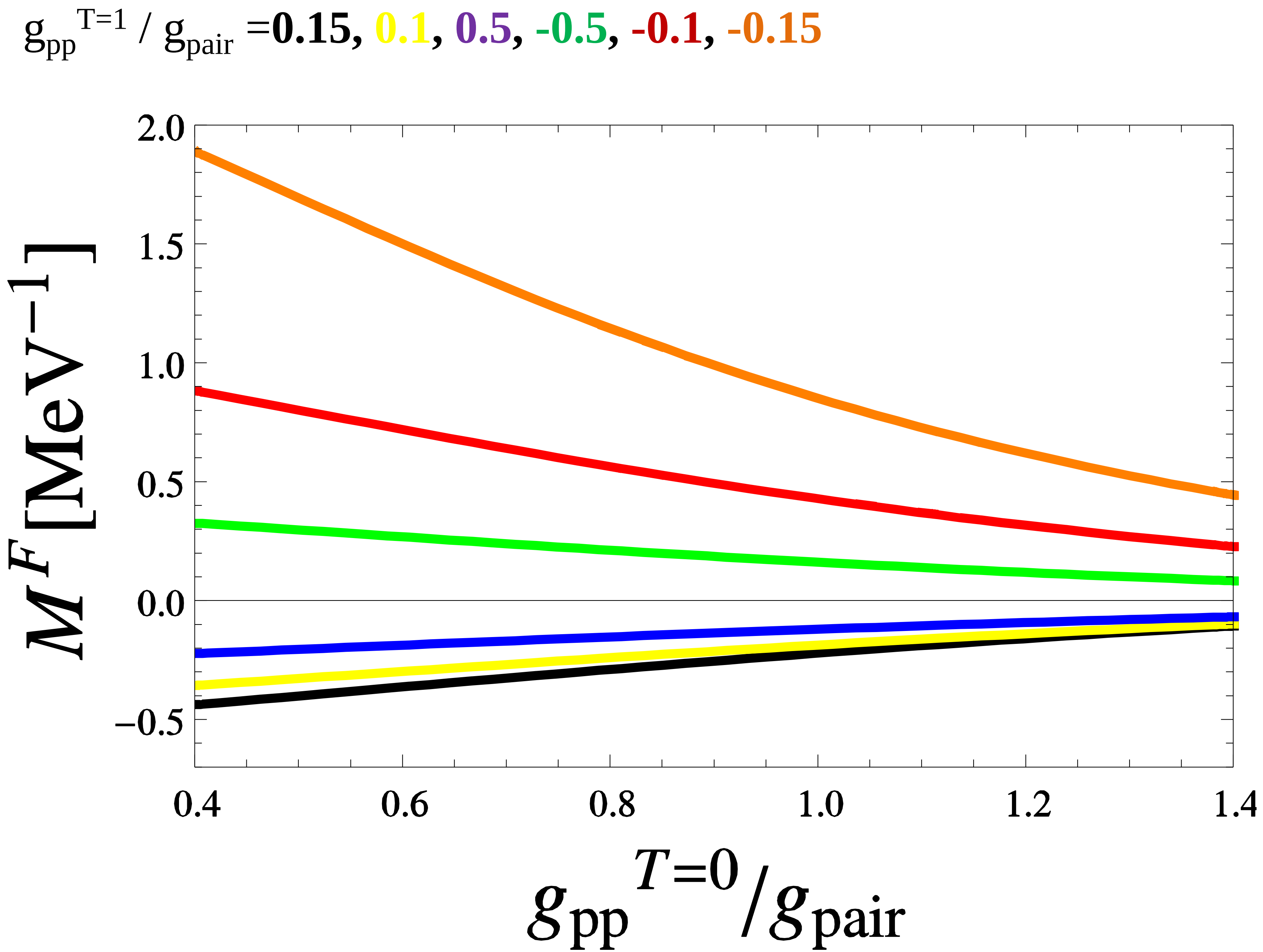
$$M_F^{2\nu} \simeq \frac{48\sqrt{\frac{33}{5}}\left(g_{pair} - g_{pp}^{T=1}\right)}{(5g_{pair} + 3g_{ph})(10g_{pair} + 6g_{ph})}.$$

$$M_{GT}^{2\nu} \simeq \frac{144\sqrt{\frac{33}{5}}}{5g_{pair} + 3g_{ph}} \frac{(10g_{pair} + 6g_{ph})(g_{pair} - g_{pp}^{T=0}) - 2g_{ph}\left(g_{pair} - g_{pp}^{T=1}\right)}{(10g_{pair} + 20g_{ph})(10g_{pair} + 6g_{ph})}$$

$g_{pair}=1.5$, $g_{ph}=1.5g_{pair}$ $\Omega=12$, N=20

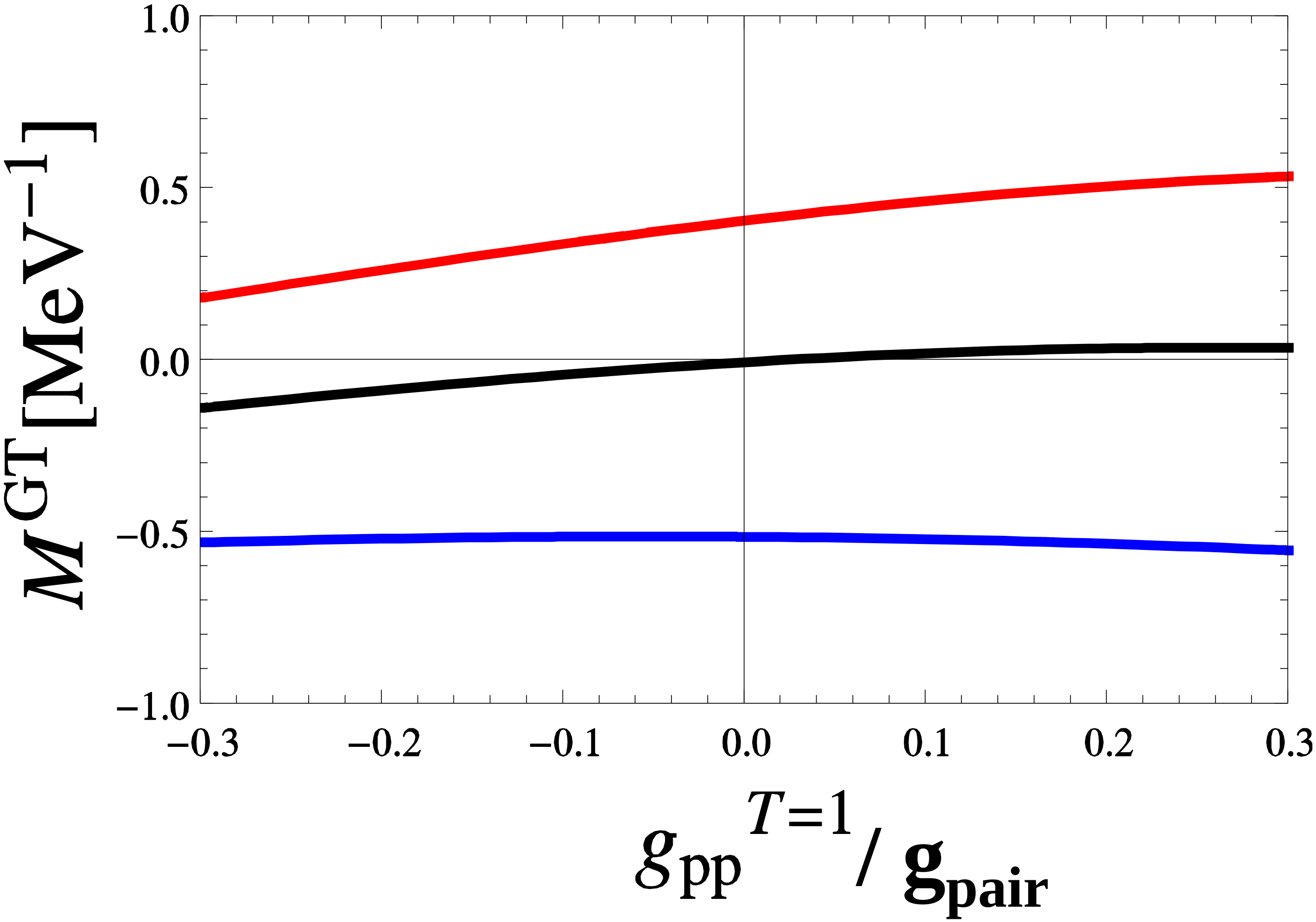
$g_{pp}{}^{T=0} / g_{pair}$ =**1**, **0.5**, **1.4**



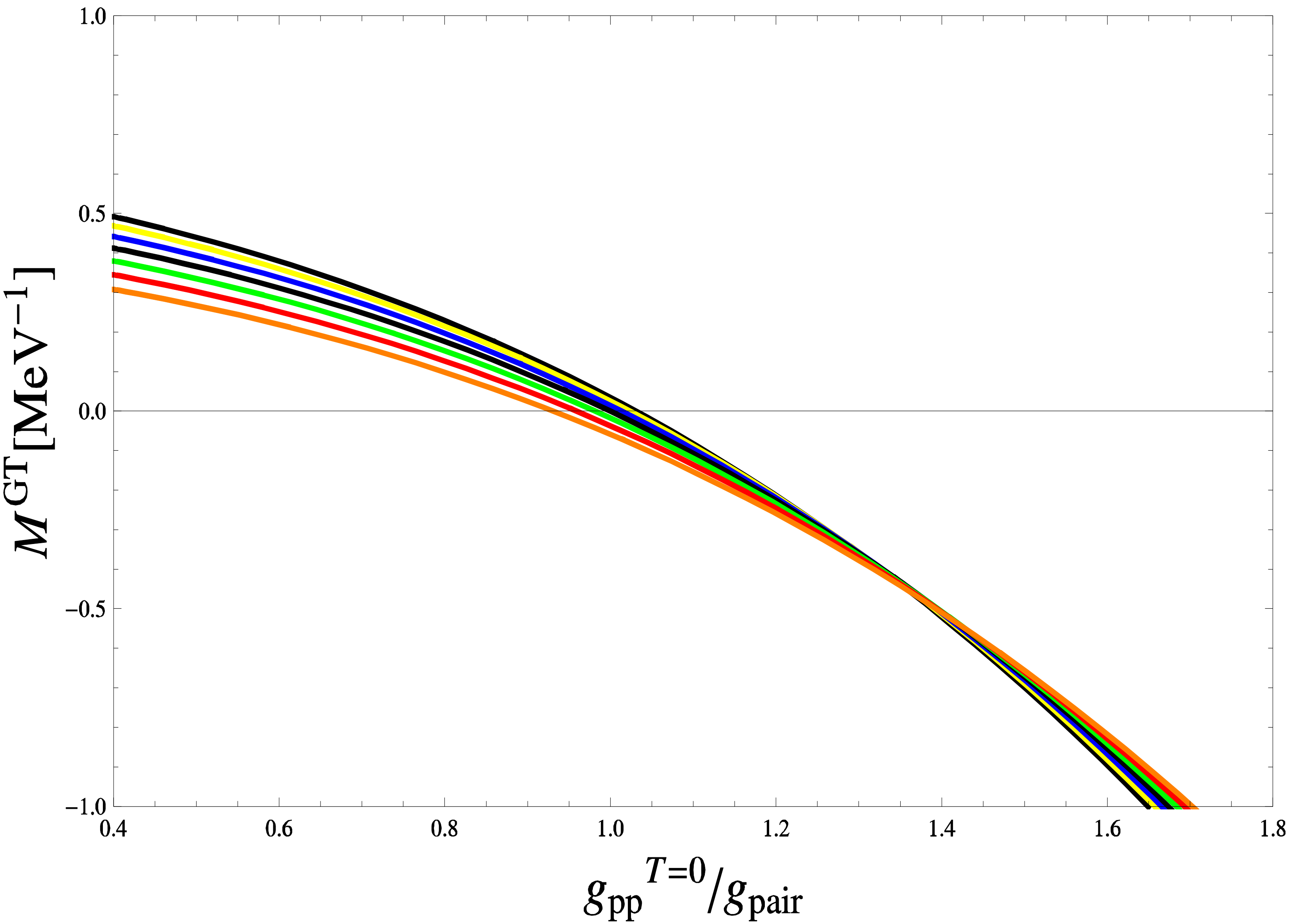


g_pp^T=1 / g_pair =0.15, 0.1, 0.5, -0.5, -0.1, -0.15
2.0
1.5
1.0
0.5
0.0
−0.5
0.4
0.6
0.8
1.0
1.2
1.4
M^F [MeV^−1]
g_pp^T=0 / g_pair

$g_{pp}{}^{T=0}$ / g_{pair} =**1**, **0.5**, **1.4**



$g_{pp}^{T=1} / g_{pair}$ = **0.15, 0.1, 0.5, 0, -0.5, -0.1, -0.15**



3.Energy - weighted sum rule involving ΔZ=2 nuclei

The energy-weighted sum rules have played an important role in the understanding of relevant properties of many-body systems.

In this contribution we shall discuss energy weighted double Fermi and double Gamow-Teller sum rules associated with ΔZ = 2 nuclei:

$$S^{ew}_{F,GT} = \sum_n (E_n - \frac{E_i + E_f}{2}) \left\langle f \left| O_{F,GT} \right| n \right\rangle \left\langle n \left| O_{F,GT} \right| i \right\rangle = \frac{1}{2} \left\langle f \left| \left[O_{F,GT}, \left[H, O_{F,GT} \right] \right] \right| i \right\rangle .$$

Here, O_F (O_{GT}) represents the Fermi (Gamow-Teller) operator. |i > is the ground state of the initial nucleus (A,Z) and |f > is the ground or excited state of the final nucleus.

The left hand side of the sum rule can be determined phenomenologically by making assumption aboat the dominance of low lying states of intermediate nucleus (SSD). Then it might be exploited to study residual interactions of nuclear Hamiltonian, namely like-nucleon pairing, particle-particle and particle-hole proton-neutron interactions.

Within the SO(5) model we obtain sum rule

$$S_F^{ew}(i,f) = \frac{1}{2}\left\langle f\left|\left[T^-,\left[H,T^-\right]\right]\right|i\right\rangle = 2\Omega(G-4\kappa)\left\langle i\left|[A^\dagger\tilde{A}]^2_2\right|f\right\rangle + 2\chi\left\langle f\left|T^-T^-\right|i\right\rangle$$

Analysis of this equation for the initial state with $T_Z=4$ and final state with $T_Z=2$, shows us that near the point of the SU(2) symmetry restoration the sum on the left hand side can be approximated by single term - SSD hypothesis is valid.

In the first-order perturbation theory with initial state |4´4> and final state |2´2> we obtain

$$S_F^{ew}(4'4,2'2) \simeq \left(16\chi + \frac{7}{3}(G+2\kappa)\right)\sqrt{\frac{2}{3}}\Omega\,(G-4\kappa)\,\frac{\left\langle 42\left|[A^\dagger\tilde{A}]^2_0\right|22\right\rangle}{(28\chi + 14/3\,(G+2\kappa))}$$

$$\times\left\langle 42\left|T^-\right|43\right\rangle\left\langle 43\left|T^-\right|44\right\rangle.$$

D. Štefánik, F. Šimkovic, K. Muto and A. Faessler, Phys. Rev. C **88**, 025503 (2013)

From the sum rule we can find again a combination of energies of involved states to be a function of pairing and particle-particle and particle-hole interactions

In first-order we obtain

$$E'_{43}-(E'_{44}+E'_{22})/2 \simeq 16\chi+\frac{7}{3}(G+2\kappa) \\ +\sqrt{\frac{1}{6}}\Omega\,(4\kappa-G)\left[2\left\langle 43\left|[A^\dagger\tilde{A}]^2_0\right|43\right\rangle-\left\langle 44\left|[A^\dagger\tilde{A}]^2_0\right|44\right\rangle-\left\langle 22\left|[A^\dagger\tilde{A}]^2_0\right|22\right\rangle\right]$$

For the sum rule including initial state |4´4> and final state |4´2> (first excited state) we obtain up to the first-order

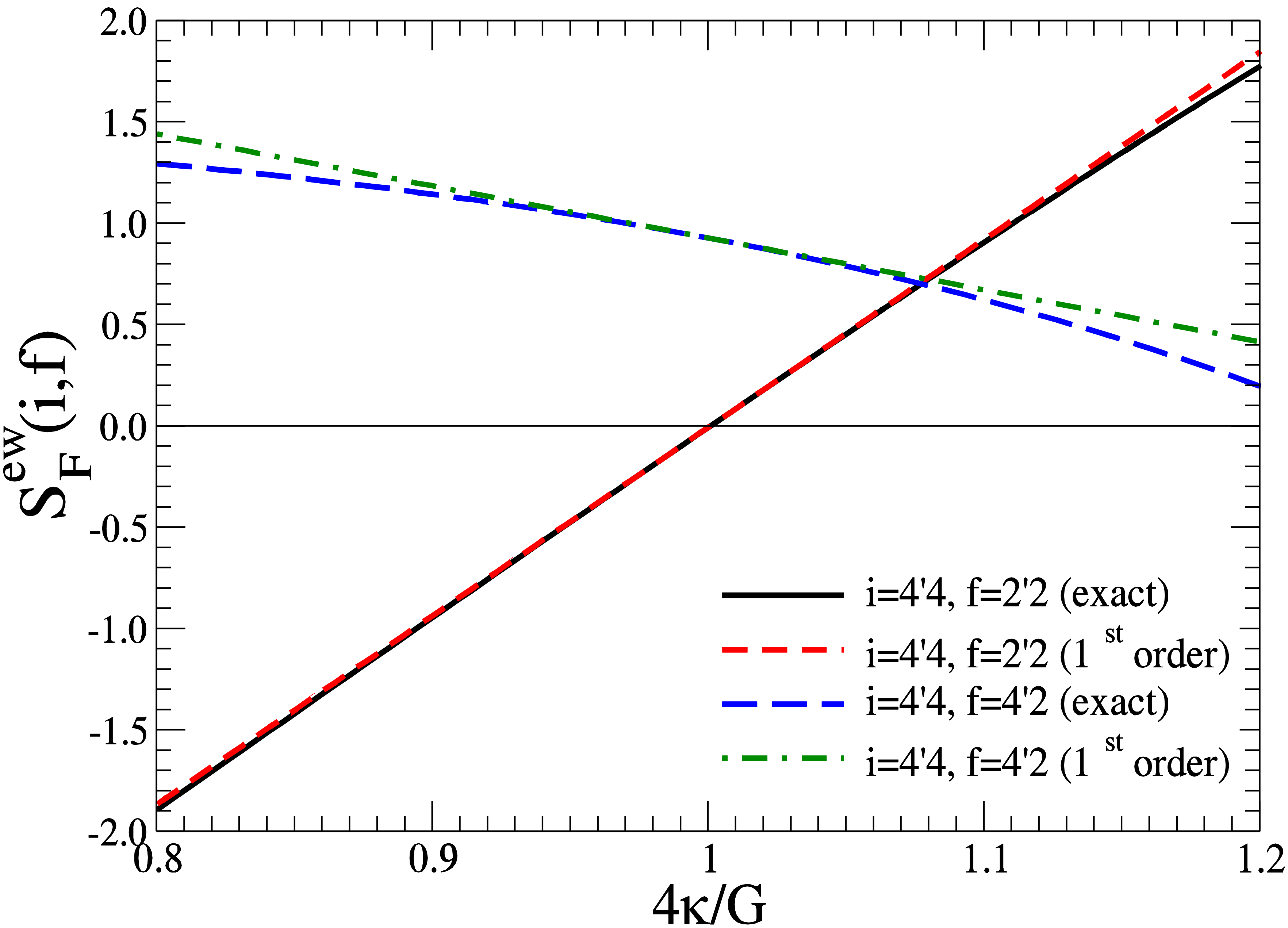
$$S^{ew}_F(4'4,4'2)\simeq\left(2\chi+\sqrt{1/6}\Omega(4\kappa-G)\left[2\left\langle 43\left|[A^\dagger\tilde{A}]^2_0\right|43\right\rangle-\left\langle 44\left|[A^\dagger\tilde{A}]^2_0\right|44\right\rangle-\left\langle 42\left|[A^\dagger\tilde{A}]^2_0\right|42\right\rangle\right]\right) \\ \times\left\langle 42\left|T^-\right|43\right\rangle\left\langle 43\left|T^-\right|44\right\rangle.$$

For a combination of energies of involved states we have

$$E'_{43} - (E'_{44} + E'_{42})/2 = 2\chi$$
$$+\sqrt{1/6}\Omega(4\kappa - G)\left(2\left\langle 43\left|[A^\dagger\tilde{A}]^2_0\right|43\right\rangle - \left\langle 44\left|[A^\dagger\tilde{A}]^2_0\right|44\right\rangle - \left\langle 42\left|[A^\dagger\tilde{A}]^2_0\right|42\right\rangle\right)$$

Thus, the energy-weighted sum rule $S_F^{ew}(4'4, 4'2)$implies another useful relation between energies of states and nucleon-nucleon interactions.

We note that a contribution from the second lowest intermediate state to the both sum rules appears only in third-order perturbation theory.



2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
0.8
0.9
1
1.1
1.2
4κ/G
$S_F^{ew}(i,f)$
i=4'4, f=2'2 (exact)
i=4'4, f=2'2 (1 st order)
i=4'4, f=4'2 (exact)
i=4'4, f=4'2 (1 st order)

Within the SO(8) model and Gamow-Teller operator we obtain next sum rule

$$S_{GT}^{ew} = \frac{1}{2}\sum_{M}(-1)^{M}\left\langle f\left|\left[\sigma_M\tau^-,\left[H,\sigma_{-M}\tau^-\right]\right]\right|i\right\rangle$$

$$= 6(g_{pp}^{T=0} - g_{pair})\left\langle f\left|A_{0,1}^{\dagger}(0,-1)A_{0,1}(0,1)\right|i\right\rangle - g_{ph}\left\langle f\left|\vec{\sigma}\tau^- \cdot \vec{\sigma}\tau^-\right|i\right\rangle - 3g_{ph}\left\langle f\left|T^-T^-\right|i\right\rangle.$$

For the Fermi transition operator T^- we obtain within the SO (8) model next sum rule

$$S_{F}^{ew} = 2\left(g_{pp}^{T=1} - g_{pair}\right)\left\langle f\left|A_{0,1}^{\dagger}(0,-1)A_{0,1}(0,1)\right|i\right\rangle.$$

Ω=12, N=20

In the case |i>=|00444´> and |f>=|00222´> and isospin conservation limit, we find that only transition through the single intermediate state with n´=4´ is relevant near the SU(4) spin-isospin symmetry restoration point. From G-T sum rule we obtain combination for energies

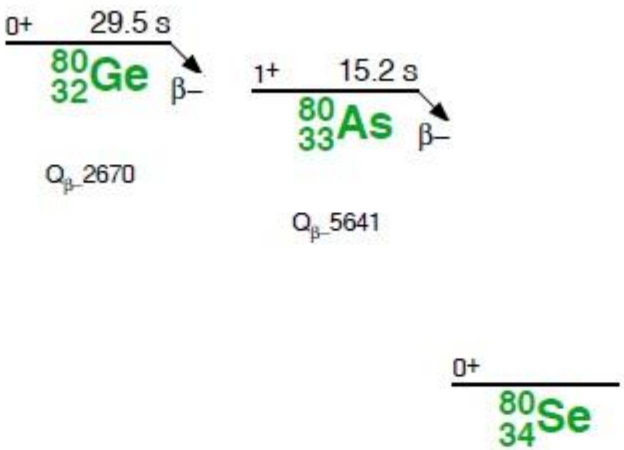
$$E'_{10334} - \frac{E'_{00444} + E'_{00222}}{2} = 5g_{pair} + 9g_{ph} + \frac{39}{5}(g_{pair} - g_{pp}^{T=0}) + \frac{64}{35}(g_{pair} - g_{pp}^{T=1}).$$

For the S_F^{ew} we can also find, that the transition thought the single state is relevant. We obtain another relation between energies

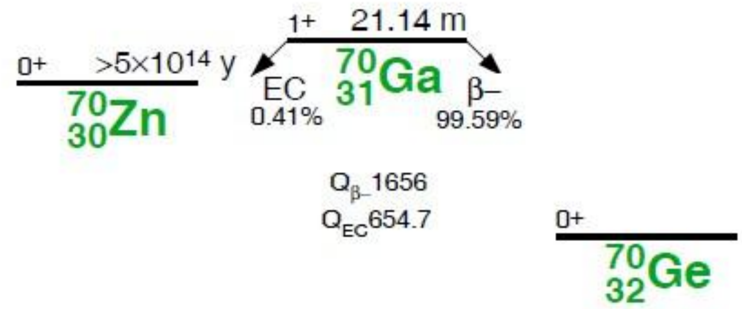
$$E'_{00434} - \frac{E'_{00444} + E'_{00222}}{2} = 5g_{pair} + 3g_{ph} - \frac{401}{35}(g_{pair} - g_{pp}^{T=1}) - \frac{192}{35}(g_{pair} - g_{pp}^{T=0}).$$

Ω=12, N=20

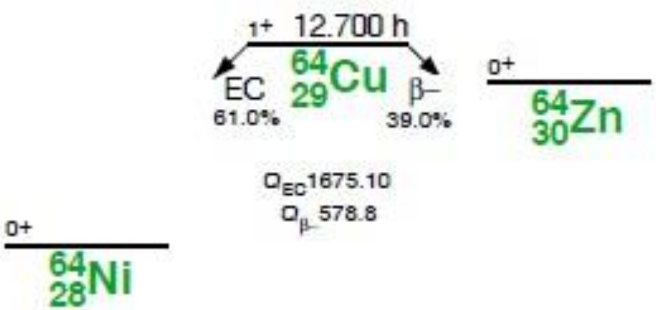
We will analyze $2E_{n=1}(1^+) - E_i(0^+) - E_f(0^+)$ for four types of processes.



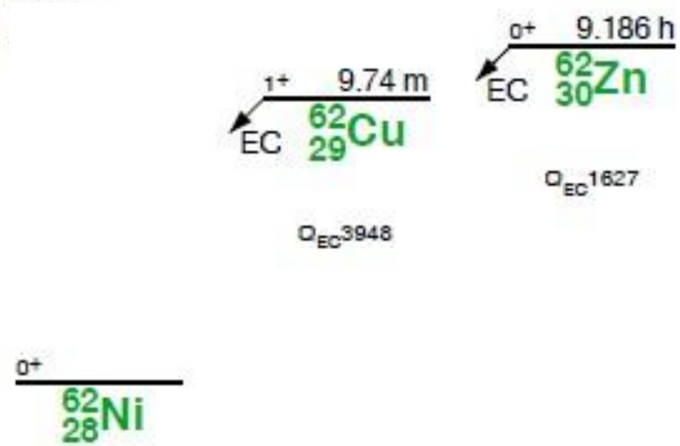
Two subsequent β^--decays



$2\nu\beta^-\beta^-$

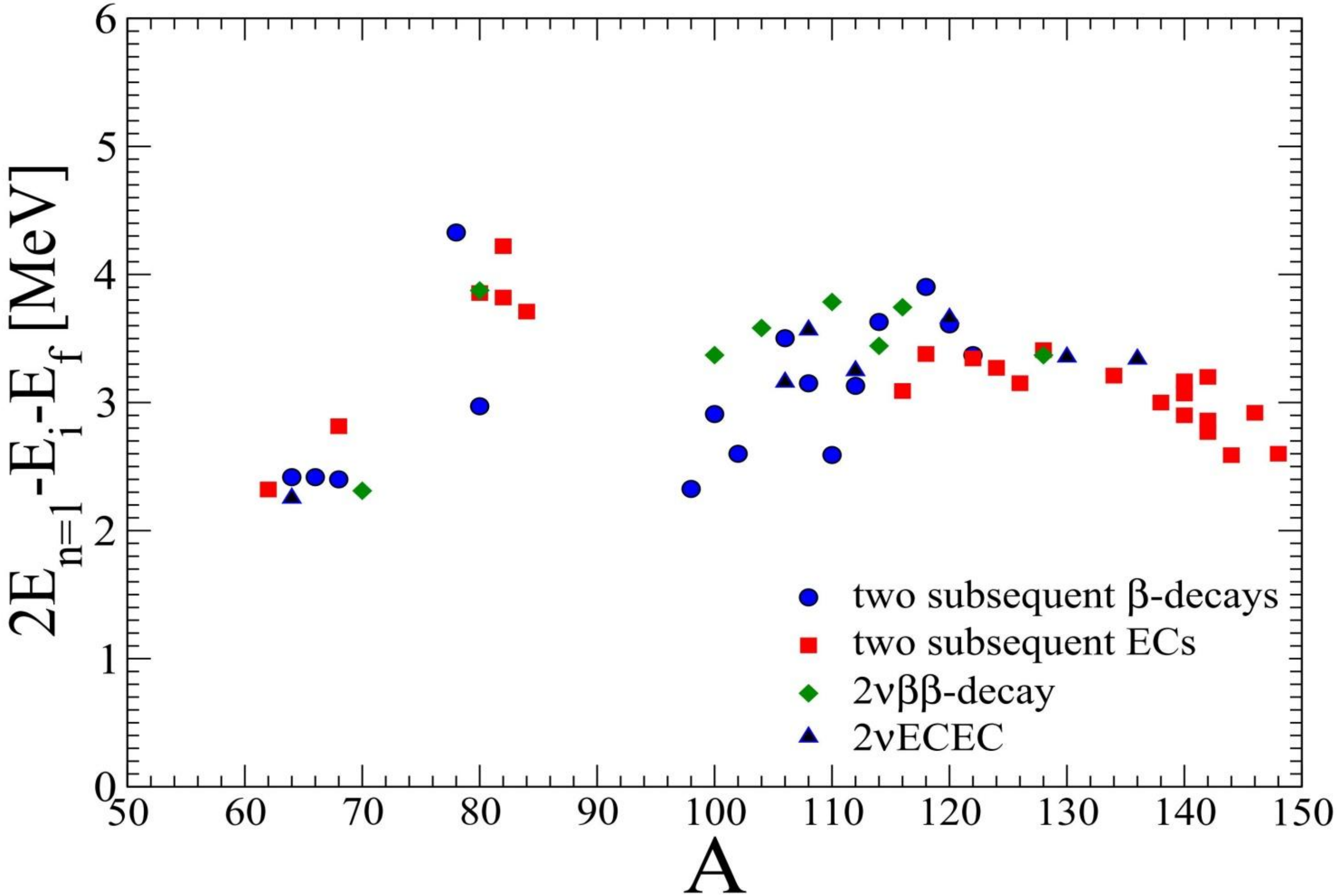


2νECEC



Two subsequent ECs

$2E_{n=1} - E_i - E_f$ throughout the periodic table of elements. Is the small spread of values of this quantity indication for dominince of low lying states (SSD)?



Study of sum rule within realistic model are needed.

Conclusions

By using the perturbative theory we have obtained explicit dependence of the Fermi and Gamow Teller nuclear matrix elements on the parameters of the schematic models based on the SO(5) and SO(8) group. We found that there is no explicit dependence on mean field parameters and pointed out the importance of different componenets of residual interactions.

We have shown that the $2\nu\beta\beta$ decay nuclear matrix element within the SO(8) model exhibits simillar dependence on the particle-particle strengths as was found in the calculation based on the QRPA with isospin symmetry restoration. The origin of the suppression of the the $2\nu\beta\beta$ decay nuclear matrix was determined, namely there is a strong cancelation of effects of pairing and residual particle-particle interactions.

The exactly solvable model allows us to test the different approximation schemes. There is a possibility to consider a more realistic schematic Hamiltonian. A two level version of SO(8), i.e., SO(8)×SO(8), might be a good place to start with(Engel J. et al., Phys. Rev. C **55,** 1781(1997)).

We found that energy weighted sum rule for $\Delta Z=2$ nuclei can be exploited to fix residual interaction of nuclear Hamiltonian. Further studies of energy-weighted sum rules connecting $\Delta Z = 2$ nuclei within a realistic nuclear model are in progress.