Inclusive and Coherent Neutrino-Nucleus Reactions

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Neutrino-Nucleus Reactions…

..high energy physics:

- Fundamental physics
- Weak interactions
- CP violation
- Neutrino mixing*
- Quark mixing**
- Majorana or Dirac?
- Mass generation

…nuclear physics:

- Neutrinos as a probe for nuclear structure
- Beta-decay
- Neutrino mixing⁺
- Weak vertices in matter
- Weak formfactors
- Chiral symmetry

*Pontecorvo–Maki–Nakagawa–Sakata matrix

**Cabibbo-Kobayashi-Maskawa matrix

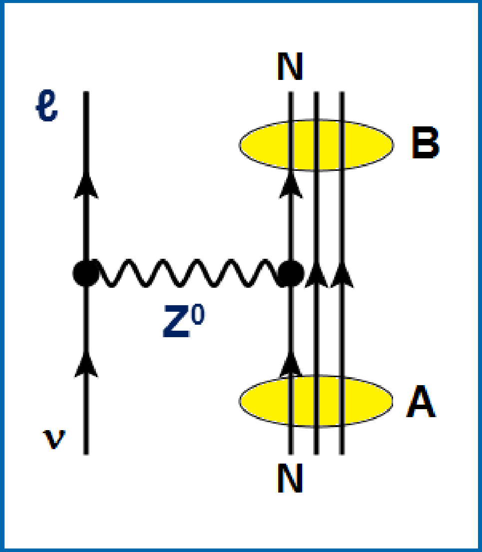
⁺Mikheyev–Smirnov–Wolfenstein effect

Agenda:

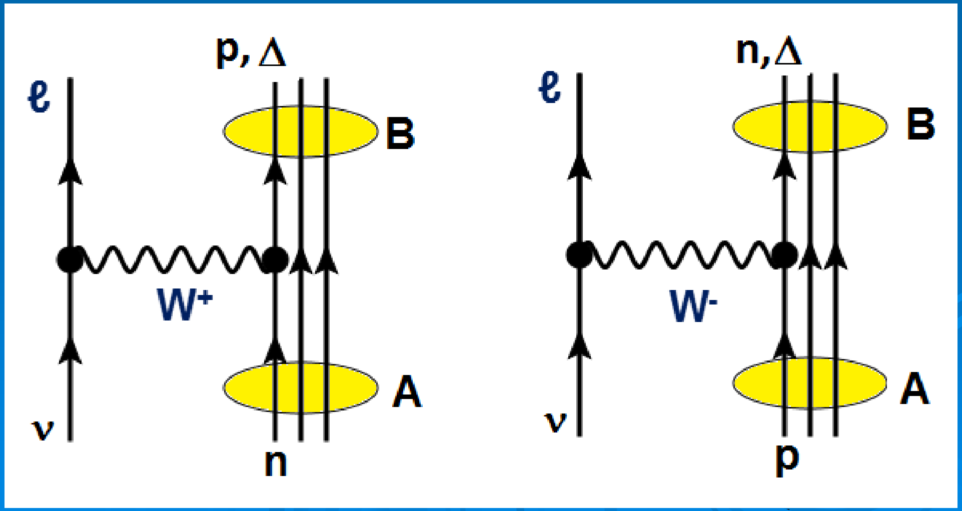
- Aspects of Neutrino-Nucleus Reactions
- Nuclear Excitations and Response Functions
- Response Functions and Electron Scattering
- Neutrino induced Pion Production

Neutrino-Nucleus Reactions

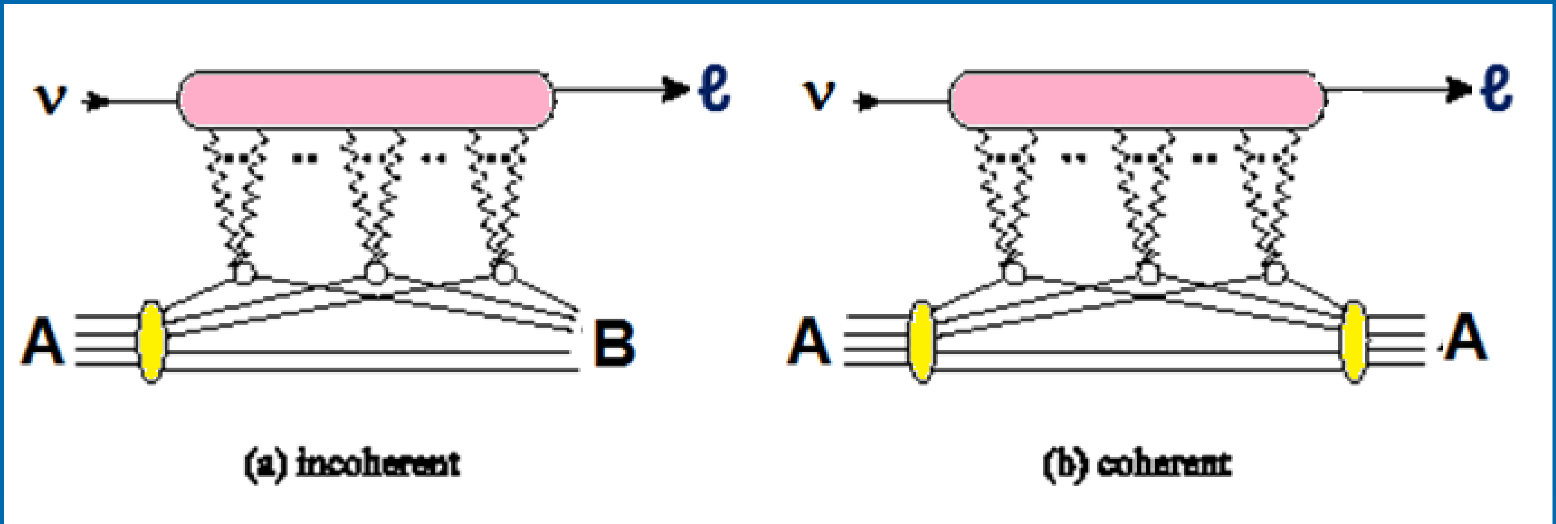
Neutral current (NC) reactions by Z^0 exchange – nuclear charge conserved



Charged current (CC) reactions by $W^{\pm}$ exchange – nuclear charge changed



- **Coherent and incoherent reactions**



Inclusive: decay of B not observed

Inclusive Neutrino-Lepton Reaction on a Nucleus

$$\nu_l(k) + A_Z \rightarrow l^-(k') + X,$$

$$\frac{d^2\sigma_{\nu l}}{d\Omega(\hat{k}')dE_l'} = \frac{|\vec{k}'|}{|\vec{k}|}\frac{G^2}{4\pi^2}L_{\mu\sigma}W^{\mu\sigma},$$

Leptonic Tensor:

$$L_{\mu\sigma} = L^s_{\mu\sigma} + \mathrm{i}L^a_{\mu\sigma} = k'_\mu k_\sigma + k'_\sigma k_\mu - g_{\mu\sigma}k\cdot k' + \mathrm{i}\epsilon_{\mu\sigma\alpha\beta}k'^\alpha k^\beta .$$

Hadronic Tensor:

$$W^{\mu\sigma} = \frac{1}{2M_i}\overline{\sum_f}(2\pi)^3\delta^4(P_f' - P - q)\ \langle f|j^\mu_{\rm cc}(0)|i\rangle\langle f|j^\sigma_{\rm cc}(0)|i\rangle^*$$

The Hadronic Tensor for νN Reactions

- given by six structure functions:

$$\frac{W^{\mu\nu}}{2M_i} = -g^{\mu\nu}W_1 + \frac{P^\mu P^\nu}{M_i^2}W_2 + i\frac{\epsilon^{\mu\nu\gamma\delta}P_\gamma q_\delta}{2M_i^2}W_3 + \frac{q^\mu q^\nu}{M_i^2}W_4 + \frac{P^\mu q^\nu + P^\nu q^\mu}{2M_i^2}W_5 + i\frac{P^\mu q^\nu - P^\nu q^\mu}{2M_i^2}W_6.$$

- Back transformation:

$$W_1 = \frac{W^{xx}}{2M_i},\ W_2 = \frac{1}{2M_i}\left(W^{00} + W^{xx} + \frac{(q^0)^2}{|\vec{q}|^2}(W^{zz} - W^{xx}) - 2\frac{q^0}{|\vec{q}|}\mathrm{Re}\,W^{0z}\right),$$

$$W_3 = -i\frac{W^{xy}}{|\vec{q}|},$$

$$W_4 = \frac{M_i}{2|\vec{q}|^2}(W^{zz} - W^{xx}),\ W_5 = \frac{1}{|\vec{q}|}\left(\mathrm{Re}\,W^{0z} - \frac{q^0}{|\vec{q}|}(W^{zz} - W^{xx})\right),$$

$$W_6 = \frac{\mathrm{Im}\,W^{0z}}{|\vec{q}|}.$$

Cross section and Structure Functions

$$\frac{d^2\sigma_{\nu l}}{d\Omega(\hat{k}')dE_l'} = \frac{|\vec{k}'|E_l'M_iG^2}{\pi^2}\Bigg\{2W_1\sin^2\frac{\theta'}{2} + W_2\cos^2\frac{\theta'}{2} - W_3\frac{E_\nu + E_l'}{M_i}\sin^2\frac{\theta'}{2} + \frac{m_l^2}{E_l'(E_l' + |\vec{k}'|)}\Bigg[W_1\cos\theta'$$

$$-\frac{W_2}{2}\cos\theta' + \frac{W_3}{2}\left(\frac{E_l' + |\vec{k}'|}{M_i} - \frac{E_\nu + E_l'}{M_i}\cos\theta'\right) + \frac{W_4}{2}\left(\frac{m_l^2}{M_i^2}\cos\theta' + \frac{2E_l'(E_l' + |\vec{k}'|)}{M_i^2}\sin^2\theta'\right) - W_5\frac{E_l' + |\vec{k}'|}{2M_i}\Bigg]\Bigg\}$$

Relation of charged current to electromagnetic current matrix elements

$$\frac{1}{\cos\theta_C}\langle p\pi^+|V^\mu_{cc+}(0)|p\rangle = \sqrt{2}\langle n\pi^0|j^\mu_{\rm em}(0)|n\rangle + \langle p\pi^-|j^\mu_{\rm em}(0)|n\rangle$$
$$\frac{1}{\cos\theta_C}\langle n\pi^+|V^\mu_{cc+}(0)|n\rangle = \sqrt{2}\langle p\pi^0|j^\mu_{\rm em}(0)|p\rangle - \langle p\pi^-|j^\mu_{\rm em}(0)|n\rangle$$

Cabbibo angle:

$$\cos\theta_C = 0.974$$

Neutrino-Nucleus Cross Sections and Response Functions

$$\nu_l(\bar{\nu}_l) + A \longrightarrow l^-(l^+) + X$$

$$\nu_l(\bar{\nu}_l) + A \longrightarrow \nu_l(\bar{\nu}_l) + X$$

$$\frac{\partial^2\sigma}{\partial\Omega\partial k'} = \frac{G_F^2 \cos^2\theta_c (k')^2}{2\pi^2} \cos^2\frac{\theta}{2} \Bigg\{ G_E^2 \left(\frac{q_\mu^2}{q^2}\right)^2 R_\tau^{NN} + G_A^2 \frac{(M_\Delta - M_N)^2}{2q^2} R_{\sigma\tau(L)}^{N\Delta} + G_A^2 \frac{(M_\Delta - M_N)^2}{q^2} \times R_{\sigma\tau(L)}^{\Delta\Delta} + \left(G_M^2 \frac{\omega^2}{q^2} + G_A^2\right)\left(-\frac{q_\mu^2}{q^2} + 2\tan^2\frac{\theta}{2}\right) \times \left[R_{\sigma\tau(T)}^{NN} + 2R_{\sigma\tau(T)}^{N\Delta} + R_{\sigma\tau(T)}^{\Delta\Delta}\right] \pm 2G_A G_M \frac{k+k'}{M_N} \times \tan^2\frac{\theta}{2}\left[R_{\sigma\tau(T)}^{NN} + 2R_{\sigma\tau(T)}^{N\Delta} + R_{\sigma\tau(T)}^{\Delta\Delta}\right] \Bigg\}$$

Nuclear Response Functions

Response Functions in Nuclear Reactions

$$\frac{d^2\sigma_{\alpha\beta}}{d\Omega dk} = -\frac{1}{\pi}\,\mathrm{Im}\left\{\left\langle A\left|T^{\dagger}_{\beta\alpha} G_A(E_\alpha - E_\beta) T_{\beta\alpha}\right|A\right\rangle\right\}$$

Nuclear Transition Operator:

$$\Rightarrow \qquad T_{\beta\alpha} = \left\langle \Psi^{(-)\dagger}_{\beta} U_{\beta\alpha} \Psi^{(+)}_{\alpha} \right\rangle \cdot \hat{O}^{(\mathrm{A})}_{\beta\alpha} = F_{\beta\alpha} \cdot \hat{O}^{(\mathrm{A})}_{\beta\alpha}$$

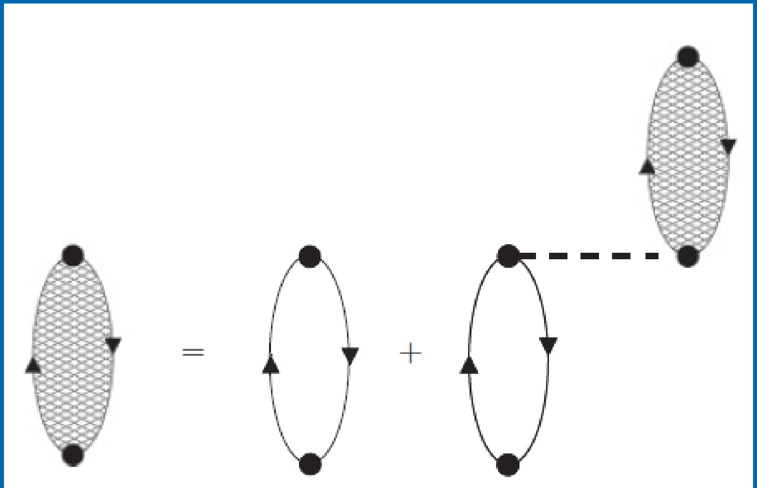
Nuclear Propagator:

$$G_A(\omega) = \sum_n \frac{\left|A_n\right\rangle\left\langle A_n\right|}{\omega - E_n + i\eta}$$

Plane Wave Approximation:

$$\Psi^{(\pm)}_{\alpha,\beta} \sim e^{i\vec{k}_{\alpha,\beta}\cdot\vec{r}}; V_{\beta\alpha} \sim G_{\beta\alpha}\delta(\vec{\mathrm{r}}_\alpha - \vec{\mathrm{r}}_\beta) \Rightarrow \mathrm{T}_{\beta\alpha} \sim G_{\beta\alpha} e^{i(\vec{k}_\beta - \vec{k}_\beta)\cdot\vec{r}} \hat{O}^{(A)}_{\beta\alpha}:$$

RPA-Polarization Propagator



Dyson Eq. for the Polarization Propagator

$$\Pi_{ab}(\omega,q) = \Pi_{ab}^{(0)}(\omega,q) + \sum_{c} \Pi_{ac}^{(0)}(\omega,q) V_{cc}(q\,|\,k_F)\Pi_{cb}(\omega,q)$$

Particle-Hole Propagator in Nuclear Matter

$$\Pi^{(0)}_{AB,ph}(q)s = -i\int \frac{d^4k}{(2\pi)^4}\, tr_s\left[\Gamma_B G_p(k+q)\Gamma_A G_h(k)\right]$$

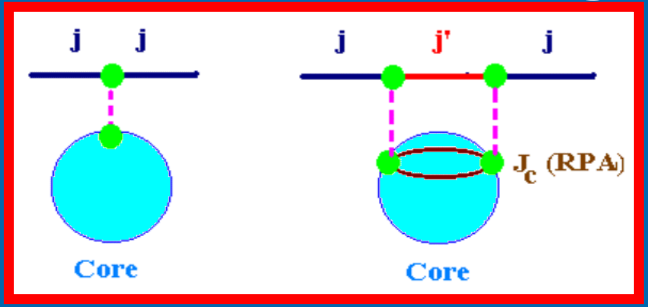
non-relativistic Lindhard Function for NN^{-1} and ΔN^{-1}:

$$\Pi^{(0)}_{AB,ph}(\omega,\vec{q},\rho) = M(\Gamma_B)\left[\Phi_{ph}(\omega,\vec{q},\rho) - \Phi_{ph}(-\omega,-\vec{q},\rho)\right]M(\Gamma_A)$$

$$\Phi_{ph}(\omega,\vec{q},\rho) = -\int \frac{d^3k}{(2\pi)^3}\left[\frac{\Theta(k_f^{(h)}-k)\,\Theta(|\vec{k}+\vec{q}|-k_f^{(p)})}{E_p(\vec{k}+\vec{q},\rho)-E_h(\vec{k},\rho)-\omega-i\eta}\right]$$

- **Analytical solutions for non-relativistic representation** (e.g. book of Fetter/Walecka; Alberico *et al.* Ann.Phys. 154 (1984))
- **Density dependence: Fermi momenta and self-energies**

Nucleon Self-Energies

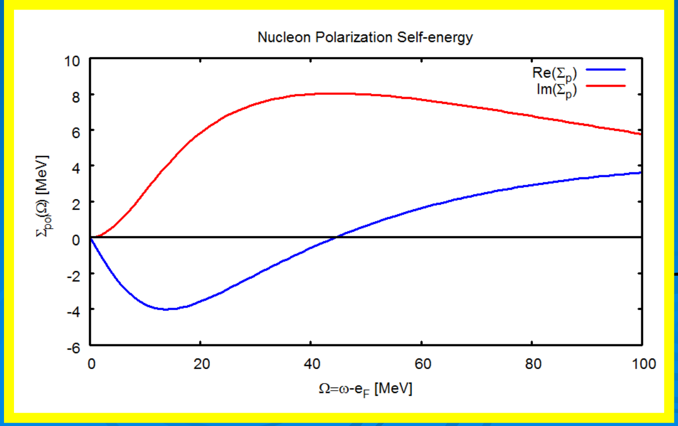


$$E_N = \frac{\vec{k}^2}{2m_N} + \tilde{\Sigma}_N(\rho, k, E_N) \sim \frac{\vec{k}^2}{2m_N^*(\rho)} + \Sigma_N(\rho, E_N)$$

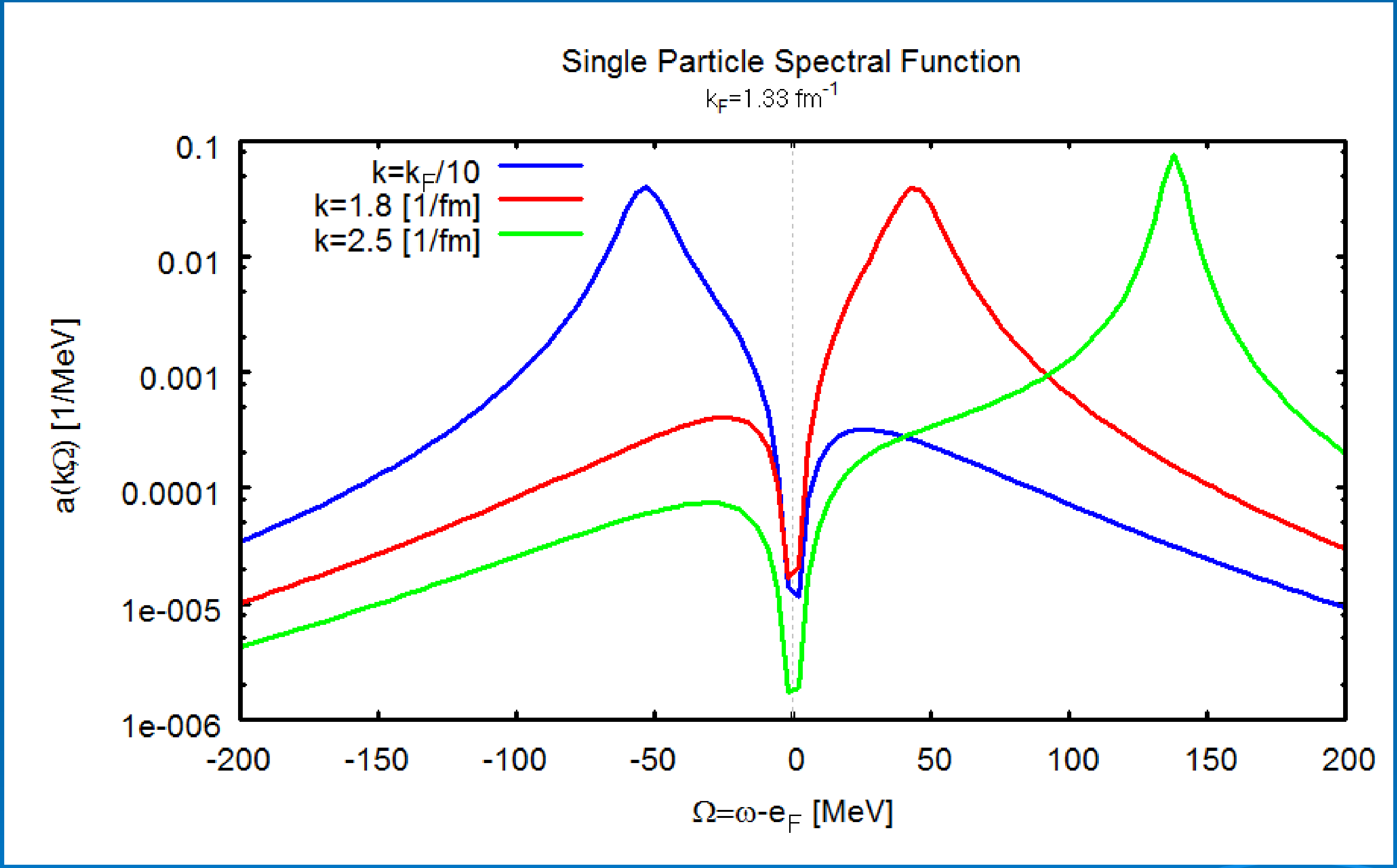
$$\Sigma_N(\rho, E) = U_{MF}(\rho) + \Sigma_{pol}(\rho, E_N)$$

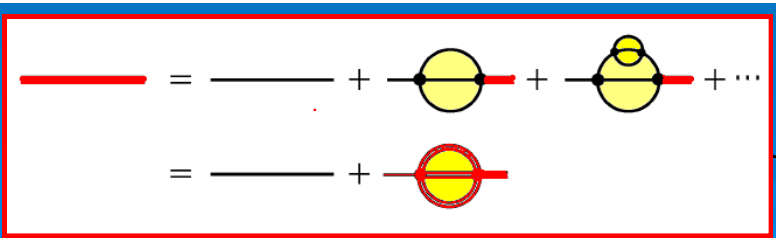
$$\Sigma_{pol}(\rho, k, E_N) = V_{pol}(\rho, k, E_N) + iW_{pol}(\rho, k, E_N)$$

$$\Gamma_N = -2\,\mathrm{Im}\,\Sigma_{pol}(\rho, k, E_N) = -2W_{pol}(\rho, k, E_N)$$



Short Range Correlations and Nucleon Spectral Functions

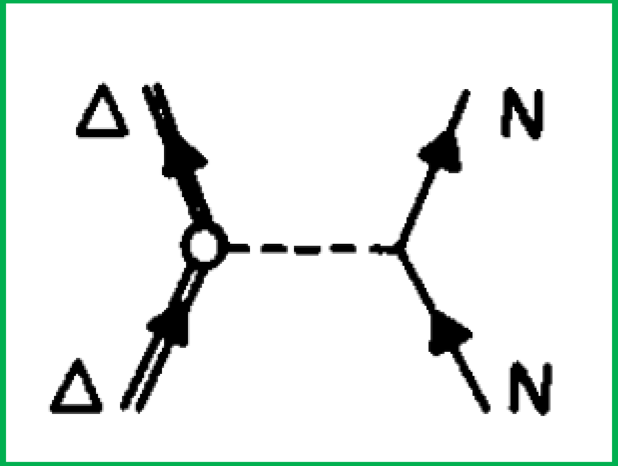


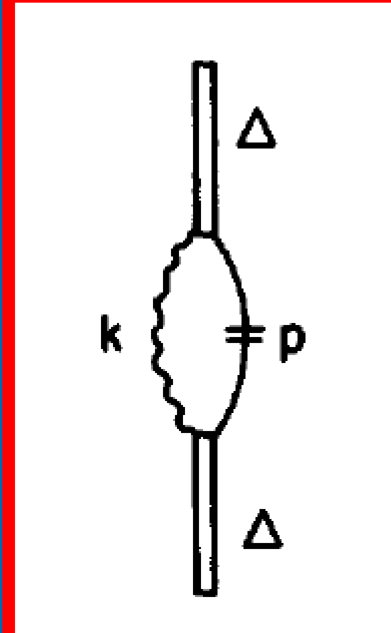


P. Konrad, H.L., Nucl.Phys. 2005/2007/2009

Delta Self-Energies

Delta Self-Energy in Nuclear Matter





Direct Self-energy → Hartree-Potential

$$U_{\Delta}^{(H)} = U_0 + U_1 \tau_{\Delta} \cdot \tau_N$$

$$U_{\Delta}^{(H)} \sim U_0 + U_1 t_z^{(\Delta)} \cdot \frac{N-Z}{A}$$

Polarization Self-Energy → dispersive (optical) potential

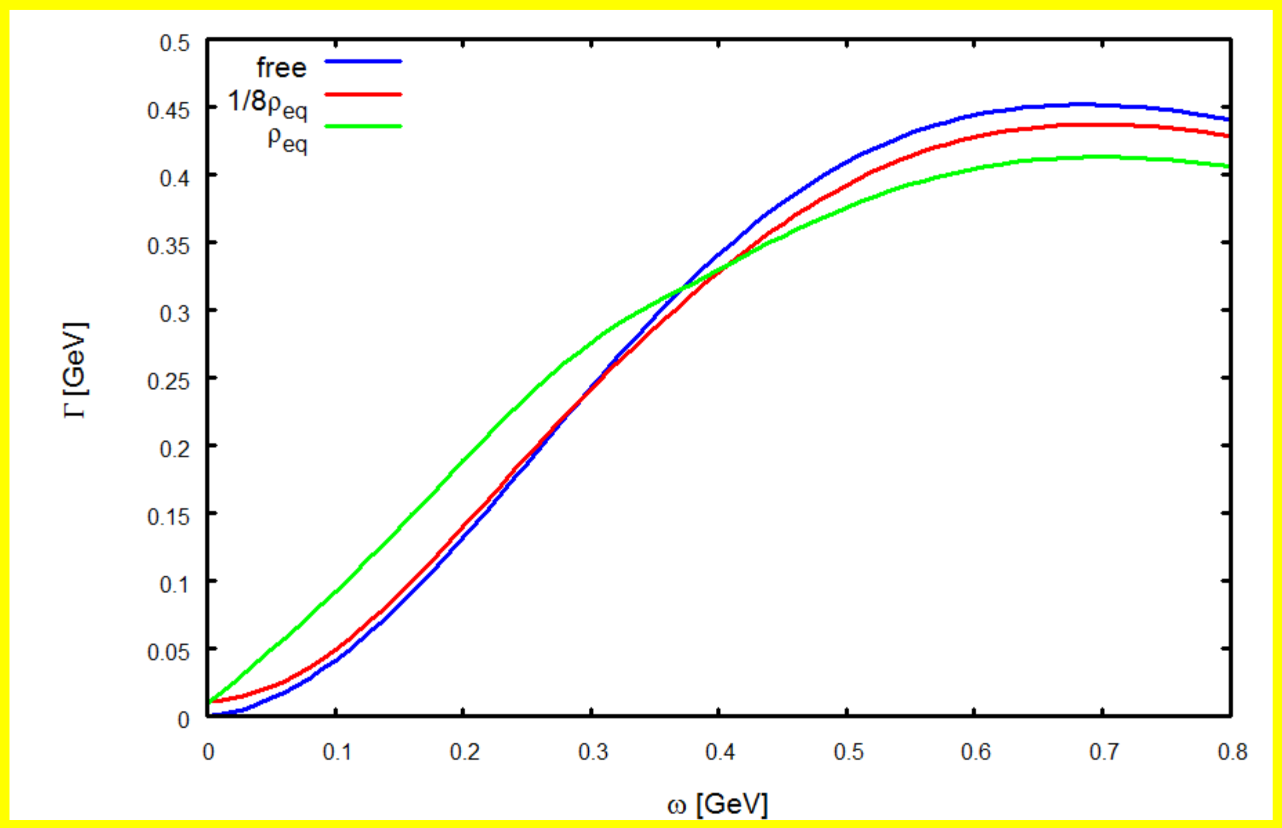
$$\Sigma_{pol}^{(\Delta)} \sim \Sigma_0 + \Sigma_1 t_z^{(\Delta)} \frac{N-Z}{A}$$

$$\Sigma_{\alpha} = V_{\alpha} - iW_{\alpha}$$

…see e.g.:

E. Oset, L.L. Salcedo, NPA 468 (1987) 631; G.E. Brown, W. Weise, Phys. Rept. 22 (1975) 279

In-Medium Delta(1232) Width



$$\Gamma_{\Delta}(p_{\Delta}^{2},\rho) = -2\,\mathrm{Im}\,\Sigma(p_{\Delta}^{2},\rho) \sim \Gamma_{free}(p_{\Delta}^{2}) + \Gamma_{Pauli}(p_{\Delta}^{2},\rho) + \Gamma_{abs}(p_{\Delta}^{2},\rho)$$

Parametrizations: J. Nieves, E. Oset, and C. Garcia-Recio, Nucl. Phys. **A554**, 554 (1993);
L. L. Salcedo, E. Oset,M. J.Vicente-Vacas, and C. Garcia-Recio, Nucl. Phys. **A484**, 557 (1988)

The Delta-Propagator

$$G_\Delta = \frac{\Lambda_{\alpha\beta}}{(w+\tilde{m}_\Delta)(w-\tilde{m}_\Delta+\frac{i}{2}\tilde{\Gamma}_\Delta)}$$

$$w = \sqrt{p_\Delta^2}$$

$$\tilde{m}_\Delta = m_\Delta + \mathrm{Re}\,\Sigma_\Delta$$

$$\tilde{\Gamma}_\Delta = \Gamma_{free} + \Gamma_{Pauli} + \Gamma_{abs}$$

Spin-3/2 Projector:

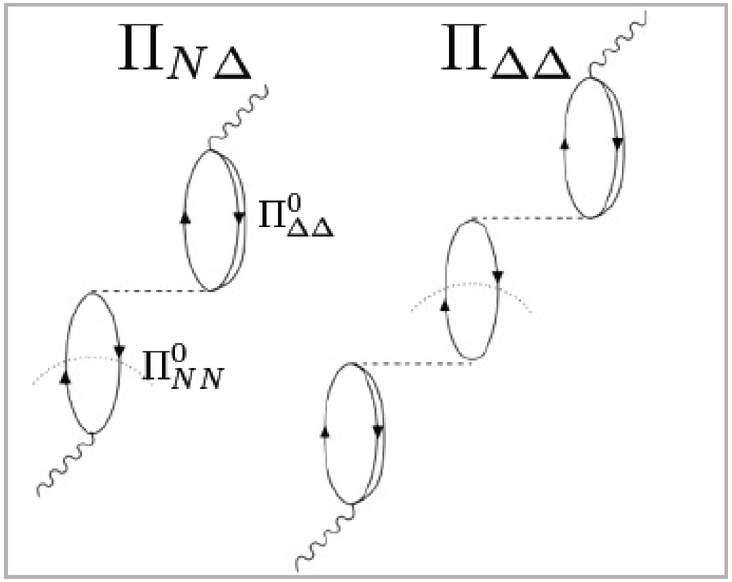
$$\Lambda_{\alpha\beta} = -(\not{p}' + M_\Delta)\left(g_{\alpha\beta} - \frac{2}{3}\frac{p'_\alpha p'_\beta}{M_\Delta^2} + \frac{1}{3}\frac{p'_\alpha\gamma_\beta - p'_\beta\gamma_\alpha}{M_\Delta} - \frac{1}{3}\gamma_\alpha\gamma_\beta\right)$$

Resonance Excitation in Nuclei: „N-Δ RPA“

$$\Pi = \Pi^0 + \Pi^0 \hat{V} \Pi$$

$$\begin{pmatrix} \Pi_{NN} & \Pi_{N\Delta} \\ \Pi_{\Delta N} & \Pi_{\Delta\Delta} \end{pmatrix} = \begin{pmatrix} \Pi^0_{NN} & 0 \\ 0 & \Pi^0_{\Delta\Delta} \end{pmatrix} + \begin{pmatrix} \Pi^0_{NN} & 0 \\ 0 & \Pi^0_{\Delta\Delta} \end{pmatrix} \begin{pmatrix} V_{NN} & V_{N\Delta} \\ V_{\Delta N} & V_{\Delta\Delta} \end{pmatrix} \begin{pmatrix} \Pi_{NN} & \Pi_{N\Delta} \\ \Pi_{\Delta N} & \Pi_{\Delta\Delta} \end{pmatrix}$$

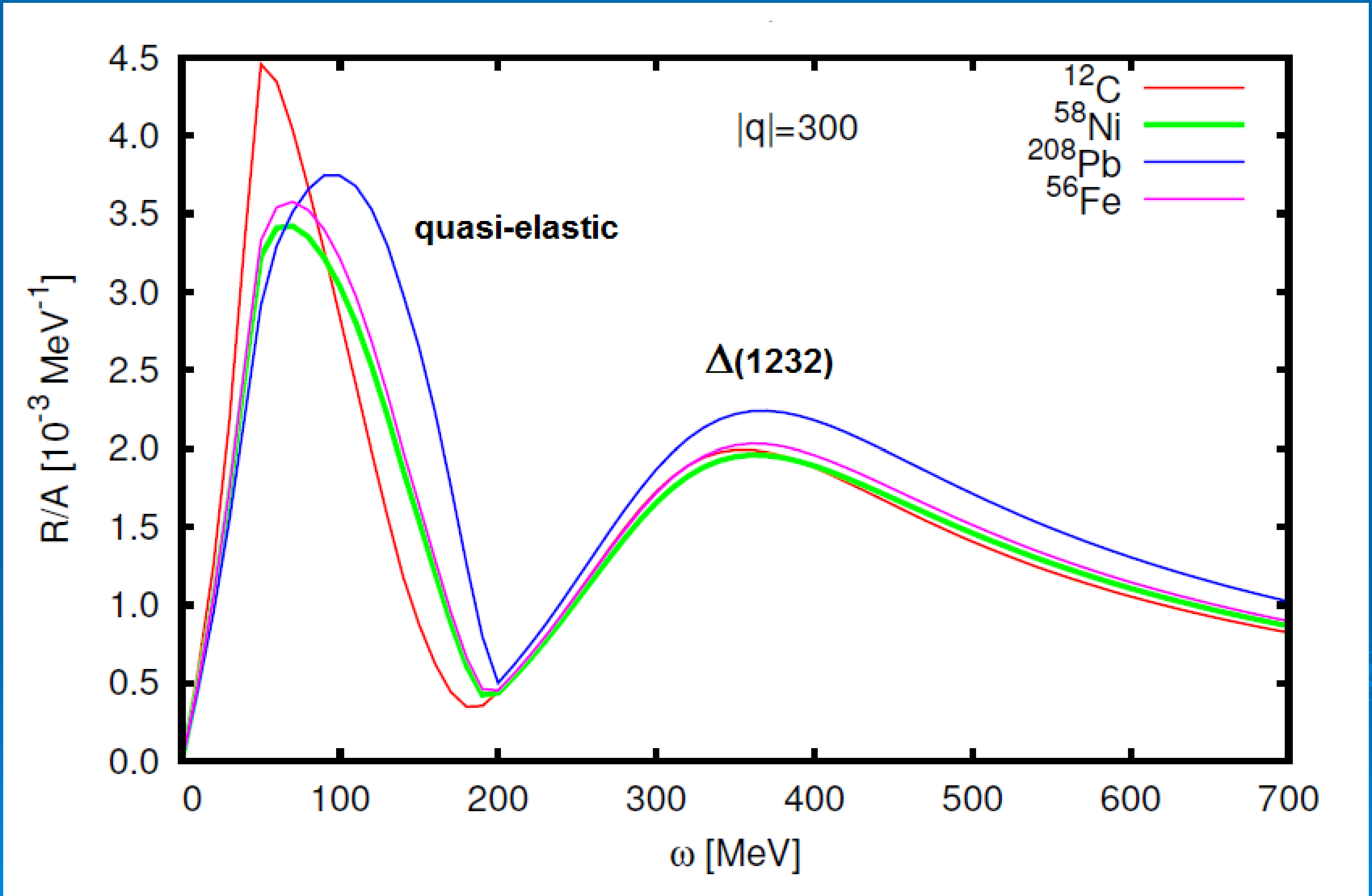
- Full RPA includes Δ-N mixing
- Non-perturbative problem
- QE-peak is influenced by intermediate Δ-hole pairs
- Structure of the spin-isospin response can give a deeper understanding of the Δ-N interaction



$$\Pi_{NN} = \chi_{\Delta N} \chi_N \Pi^0_{NN}$$

Nuclear Response Functions

Response Functions in β-Stable Nuclei: RPA results for $T_a=\tau_-$ (pn^{-1} transitions)



A. Fedoseew, H.L.

Quasielastic Electron Scattering

Quasi-Elastic Electron Scattering

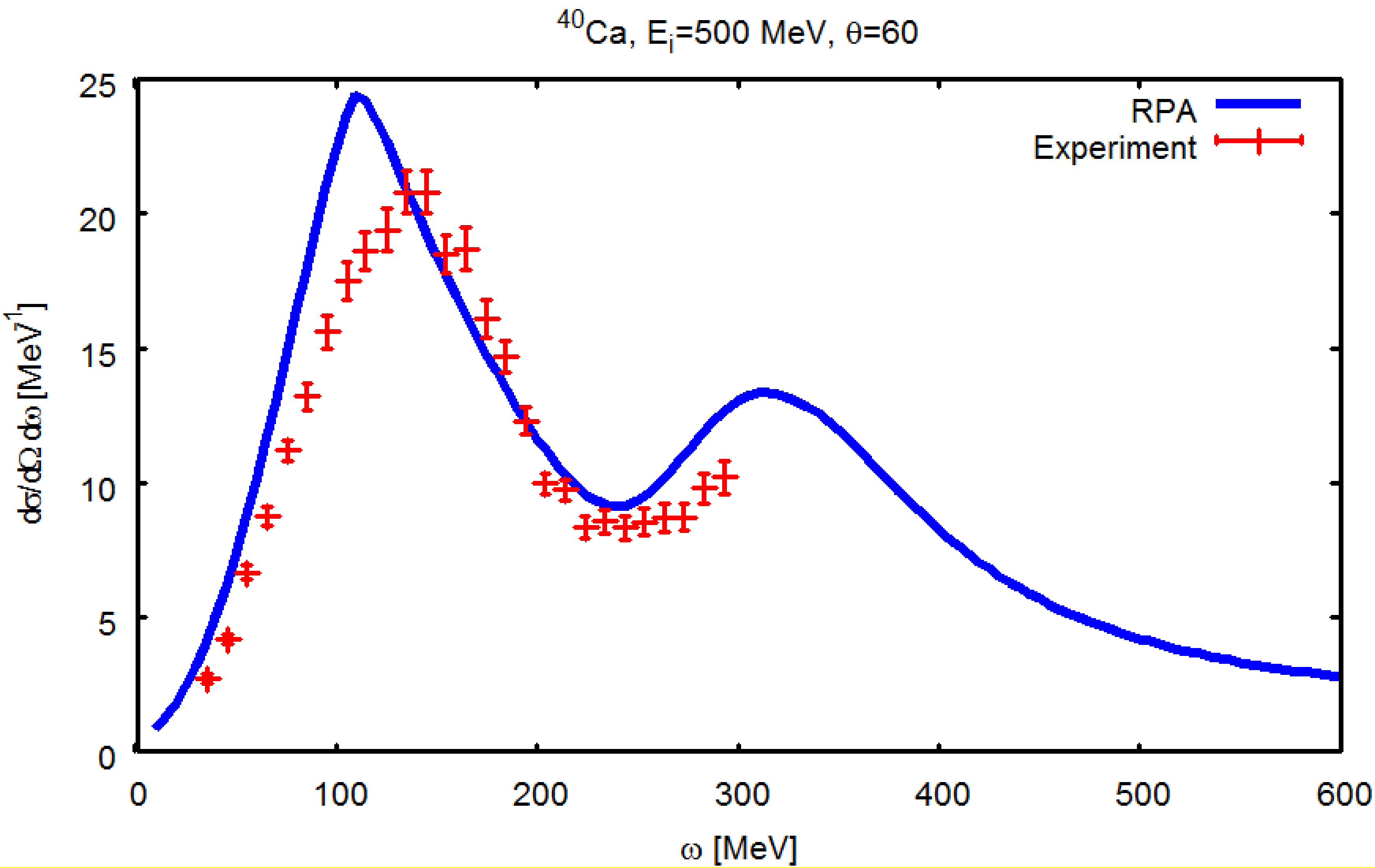
$$\frac{d^2\sigma}{d\omega\, d\Omega} = \sigma_{Mott}\left[\left(\frac{Q^2}{q^2}\right)^2 R_L(q,\omega) + \left(\frac{Q^2}{2q^2} + \tan^2\frac{\theta}{2}\right) R_T(q,\omega)\right]$$

$$\sigma_M = \frac{\alpha^2 \cos^2(\theta/2)}{4\varepsilon_i^2 \sin^4(\theta/2)}$$

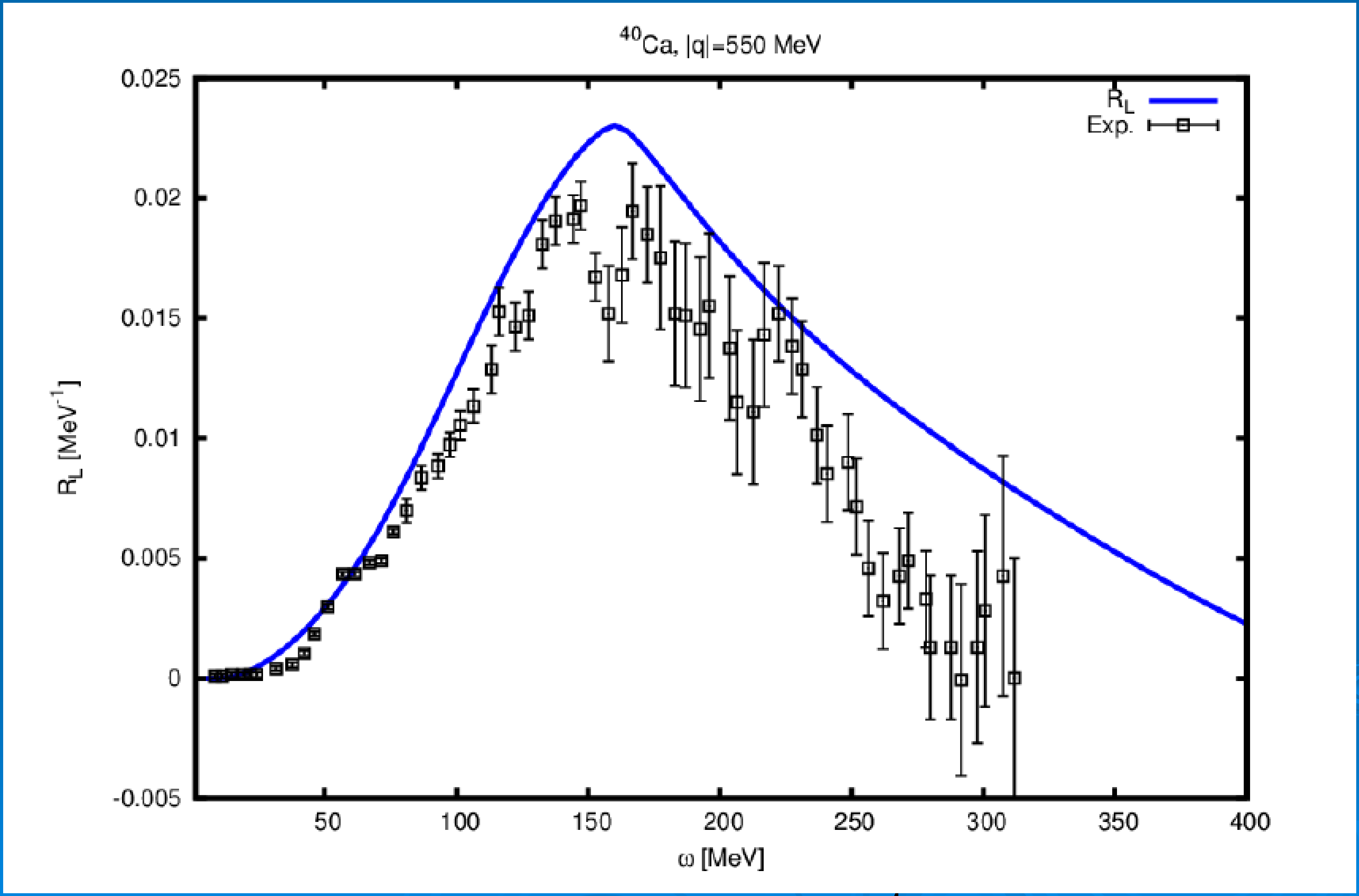
$$\hat{O} \sim \vec{\sigma}\cdot\vec{q} : R_L = -\frac{1}{\pi}\,\mathrm{Im}\,\Pi^{(S=1,M=0)}$$

$$\hat{O} \sim \vec{\sigma}\times\vec{q} : R_T = -\frac{1}{\pi}\,\mathrm{Im}\,\Pi^{(S=1,M=\pm 1)}$$

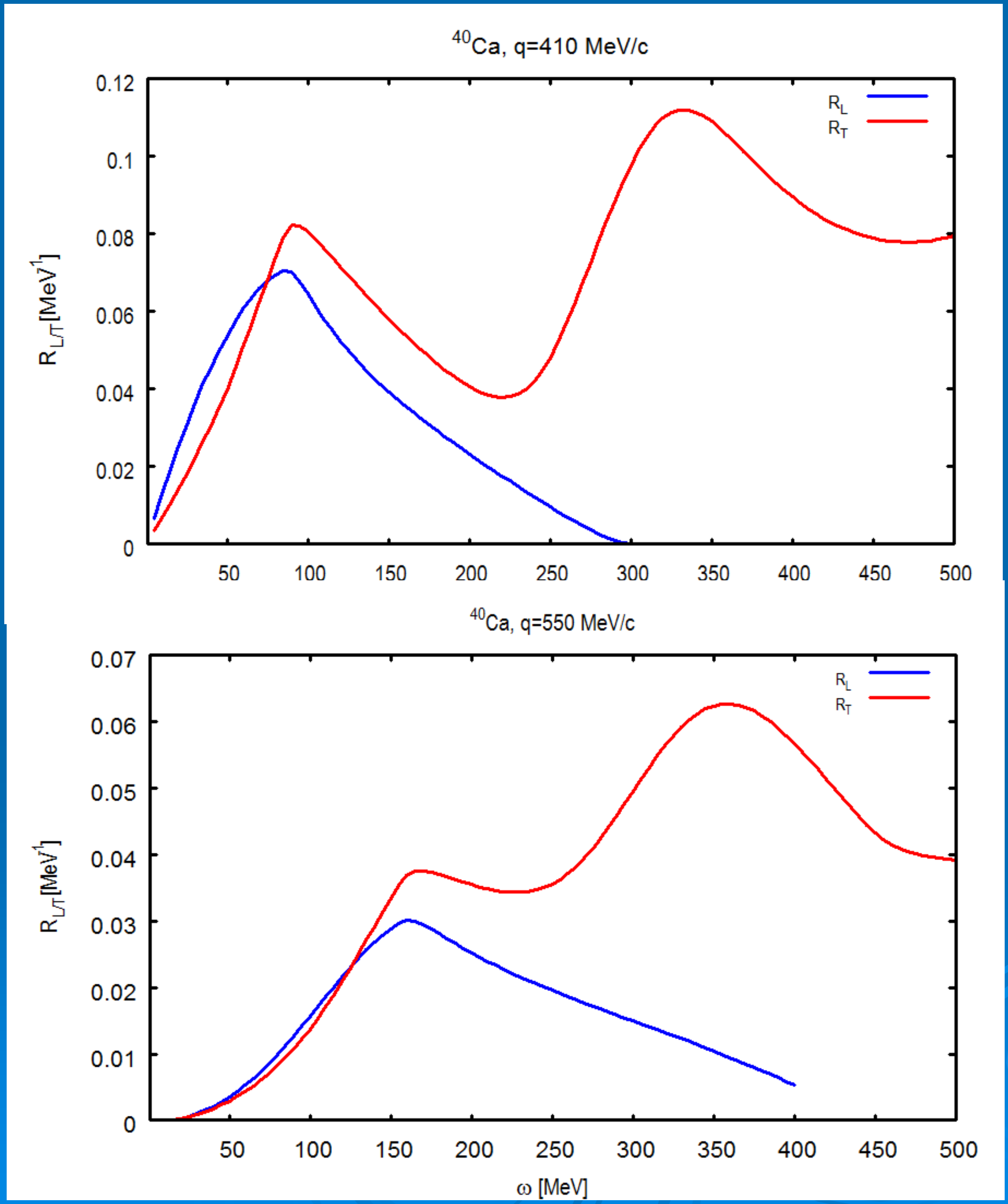
…including the D(1232):



Quasi-elastic Electron Sacttering



Longitudinal (R_L) and Transverse (R_T) Response Functions ^{40}Ca



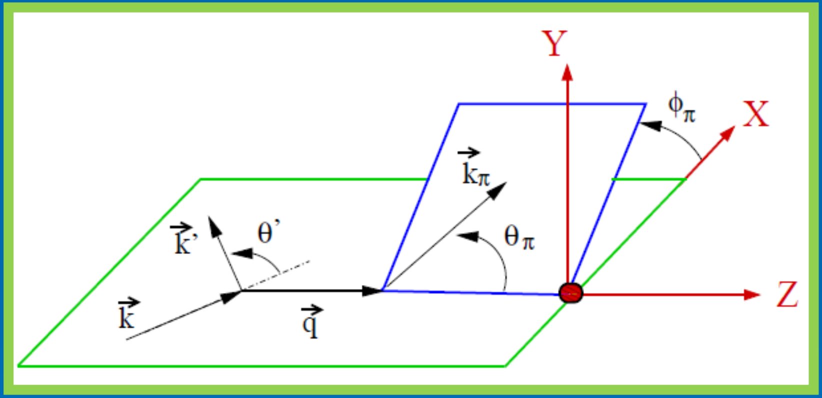
Neutrino-induced Pion Production

Neutrino-induced Pion Production

$$\nu_l(k) + N(p) \to l^-(k') + N(p') + \pi(k_\pi)$$

Cross Section and Kinematics of Pion Production:

$$\frac{d^5\sigma_{\nu_l l}}{d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)} = \frac{|\vec{k}'|}{|\vec{k}\,|}\frac{G^2}{4\pi^2}\int_0^{+\infty}\frac{d|\vec{k}_\pi||\vec{k}_\pi|^2}{E_\pi}L^{(\nu)}_{\mu\sigma}\,(W^{\mu\sigma}_{\mathrm{CC}\pi})^{(\nu)}$$



...constraints by Lorentz-invariance (A...E: real structure functions):

$$\frac{d^5\sigma_{\nu_l l}}{d\Omega(\hat{k}')dE'd\Omega(\hat{k}_\pi)} = \frac{|\vec{k}'|}{|\vec{k}\,|}\frac{G^2}{4\pi^2}\{A + B\cos\phi_\pi + C\cos 2\phi_\pi + D\sin\phi_\pi + E\sin 2\phi_\pi\}$$

Pion Production and Hadronic Tensor

$$(W^{\mu\sigma}_{\mathrm{CC}\pi})^{(\nu)} = \frac{1}{4M}\overline{\sum_{\mathrm{spins}}}\int \frac{d^3p'}{(2\pi)^3}\frac{1}{2E'_N}\delta^4(p'+k_\pi-q-p)\langle N'\pi|j^\mu_{\mathrm{cc}+}(0)|N\rangle\langle N'\pi|j^\sigma_{\mathrm{cc}+}(0)|N\rangle^*$$

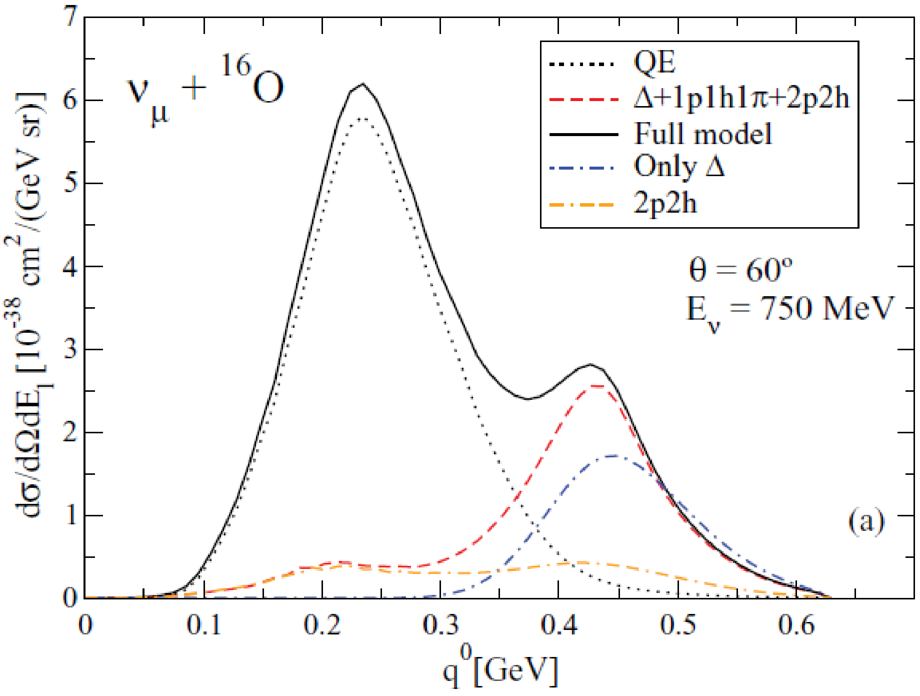
Interdependencies of CC M.E. → 2 independent M.E. (Wigner-Eckardt Theorem)

$$\begin{aligned}
\langle p\pi^0|j^\mu_{\mathrm{cc}+}(0)|n\rangle &= -\frac{1}{\sqrt{2}}\left[\langle p\pi^+|j^\mu_{\mathrm{cc}+}(0)|p\rangle - \langle n\pi^+|j^\mu_{\mathrm{cc}+}(0)|n\rangle\right]\\
\langle p\pi^-|j^\mu_{\mathrm{cc}-}(0)|p\rangle &= \langle n\pi^+|j^\mu_{\mathrm{cc}+}(0)|n\rangle\\
\langle n\pi^-|j^\mu_{\mathrm{cc}-}(0)|n\rangle &= \langle p\pi^+|j^\mu_{\mathrm{cc}+}(0)|p\rangle\\
\langle n\pi^0|j^\mu_{\mathrm{cc}-}(0)|p\rangle &= -\langle p\pi^0|j^\mu_{\mathrm{cc}+}(0)|n\rangle = \frac{1}{\sqrt{2}}\left[\langle p\pi^+|j^\mu_{\mathrm{cc}+}(0)|p\rangle - \langle n\pi^+|j^\mu_{\mathrm{cc}+}(0)|n\rangle\right]
\end{aligned}$$

Anti-neutrino Reactions:

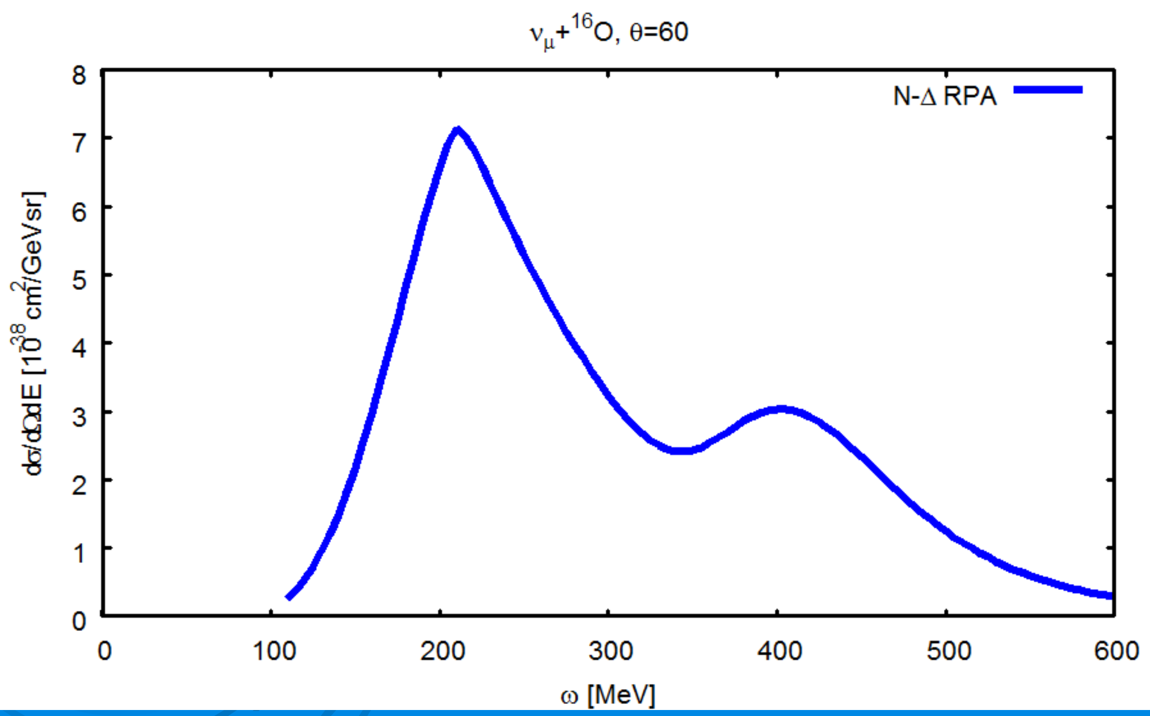
$$L^{(\bar\nu)}_{\mu\sigma} = L^{(\nu)}_{\sigma\mu} \quad , \quad j^\mu_{\mathrm{cc}-} = j^{\mu\dagger}_{\mathrm{cc}+}$$

Inclusive CC cross section



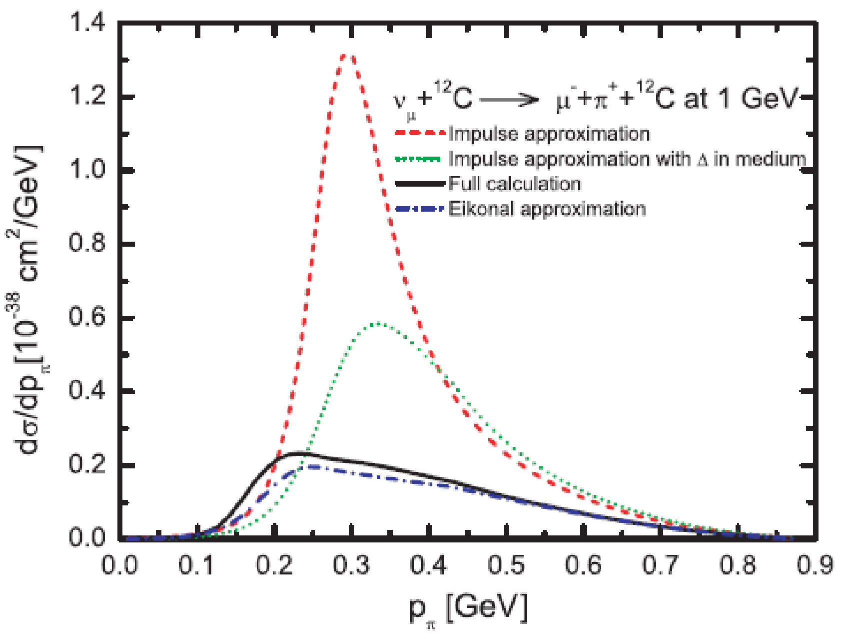
J. Nieves, I. Ruiz Simo, M. J. Vicente Vacas
PRC **83**, 045501 (2011)

Gießen
N-Δ RPA



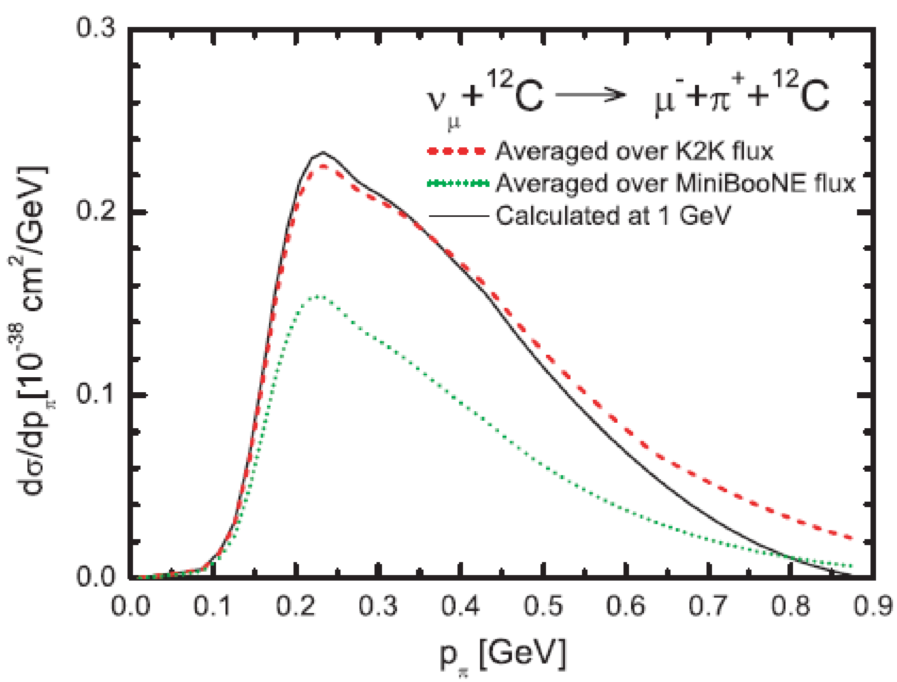
CC coherent pion production

L. Alvarez-Ruso et al, PRC 75:055501 (2007); PRC(0:019906(E) (2009)



Momentum Distribution

Cross Section



Hadronic Tensor:

$$W^{\mu}_{N\pi} = -\frac{i}{2}\int d^3r\, e^{i(\vec{q}-\vec{p}_\pi)\cdot\vec{r}}\left[\rho_p(r)+\frac{\rho_n(r)}{3}\right] \times \sqrt{3}\,\frac{f^*}{m_\pi}F(p')\,\tilde{D}(p')\,p_\pi^\alpha\,\mathrm{Tr}\{\bar{u}(0)\Lambda_{\alpha\beta}\mathcal{A}^{\beta\mu}u(0)\}$$

Summary and Outlook

- Neutrino-nucleus interactions
- Quantum mechanical approach
- Theory of nuclear response functions
- Cross sections in local density approximation
- Higher resonances?
- Form Factors?
- Dependence on ph-interaction?
- „Finite System" RPA?

- Credits to
- Andreas Fedoseew and Julian Georg