Attempts to explain neutrino masses and mixing

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Marek Zralek
University of Silesia, Katowice, Poland

Attempts to explain neutrino masses and mixing

(abstract)

Explanation of the origin of mass of matter is one of the central problems of physics. Certainly this is the case in classical physics (space, time, mass). In the quantum and relativistic world, mass becomes part of the conserved energy. Currently we can explain more than 96% of mass of surrounding us matter. The remaining 4% of mass is associated directly with the operation of the Higgs field which is responsible for the mass of elementary fermions - electrons, "up" and "down" quarks. So far, these masses cannot be calculated from first principles.

The lectures is devoted to attempts of explanation the leptons mixing. To explain the lepton masses, GUT models must be included. There are good reasons to start with leptons. Strong mixing between them give hope for clarification the relationship between lepton masses and mixing, and we hope that later also for all quarks.

CONTENTS

1) Introduction – why the problem is important? – origin of mass of the visible matter in the Universe,

2) The problem of mass of elementary fermions,

3) The problem of neutrino mass and mixing – various horizontal symmetry models,

4) Conclusion - many unknowns remain…

1) Introduction – why the problem is important? – origin of mass of the visible matter in the Universe

Definition of mass:

The mass of the body is its total energy (divided by c^2) measured in its rest frame

$$m = \frac{(E)_{rest}}{c^2} = \frac{(E^{rest}_{kinetic} + E_{interaction}) + \text{Particles Mass}}{c^2}$$

$$\Delta = \frac{E^{rest}_{kinetic} + E_{interaction}}{\text{Particles Mass}}$$

1) The heated body has a greater mass, e.g. (t_2-t_1 = 60^0 C):

$$\Delta_{t_1}^{t_2}(M_{H_2O}) = \frac{5k(t_2 - t_1)}{2c^2 M_{H_2O}} = 7.65 \otimes 10^{-13}$$

2) Any chemical bonds in the substance reduced the mass of the whole body, e.g. for carbon monoxide:

$$\frac{|\Delta V|}{M_{CO}} = 4.26 \otimes 10^{-10}.$$

3) The atomic forces binding electrons reduce the mass of an atom, e.g. for hydrogen:

$$M_H = (M_p + m_e - 13.6\ eV/c^2) = 9.38 \otimes 10^8 eV/c^2$$

$$\frac{13.6\ eV/c^2}{M_H} = 1.45 \otimes 10^{-8}$$

It's easy to understand why the mass for so many years was considered as additive and conserved quantity

4) Electrons contribution to the mass of atoms, e.g.

$$\frac{m_e}{M_H} = 5.45 \otimes 10^{-4}$$

With the accuracy of one per thousand any body mass is the mass of its atomic nuclei

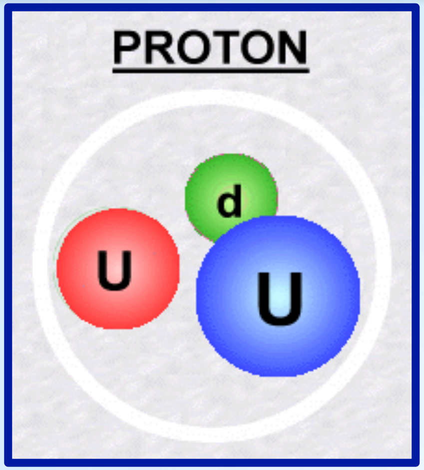
5) The mass of atomic nuclei is the mass of all nucleons reduced by the binding energy, e.g.

$$\mathrm{M}_\mathrm{D} = (\mathrm{m}_p + m_n - 2.224\ MeV / c^2),$$

$$\frac{2.224\ MeV / c^2}{\mathrm{M}_D} \approx 1.2 \otimes 10^{-3}$$

Mass becomes not-additive and non-conserved quantity

Proton and neutron



$$m_p = 938.3 \text{ MeV}$$

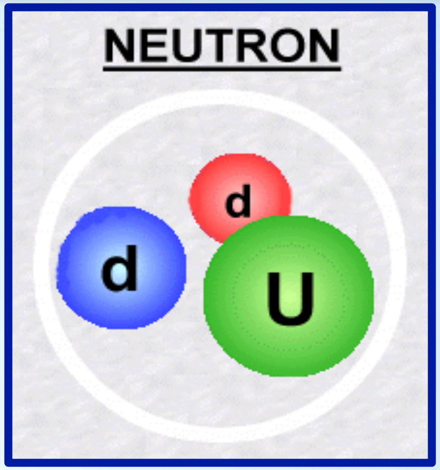
$$m_u = 2.17 \pm 0.14 \ MeV$$

$$(= 2.3^{+0.7}_{-0.5} \ MeV)$$

$$m_d = 4.84 \pm 0.19 \ MeV$$

$$(= 4.8^{+0.5}_{-0.3} \ MeV)$$

Physics Letters B 701 (2011) 265-268
(PDG, Phys. Rev. D86 (2012) 010001)
Current quark masses



$$m_n = 939.6 \text{ MeV}$$

$$\frac{m_p - (2m_u + m_d)}{m_p} 100\% \approx 99.0\%$$

$$\frac{m_n - (2m_d + m_u)}{m_n} 100\% \approx 98.8\%$$

Masses of the u and d quarks are unknown (are know only from matching), but proton and neutron masses can be calculated in frame of the lattice QCD. Even assuming vanishing quarks masses, nucleon masses are reproduced with 96% precision.

Physics Letters B 701 (2011) 265–268

„Lattice QCD at the physical point: Light quark masses”
Budapest–Marseille–Wuppertal Collaboration

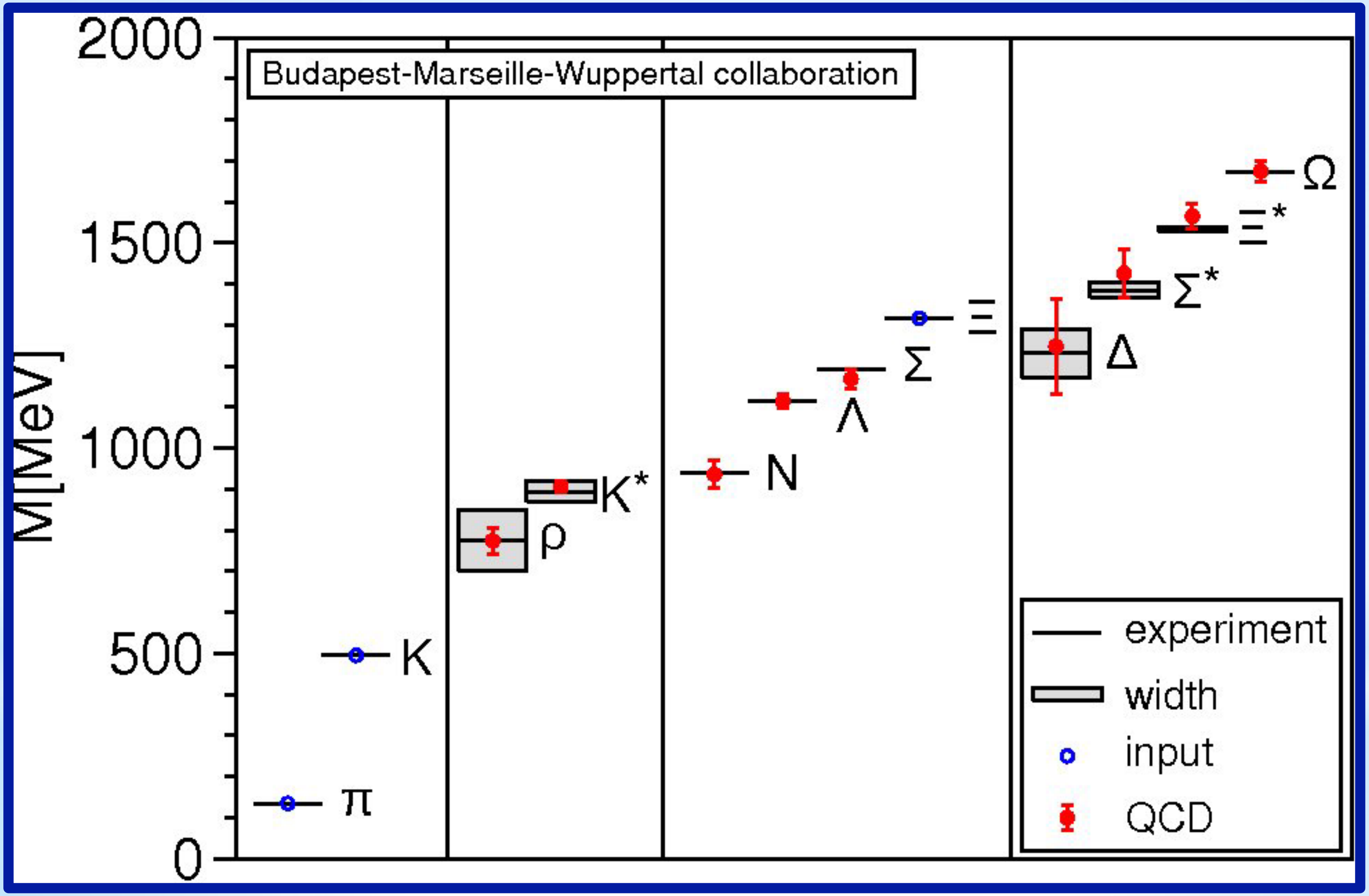
S. Durr, Z. Fodor, C. Hoelbling, S.D. Katzd, S. Kriega, T. Kurtha, L. Lellouch,
T. Lippert, K.K. Szabo, G. Vulvert

Ordinary matter is described by six fundamental parameters: three couplings

$$\left.\begin{array}{c}\text{(gravitational,}\\ \text{electromagnetic and}\\ \text{strong)}\end{array}\right\}\ \text{Known very well}$$

and three masses:
the electron’s (m_e) and those of the up (m_u) and down (m_d) quarks. By quantum fluctuation also (m_s)

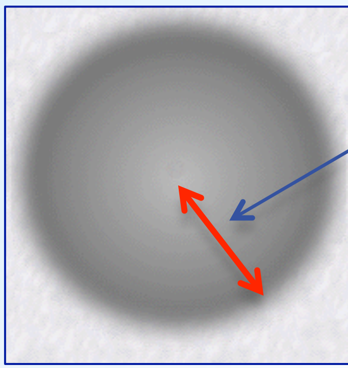
The basic part to the mass of matter are given by protons and neutrons



S. Dürr, Z. Fodor, J. Frison, C. Hoelbling, R. Homann, S. D. Katz, S. Krieg, T. Kurth, L. Lellouch, T. Lippert, K. K. Szabo, G. Vulvert, Science 322 (2008).

We understand the origin of almost all of the visible mass of matter in the universe

It seems that without much prejudice we can assume that electrons are massless, unfortunately we can not do that:



$$a_B = \frac{\hbar}{m_e c \alpha}$$

Size and stability of matter depends on electron mass.

To explain fully the origin of mass and structure of the visible matter we should know at least masses of electron and two quarks, „up” and „down”.

2) The problem of mass of elementary fermions

Possible ways to resolve the problem of quarks and leptons masses

(With the current state of knowledge)

I) Partial answer - find the symmetry which connect masses and mixing angles for quarks and leptons - Horizontal symmetries,GUT

II) Quarks and leptons are composite objects – Preons theory

?

III) Unification at very high scale, e.g. String theory

?

In Particle Physics SYMMETRY plays a fundamental role:

Isospin, Eightfold way,

Gauge symmetry

Supersymmetry, Superstring.

So it is natural to expect:

Symmetry will be able to open the door to

GENERATION PROBLEM

Symmetry gives relations between **Yukawa couplings** and **Higgs vacuum expectation values**

↓

Relation between masses of quarks and leptons and between mixing angles (CP violating phases)

The most successful principle in Particle Physics

Gauge Symmetry

$$SU(2)_L \times U(1)_Y$$

Must be broken (Spontaneously) to get:

$\boxed{\text{Masses for } W^{\pm}, Z_0, H, \text{ all fermions}}$

Yukawa Lagrangian (L_Y) and scalar Higgs potential (Y_H) are gauge symmetric (before SSB),

but up to now:

We do not know of any fundamental principle (as gauge symmetry) which allows to constrain L_Y and Y_H to explain the fermion masses and mixing angles.

In order to obtain restriction for L_Y we try to find:

Flavour (or Family, or Horizontal) symmetry

So we will consider the symmetry structure:

$$\boxed{G = G_{gauge} \times G_{flavour}}$$

where: $$\boxed{G_{gauge} = SU(3)_c \times SU(2)_L \times U(1)_Y}$$

or: $$\boxed{G_{gauge} = G_{GUT}}$$

But $G_{flavour}$ is unknown

Masses of

- ✧ down quarks in different generation
- ✧ charged fermions in different generation
- ✧ up quarks in the same generations

significantly differ

Flavour Symmetry must be broken very strongly

Quarks mixing is very small → It can be perturbation

Neutrinos mixing is large → Can be used to find out some unperturbed and unbroken horizontal symmetry

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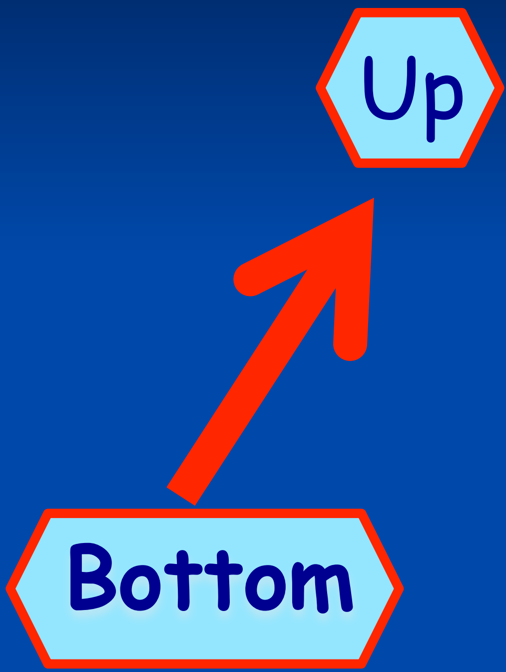
Regularity of the tribimaximal mixing of leptons

Regularity in the Balmer series of the Hydrogen atom

There are two approaches to the generation problem



(Symmetry,
Higgs bosons)

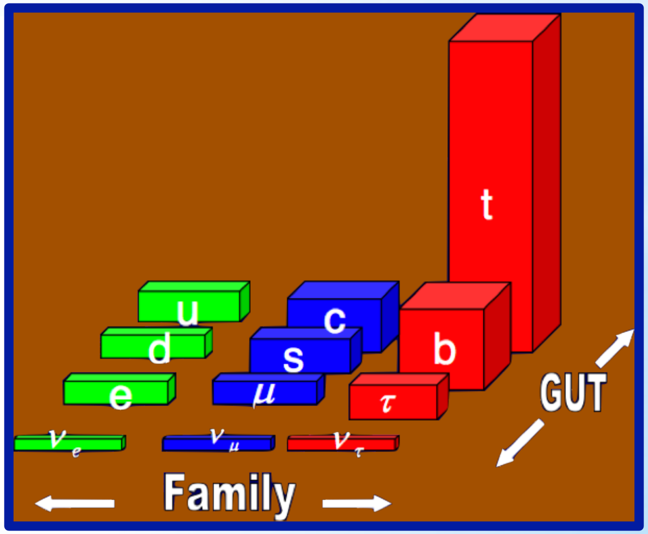
Neutrino masses and the
PMNS matrix elements

It is almost sure that the Higgs particle was discovered (LHC; ATLAS, CMS) and the Higgs mechanism is correct.

Fermions acquire mass by the interaction with the Higgs field.

Mixing between fermions appears because there is a mismatch between the fermion flavour and mass eigenstates.

Many models have been constructed, which in different ways and at different energy levels, introduce a family symmetry



3) The problem of neutrino mass and mixing - various horizontal symmetry models

A lot of work appeared on the subject, see e.g.

G. Altarelli, arXiv:0905.3265 [hep-ph], arXiv:0905.2350 [hep-ph], E. Ma and G. Rajasekaran, Phys. Rev. D 64, 113012 (2001,) K. S. Babu, E. Ma and J. W. F. Valle, Phys. Lett. B 552, 07 (2003), M. Hirsch, J. C. Romao, S. Skadhauge, J. W. F. Valle and A. Villanova del Moral, arXiv:hep-ph/0312244, M. Hirsch, J. C. Romao, S. Skadhauge, J. W. F. Valle and A. Villanova del Moral, Phys. Rev. D 69, 093006 (2004), E. Ma, Phys. Rev. D 70, 031901 (2004), New J. Phys. 6, 104 (2004), Mod. Phys. Lett. A 20, 2601 (2005), Phys. Rev. D 72, 037301 (2005), S. L. Chen, M. Frigerio and E. Ma, Nucl. Phys. B 724, 423 (2005), M. Hirsch, A. Villanova del Moral, J. W. F. Valle and E. Ma, Phys. Rev. D 72, 091301 (2005), [Erratum-ibid. D 72,19904 (2005)], K. S. Babu and X. G. He, arXiv:hep-ph/0507217, A. Zee, Phys. Lett. B 630, 58 (2005), X. G. He, Y. Y. Keum and R. R. Volkas, JHEP 0604, 039 (2006), B. Adhikary, B. Brahmachari, A. Ghosal, E. Ma and M. K. Parida, Phys. Lett. B 638, 345 (2006), E. Ma, Phys. Rev. D 73, 057304 (2006), Mod. Phys. Lett. A 21, 2931 (2006), Mod. Phys. Lett. A 22, 101 (2007), S. F. King and M. Malinsky, Phys. Lett. B 645, 351 (2007), S. Morisi, M. Picariello and E. Torrente-Lujan, Phys. Rev. D 75, 075015 (2007), F. Yin, Phys. Rev. D 75, 073010 (2007), F. Bazzocchi, S. Kaneko and S. Morisi, JHEP 0803, 063 (2008), F. Bazzocchi, S. Morisi and M. Picariello, Phys. Lett. B 659, 628 (2008), M. Honda and M. Tanimoto, Prog.Theor. Phys. 119, 583 (2008), B. Brahmachari, S. Choubey and M. Mitra, Phys. Rev. D 77, 073008 (2008), [Erratum-ibid. D 77, 119901 (2008)], B. Adhikary and A. Ghosal, Phys. Rev. D 78, 073007 (2008), A. Ghosal, arXiv:hep-ph/0612245, B. Adhikary and A. Ghosal, Phys. Rev. D 75, 073020 (2007), G. Altarelli, F. Feruglio and C. Hagedorn, JHEP 0803, 052 (2008), F. Bazzocchi, S. Morisi, M. Picariello and E. Torrente-Lujan, J. Phys. G 36, 015002 (2009), M. Hirsch, S. Morisi and J. W. F. Valle, Phys. Rev. D 78, 093007 (2008), P. H. Frampton and S. Matsuzaki, arXiv:0806.4592 [hep-ph], C. Csaki, C. Delaunay, C. Grojean and Y. Grossman, JHEP 0810, 055 (2008), F. Bazzocchi, M. Frigerio and S. Morisi, Phys. Rev. D 78, 11601816 (2008), S. Morisi, arXiv:0901.1080 [hep-ph], P. Ciafaloni, M. Picariello, E. Torrente-Lujan and A. Urbano, Phys. Rev. D 79, 116010 (2009), M. C. Chen and S. F. King, JHEP 0906, 072 (2009), G. Altarelli and F. Feruglio, Nucl. Phys. B 720, 64 (2005), G. Altarelli and F. Feruglio, Nucl. Phys. B 741, 215 (2006), G. Altarelli, F. Feruglio and Y. Lin, Nucl. Phys. B 775, 31 (2007), Y. Lin, Nucl. Phys. B 813, 91 (2009), Y. Lin, arXiv:0903.0831 [hep-ph], G. Altarelli and D. Meloni, J. Phys. G 36, 085005 (2009).

C. H. Albright, arXiv:0905.0146 [hep-ph], E. Ma, Phys. Rev. D 44, 587 (1991), Y. Koide, Phys.Rev. D 60, 077301 (1999), M. Tanimoto, Phys. Lett. B 483, 417 (2000), J. Kubo, Phys. Lett.B 578, 156 (2004), [Erratum-ibid. B 619, 387 (2005)], F. Caravaglios and S. Morisi, arXiv:hep- ph/0503234, S. Morisi and M. Picariello, Int. J. Theor. Phys. 45, 1267 (2006), P. F. Harrison19 and W. G. Scott, Phys. Lett. B 557, 76 (2003), W. Grimus and L. Lavoura, JHEP 0508, 013 (2005), R. N. Mohapatra, S. Nasri and H. B. Yu, Phys. Lett. B 639, 318 (2006), N. Haba and K. Yoshioka, Nucl. Phys. B 739, 254 (2006), C. Y. Chen and L. Wolfenstein, Phys. Rev. D 77, 093009 (2008), S. Kaneko, H. Sawanaka, T. Shingai, M. Tanimoto and K. Yoshioka, arXiv:hep-ph/0703250, Y. Koide, Eur. Phys. J. C 50, 809 (2007), Phys. Rev. D 73, 057901 (2006), T. Teshima, Phys. Rev. D 73, 045019 (2006), L. Lavoura and E. Ma, Mod. Phys. Lett. A 20, 1217 (2005), T. Araki, J. Kubo and E. A. Paschos, Eur. Phys. J. C 45, 465 (2006), N. Haba, A. Watanabe and K. Yoshioka, Phys. Rev. Lett. 97, 041601 (2006), F. Feruglio and Y. Lin, Nucl. Phys. B 800, 77 (2008), A. S. Joshipura and S. D. Rindani, Eur. Phys. J. C 14, 85 (2000), R. N. Mohapatra, A. Perez-Lorenzana and C. A. de Sousa Pires, Phys. Lett. B 474, 355 (2000), Q. Sha and Z. Tavartkiladze, Phys. Lett. B 482, 145 (2000), L. Lavoura, Phys. Rev. D 62, 093011 (2000), W. Grimus and L. Lavoura, Phys. Rev. D 62, 093012 (2000), T. Kitabayashi and M. Yasue, Phys. Rev. D 63, 095002 (2001), A. Aranda, C. D. Carone and P. Meade, Phys. Rev. D 65, 013011 (2002), K. S. Babu and R. N. Mohapatra, Phys. Lett. B 532, 77 (2002), Phys. Lett. B 536, 83 (2002), H. S. Goh, R. N. Mohapatra and S. P. Ng, Phys. Lett. B 542, 116 (2002), G. K. Leontaris, J. Rizos and A. Psallidas, Phys. Lett. B 597, 182 (2004), S. Dev, R. R. Gautam, L. Singh, Phys.Lett. B708 (2012) 284-289 [arXiv : 1201.3577 [hep-ph]], S. Zhou,Phys. Lett. B 704, 291 (2011) [arXiv:1106.4808 [hep-ph]], S. Dev, S. Gupta, R. R. Gautam Phys.Lett.B702:28-33,2011 [arXiv : 1106.3878 [hep-ph]].

- Modes which introduce family symmetry G_F at GUT scale
  - GUT Symmetry(e.g. SU(5), SO(10),…) x Family Symmetry G_F

has chance to solve problem of fermion mass and mixing

- Models where family symmetry is represented by continuous group:
  - U(1)
  - SU(2) ≈ SO(3)
  - SU(3)
- Models based on discrete family symmetry
  - S_3(equilateral triangle)
  - A_4(tetrahedron)
  - T' (double tetrahedron)
  - S_4(cube)
  - A_5(dodecahedron)
  - Δ(24)
  - Δ(150)
  - Δ(348)
  - Σ(168)…

  …….. **up to order = 511**

Only fermion mixing

Example - discrete family groups

Dublets i singlets of the $SU_L(2) \times U_Y$ group

$$Q_{\alpha L} = \begin{pmatrix} u_{\alpha L} \\ d_{\alpha L} \end{pmatrix} \begin{matrix} \longleftrightarrow u_{\alpha R} \\ \longleftrightarrow d_{\alpha R} \end{matrix} \qquad \begin{matrix} u_{1L} \equiv u_L; \; u_{2L} \equiv c_L; \; u_{3L} \equiv t_L \\ d_{1L} \equiv d_L; \; \mathrm{d}_{2L} \equiv s_L; \; \mathrm{d}_{3L} \equiv b_L \end{matrix}$$

$$L_{\alpha L} = \begin{pmatrix} \nu_{\alpha L} \\ l_{\alpha L} \end{pmatrix} \begin{matrix} \longleftrightarrow \color{red}{\nu_{\alpha R}} \\ \longleftrightarrow l_{\alpha R} \end{matrix} \qquad \begin{matrix} \nu_{1L} \equiv \nu_{eL}; \; \nu_{2L} \equiv \nu_{\mu L}; \; \nu_{3L} \equiv \nu_{\tau L} \\ l_{1L} \equiv e_L; \; l_{2L} \equiv \mu_L; \; l_{3L} \equiv \tau_L \end{matrix}$$

$$\bar{Q}_{\alpha L} \equiv Q^{+}_{\alpha L} \gamma^0 = \left(\bar{u}_L, \bar{d}_L \right)$$ and so on.

$$-iL_{kin} = \sum_\alpha \bar{Q}_{\alpha L} \gamma^\mu \partial_\mu Q_{\alpha L} + \sum_\alpha \bar{L}_{\alpha L} \gamma^\mu \partial_\mu L_{\alpha L} +$$

$$+ \sum_\alpha \bar{u}_{\alpha R} \gamma^\mu \partial_\mu u_{\alpha R} + \sum_\alpha \bar{d}_{\alpha R} \gamma^\mu \partial_\mu d_{\alpha R} + \sum_\alpha \bar{l}_{\alpha R} \gamma^\mu \partial_\mu l_{\alpha R} + \color{red}{\sum_\alpha \bar{\nu}_{\alpha R} \gamma^\mu \partial_\mu \nu_{\alpha R}}$$

In the Standard Model

$$\Phi = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} = e^{i\frac{\vec{\tau}\vec{\rho}}{v}} \begin{pmatrix} \frac{1}{\sqrt{2}}(v+H) \\ 0 \end{pmatrix}$$

$$\tilde{\Phi} = \begin{pmatrix} \phi^+ \\ -\phi^{0*} \end{pmatrix} = e^{-i\frac{\vec{\tau}\vec{\rho}}{v}} \begin{pmatrix} 0 \\ -\frac{1}{\sqrt{2}}(v+H) \end{pmatrix}$$

$$\left(\partial_\mu \Phi\right)^+ \left(\partial^\mu \Phi\right) \to \left(D_\mu \Phi\right)^+ \left(D^\mu \Phi\right) \qquad D_\mu = \partial_\mu - ig\frac{\vec{\tau}}{2}\vec{W}_\mu - ig'YB_\mu$$

$$\boxed{L_{Higgs} = \left(D_\mu \Phi\right)^+ \left(D^\mu \Phi\right) - \frac{1}{2}V(\Phi^+\Phi)} \qquad \vec{W}_\mu = \left(W_{1\mu}, W_{2\mu}, W_{3\mu}\right)$$

$$V_H(\Phi^+\Phi) = \lambda\left(\Phi^+\Phi - \frac{1}{2}v^2\right)^2$$

(λ, v are free parameters)

SSB → $v^2 > 0;\quad Nambu - Goldston\ realization;\quad \Phi^+\Phi = \frac{1}{2}v^2$

$$L_Y = -\sum_{\alpha,\beta}\left(h^u_{\alpha,\beta}\left[\bar{Q}_{\alpha L}\Phi u_{\beta R}\right] + h^d_{\alpha,\beta}\left[\bar{Q}_{\alpha L}\tilde{\Phi} d_{\beta R}\right] + h^l_{\alpha,\beta}\left[\bar{L}_{\alpha L}\tilde{\Phi} l_{\beta R}\right] + h^\nu_{\alpha,\beta}\left[\bar{L}_{\alpha L}\Phi \nu_{\beta R}\right]\right) + h.c.$$

$$\langle\tilde{\Phi}\rangle = -\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$$

Unknown Yukawa parameters

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}v\\ 0\end{pmatrix}$$

$$M^u_{\alpha,\beta} = \frac{v h^u_{\alpha,\beta}}{\sqrt{2}} \qquad M^d_{\alpha,\beta} = -\frac{v h^d_{\alpha,\beta}}{\sqrt{2}} \qquad M^l_{\alpha,\beta} = -\frac{v h^l_{\alpha,\beta}}{\sqrt{2}} \qquad M^\nu_{\alpha,\beta} = \frac{v h^\nu_{\alpha,\beta}}{\sqrt{2}}$$

$$L_{Mass} = -\sum_{\alpha,\beta}\left(\left[\bar{u}_{\alpha L} M^u_{\alpha,\beta} u_{\beta R}\right] + \left[\bar{d}_{\alpha L} M^d_{\alpha,\beta} d_{\beta R}\right] + \left[\bar{L}_{\alpha L} M^l_{\alpha,\beta} l_{\beta R}\right] + \left[\bar{\nu}_{\alpha L} M^\nu_{\alpha,\beta} \nu_{\beta R}\right]\right) + h.c$$

For example:

$$U^{u+}_L M^u U^u_R = M^u_{diagonal} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

Where $M^{u,d,l,\nu}$ are complex 3x3 matrices; symmetric for Majorana neutrinos

Generally there is possibility to introduce more Higgs particles which form:

N_s singlets: H_m $m = 1,2,\ldots,N_s$

N_d doublets: Φ_i $i = 1,2,\ldots,N_d$ $\Phi_i = \begin{pmatrix} \phi_i^0 \\ \phi_i^- \end{pmatrix}$

N_t triplets: Δ_n $n = 1,2,\ldots,N_t$ $\Delta_n = \begin{pmatrix} \Delta_n^+ & \sqrt{2}\Delta_n^{++} \\ \Delta_n^0 & -\Delta_n^+ \end{pmatrix}$

Then Yukawa Lagrangian and Higgs potential are more complicated:

$$L_Y = f(H_m, \Phi_i, \Delta_n) \qquad V_H = g(H_m, \Phi_i, \Delta_n)$$

Let us denote:

$$\boxed{\psi_\alpha = \{u_{\alpha L}, \mathrm{d}_{\alpha L}, u_{\alpha R}, \mathrm{d}_{\alpha R}; \; l_{\alpha L}, \nu_{\alpha L}, l_{\alpha R}, l_{\alpha R}, \nu_{\alpha R} \; \nu_{\alpha R}\}}$$

For each field ψ_α and for each flavour symmetry „p” there exist 3 × 3 matrices (representation of G_F):

$$\boxed{A_p^{\psi}}$$

For each multiplets of Higgs fields there exist appropriate dimensional representation of G_F (e.g. for N_d doublets should exist N_d x N_d matrices:

$$\boxed{A_p^{\Phi}}$$

Kinetic energy and Higgs potential are invariant. Let us consider the symmetry for the neutrino Yukawa interaction:

$$L_Y = -h^{\nu}_{\alpha,\beta}\left[\bar{l}_{\alpha L}\Phi\nu_{\beta R}\right] \Rightarrow -\sum_{i=1}^{N_d}\bar{l}_{\alpha L}(h^{\nu}_i)_{\alpha,\beta}\Phi_i\nu_{\beta R} \to L'_Y$$

$$\boxed{L'_{\alpha L} = \sum_{\gamma=e,\mu,\tau}(A^L_p)_{\alpha,\chi}L_{\chi L}} \quad \boxed{\Phi'_i = \sum_{k=1}^{N_d}(A^{\Phi}_p)_{i,k}\Phi_k} \quad \boxed{\nu'_{\beta R} = \sum_{\delta=e,\mu,\tau}(A^{\nu}_p)_{\beta,\delta}\nu_{\delta R}}$$

$$L'_Y = -\sum_{i=1}^{N_d}[(A^{L*}_p)_{\alpha,\chi}\bar{L}_{\chi L}](h^{\nu}_i)_{\alpha,\beta}[\sum_{k=1}^{N_d}(A^{\Phi}_p)_{i,k}\Phi_k][(A^{\nu}_p)_{\beta,\delta}\nu_{\delta R}] =$$

where

$$= -\sum_{k=1}^{N_d}\bar{L}_{\chi L}(\tilde{h}^{\nu}_k)_{\chi,\delta}\Phi_k\nu_{\delta R}$$

$$(\tilde{h}^{\nu}_k)_{\chi,\delta} = \sum_{i=1}^{N_d}(A^{L+}_p)_{\chi,\alpha}(h^{\nu}_i)_{\alpha,\beta}(A^{\Phi}_p)_{i,k}(A^{\nu}_p)_{\beta,\delta} = \sum_{i=1}^{N_d}\left((A^{L+}_p)(h^{\nu}_i)(A^{\Phi}_p)_{i,k}(A^{\nu}_p)\right)_{\chi,\delta}$$

Symmetry $\Rightarrow$ $L_Y = L'_Y$ $\Rightarrow$ $(\tilde{h}^{\nu}_i)_{\chi,\delta} = (h^{\nu}_i)_{\chi,\delta}$ and

$$\boxed{\sum_{k=1}^{N_d}\left(A^{L+}_p(h^{\nu}_k)(A^{\Phi}_p)_{k,i}A^{\nu}_p\right)_{\chi,\delta} = (h^{\nu}_i)_{\chi,\delta}}$$

For neutrino mass matrix (e.g. for two Higgs doublets):

$$M^{\nu}_{\alpha,\beta} = \frac{1}{\sqrt{2}}\left(v_1 (h^{\nu}_1)_{\alpha,\beta} + v_2 (h^{\nu}_2)_{\alpha,\beta}\right) \qquad \Phi_i = e^{i\frac{\vec{\tau}\vec{\rho}_i}{v_i}} \begin{pmatrix} \frac{1}{\sqrt{2}}(v_i + H_i) \\ 0 \end{pmatrix}$$

$$v_2 \geq 1\ \mathrm{TeV}$$

$$M^{\nu'}_{\alpha,\beta} = \frac{1}{\sqrt{2}} \sum_{i=1}^{N_d} v_i (\tilde{h}^{\nu}_i)_{\alpha,\beta} = \frac{1}{\sqrt{2}} \sum_{i,k=1}^{N_d} v_i \left(A^{L+}_p (h^{\nu}_k)(A^{\Phi}_p)_{k,i} A^{\nu}_p \right)_{\alpha,\beta} \Rightarrow$$

$$\boxed{M^{\nu'} = A^{L+}_p \left(\frac{1}{\sqrt{2}} \sum_{i,k=1}^{N_d} v_i (h^{\nu}_k)(A^{\Phi}_p)_{k,i} \right) A^{\nu}_p = M^{\nu}}$$

If there is only one Higgs fields:

$$M^{\nu'} = A^{L+}_p M^{\nu} A^{\nu}_p = M^{\nu}$$

If there is only one 3-dimensional irreducible representation of the G_F group:

$$A^{L}_p = A^{\nu}_p = A_p \qquad \boxed{A^{+}_p M^{\nu} A_p = M^{\nu}} \quad \Leftrightarrow \quad \boxed{\left[M^{\nu}, A_p \right] = 0}$$

There was great interest in explaining the structure of lepton mixing matrix

Especially in a situation, where

- is only one Higss field, and
- all multiplets (L,ν) transform according to one three-dimensional representation, then

$$\boxed{\left[M^{\nu},A_{p}\right]=0}$$

In many cases there exist group generators, that for any A_p :

$$A_{p}=G_{1}^{a}G_{2}^{b}G_{3}^{c}$$

For finite groups:

$$G_{1}^{n}=1,\quad G_{1}^{m}=1,\quad G_{1}^{r}=1$$

Then mass matrix commutates with the group generators:

$$\left[M^{\nu}, G_i\right] = 0$$

From this relations it follows that the neutrino mass matrix has the same eigenvectors as generators.

In the base where charged lepton mass matrix is diagonal, PMNS mixing matrix diagonalizes neutrino mass matrix

If $M^{l} = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \Rightarrow U^{l} = I \Rightarrow U_{PMNS} = U^{l+}U^{\nu} = U^{\nu} \equiv U$

Then $\boxed{U^{T} M^{\nu} U = \begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_3} \end{pmatrix}}$

$$\mathbf{u}_1 = \begin{pmatrix} u_{11} \\ u_{21} \\ u_{31} \end{pmatrix} \qquad \mathbf{u}_2 = \begin{pmatrix} u_{12} \\ u_{22} \\ u_{32} \end{pmatrix} \qquad \mathbf{u}_3 = \begin{pmatrix} u_{13} \\ u_{23} \\ u_{33} \end{pmatrix} \qquad U_{PMNS} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{pmatrix};$$

From symmetry it follows that the group generators are equal:

$$G_1 = \mathbf{u}_1\mathbf{u}_1^+ - \mathbf{u}_2\mathbf{u}_2^+ - \mathbf{u}_3\mathbf{u}_3^+;$$
$$G_1 = -\mathbf{u}_1\mathbf{u}_1^+ + \mathbf{u}_2\mathbf{u}_2^+ - \mathbf{u}_3\mathbf{u}_3^+;$$
$$G_3 = -\mathbf{u}_1\mathbf{u}_1^+ - \mathbf{u}_2\mathbf{u}_2^+ + \mathbf{u}_3\mathbf{u}_3^+$$

So, in the **Bottom –up approach:**

Experiment $\longrightarrow$ U_{PMNS} $\longrightarrow$ $\mathbf{u}_i$ $\longrightarrow$ G_i $\longrightarrow$ symmetry group G_F

In the **Top-down approach**:

Symmetry group G_F $\longrightarrow$ generators G_i $\longrightarrow$ eigenvectors $\mathbf{u}_i$ $\longrightarrow$ U_{PMNS}

Just after discovery of neutrino oscillation:

Two maximal mixing angles

$$\boxed{\theta_{13} \approx 0; \quad \theta_{23} \approx 45^0; \quad \theta_{12} \approx 45^0}$$

Neutrino BI-MAXIMAL mixing

$$U_{BM} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix}$$

Vissani F., arXiv: hep-ph/9708483;
Barger V. D. et al., Phys. Lett.B, 437 (1998) 107;
Nomura Y. and Yanagida T., Phys. Rev.D, 59 (1999) 017303;
Altarelli G. and Feruglio F., JHEP, 11 (1998) 021.

At 2002 better data $\theta_{12} < 45^0$ and:

$$\boxed{\theta_{13} \approx 0; \quad \theta_{23} \approx 45^0; \quad \theta_{12} = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 35.3^0}$$

Neutrino TRI-BIMAXIMAL mixing

$$U_{TBM} = \begin{pmatrix} \sqrt{2}/\sqrt{3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Tri- Bi- - maximal

Harrison P. F., Perkins D. H. and Scott W. G., Phys. Lett. B, 530 (2002) 167;
Xing Z.-z., Phys. Lett.B, 533 (2002) 85.

Presented U_{PMNS} mixing matrix for TBM corresponds to generators:

$$G_1 = \frac{1}{3}\begin{pmatrix} 1 & -2 & 2 \\ -2 & -2 & -1 \\ 2 & -1 & -2 \end{pmatrix}$$

$$G_2 = \frac{1}{3}\begin{pmatrix} -1 & 2 & -2 \\ 2 & -1 & -2 \\ -2 & -2 & -1 \end{pmatrix}$$

$$G_3 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

These are the generators of the group A_4, even permutations of four elements

But present experimental data:

$$\theta_{13} = 9.12^0 \pm 0.63^0 \qquad \theta_{12} = 33.9^0 \pm 1.0^0 \qquad \theta_{23} \in \{38.5^0 - 45.0^0\}$$

$$\theta_{13}^{TBM} = 0^0; \qquad \theta_{12}^{TBM} = 35.3^0; \qquad \theta_{23}^{TBM} = 45.0^0;$$

A	B	C	D	E	F	G	H
[12, 3]	A_4, T	∘	[0.577, 0.577, 0.577]	2.85	*aN*	2	
[21, 1]	T_7	×					
[24, 12]	S_4, O, $\Delta(24)$	•	[0.816, 0.408, 0.408]	3.65	*bN*	1	×
			[0.500, 0.707, 0.500]	4.95	*bI*	2	×
[27, 3]	$\Delta(27)$	×					
[39, 1]	T_{13}	×					
[48, 3]	$\Delta(48)$	∘					
[54, 8]	$\Delta(54)$	*p*	[0.500, 0.707, 0.500]	4.95	*bI*	2	×
[57, 1]	T_{19}	×					
[60, 5]	A_5, I, $\Sigma(60)$	∘	[0.526, 0.602, 0.602]	3.68	*aN*	2	×
[75, 2]	$\Delta(75)$	×					
[81, 9]		×					
[84, 11]		∘					
[93, 1]	T_{31}	×					
[96, 64]	$\Delta(96)$	•					
[108, 15]	$\Sigma(36\varphi)$	*p*	⋯	⋯	⋯	⋯	
[108, 22]	$\Delta(108)$	∘					
[111, 1]	T_{37}	×					
[129, 1]	T_{43}	×					
[147, 1]	T_{49}	×					
[147, 5]	$\Delta(147)$	×					

A	B	C	D	E	F	G	H
[150, 5]	$\Delta(150)$	*p*	[0.812, 0.332, .0480]	0.018	*aN*	1	
			[0.812, 0.480, 0.332]	0.086	*aI*	1	
			[0.500, 0.707, 0.500]	4.95	*bI*	2	×
			[0.170, 0.607, 0.777]	1.25	*bN*	3	
[156, 14]		∘					
[162, 14]		*p*	[0.804, 0.279, 0.525]	1.41	*aN*	1	
			[0.804, 0.525, 0.279]	3.05	*aI*	1	×
			[0.500, 0.707, 0.500]	4.95	*bI*	2	×
[168, 42]	$\Sigma(168)$, $PSL(3,2)$	•	[0.815, 0.363, 0.452]	0.267	*bN*	1	
			[0.815, 0.452, 0.363]	0.269	*bI*	1	
[183, 1]	T_{61}	×					
[189, 8]		×					
[192, 3]	$\Delta(192)$	∘					
[201, 1]	T_{67}	×					
[216, 88]	$\Sigma(72\varphi)$	*p*	⋯	⋯	⋯	⋯	
[216, 95]	$\Delta(216)$	•					
[219, 1]	T_{73}	×					
[228, 11]		∘					
[237, 1]	T_{79}	×					
[243, 26]	$\Delta(243)$	×					
[273, 3]	T_{91}	×					
[273, 4]	T'_{91}	×					
[291, 1]	T_{97}	×					
[294, 7]	$\Delta(294)$	*p*	[0.814, 0.460, 0.354]	1.16	*aI*	1	
			[0.814, 0.354, 0.460]	0.312	*bI*	1	
			[0.796, 0.241, 0.555]	4.63	*aN*	1	×
			[0.500, 0.707, 0.500]	4.95	*bI*	2	×
			[0.122, 0.638, 0.760]	5.80	*bI*	3	×

A	B	C	D	E	F	G	H
[300, 43]	$\Delta(300)$	∘					
[309, 1]	T_{103}	×					
[324, 50]		∘					
[327, 1]	T_{109}	×					
[336, 57]		∘					
[351, 8]		×					
[363, 2]	$\Delta(363)$	×					
[372, 11]		∘					
[381, 1]	T_{127}	×					
[384, 568]	$\Delta(384)$	•	[0.810, 0.312, 0.497]	0.188	*aN*	1	
			[0.810, 0.497, 0.312]	0.287	*aI*	1	
[399, 3]	T_{133}	×					
[399, 4]	T'_{193}	×					
[417, 1]	T_{139}	×					
[432, 103]	$\Delta(432)$	∘					
[444, 14]		∘					
[453, 1]	T_{151}	×					
[471, 1]		×					
[486, 61]	$\Delta(486)$	*p*	[0.804, 0.279, 0.525]	1.41	*aN*	1	
			[0.804, 0.525, 0.279]	3.05	*aI*	1	×
			[0.500, 0.707, 0.500]	4.95	*bI*	2	×
[489, 1]	T_{163}	×					
[507, 1]	T_{169}	×					
[507, 5]	$\Delta(507)$	×					

C.S.Lam, „Finite symmetry of leptonic mixing matrix", Phys. Rev. D 87, 013001 (2013)

No finite group up to order = 511 can be full symmetry group of the lepton mixing !!!

For models with two Higgs doublets
(2HDM, Supersymmetric models)

Two Higgs matrices: h_1^ν, h_2^ν

For Majorana neutrinos they are symmetric.

Symmetry:

$$\begin{cases} A_p^{L+}(h_1^\nu)(A_p^\Phi)_{1,1}A_p^\nu + A_p^{L+}(h_2^\nu)(A_p^\Phi)_{2,1}A_p^\nu = h_1^\nu \\ A_p^{L+}(h_1^\nu)(A_p^\Phi)_{1,2}A_p^\nu + A_p^{L+}(h_2^\nu)(A_p^\Phi)_{2,2}A_p^\nu = h_2^\nu \end{cases}$$

For Majorana neutrinos set of 12 equations must be solved.

Neutrino mass matrix: $$\boxed{M_{\alpha,\beta}^\nu = \frac{1}{\sqrt{2}}\left(v_1(h_1^\nu)_{\alpha,\beta} + v_2(h_2^\nu)_{\alpha,\beta}\right)}$$

$v_1 \approx 246$ GeV

$v_2 \geq 1$ TeV

More Higgs particles

* There are two Higss fields and
* all multiplets (L,ν) transform according to one three-dimensional representation, then

$$A_p^L = A_p^\nu \equiv A \qquad (A_p^\Phi)_{i,k} \equiv a_{ik}$$

$$(h_1^\nu)a_{11} + (h_2^\nu)a_{21} = A\, h_1^\nu A^+ \Rightarrow (h_2^\nu) = \frac{1}{a_{21}}\left(A\, h_1^\nu A^+ - (h_1^\nu)a_{11}\right)$$

$$(h_1^\nu)a_{12} + (h_2^\nu)a_{22} = A\, h_2^\nu A^+ \Rightarrow (h_1^\nu) = \frac{1}{a_{12}}\left(A\, h_2^\nu A^+ - (h_2^\nu)a_{22}\right)$$

$$(h_1^\nu) = \frac{1}{a_{12}a_{21}}\left(A^2 h_1^\nu A^{2+} - \left(A\, h_1^\nu A^+\right)\left(a_{11}+a_{22}\right) + (h_1^\nu)a_{11}a_{22}\right)$$

$$A^2 h_1^\nu A^{2+} - \left(A\, h_1^\nu A^+\right)\left(a_{11}+a_{22}\right) + (h_1^\nu)\left(a_{11}a_{22} - a_{12}a_{21}\right) = 0$$

Symmetry breaking

$$L_Y = -\sum_{\alpha,\beta}\left(h^l_{\alpha,\beta}\left[\bar{L}_{\alpha L}\tilde{\Phi} l_{\beta R}\right] + h^\nu_{\alpha,\beta}\left[\bar{L}_{\alpha L}\Phi \nu_{\beta R}\right]\right) + h.c.$$

$$\Downarrow$$

$$L_Y = -\sum_{\alpha,\beta}\left(-h^l_{\alpha,\beta}\left[\bar{l}_{\alpha L}(\varphi^{0*}) l_{\beta R}\right] + h^\nu_{\alpha,\beta}\left[\bar{\nu}_{\alpha L}\varphi^0 \nu_{\beta R}\right]\right) + h.c$$

$$l'_{\alpha L} = \sum_{\gamma=e,\mu,\tau}(A^l_L)_{\alpha,\chi} l_{\chi L} \qquad l'_{\alpha R} = \sum_{\gamma=e,\mu,\tau}(A^l_R)_{\alpha,\chi} l_{\chi P}$$

$$\nu'_{\alpha L} = \sum_{\gamma=e,\mu,\tau}(A^\nu_L)_{\alpha,\chi} \nu_{\chi L} \qquad \nu'_{\alpha R} = \sum_{\gamma=e,\mu,\tau}(A^\nu_R)_{\alpha,\chi} \nu_{\chi P}$$

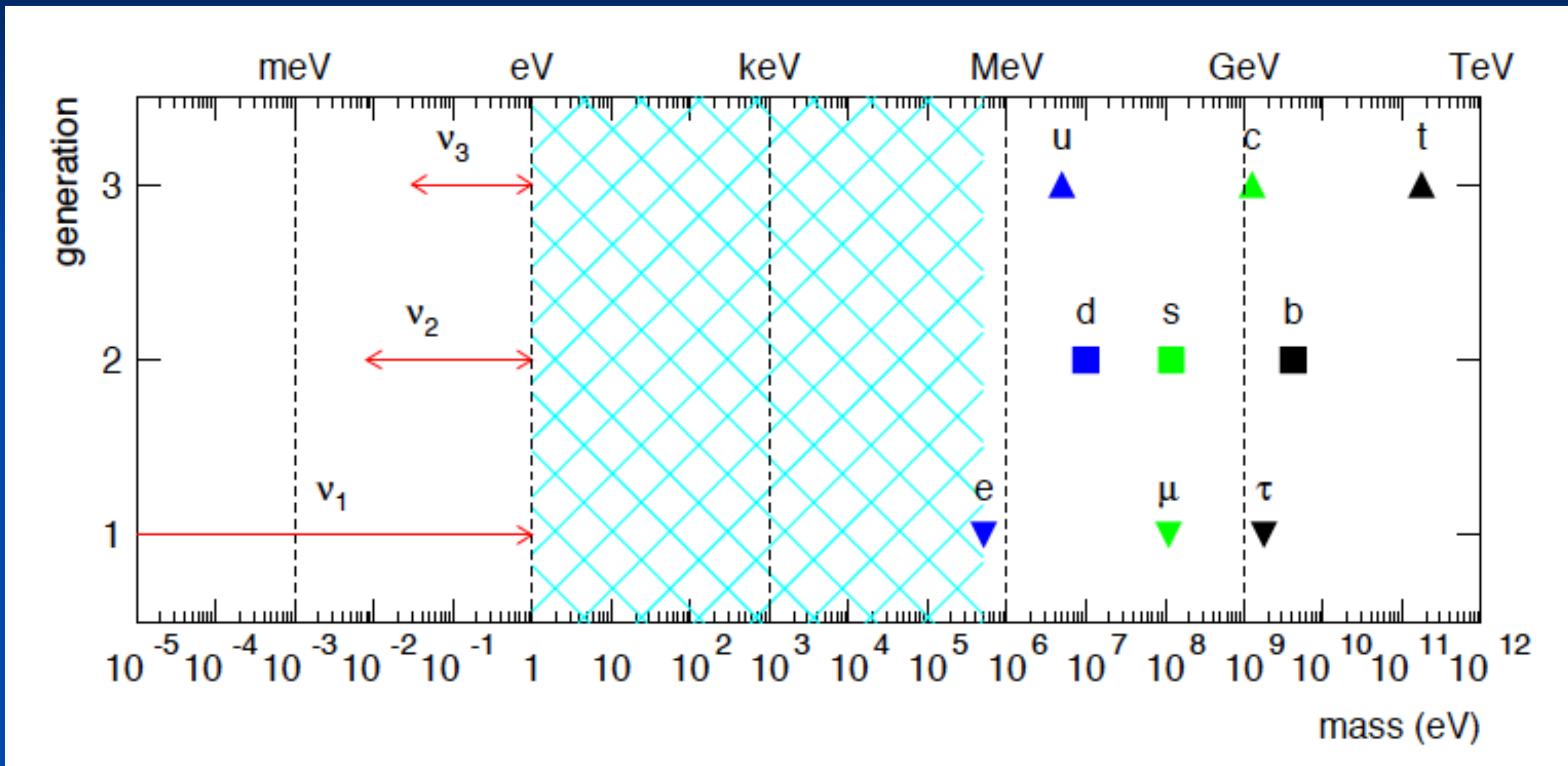
4) Conclusions - many unknowns remain…

Problem of fermion mass and their mixing is far from being solved

- Simple models of family symmetry (with three neutrino flavours) with one Higgs particle can not satisfactorily explain the lepton mixing.
- Models with more Higgs fields (not necessarily doublets) should be considered.
- The existence of one or more sterile neutrinos must be explained experimentally, then models with three active and some number of sterile neutrinos need to be consider.
- The difference of lepton and quark mixings is probably related directly to smallness of neutrino mass and probably its Majorana character.
- GUT models with see-saw mechanism, extra dimensions or R parity violation supersymmetry should give some information about quarks and leptons masses and mixing.
- More precise information on neutrino masses, mixing angles and phase (phases) of CP violation in the lepton sector are very much needed.

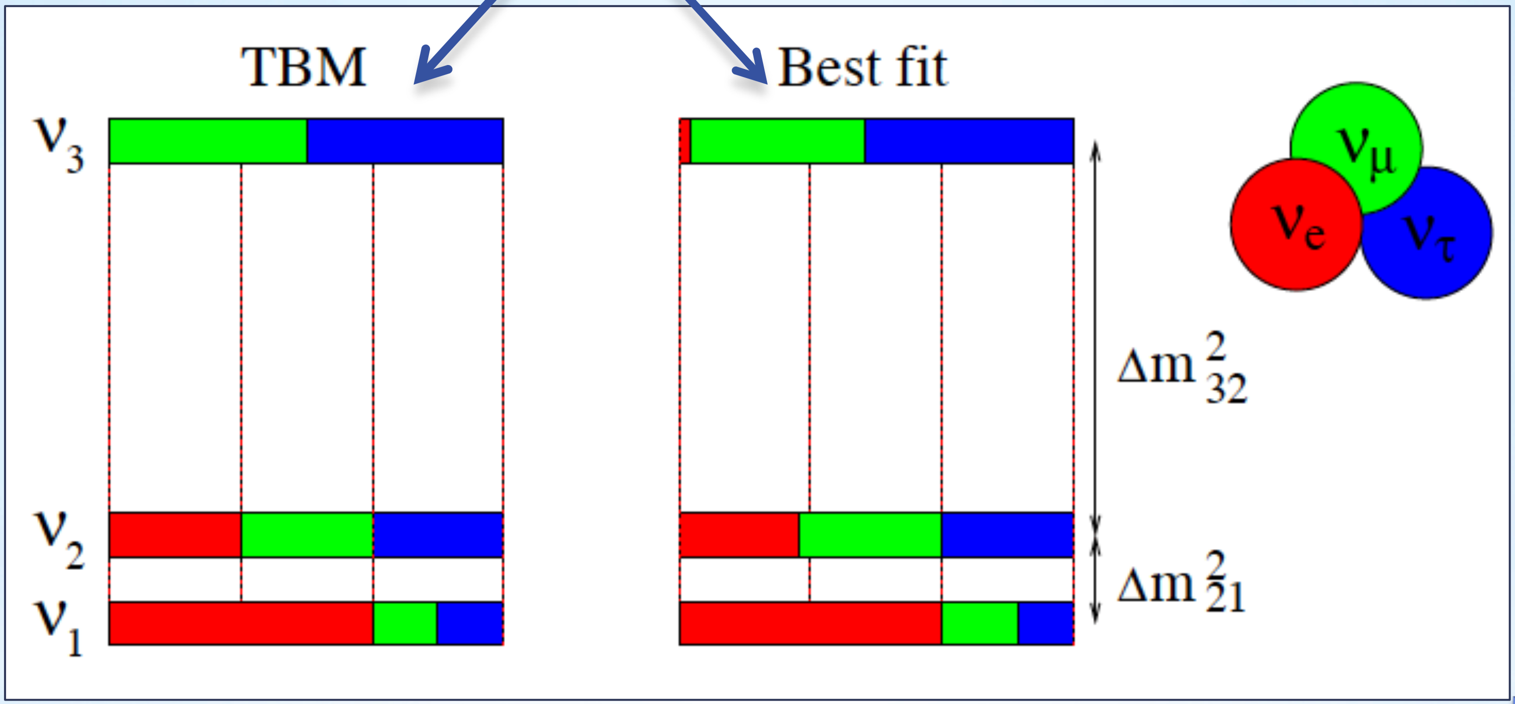
10^6 eV ----- Desert

0.5 eV 0.5 MeV

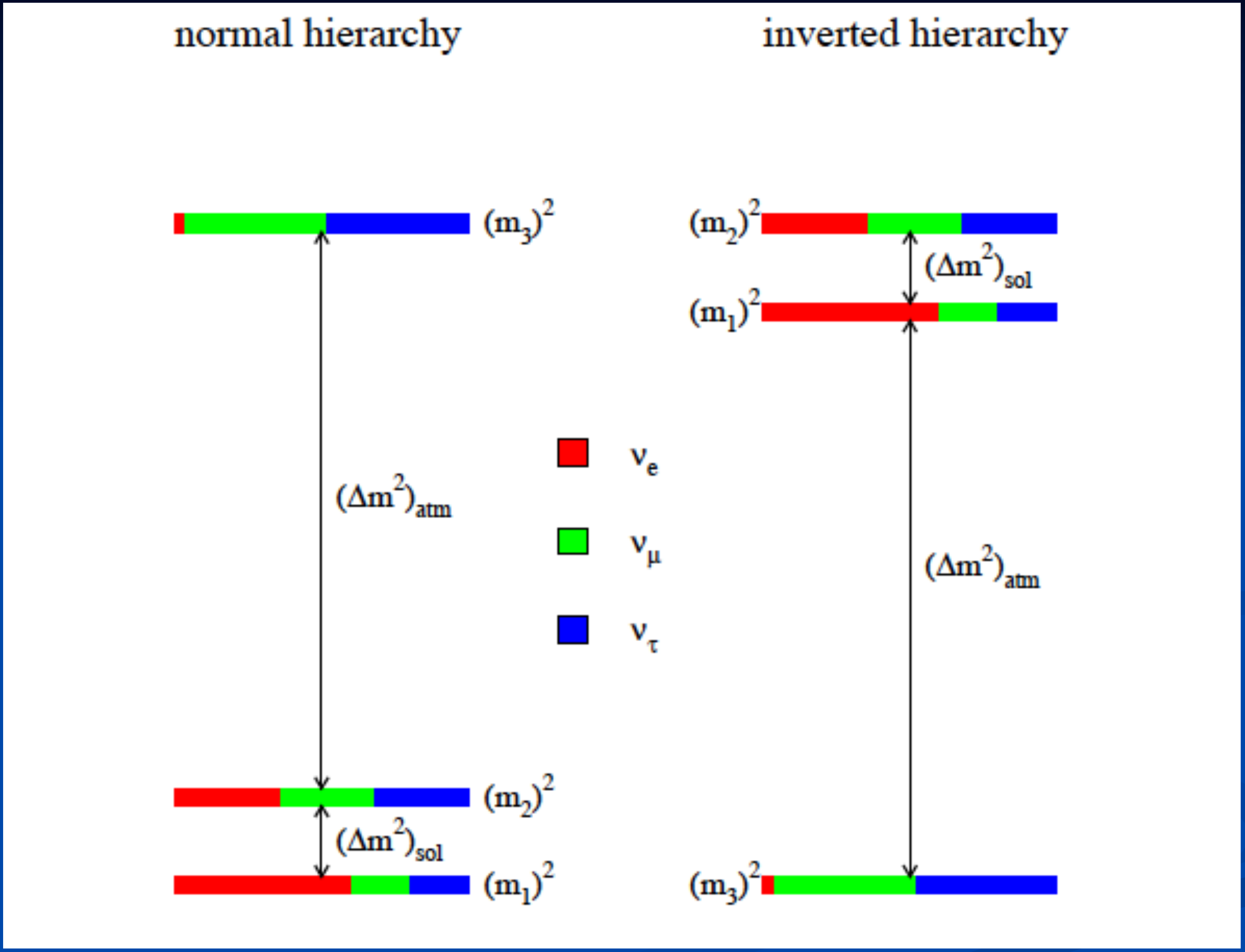


From A. De Gouvêa hep-ph/0411274

ν_μ and ν_τ flavors share ν_3 equally (bi-maximal mixing)



In ν_2 all tree flavours are present with the same weight (tri-bi-maximal (TBM) mixing)



Experimental technologies have now advanced to the point that sensitivity to the inverted hierarchy mass scale will soon be achieved.