Ingredients for Double Beta Decay

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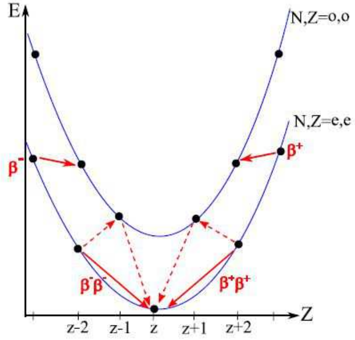
Introduction

- Mass parabola for isobaric nuclei with even $\boldsymbol{A}$

- Splits into two parabolas due to the nuclear pair energy: odd-odd, even-even
- Favoured decay: Single β-decay
  - Changes the nuclear charge $\boldsymbol{Z}$ by a value of $\pm\mathbf{1}$
  - Can only occur if the energy of the daughter nucleus is smaller than the energy of the parent nucleus
  - If not, two consecutive β-decays as single process are possible



Introduction

- Nucleus (A, Z) decays to nucleus $(A, Z \pm 2)$ by emitting two electrons or positrons + other light particles
- For processes allowed by the standard model the half-life is

$$\left[\tau_{1/2}^{2\nu}\right]^{-1} = G_{2\nu} g_A^4 |m_e c^2 M^{(2\nu)}|^2$$

- and for neutrinoless modes

$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu} g_A^4 |M^{(0\nu)}|^2 |f(m_i, U_{ei})|^2$$

$$\propto |\langle m_\nu \rangle|^2 \text{ and/or } |\langle m_{\nu_h}^{-1} \rangle|^2$$

- $G_{2\nu}$ and $G_{0\nu}$ are the phase space factors
- g_A is the axial vector coupling constant (effective value essentially model dependent!)
- $M^{(2\nu)}$ and $M^{(0\nu)}$ are the nuclear matrix elements
- $f(m_i, U_{ei})$ contains the physics beyond standard model

For both processes, two crucial ingredients are the phase space factors and the nuclear matrix elements!

Ingredient #1: Phase Space Factors

- Starting from differential decay rate

$$dW_{0\nu} = \left(a^{(0)} + a^{(1)} \cos\theta_{12}\right) w_{0\nu} d\epsilon_1 d(\cos\theta_{12})$$

  - $w_{0\nu} \propto \epsilon_1, \epsilon_2$
  - $a^{(i)} \propto |\langle m_\nu \rangle|^2$ and/or $|\langle m_{\nu_h}^{-1} \rangle|^2$, $|M^{(0\nu)}|^2$, and $f_{11}^{(i)}$ (combination of emitted electron wave functions)

- And using the factorized form

$$\left[\tau_{1/2}^{0\nu}\right]^{-1} = G_{0\nu} g_A^4 |M^{(0\nu)}|^2 |f(m_i, U_{ei})|^2$$

- PSF can be written as

$$G_{0\nu}^{(i)} = \frac{1}{2R^2 \ln 2} \int_{m_e c^2}^{Q_{\beta\beta}+m_e c^2} f_{11}^{(i)} w_{0\nu} d\epsilon_1$$

Ingredient #1: Phase Space Factors

- The key ingredient for the evaluation of phase space factors (and thus double beta decay) are the electron wave functions
- Depending on mode, we use positive/negative energy Dirac central field scattering wave functions (β^-/β^+ decay), or positive energy central field bound state wave functions (EC), consisting on radial functions g_κ and f_κ, and spherical spinors χ_κ
- In all cases g_κ and f_κ satisfy the radial Dirac equations:

$$\begin{cases} \dfrac{dg_\kappa(r)}{dr} = -\dfrac{\kappa}{r} g_\kappa(r) + \dfrac{\epsilon - V + m_e c^2}{c\hbar} f_\kappa(r) \\ \dfrac{df_\kappa(r)}{dr} = -\dfrac{\epsilon - V - m_e c^2}{c\hbar} g_\kappa(r) + \dfrac{\kappa}{r} f_\kappa(r) \end{cases}$$

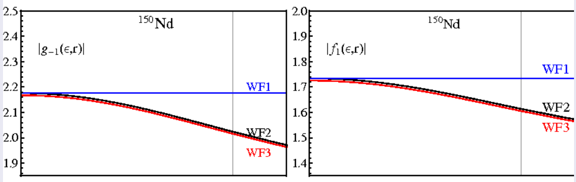
- Numerical solution by Salvat *et al.*, Comput. Phys. Commun. 90, 151 (1995)

Ingredient #1: Phase Space Factors

- To simulate realistic situation, we take into account the finite nuclear size and the electron screening
- Thomas-Fermi screening function, $\varphi(r) = Z_{eff}/Z_d$, obtained by Majorana solution of Thomas Fermi equation (Esposito, Am. J. Phys. 70, 852 (2002))
- Boundary conditions take into account the fact that final atom is charged ion or neutral atom depending on mode
- Comparison with previous calculations: Considerable difference

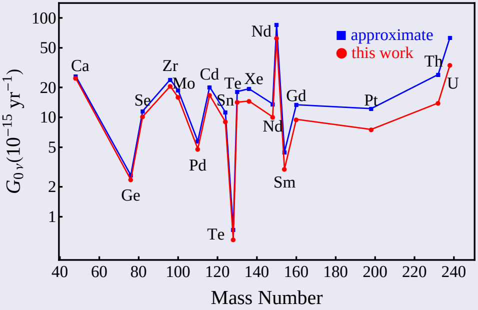
Example of obtained radial wave functions: ^{150}Nd decay, $Z_d = 62$ at $\epsilon = 2.0$MeV, $R(150) = 6.38$fm

$e_s^{S_{1/2}}(\epsilon, r)$

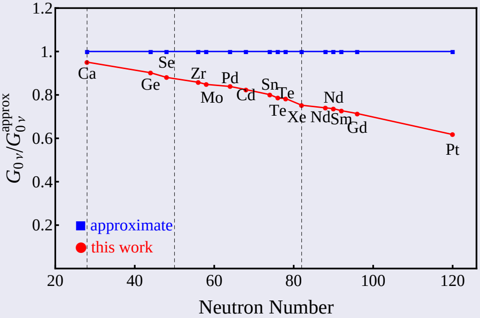


WF1 = Leading finite size Coulomb (previous studies)
WF2 = Exact finite size Coulomb
WF3 = Exact finite size Coulomb & electron screening

Ingredient #1: Phase Space Factors



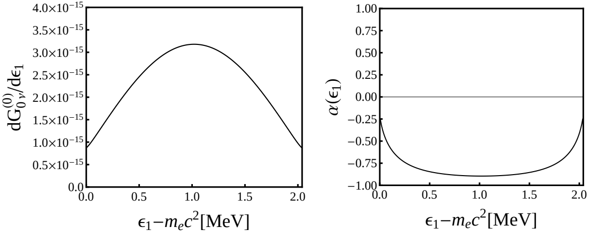
- Current $\mathbf{0\nu\beta^-\beta^-}$ PSFs (red) compared to previous calculations (blue)



- Relative difference: $\boldsymbol{G_{0\nu}/G_{0\nu}^{approx}}$
- Difference:
  few % for Ca,
  $\sim$ **30%** for Nd,
  and already 60% for Pt
- Does affect half-life and average neutrino mass predictions!

Ingredient #1: Phase Space Factors

- These results have been confirmed by independent calculations of Stoica *et al.* PRC **88**, 037303 (2013)
- Their conclusion: An inadequate numerical treatment can change significantly the results
- Now: Very detailed (1keV/100eV increments) calculations using Yale supercomputer to
  1. Minimize uncertainty coming from numerical integration
  2. Offer adequate numerical accuracy for single electron, summed energy and two-dimensional spectra, and angular correlations to be used in the analysis of experimental data



- Example ^{76}Ge $\rightarrow ^{76}$Se decay: (left) single electron spectrum, $\frac{dW_{0\nu}}{d\epsilon_1} = \mathcal{N}_{0\nu}\frac{dG^{(0)}_{0\nu}}{d\epsilon_1}$, (right) angular correlations, $\alpha(\epsilon_1) = \frac{dG^{(1)}_{0\nu}/d\epsilon_1}{dG^{(0)}_{0\nu}/d\epsilon_1}$

Ingredient #1: Phase Space Factors

- These files scan be downloaded as numerical tables for all the nuclei of interest at nucleartheory.yale.edu

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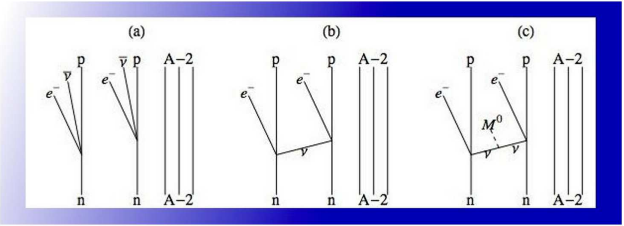


Welcome to the *Yale Nuclear Theory: Double beta decay* site!

At the moment this site is dedicated to research and results related to double beta decay, even though the Yale nuclear theory group is working with a variety of other fascinating topics as well. Dr J. Kotila is the author of the work presented in these pages and the main purpose of this site is to provide easy access to theoretically calculated single electron spectra, summed electron spectra, angular correlation, and two-dimensional electron spectra files. For further information and questions we encourage you to contact Dr J. Kotila.

Ingredient #1: Phase Space Factors

Estimate of uncertainties introduced to PSF

0ν	Q-value	$3 \times \delta Q/Q$
	Radius	7%
	Screening	0.10%

- Most Q-values very accurately known
- Error coming from radius, $R = r_0 A^{1/3}$, can be significantly reduced by adjusting r_0 for each nucleus instead of using $r_0 = 1.2$ fm:

$$\frac{3}{5} r_0^2 A^{2/3} = \langle r^2 \rangle_{exp}$$

  where $\langle r^2 \rangle_{exp}$ is obtained from electron scattering and/or muonic x-rays.
- The screening error is estimated to be 10% of the Thomas-Fermi contribution, known to overestimate the electron density at the nucleus

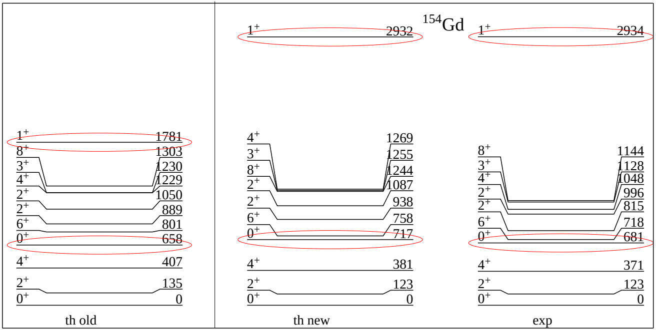
Ingredient #2: Nuclear Matrix Elements

- Nuclear matrix elements from Interacting Boson Model (IBM-2)
  - Describes even-even nuclei in terms of correlated pairs of nucleons treated as bosons with $\boldsymbol{L = 0}$ (s-boson) and $\boldsymbol{L = 2}$ (d-boson) that are associated with pairs of valence fermions
  - The model benefits from proximity to the phenomenological geometric approach while still linking to the microscopic foundations
- Can be used in any nucleus and thus all nuclei of interest can be calculated within the same model
- Realistic and well checked wave functions
  - Level energies, B$\boldsymbol{(E2)}$ and B$\boldsymbol{(M1)}$ values, quadrupole and dipole moments, etc.
- Shell effects: The matrix elements are smaller at the closed shells than in the middle of the shell
- Deformation effects: Deformation effects always decrease the matrix elements
- Isospin restoration reduces matrix elements

Ingredient #2: Nuclear Matrix Elements

Example of wave functions: ^{154}Gd

- Shape transitional region ⇒ rapid changes of nuclear deformation
- Old calculation: No experimental information about $\mathbf{1^+}$ scissors mode
- New experimental data ⇒ parameters of Majorana operator can be fitted
  - Little effect on low-lying full-symmetric states
  - BUT significant effect on the $\mathbf{0_2^+}$ state wave function ⇒ new $\boldsymbol{M^{0\nu}(0_2^+) = 0.37}$ (old $\boldsymbol{M^{0\nu}(0_2^+) = 0.02}$)



Ingredient #2: Nuclear Matrix Elements

ISOSPIN RESTORATION

- For $\beta\beta$ decay the fermionic transition operators can be written in compact form

$$V^{(\lambda)}_{s_1,s_2} = \frac{1}{2}\sum_{n,n'} \tau^+_n \tau^+_{n'} \left[\Sigma^{s_1}_n \times \Sigma^{s_2}_{n'}\right]^{(\lambda)} \cdot V(r_{nn'}) C^{(\lambda)}(\Omega_{nn'})$$

  - For $s = 0$, $\Sigma^{(0)} = 1$, and for $s = 1$, $\Sigma^{(1)} = \vec{\sigma}$
  - $V(r)$ generic radial form (depends on the model of double-β decay), and $C^{(\lambda)} = \sqrt{4\pi/(2\lambda+1)}Y^{(\lambda)}$
- Fermi term: $\lambda = 0$, $s_1 = s_2 = 0$
- Gamow-Teller term: $\lambda = 0$, $s_1 = s_2 = 1$ (+ additional factor $(-)^{s_1}\sqrt{2s_1+1} = -\sqrt{3}$)
- Tensor term: $\lambda = 2$, $s_1 = s_2 = 1$, (+ additional factor $\sqrt{\frac{2}{3}}$)

ISOSPIN RESTORATION

- Writing $V^{(\lambda)}_{s_1,s_2}$ in second quantized form, we find

$$
\begin{aligned}
V^{(\lambda)}_{s_1,s_2} \propto R^{(k_1,k_2,\lambda)}&(n_1,l_1,n_2,l_2,n'_1,l'_1,n'_2,l'_2) \\
&= \int_0^\infty v_\lambda(p)p^2 dp \\
&\times \int_0^\infty R_{n_1 l_1}(r_1) R_{n'_1 l'_1}(r_1) j_{k_1}(pr_1) r_1^2 dr_1 \\
&\times \int_0^\infty R_{n_2 l_2}(r_1) R_{n'_2 l'_2}(r_1) j_{k_2}(pr_2) r_2^2 dr_1
\end{aligned}
$$

- The offending isospin violating NME is the Fermi NME in $2\nu\beta\beta$, for which
  - $V(r) = 1$, $v_\lambda(p) = \frac{\delta p}{p^2}$, $\lambda = 0$, $s_1 = s_2 = 0$
    $\Rightarrow$The radial integral in the definition of the two-body matrix element between two fermion states is just overlap integral between initial and final states
    $R^{(0,0,0)}_{2\nu}(n_1,l_1,n_2,l_2,n'_1,l'_1,n'_2,l'_2) \equiv \delta_{n_1,n'_1}\delta_{l_1,l'_1}\delta_{n_2,n'_2}\delta_{l_2,l'_2}$

ISOSPIN RESTORATION

- Isospin restoration is obtained by using

$$\underline{\boldsymbol{2\nu\beta\beta:}}\quad \boldsymbol{R^{(k_1,k_2,\lambda)}(n_1,l_1,n_2,l_2,n_1',l_1',n_2',l_2') - \delta_{k_1,0}\delta_{k_2,0}\delta_{k,0}\delta_{\lambda,0}\delta_{j_1,j_1'}\delta_{j_2,j_2'}\delta_{n_1,n_1'}\delta_{l_1,l_1'}\delta_{n_2,n_2'}\delta_{l_2,l_2'}}$$

  - Fermi NME vanish for $\boldsymbol{2\nu\beta\beta}$

$$\underline{\boldsymbol{0\nu\beta\beta:}}\quad \boldsymbol{R^{(k_1,k_2,\lambda)}(n_1,l_1,n_2,l_2,n_1',l_1',n_2',l_2') - \delta_{k_1,0}\delta_{k_2,0}\delta_{k,0}\delta_{\lambda,0}\delta_{j_1,j_1'}\delta_{j_2,j_2'}\delta_{n_1,n_1'}\delta_{l_1,l_1'}\delta_{n_2,n_2'}\delta_{l_2,l_2'} \times R_{0\nu}^{(0,0,0)}(n_1,l_1,n_2,l_2,n_1',l_1',n_2',l_2')}$$

  - Fermi NME for $\boldsymbol{0\nu\beta\beta}$ is reduced by subtraction of the monopole term in the expansion of the matrix element multipoles

Ingredient #2: Nuclear Matrix Elements

ISOSPIN RESTORATION

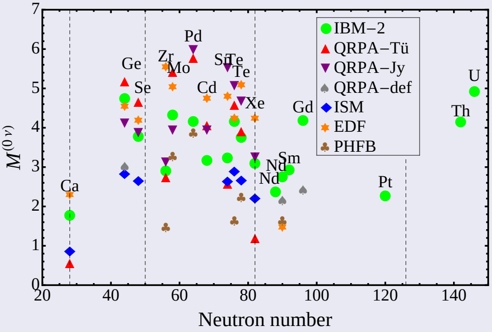
	$\chi_F = (g_V/g_A)^2 M_F^{(0\nu)}/M_{GT}^{(0\nu)}$		
Decay	IBM-2	QRPA	ISM
^{48}Ca	-0.10(-0.42)	-0.32(-0.93)	
^{76}Ge	-0.09(-0.38)	-0.21(-0.34)	-0.12
^{82}Se	-0.10(-0.42)	-0.23(-0.35)	-0.11
^{96}Zr	-0.08(-0.08)	-0.23(-0.38)	
^{100}Mo	-0.08(-0.08)	-0.30(-0.30)	
^{110}Pd	-0.07(-0.07)	-0.27(-0.33)	
^{116}Cd	-0.07(-0.07)	-0.30(-0.30)	
^{124}Sn	-0.12(-0.35)	-0.27(-0.40)	
^{128}Te	-0.12(-0.34)	-0.27(-0.38)	-0.15
^{130}Te	-0.12(-0.34)	-0.27(-0.39)	-0.15
^{136}Xe	-0.11(-0.33)	-0.25(-0.38)	-0.15
^{148}Nd	-0.12(-0.12)		
^{150}Nd	-0.10(-0.10)		
^{154}Sm	-0.09(-0.09)		
^{160}Gd	-0.07(-0.07)		
^{198}Pt	-0.10(-0.10)		

- Considerable reduction obtained!
- Similar prescription has been used for QRPA (Simkovic *et al.*, PRC **87** 045501 (2013))

Ingredient #2: Nuclear Matrix Elements

Full matrix element:

$$M^{(0\nu)} = M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)} + M_T^{(0\nu)}$$



- Comparison of IBM-2, QRPA, ISM, EDF, and PHFB matrix elements for light neutrinos with Argonne/UCOM SRC
- IBM-2/QRPA/ISM similar trend
- IBM-2 NMEs in (surprisingly) good agreement with QRPA NMEs
- The ISM is a factor of (approximately) two smaller than both the IBM-2 and QRPA in the lighter nuclei and the difference is smaller for heavier
  - Effective value of g_A?
- EDF and PHFB: different behavior than IBM-2/QRPA/ISM
  - Treatment of shell effects?

Ingredient #2: Nuclear Matrix Elements

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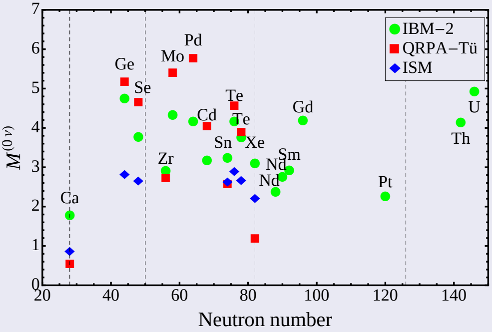

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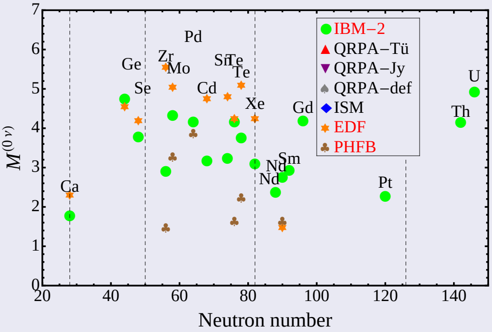

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Ingredient #2: Nuclear Matrix Elements

Estimate of error

- Sensitivity to input parameter changes
  - Single particle energies: 10%
  - Strengths of interactions: 5%
  - Oscillator parameter (SP wave functions): 5%
  - Closure energy in the neutrino potential: 5%
  - Nuclear radius (If NMEs in dimensionless units): 5%
- Sensitivity to model assumptions
  - Truncation to S-D space: 1% (spherical), 10% (deformed)
  - Isospin purity: 2%
  - Special case, ^{48}Ca decay: the sensitivity to model assumptions may be as high as 20% (addition) or 16% (quadrature)
- Sensitivity to operator assumptions
  - Form of the transition operator: 5%
  - Finite nuclear size: 1%
  - Short range correlations (SRC): 5%

- The total error estimate is 16-19% (30% for ^{48}Ca)

Ingredient #2: Nuclear Matrix Elements

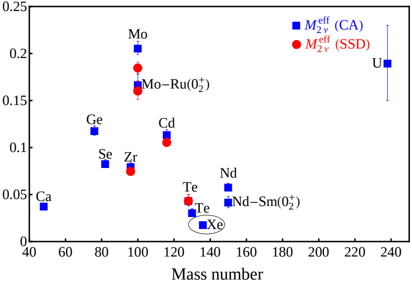
	$M_h^{(0\nu)}$		
Decay	IBM-2	QRPA-Tü	ISM
$^{48}Ca \rightarrow ^{48}Ti$	42.2		47.5
$^{76}Ge \rightarrow ^{76}Se$	97.1	233	138
$^{82}Se \rightarrow ^{82}Kr$	76.6	226	127
$^{96}Zr \rightarrow ^{96}Mo$	98.8		
$^{100}Mo \rightarrow ^{100}Ru$	165	250	
$^{110}Pd \rightarrow ^{110}Cd$	155		
$^{116}Cd \rightarrow ^{116}Sn$	110		
$^{124}Sn \rightarrow ^{124}Te$	75.2		
$^{128}Te \rightarrow ^{128}Xe$	95.8		
$^{130}Te \rightarrow ^{130}Xe$	87.1	234	
$^{136}Xe \rightarrow ^{136}Ba$	68.9		
$^{148}Nd \rightarrow ^{148}Sm$	102		
$^{150}Nd \rightarrow ^{150}Sm$	116		
$^{154}Sm \rightarrow ^{154}Gd$	113		
$^{160}Gd \rightarrow ^{160}Dy$	155		
$^{198}Pt \rightarrow ^{198}Hg$	103		
$^{232}Th \rightarrow ^{232}U$	160		
$^{238}U \rightarrow ^{238}Pu$	189		

- For completenes: Comparison of IBM-2, QRPA, and ISM matrix elements for heavy neutrinos with Argonne/UCOM SRC
  - Other available calculations very limited
  - BUT: IBM-2/QRPA/ISM seem to have similar trend
  - Note: Error estimate in this case is 50% (58% in 48Ca) mostly coming from SRC
    - The neutrino potential for heavy neutrino exchange is a contact interaction in configuration space and thus strongly influenced by SRC

Ingredient #3: effective value of g_A

Maximally quenched value from $2\nu\beta^-\beta^-$ experiments:

Nucleus	$\tau^{2\nu}_{1/2}$ (10^{18} y) exp*
^{48}Ca	44^{+6}_{-5}
^{76}Ge	1500 ± 100
^{82}Se	92 ± 7
^{96}Zr	23 ± 2
^{100}Mo	7.1 ± 0.4
^{100}Mo-^{100}Ru(0^+_2)	590^{+80}_{-60}
^{116}Cd	28 ± 2
^{128}Te	1900000 ± 400000
^{130}Te	680^{+120}_{-110}
^{136}Xe	2110 ± 250
^{150}Nd	8.2 ± 0.9
^{150}Nd-^{150}Sm(0^+_2)	133^{+45}_{-26}
^{238}U	2000 ± 600



- $|M^{eff}_{2\nu}|^2$ is obtained from the measured half-life by

$$|M^{eff}_{2\nu}|^2 = \left[\tau^{2\nu}_{1/2} \times G_{2\nu}\right]^{-1}$$

- Extracted dimensionless quantity

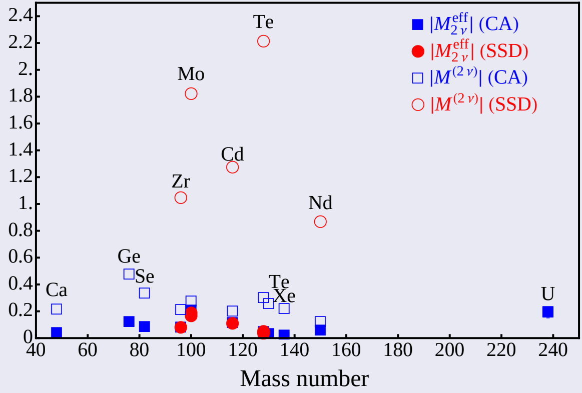
$$|M^{eff}_{2\nu}|^2 = g_A^4 |(m_e c^2) M^{(2\nu)}|^2$$

* A.S. Barabash, Phys. Rev. C **81**, 035501 (2010).

Smallest $M^{eff}_{2\nu}$ for ^{136}Xe, the newest one measured!

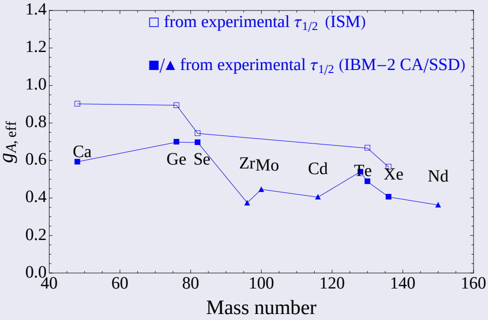
Ingredient #3: effective value of g_A

- Now, if we add to the same figure the theoretical IBM-2 matrix elements $|M^{(2\nu)}| = |\frac{M_{GT}^{(2\nu)}}{\tilde{A}_{GT}} - \left(\frac{g_V}{g_A}\right)^2 \frac{M_F^{(2\nu)}}{\tilde{A}_F}|$ which DO NOT include the factor g_A^2...
- ... but they are still much larger than $M_{2\nu}^{eff}$
- $g_{A,eff} < 1.0$, at least in the case of $2\nu\beta^-\beta^-$!



Ingredient #3: effective value of g_A

- g_A is renormalized in nuclei
- renormalization depends on the size of the model space
  - IBM-2: small model space
  - ISM: large model space
- $g_{A,eff}$ can be extracted comparing $|M_{2\nu}^{eff}|$ and $|M_{2\nu}|$



* ISM NMEs from E. Caurier *et al.*, Int. J. Mod. Phys. E **16**, 552 (2007).

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  - IBM-2: small model space
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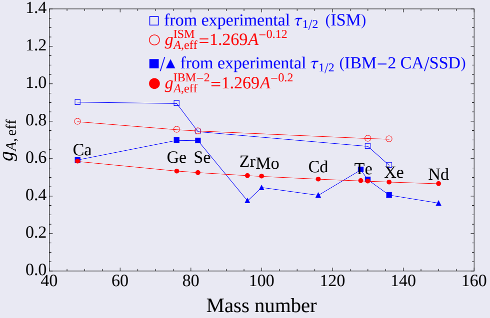
- $g_{A,eff}$ can be extracted comparing $|M_{2\nu}^{eff}|$ and $|M_{2\nu}|$
- Assumption: $g_{A,eff}$ is a smooth function of A



* ISM NMEs from E. Caurier *et al.*, Int. J. Mod. Phys. E **16**, 552 (2007).

- Parametrization:

  $g_{A,eff} = 1.269A^{-\gamma}$
  - IBM-2: $\gamma = 0.2$
  - ISM: $\gamma = 0.12$
- Similar values found for IBM-2 by analyzing β^-/EC, Yoshida and Iachello, PTEP **2013**, 043D01 (2013)
- Similar values found for QRPA by J. Engel *et al.*, PRC **89**, 064308 (2014).

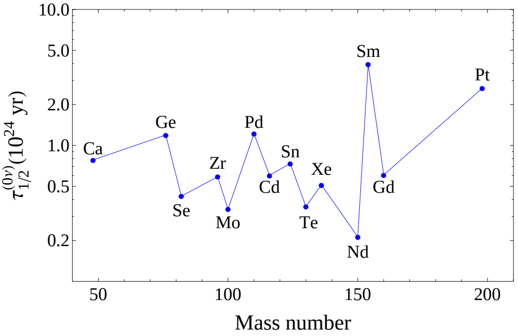
Ingredient #3: effective value of g_A

Effective value of g_A is work in progress, since:

- The closure approximation may not be good for $2\nu\beta^-\beta^-$ and one needs to evaluate explicitly the matrix elements to and from the individual intermediate odd-odd nucleus
  - If this is the case phase space factors can not be exactly separated from the nuclear matrix elements
- Is the renormalization of g_A the same in $2\nu\beta\beta$ as in $0\nu\beta\beta$?
  - If not, how to estimate $g_{A,eff}$?
- Quenching coming from other sources than size of the model space?
- Half-life predictions with maximally quenched g_A are ~ 40 times longer due to the fact that g_A enters the equations to the power of 4!

Half-Life Predictions: $0\nu\beta^-\beta^-$

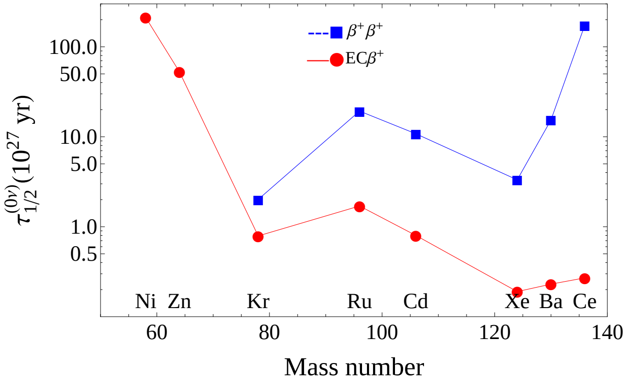
- Keep this in mind but for now predictions are calculated with g_A=1.269 (and $|\langle m_\nu \rangle| = 1$eV)



- Judging by the half-life, best candidates ^{150}Nd, ^{100}Mo, and ^{130}Te, where half-lives $\sim 10^{23}$yr

Half-Life Predictions: $0\nu\beta^+\beta^+$ and $0\nu EC\beta^+$

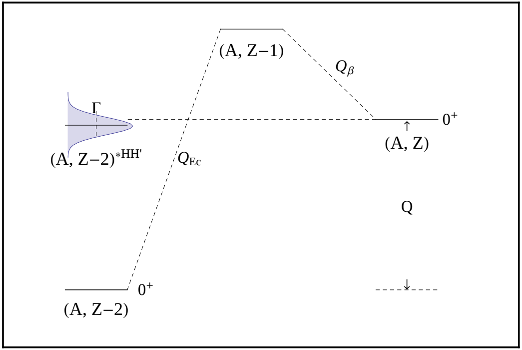
- $\beta^+\beta^+$, $EC\beta^+$ available kinetic energy much smaller $\Rightarrow$ much smaller phase space $\Rightarrow$ much longer half-lives



- Best candidates $0\nu EC\beta^+$ in ^{124}Xe, ^{130}Ba, and ^{136}Ce, where half-lives $\sim 10^{26}$ for g_A=1.269, $|\langle m_\nu\rangle| = 1eV$
  - Compared to $0\nu\beta^-\beta^-$ hardly detectable
  - BUT ^{130}Ba nucleus where $2\nu ECEC$ observed in geochemical experiments

Half-Life Predictions: Resonantly Enhanced $0\nu ECEC$

- $\boldsymbol{0\nu ECEC}$ available energy larger, but since all the energies are fixed, additional requirement that Q-value matches the state energy
- Resonance enhancement:



$$\left[\tau_{1/2}^{ECEC}(0^+)\right]^{-1} = g_A^4 G_{0\nu}^{ECEC} \left|M_{ECEC}^{0\nu}\right|^2 \left|f(m_i, U_{ei})\right|^2 \frac{(m_e c^2)\Gamma}{\Delta^2 + \Gamma^2/4},$$

where $\Delta = |Q - B_{2h} - E|$ is the degeneracy parameter, and Γ is the two-hole width

- Many candidates, such as ^{112}Sn, ^{130}Ba, and ^{136}Ce, ruled out by recent high precision Q-value measurements

Half-Life Predictions: Resonantly Enhanced $0\nu ECEC$

- Best candidates at the moment ^{152}Gd, and ^{180}W

Decay	Δ(keV)	Γ(keV)	$(m_e c^2)$F	$\tau_{1/2}(10^{27})$yr
^{124}Xe	1.86	0.0198	2.92	1520
^{152}Gd	0.91	0.023	14.38	8.03
^{156}Dy	0.54	0.0076	13.52	2890
^{164}Er	6.81	0.0086	0.095	1880
^{180}W	11.24	0.072	0.29	3.44

- For ^{152}Gd, ^{164}Er and ^{180}W decay resonance state is ground state, for ^{156}Dy and ^{124}Xe resonance state is excited 0^+-state $\Rightarrow$ importance of reliable wave functions!
- Half-lives $> 10^{27}$ for $|\langle m_\nu \rangle| = 1eV$ and $g_A = 1.269$
  - Compared to $0\nu\beta^-\beta^-$ hardly detectable

Limits on Average Neutrino Mass

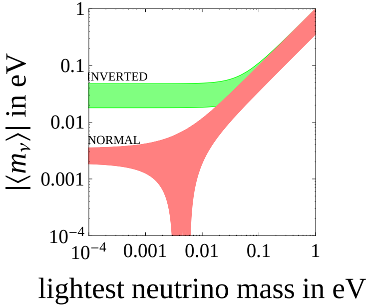
Remember:

$$\left[\tau^{0\nu}_{1/2}\right]^{-1} = G_{0\nu} g_A^4 |M^{(0\nu)}|^2 |f(m_i, U_{ei})|^2$$

- Light neutrinos:

$$f(m_i, U_{ei}) = \frac{\langle m_\nu \rangle}{m_e} = \frac{1}{m_e} \sum_{k=light} (U_{ek})^2 m_k$$

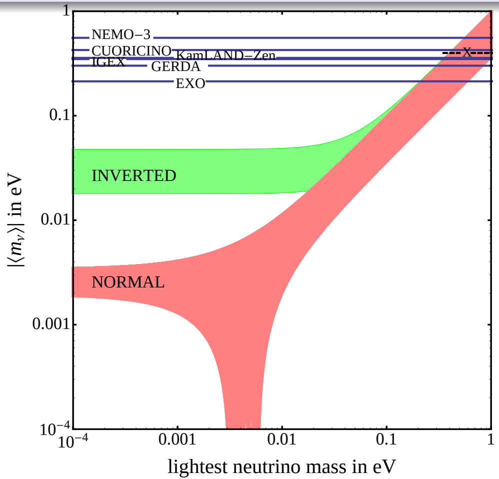
- Advance: The average light neutrino mass is now well constrained by atmospheric, solar, reactor and accelerator neutrino oscillation experiments



- Heavy neutrinos:

$$f(m_i, U_{ei}) = |\eta| = m_p \left\langle m_{\nu_h}^{-1} \right\rangle = m_p \sum_{k=heavy} (U_{ek_h})^2 \frac{1}{m_{k_h}}$$

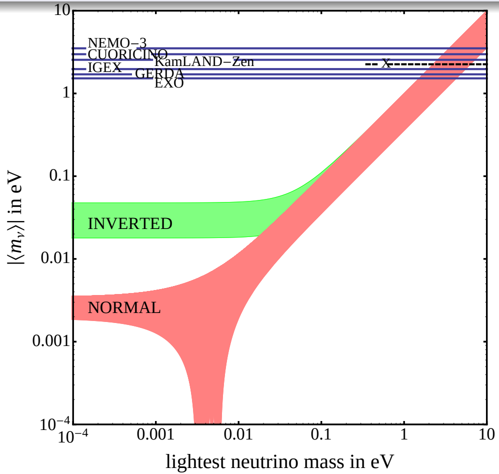
- Current limits to $\langle m_\nu \rangle$ from CUORICINO, IGEX, NEMO-3, KamLAND-Zen, EXO, and GERDA $0\nu\beta\beta$ experiments



IGEX: C. E. Aalseth *et al.*, Phys. Rev. D **65**, 092007 (2002), NEMO-3: R. Arnold, *et al.*, Nucl. Phys. A **765**, 483 (2006), CUORICINO: C. Arnaboldi *et al.*, Phys. Rev. C **78**, 035502 (2008), KamLAND-Zen: A. Gando *et al.*, Phys. Rev. C **85**, 045504 (2012), EXO: M. Auger *et al.*, Phys. Rev. Lett. **109**, 032505 (2012), GERDA: M. Agostini *et al.* (GERDA collaboration) arXiv:1307.4720v1 [nucl-ex] (2013), X: H.V. Klapdor-Kleingrothaus *et al.*, Phys. Lett. B **586**, 198 (2004),

Limits on Average Neutrino Mass

- Same figure as the previous one with maximally quenched g_A. In this case even the inverted mass hierarchy is still far away.



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$0\nu\beta\beta$: Remarks

- The renormalization of g_A is not known for $0\nu\beta\beta$
  - This is very important issue since g_A enters the calculation to the fourth power!
- If both light and heavy neutrino exchange contribute, the half-lives are given by

$$[\tau_{1/2}^{0\nu}]^{-1} = G_{0\nu}^{(0)} \left| M_{0\nu} \frac{\langle m_\nu \rangle}{m_e} + M_{0\nu_h} \eta \right|^2$$

  - There is a possibility of interference between light and heavy neutrino exchange

Conclusions & Outlook

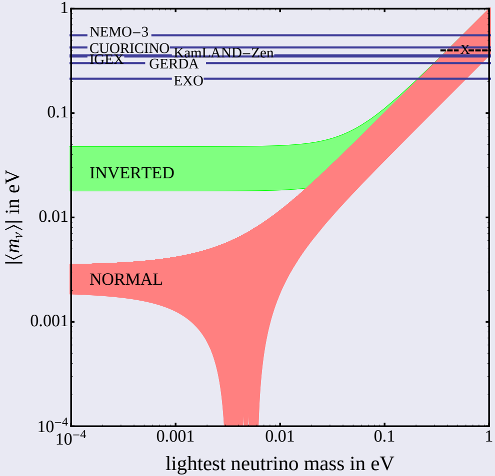
Conclusions

- We have calculated phase space factors and NMEs needed for the description of double beta decay and competing modes and analyzed the results
  - This includes two neutrino modes, as well as the exchange of heavy neutrinos
- Effective value of g_A is work in progress and first results suggest considerable quenching
- Based on our results $0\nu\beta^-\beta^-$ is the likeliest mode to be observed, even if $0\nu ECEC$ is resonantly enhanced

Outlook

- Additional improvements may be included to PSFs if needed (P-wave contribution, finite extent of nuclear surface, etc.)
- Deeper analysis about the similarities and differences of IBM-2 and QRPA results $\Rightarrow$ more reliable NMEs
- Investigation of Majoron emission and right-handed current modes of DBD

THANK YOU!



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