EFFECTIVE FIELD THEORY FOR LATTICE NUCLEI

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LISC, Interdisciplinary Laboratory for Computational Science

International School of Nuclear Physics, 36th Course,
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UNIVERSITY OF TRENTO - Italy

INFN

LISC Laboratorio Interdisciplinare di Scienza Computazionale

Collaboration

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N. Barnea, D. Gazit
J. Kirscher

האוניברסיטה העברית בירושלים
The Hebrew University of Jerusalem

Trento, Italy
L. Contessi

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Orsay, France
U. van Kolck

cnrs IN2P3 Les deux infinis

ANL, USA
A. Lovato

Argonne NATIONAL LABORATORY

PUNCHLINES

- This talk is about nuclei that do not exist neither in the laboratory nor in the cosmos, but only in our minds (and possibly in our computers...).
- We want to understand if it is possible to infer some systematics and some general behaviors of "nuclear physics" in a regime where the pion mass is *unphysical*, but such that *unquenched Lattice QCD simulations are possible*.
- Extrapolation to the physical case is very tricky and is not the main aim of this work at the moment.

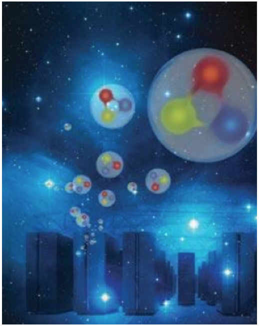

Image from M. Savage website (UW/INT)

LQCD - The Single Baryon Case

Lattice QCD

- QCD is the fundamental theory for nuclear physics.
- It is formulated in terms of quarks and gluons.
- At low energy QCD is non-perturbative $\longrightarrow$ lattice simulations (LQCD).
- Neutron and proton masses are predictions.
- Same for pion masses.

Xui-Lei Ren *et* al., PRD **87** 074001 (2013)
L. Alvarez-Ruso *et* al., ArXiv hep-ph: 1304.0483 (2013)

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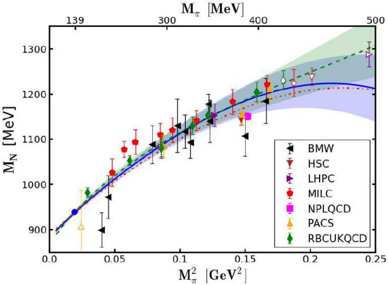

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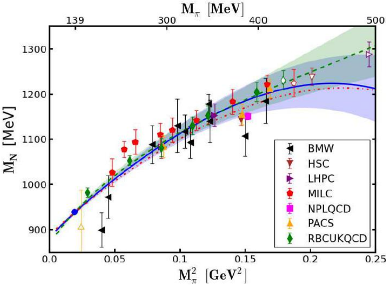

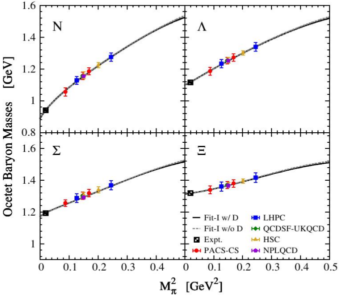

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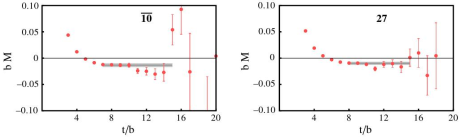
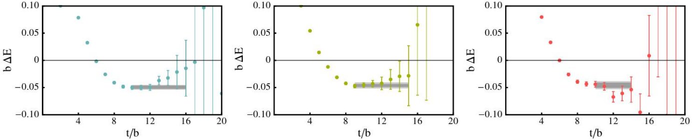
LQCD - Multi Baryon Configurations

Deuteron ($\overline{10}$) and dineutron (27) simulations



Triton simulations with different lattice sizes ($24^3 \times 48, 32^3 \times 48, 48^3 \times 64$)



- LQCD simulations with $SU_f(3)$ symmetry
- Large pion mass $m_{pi} = 800$MeV
- Results with smaller m_π are already available.

NPLQCD Collaboration, PRD **87** 034506 (2013)

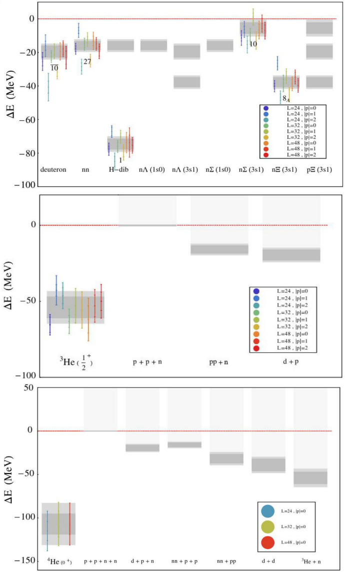
LQCD - Few-Body Baryon Spectra

2-body system - Deutron, dineutron,...

3-body system - ^{3}He, triton

4-body system - ^{4}He

NPLQCD Collaboration, PRD **87** 034506 (2013)



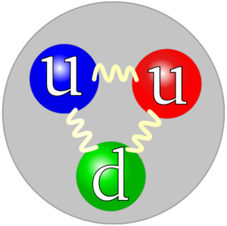
EFT in Nuclear Physics

Effective Field Theory

- At this point LQCD simulations for $A \geq 2$ nuclei are still far away from the physical value of the pion mass.
- Currently no reliable and/or usable NN interactions can be derived from lattice simulations.
- Contemporary nuclear theory is based on Effective Field Theory $\longrightarrow$ phenomenology.
- Quark and Gluon degrees of freedom are replaced by baryons and mesons.

$$\mathcal{L}_{QCD}(q, G) \longrightarrow \mathcal{L}_{Nucl}(N, \pi, \ldots)$$

- The $\mathcal{L}_{Nucl}(N, \pi, \ldots)$ is constructed to retain QCD symmetries.
- $\mathcal{L}_{Nucl}(N, \pi, \ldots)$ is an expansion in low momentum Q.
- Contains all terms compatible with QCD up to a "given order".
- The low-energy coupling constants (LECs) are explicit function of the cutoff Λ.



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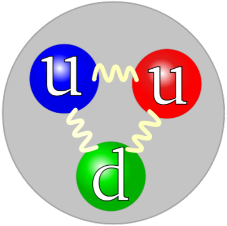

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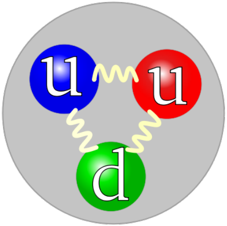

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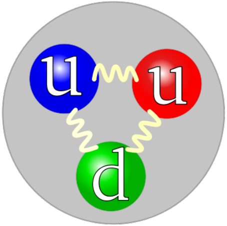

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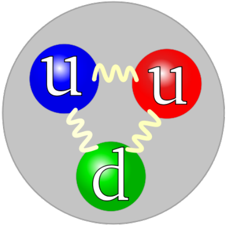

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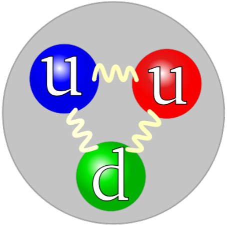

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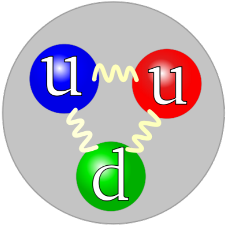

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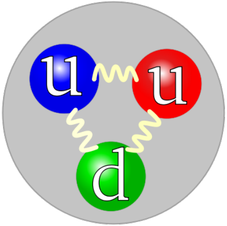


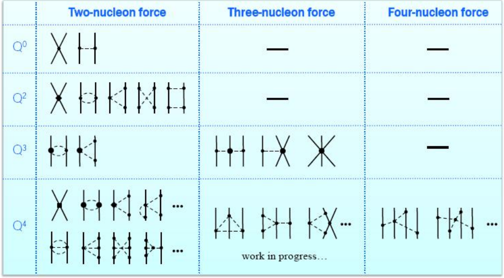
Effective Field Theory potentials

Low Energy Constants

- There are 2 free parameters in LO, 7 at NLO, ...
- NNN and NNNN forces come in naturally at orders N2LO and N3LO.
- The NNN force contains 2 free parameters

χ^2/datum for the reproduction of the 1999 np database

Bin (MeV)	# of data	N^{3}LO	NNLO	NLO	AV18
0–100	1058	1.06	1.71	5.20	0.95
100–190	501	1.08	12.9	49.3	1.10
190–290	843	1.15	19.2	68.3	1.11
0–290	2402	1.10	10.1	36.2	1.04



2 nucleon force ≫ 3 nucleon force ≫ 4 nucleon force ...

$$
\begin{aligned}
V &= -\left(\frac{g_A}{2f_\pi}\right)^2 \frac{(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})}{q^2+m_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \\
&+ C_S + C_T\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 \\
&+ V_{NLO} + V_{N2LO} + \ldots
\end{aligned}
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D. R. Entem and R. Machleidt, PRC **68**, 041001(R) (2003).
Epelbaum *et* al., EPJA **19**, 401 (2004), NPA **747**, 362 (2005).

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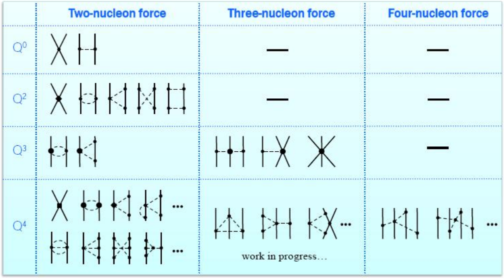

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EFT for Lattice Nuclei

Energy Scales

- Nucleon mass M_n, and the difference with the mass of the Δ baryon $\delta M = M_\Delta - M_n$
- The pion mass m_π, pion exchange momentum $q_\pi = m_\pi/\hbar c$, and energy

$$E_\pi = \frac{\hbar^2 q_\pi^2}{M_n} = \frac{m_\pi}{M_n} m_\pi$$

- Nuclear binding energy B/A

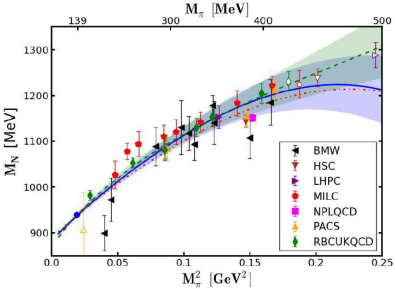
Scale	Nature	LQCD@m_π=500MeV	LQCD@m_π=800MeV
M_n	940 MeV	1300 MeV	1600 MeV
δM	300 MeV	300 MeV	180 MeV
m_π	140 MeV	500 MeV	800 MeV
E_π	20 MeV	200 MeV	400 MeV
B/A	10 MeV	15 Mev	25 MeV

Conclusions

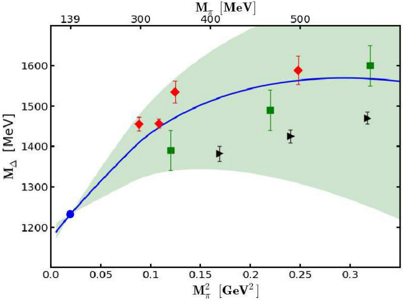
- For the nature case $\mathcal{L} \longrightarrow \mathcal{L}_{EFT}(N, \pi)$
- For lattice nuclei at $m_\pi \geq 400$MeV, $E_\pi \gg B/A$
- In this case $\not\pi$EFT is the natural theory $\mathcal{L} \longrightarrow \mathcal{L}_{EFT}(N)$

The nucleon Δ mass difference

Nucleon mass - n,p



Δ mass

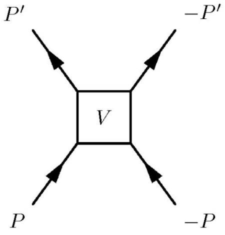


L. Alvarez-Ruso *et* al., ArXiv hep-ph: 1304.0483 (2013)

$\pi\!\!\!/$EFT for Lattice Nuclei

- We write all possible terms in the Lagrangian $\mathcal{L}$ ordered by the *number of derivatives*:

$$\begin{aligned}\mathcal{L} \quad = \quad & N^\dagger\left(i\partial_0 + \frac{\vec{\nabla}}{2M}\right)N - a_1 N^\dagger N N^\dagger N - a_2 N^\dagger \boldsymbol{\sigma} N \cdot N^\dagger \boldsymbol{\sigma} N \\ & -a_3 N^\dagger \boldsymbol{\tau} N \cdot N^\dagger \boldsymbol{\tau} N - a_4 N^\dagger \boldsymbol{\sigma}\boldsymbol{\tau} N \cdot N^\dagger \boldsymbol{\sigma}\boldsymbol{\tau} N + \ldots \\ & -d_1 N^\dagger \boldsymbol{\tau} N \cdot N^\dagger \boldsymbol{\tau} N N^\dagger N\end{aligned}$$



- Higher order terms include more derivatives.
- Naively, the order goes as the number of derivatives.
- The 3-body term appears at LO to avoid the Thomas collapse.
- Due to Fermi symmetry the number of terms can be cut by half.
- The coefficients depend on the cutoff Λ.

Some further wishes (to be explained later)

- The potential needs to be local.
- Avoid 3-body spin-isospin operators.

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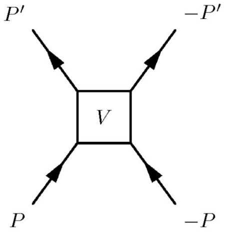

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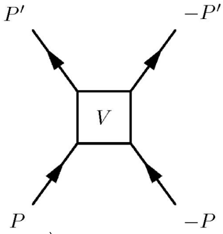
$\not\pi$EFT Potential at NLO



- At LO the $\not\pi$EFT potential takes the form

$$V_{LO}^{2b} = a_1 + a_2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + a_3\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + a_4 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$$

- The leading order also contains a 3-body term of the form

$$V_{LO}^{3b} = D_1 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

- At NLO the $\not\pi$EFT potential takes the form

$$\begin{aligned} V_{NLO}^{2b} \;=\;& b_1 q^2 + b_2 q^2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + b_3 q^2\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + b_4 q^2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \\ +\;& b_5 k^2 + b_6 k^2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + b_7 k^2\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + b_8 k^2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \\ +\;& b_9 i \frac{1}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\boldsymbol{k} \times \boldsymbol{q}) + b_{10}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 i \frac{1}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\boldsymbol{k} \times \boldsymbol{q}) \\ +\;& b_{11} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}) + b_{12}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}) \\ +\;& b_{13} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{k})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{k}) + b_{14}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{k})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{k}) \end{aligned}$$

- The incoming particle have relative momentum $\boldsymbol{p}$, the outgoing $\boldsymbol{p}'$.
- The momentum transfer $\boldsymbol{q} = \boldsymbol{p}' - \boldsymbol{p}$, and $\boldsymbol{k} = (\boldsymbol{p}' + \boldsymbol{p})$
- Nonlocalities are associated with $\boldsymbol{k}$.
- The power counting changes for a shallow s-wave dimer.

J. Kirscher, H. W. Griesshammer, D. Shukla, H. M. Hofman, arXiv: 0903.5583

A. Gezerlis, I. Tews, E. Epelbaum, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, PRL **111**, 032501 (2013).

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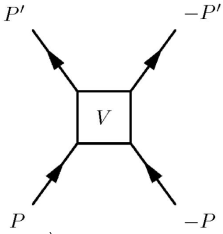

$$\begin{aligned} V_{NLO}^{2b} \quad = \quad & b_1 q^2 + b_2 q^2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + b_3 q^2\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + b_4 q^2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \\ + \quad & b_5 k^2 + b_6 k^2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + b_7 k^2\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + b_8 k^2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \\ + \quad & b_9 i \frac{1}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\boldsymbol{k} \times \boldsymbol{q}) + b_{10}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 i \frac{1}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\boldsymbol{k} \times \boldsymbol{q}) \\ + \quad & b_{11} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}) + b_{12}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}) \\ + \quad & b_{13} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{k})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{k}) + b_{14}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{k})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{k}) \end{aligned}$$

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- Nonlocalities are associated with $\boldsymbol{k}$.
- The power counting changes for a shallow s-wave dimer.

J. Kirscher, H. W. Griesshammer, D. Shukla, H. M. Hofman, arXiv: 0903.5583

A. Gezerlis, I. Tews, E. Epelbaum, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, PRL **111**, 032501 (2013).

$\not\pi$EFT Potential at NLO

- At LO the $\not\pi$EFT potential takes the form

$$V_{LO}^{2b} = a_1 + a_2\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + a_3\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + a_4 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$$

- The leading order also contains a 3-body term of the form

$$V_{LO}^{3b} = D_1 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

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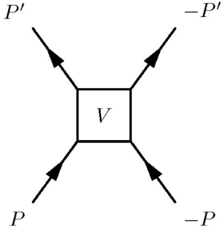

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- Due to antisymmetrization of the nuclear wave function

$$\boxed{V_{LO}^{2b} = C_1^{LO} + C_2^{LO}\,\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2}$$

- The leading order also contains a 3-body term of the form

$$\boxed{V_{LO}^{3b} = D_1^{LO}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}$$

- Using the freedom to choose these parameters we set

$$\begin{aligned}
V_{NLO}^{2b} =\ & C_1^{NLO}q^2 + C_2^{NLO}q^2\,\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + C_3^{NLO}q^2\,\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + C_4^{NLO}q^2(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \\
& + C_5^{NLO} i\frac{1}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\boldsymbol{k} \times \boldsymbol{q}) + C_6^{NLO}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}) \\
& + C_7^{NLO}\,\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2(\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q})
\end{aligned}$$

- The antisymmetric potential V_{NLO} contains
  1. LO 2-body: 2 parameters.
  2. LO 3-body: 1 parameter.
  3. NLO: 7 parameters.
- At the moment we consider only LO.

Coordinate space

- We introduce a Gaussian cutoff in $\boldsymbol{q}$

$$F_\Lambda(q) = \left(\frac{\sqrt{4\pi}}{\Lambda}\right)^3 e^{-q^2/\Lambda^2} \Longrightarrow F_\Lambda(r) = e^{-\Lambda^2 r^2/4}$$

- The potential matrix elements can be evaluated now

$$\begin{aligned} V(\boldsymbol{r},\boldsymbol{r}') &= N\langle \boldsymbol{r}| \int d\boldsymbol{k} d\boldsymbol{q} V(\boldsymbol{k},\boldsymbol{q}) f_\Lambda(\boldsymbol{q}) |\boldsymbol{r}'\rangle \\ &= N'\, V(-i\nabla_y, -i\nabla_x)\, e^{-\Lambda^2 x^2/4} \delta(\boldsymbol{y}) \end{aligned}$$

where

$$\boldsymbol{x} = \frac{1}{2}(\boldsymbol{r}+\boldsymbol{r}') \quad ; \quad \boldsymbol{y} = \frac{1}{2}(\boldsymbol{r}'-\boldsymbol{r})$$

- The LO potential contains no momentum dependence therefore

$$\boxed{V_{LO}^{2b}(r) = \left(C_1^{LO} + C_2^{LO}\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\right) e^{-\Lambda^2 r^2/4}}$$

- With our choice of parameterization also V_{NLO} is local.
- The 3-body term takes the form

$$\boxed{V_{LO}^{3b} = D_1^{LO} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 e^{-\Lambda^2 (r_{13}^2 + r_{23}^2)}}$$

The Hamiltonian

At leading order the coordinate space Hamiltonian is

$$\begin{aligned} H &= -\sum_i \frac{\hbar^2}{2M_n}\nabla_i^2 + \sum_{i<j}\left(C_1^{LO}(\Lambda) + C_2^{LO}(\Lambda)\,\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right) e^{-\Lambda^2 r_{ij}^2} \\ &+ \sum_{i<j<k}\sum_{cyc} D_1^{LO}(\Lambda)\left(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\right) e^{-\Lambda^2 (r_{ik}^2 + r_{jk}^2)} \end{aligned}$$

In the 3-body term the notation $\sum_{cyc}$ stands for cyclic permutation of particles (ijk).

We use four methods

1. Numerov, $A = 2$
2. The Effective Interaction Hypershperical Harmonics (EIHH) method, $6 \geq A \geq 3$
3. (Refined) Resonating Group Model (RRGM) for $N \leq 12$
4. The Auxiliary Field Diffusion Monte-Carlo (AFDMC) method, $A \geq 2$

Auxiliary Field Diffusion Monte Carlo

- In order to extend the prediction to heavier mass nuclei we employ a QMC algorithm, namely AFDMC.
- The Fermion ground state of the Hamiltonian is computed by means of imaginary time projection with constraints to avoid the sign problem.

$$\langle R, S | 0_A \rangle \sim \lim_{\tau \to \infty} \langle R, S | e^{-(\tilde{H} - E_0)\tau} | R' S' \rangle \langle R' S' | \Psi \rangle$$

- A population of points $|R, S\rangle$ in the coordinate/spin space evolved in imaginary time.
- $\tilde{H}$ is a modified Hamiltonian defined on a set of artificial boundary conditions imposed to obtain a finite overlap with an antisymmetric function.

Auxiliary Field Diffusion Monte Carlo

- The propagator contains operators that span a multicomponent wavefunction with order 4^A components.
- This is the approach used in GFMC calculations.
- In AFDMC we make use of a basis set in which the single particle wavefunctions take the form

$$\phi(\boldsymbol{r}_i, s_i) = \begin{pmatrix} \alpha_i \\ \beta_i \end{pmatrix} \varphi_n(\boldsymbol{r}_i)$$

- Linearization of the operators is achieved through the Hubbard-Stratonovitch transform.
- As an example, the propagator corresponding to $\hat{V} = v_{\sigma \cdot \sigma}(r_{12}) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$ is:

$$e^{-\hat{V}\Delta\tau} = \frac{1}{\sqrt{\pi}} \int dx_1\, e^{-\frac{x_1^2}{2}} e^{-x_1 \sqrt{-\Delta\tau v_{\sigma \cdot \sigma}(r_{12})} \sigma_1} \times \\ \times \int dx_2 e^{-\frac{x_2^2}{2}} e^{-x_2 \sqrt{-\Delta\tau v_{\sigma \cdot \sigma}(r_{12})} \sigma_2}$$

Auxiliary Field Diffusion Monte Carlo

PRO:

- Very favorable scaling (A^3 instead of 4^A)
- No problem in computing nuclei up to $A = 90$

CON:

- Limited freedom on the choice of the trial functions $\longrightarrow$ some possible ambiguities in the results.
- No upper bound property
- No easy way to include spin-orbit and three body forces in the propagator for finite nuclei (no problems in pure neutron systems instead)

The "Experimental" data

There are 4 input parameters in our model:

1. The nucleon mass M_n
2. LECs: $C_1^{LO}(\Lambda),\ C_2^{LO}(\Lambda),\ D_1^{LO}(\Lambda)$

Table : Masses [MeV]

particle	LQCD	Nature
π	805.9 $\pm$ 8.9	135.0-139.6
n	1634.0 $\pm$ 18.0	939.6
p	1634.0 $\pm$ 18.0	938.3

Table : Binding energies [MeV]

nuclei	LQCD	Nature
D	-19.5 $\pm$ 4.8	-2.224
nn	-15.9 $\pm$ 3.8	-
^{3}H	-53.9 $\pm$ 10.7	-8.48
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NPLQCD Collaboration, PRD **87** 034506 (2013)

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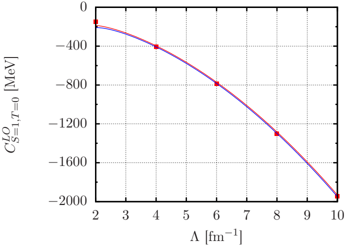
The 2-body LECs

- The 2-body potential is diagonal in the S, T basis.

$$V^{LO}_{S,T}(r;\Lambda) = \langle S,T|V^{LO}(r;\Lambda)|S,T\rangle$$
$$= \left\{C^{LO}_1(\Lambda) + [2S(S+1)-3]C^{LO}_2(\Lambda)\right\} F_\Lambda(r)$$
$$\equiv C^{LO}_{ST}(\Lambda) F_\Lambda(r)$$



- $C^{LO}_{ST}(\Lambda)$ are fitted to the D, nn B.E.
- We expect to get

$$a_s \approx 1/\sqrt{m_N B}$$

- For LQCD@$m_\pi = 800MeV$

$$a^{LO}_{S=0,T=1} = 1.4 \pm 0.20 \text{ fm}$$
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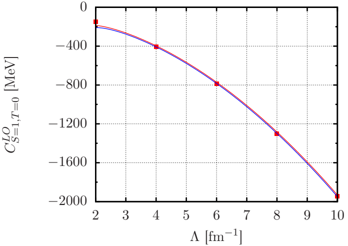

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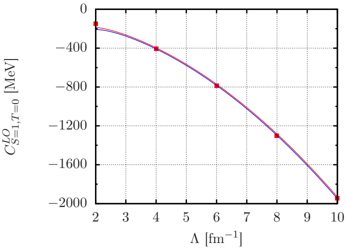

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Calibration of D_1

^{4}He binding energy without NNN force

Λ [fm^{-1}]	EIHH [MeV]	AFDMC [MeV]
2.0	-256.8	-256.9
4.0	-478.3	-478.2
6.0	-767.1	-766.4
8.0	-1122.9	-1120.8
LQCD	-107.0±24.2	

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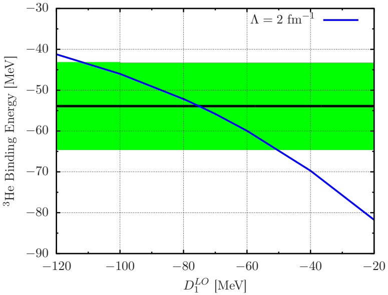

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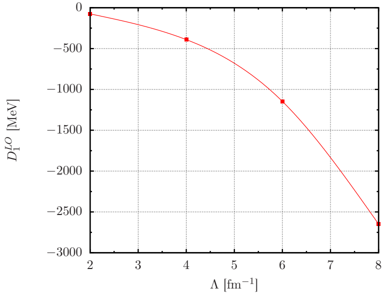

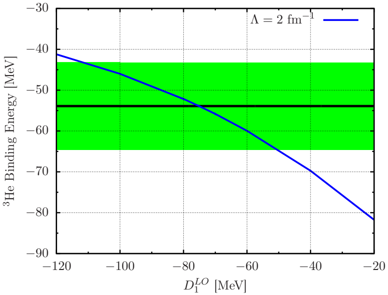

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Few Predictions

The binding energy of ^{4}He

Λ [fm^{-1}]	EIHH [MeV]	AFDMC [MeV]
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4.0	-93.6(1)	-93.3(2)
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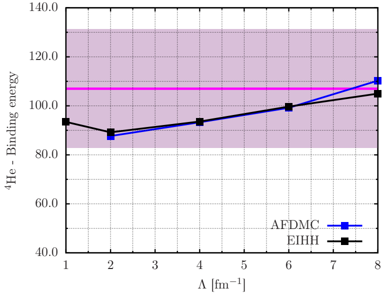

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- The radii of D, ^{3}He, ^{4}He exhibit strong cutoff dependence.
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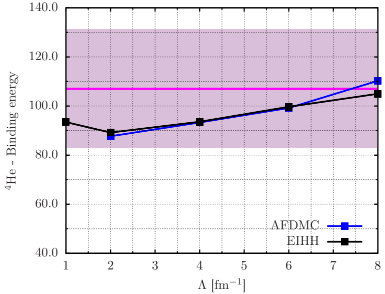
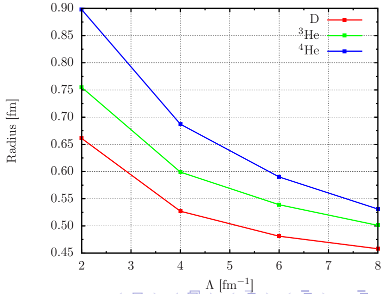
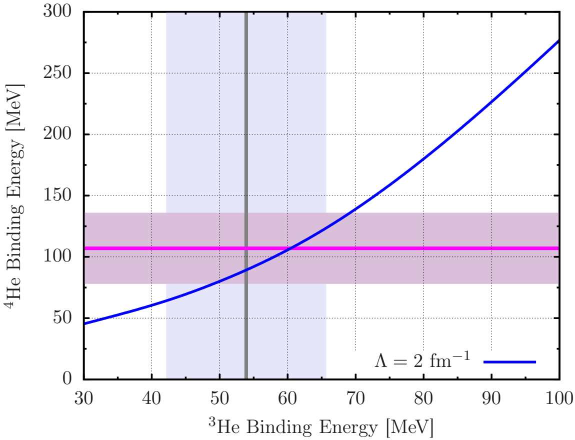




The Tjon line(?)



Conclusion

The EFT error due to uncertainty in the binding energy of ^{3}He is roughly as large as the ^{4}He measurment error.

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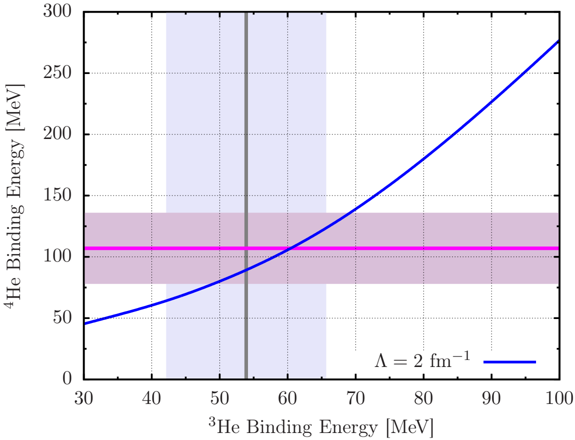

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Few more predictions

Binding energies of the Light nuclei

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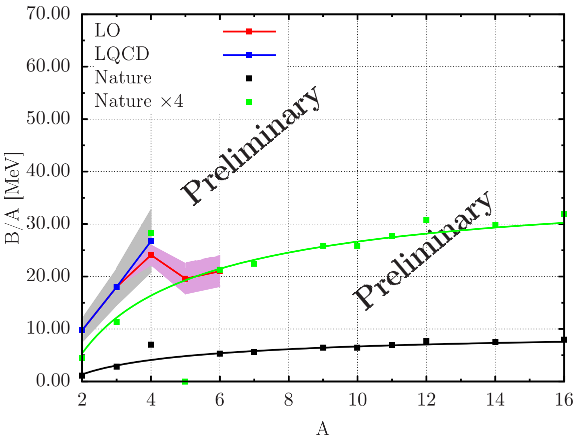
^{3}n, ^{3}p		unbound
^{4}He	-107.0 $\pm$ 24.2	-89.2
^{4}He $J^\pi = 2^+$		-66 (?)
^{5}He		-98.2
^{6}Li		-121.(3)

Saturation Energy on the Lattice



Even more predictions...

We were able to perform calculations of ^{16}O. AFDMC calculations are non-trivial, especially at large Λ due to the large cancelations in the attractive (2-body) and repulsive (3-body) components in the potential.

Table : Binding energies of ^{16}O from FDMC calculations (MeV)

	Λ (fm^{-1})			
m_π (MeV)	2	4	6	8
805	346.6(3)	334.9(6)	324(1)	305(1)
510	115.5(4)	108.0(4)	105.8(8)	100(1)
140	61.59(2)	56.9(6)	48(1)	45(2)

Comments

- The three-body force used here has no isospin dependence (should be equivalent, but there might be some redundance).
- As it stands, ^{16}O is unbound with respect to breakup in 4(^{4}He). However, at $m_\pi = 805$MeV the difference $BE(^4\text{He}) - BE(^{16}\text{O})/4$ can be as small as 1.4MeV.
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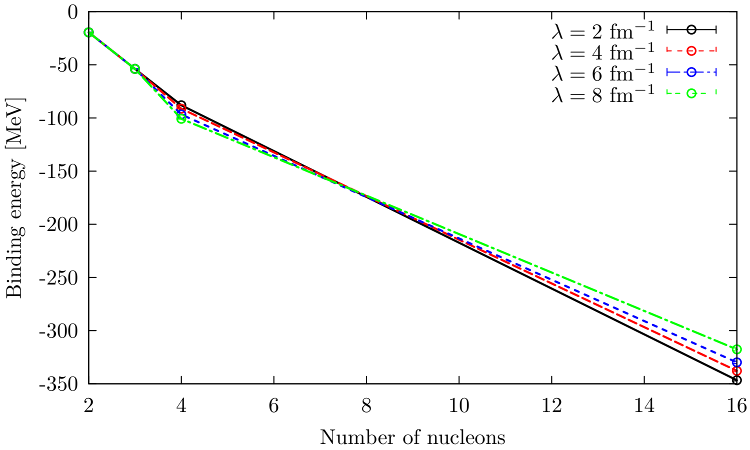
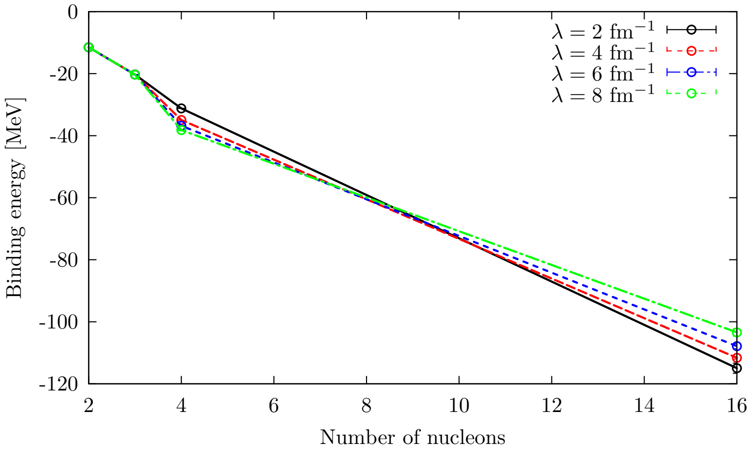
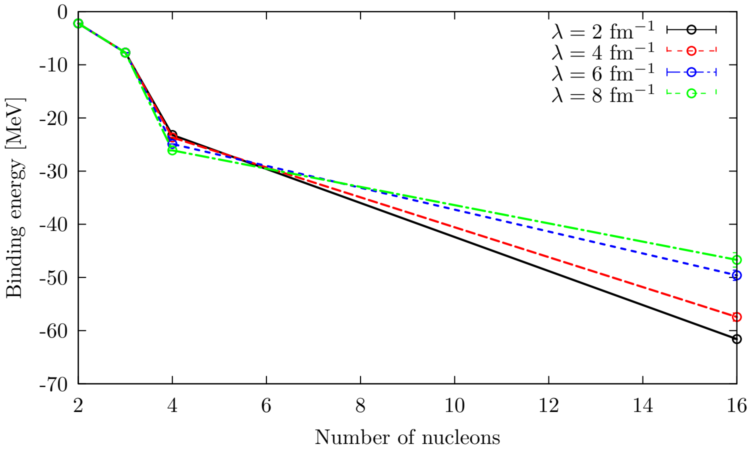
Summary of AFDMC results for m_π=805 MeV



Summary of AFDMC results for m_π=510 MeV



Summary of AFDMC results for m_π=140 MeV



Summary and Conclusions

1. Lattice QCD simulations of few-nucleon systems open up a new front in nuclear physics.
2. $\not{\pi}$EFT is the appropriate theory to study these Lattice Nuclei, down to rather small pion masses.
3. Fitted to recent LQCD data we found that $\not{\pi}$EFT@LO reproduces the ^{4}He binding energy for $m_\pi = 805$MeV and $m_\pi = 510$MeV nucleons.
4. The np, nn scattering lengths for this system are about a factor of 2 too small in comparison with recent NPLQCD calculations.
5. At LO we see probelms with the calculated nuclear radii.
6. More LQCD data are avilable to go behyond LO.

Ongoing work

- Begin the journey to include NLO in calculations.
- More LQCD data at different m_π to understand the systematics of some properties (e.g. evolution of the LEC's, BEs, saturation,...)
- Larger ($A = 40$) calculations, and better wavefunctions in AFDMC (e.g. from RRG to check the cluster structyre in ^{16}O)
- Redo the calculations using a recently developed QMC algorithm in momentum space: systematic comparison of non-local and local formulation (effect of regulators?) as a function of the mass.