Universal aspects halo nuclei

Tobias Frederico
Instituto Tecnológico de Aeronáutica
São José dos Campos – Brazil
tobias@ita.br

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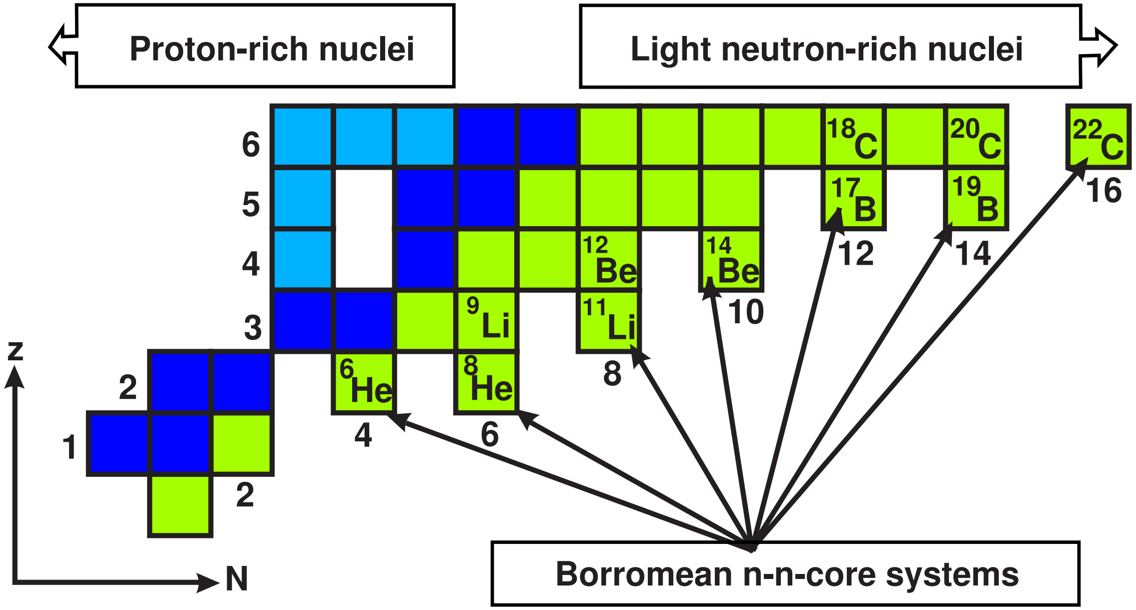
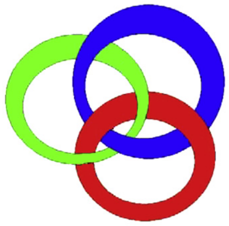
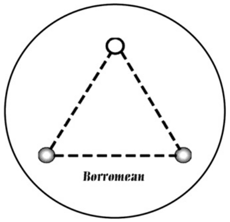
Light-neutron rich nuclei




Borromean rings



Two-body unbound -----
Three-body bound

^{11}Li, ^{14}Be, ^{20}C, ^{22}C

C.A. Bertulani, Nuclear Physics in a Nutshell, Princeton University Press, 2007.

TF, Delfino, Tomio, Yamashita, “Universal aspects of light halo nuclei
Prog. Part. Nucl. Phys. 67 (2012) 939”

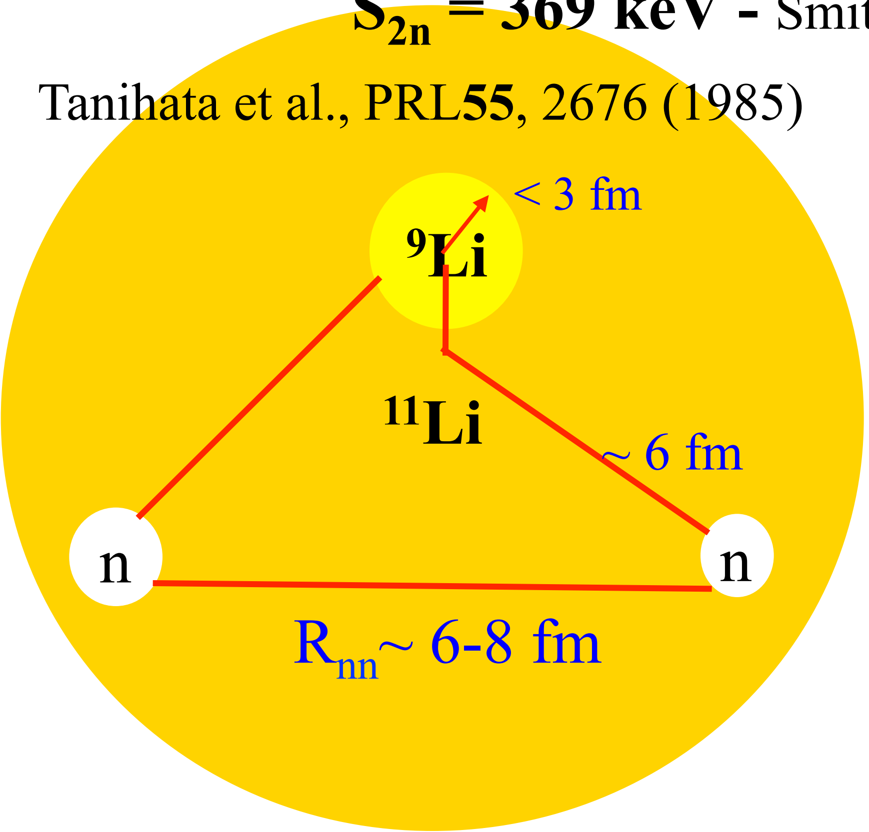
Tanihata,Savajols Kanungo. “Recent experimental progress in nuclear halo
structure studies Prog. Part. Nucl. Phys. 68 (2012) 215”

Zinner, Jensen. ”Comparing and contrasting nuclei and cold atomic gases”.
J. Phys. G: Nucl. Part. Phys. 40 (2013) 053101

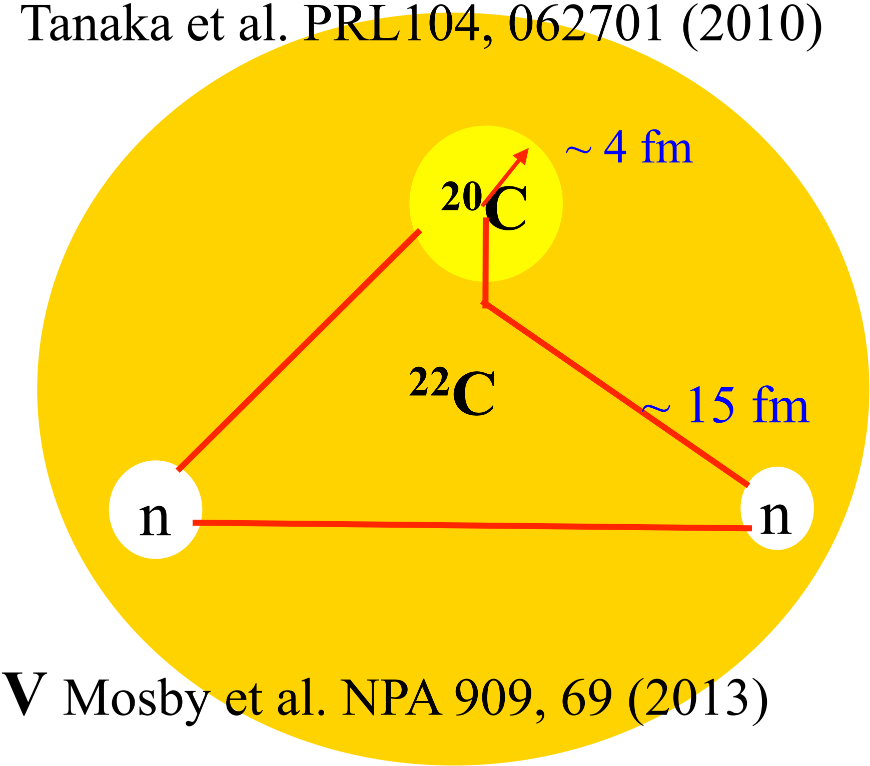
Two-neutron weakly bound s-wave three-body halo nuclei

S_{2n} = 369 keV - Smith et al. PRL101, 202501 (2008)

Tanihata et al., PRL**55**, 2676 (1985)



Tanaka et al. PRL104, 062701 (2010)



S_{2n} < 70 keV Mosby et al. NPA 909, 69 (2013)

Weakly bound quantum systems

$$(E - H_0)\psi = 0$$

- Almost everywhere the wf is an eigenstate of H_0 - short-range force
- Physics: symmetry, scales and dimension (& mass ratios)

→ Universality (model independence)

Generalization: “The few scales of nuclei and nuclear matter”
Delfino, TF, Timóteo, Tomio. PLB 634 (2006) 185

Configuration space two-neutron halo wave function (2n spin singlet)

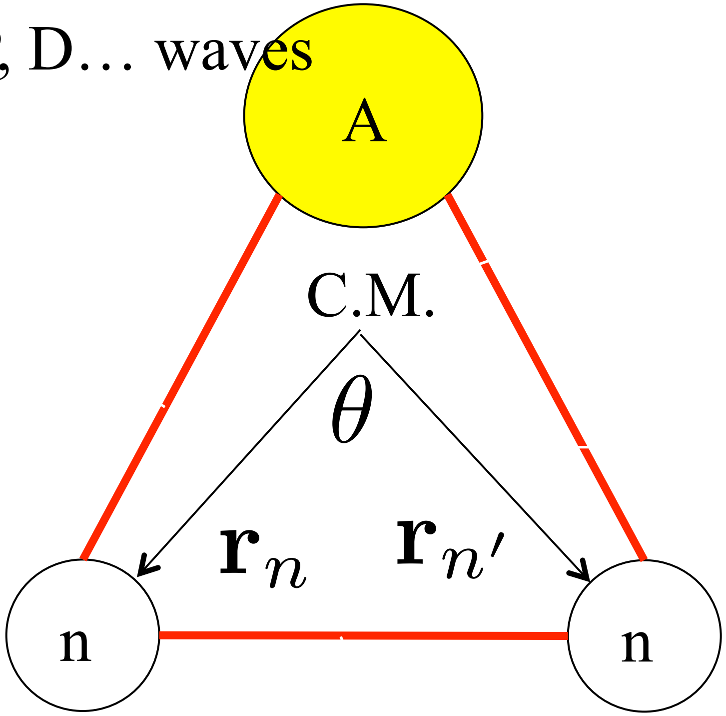
$$H\Psi = \left[-\sum_{i=1}^{3} \frac{\hbar^2}{2m_i} \nabla_i^2 + \lambda_{jk}\delta(\mathbf{R}_{jk}) \right] \Psi = -S_{2n}\Psi \quad \text{(C.M.)}$$

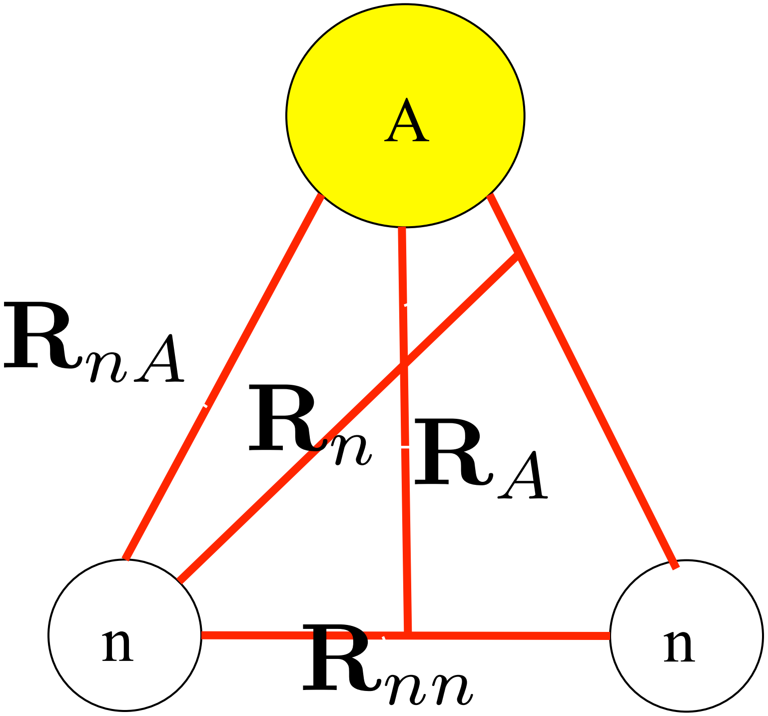
$$\Psi(\mathbf{r}_n, \mathbf{r}_{n'}) = \int d\mathbf{q} \, \frac{e^{-\kappa_{nn}\,|\mathbf{R}_{nn}|}}{|\mathbf{R}_{nn}|} \, e^{\imath \mathbf{q}\cdot\mathbf{R}_A} \chi_A(\mathbf{q}) + \int d\mathbf{q} \, \frac{e^{-\kappa_{nA}\,|\mathbf{R}_{nA}|}}{|\mathbf{R}_{nA}|} \, e^{\imath \mathbf{q}\cdot\mathbf{R}_n} \chi_n(\mathbf{q}) + \cdots$$

$$\Psi(|\mathbf{r}_n|, |\mathbf{r}_{n'}|, \cos\theta)$$

$$\kappa_{nn} = \sqrt{2\mu_{nn}\left(S_{2n} + \frac{q^2}{2\mu_A}\right)} \text{ and } \kappa_{nA} = \sqrt{2\mu_{nA}\left(S_{2n} + \frac{q^2}{2\mu_n}\right)}$$

S, P, D… waves





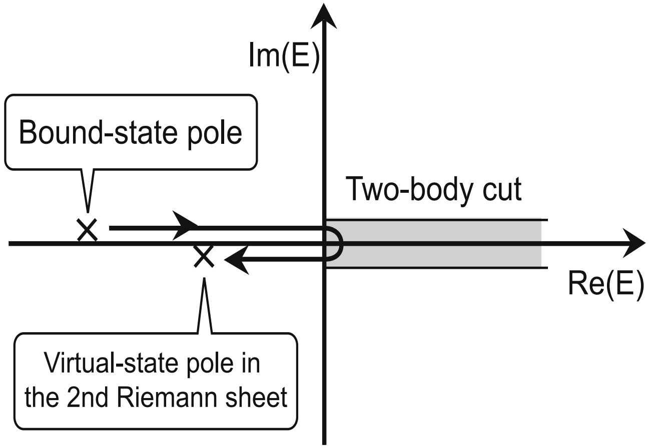
Two-body s-wave phase-shift (large scatt. lenghts)

$$k\cot(\delta) = -\frac{1}{a} + \frac{r_0}{2}k^2 + \cdots$$

$$|a| >> r_0$$

$a > 0$

$a < 0$



- $^1S_0$ nn state $E_{virtual}$= -143 keV (a = -17fm)
- S-wave n-core state: virtual (^{10}Li ~ -25 keV) or bound (^{19}C ~ 500 keV)

Three-boson system

Subtle three-body phenomenum in L=0:

Thomas collapse (1935)	**Efimov effect (1970)**
$r_o \rightarrow 0$	$\lvert a \rvert \rightarrow \infty$
Route to collapse?	infinitely many bound states condensing at E=0

Thomas-Efimov effect!

$\lvert a \rvert / r_o \rightarrow \infty$

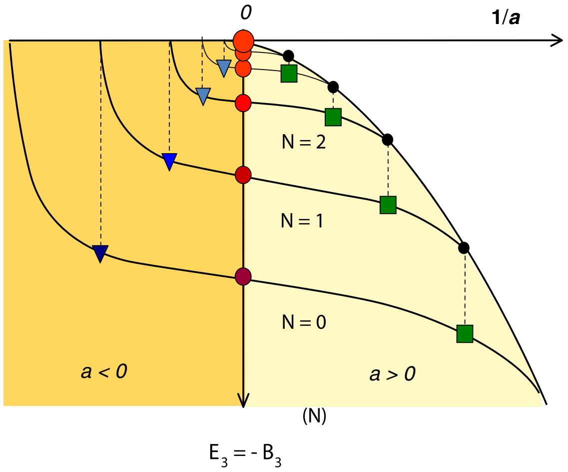
Adhikari, Delfino,TF,Goldman,Tomio, PRA37 (1988) 3666

One three-body scale is necessary to represent short-range physics !!!!
& discrete scaling

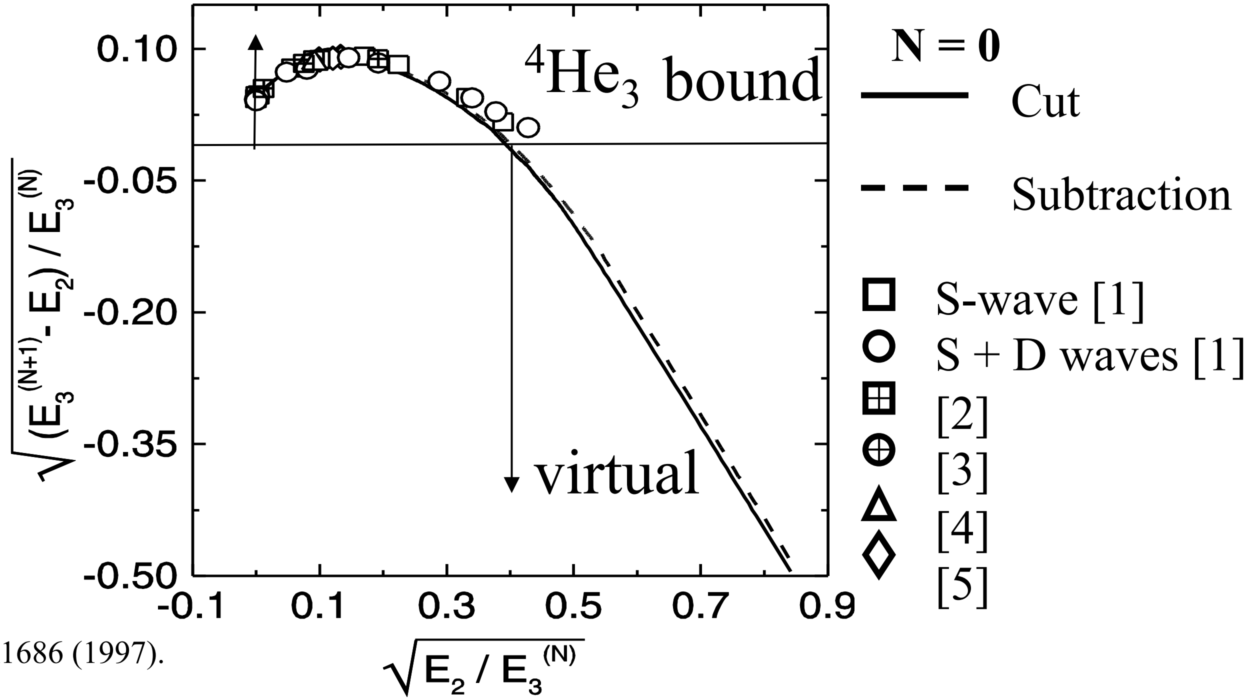
Jensen, Riisager, Fedorov, Garrido, RMP76, 215 (2004)
Braaten, Hammer Phys. Rep.428, 259 (2006)

Efimov States – Bound and virtual states (3 identical bosons)

Correlations between observables: Jensen, Fedorov, Yamashita, Hammer, Platter, Gattobigio, Kievsky, Kolganova, Van Kolck, Bedaque, Phillips,...



Scaling plot from zero range force



[1] Cornelius, Glöckle. *JCP* **85**, 1 (1996).
[2] Huber. *PR***A31**, 3981 (1985).
[3] Barletta, Kievsky. *PR* **A64**, 042514 (2001).
[4] Fedorov, Jensen. *JP***A34**, 6003 (2001).
[5] Kolganova, Motovilov, Sofianos. PRA **56**, R1686 (1997).

- *Scaling limit:* T. Frederico, LT, A. Delfino and E. A. Amorim, PRA**60**, R9 (1999)
- *Limit cycle*: Mohr et al Ann.Phys. 321 (2006)225
- *Correlation between observables: Phillips Plot* $^2a_{nd}$ v.s. E_{triton}

Range correction: Thogersen, Fedorov, Jensen PRA78(2008)020501(R)

For 2n+core (one 3B short-range scale)

Halo Nuclei and Efimov physics (n+n+core)

Fedorov, Jensen, Riisager, “Efimov states in halo nuclei” PRL73 (1994) 2817. ^{14}Be ^{18}C ^{20}C

Mazumdar, Bhasin, “Efimov effect in the nuclear halo 14Be nucleus” PRC 56 (1997) R5

Amorim, TF, Tomio "Universal aspects of Efimov states and light halo nuclei", PRC 56, R2378 (1997)

Mazumdar, Arora, Bhasin, ”Three-body analysis of the occurrence of Efimov states in 2n halo nuclei such as ^{19}B, ^{22}C, and ^{20}C”, PRC61 (2000) 051303

Halo Nuclei and EFT

Bertulani, Hammer, van Kolck, "Effective field theory for halo nuclei: shallow p-wave states", NPA712 (2002) 37

Halo Nuclei, EFT and Efimov physics

Hammer, Platter, ”Efimov States in Nuclear and Particle Physics”, Annu. Rev. Nucl. Part. Sci. 60 (2010) 207

[redacted]

PHYSICAL REVIEW C **81**, 021302(R) (2010)

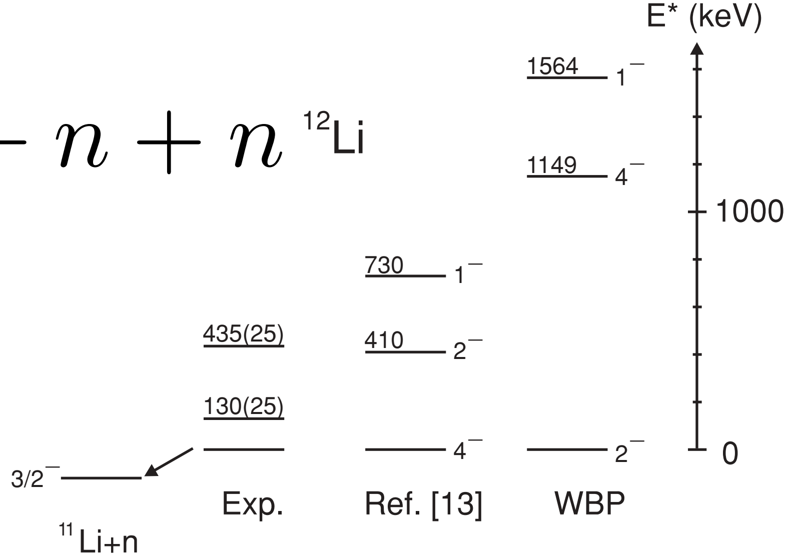
First observation of excited states in ^{12}Li

(n+n+n+core)

C. C. Hall,[1] E. M. Lunderberg,[1] P. A. DeYoung,[1,*] T. Baumann,[2] D. Bazin,[2] G. Blanchon,[3] A. Bonaccorso,[4] B. A. Brown,[2,5]
J. Brown,[6] G. Christian,[2,5] D. H. Denby,[1] J. Finck,[7] N. Frank,[2,5,†] A. Gade,[2,5] J. Hinnefeld,[8] C. R. Hoffman,[9,10] B. Luther,[11]
S. Mosby,[2,5] W. A. Peters,[2,5,‡] A. Spyrou,[2,5] and M. Thoennessen[2,5]

The neutron-unbound ground state and two excited states of ^{12}Li were formed by the two-proton removal reaction from a 53.4-MeV/u ^{14}B beam. The decay energy spectrum of ^{12}Li was measured with the Modular Neutron Array (MoNA) and the Sweeper dipole superconducting magnet at the National Superconducting Cyclotron Laboratory. Two excited states at resonance energies of 250 ± 20 keV and 555 ± 20 keV were observed for the first time and the data are consistent with the previously reported s-wave ground state with a scattering length of $a_s = -13.7$ fm.

$$^{12}Li = ^{9}Li + n + n + n$$



Four-boson scale with s-wave zero-range potential:
Hadizadeh, Yamashita,Tomio, Delfino, TF, Phys. Rev. Lett. 107, 135304 (2011)

BUT Pauli principle kills sensitivity to the 4-body scale!

Scales for L=0 n-n-c system with s-wave zero-range interaction

E_{nn} Energy of the virtual nn system

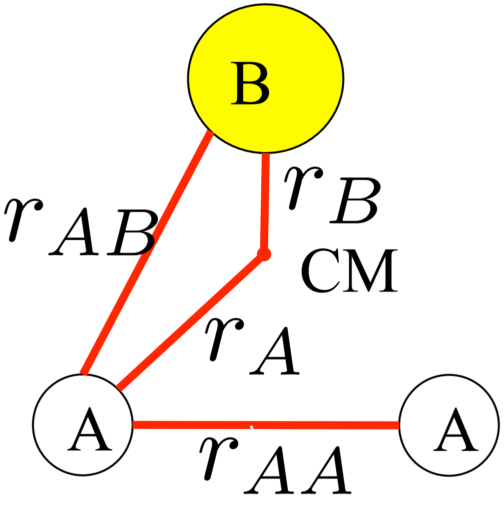
E_{nc} Energy of the bound/virtual nc system

$B_N = |E_3^{(N)}|$ Energy of the Nth state of the nnc system

A = mass of the core

Root mean square radii

Universal Scaling functions (model independent)



$$\sqrt{\langle r_{A\gamma}^2\rangle |E_3|} = R_{A\gamma}\left(\pm\sqrt{\frac{E_{AA}}{E_3}}, \pm\sqrt{\frac{E_{AB}}{E_3}}, \mathrm{A}\right)$$

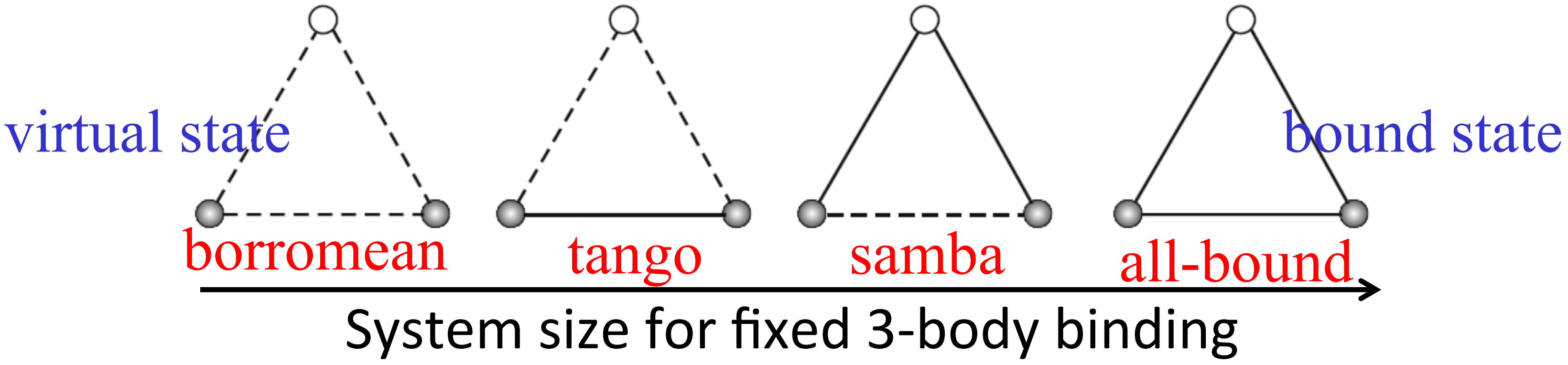
$$\sqrt{\langle r_{\gamma}^2\rangle |E_3|} = R_{\gamma}^{CM}\left(\pm\sqrt{\frac{E_{AA}}{E_3}}, \pm\sqrt{\frac{E_{AB}}{E_3}}, \mathrm{A}\right)$$

Build constraints!

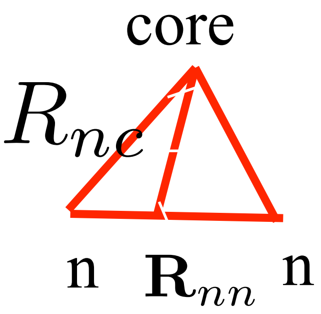
$\gamma = A$ or B

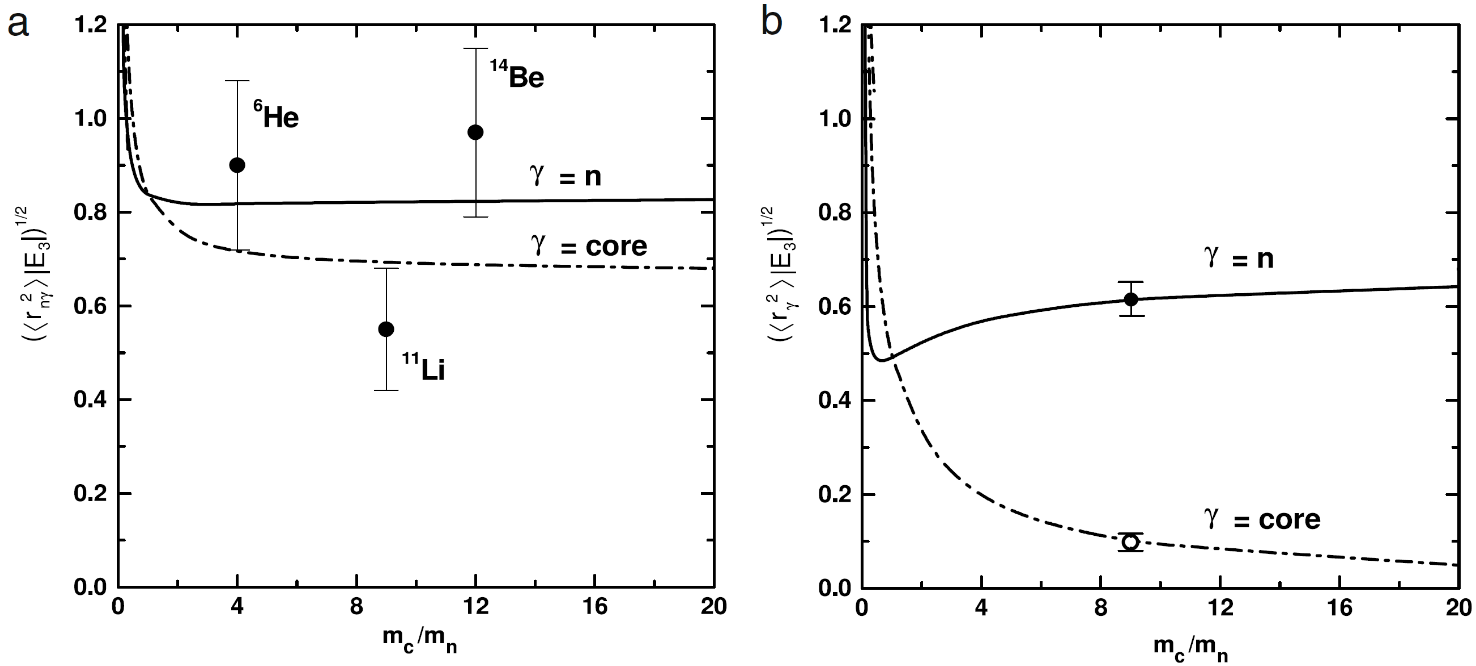
+ two-body bound state

- two-body virtual state



Root mean square radii: Core+neutron+neutron



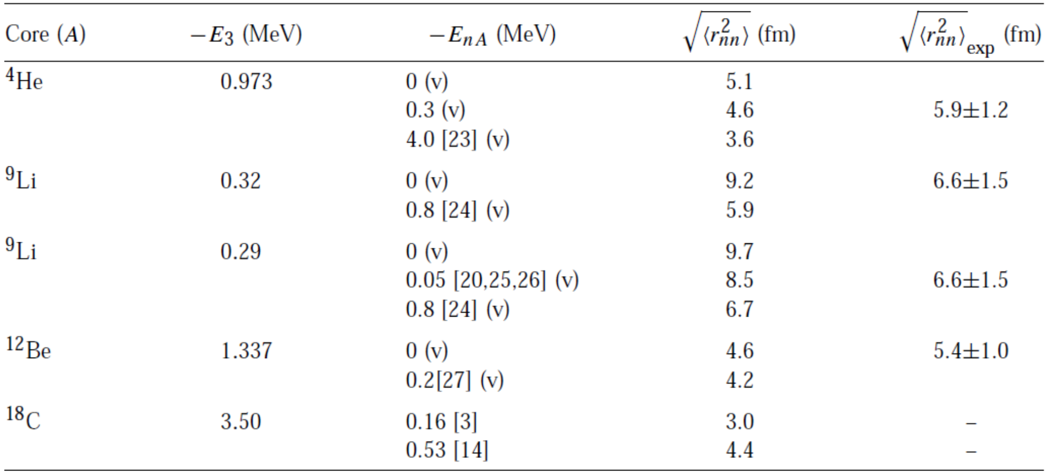


The experimental values of the charge radius of ^{9}Li and ^{11}Li are given in [4] as 2.217(35) and 2.467(37) fm, respectively, such that $\sqrt{\langle r_{ch}^2(^{11}\mathrm{Li})\rangle - \langle r_{ch}^2(^{9}\mathrm{Li})\rangle} = 1.08(11)$ fm. A neutron halo radius of 6.54(38) fm was obtained from the extracted matter radius in the experiment performed by [3]. Together with $S_{2n} = 369.15(65)$ keV, reported in [176] for ^{11}Li, the experimental value of the root-mean-square distance of ^{9}Li in respect to the center-of-mass of ^{11}Li ($\sqrt{\langle r_c^2\rangle}$) in units of $\hbar/\sqrt{m_n S_{2n}}$, is 0.10(1) and the halo radius ($\sqrt{\langle r_n^2\rangle}$) in such units is 0.617(36), these values should be compared with the theoretical results extracted from Fig. 21, of 0.10 and 0.61, respectively. The agreement with the experimental supports the model assumptions.

[3] P. Egelhof, et al., Eur. J. Phys. A 15 (2002) 27.
[4] R. Sánchez, et al., Phys. Rev. Lett. 96 (2006) 033002.
[176] M. Smith, et al., Phys. Rev. Lett. 101 (2008) 202501.

Root mean square radii: Core+neutron+neutron

Exp: F.M. Marqués, et al., Phys. Lett. B 476 (2000) 219

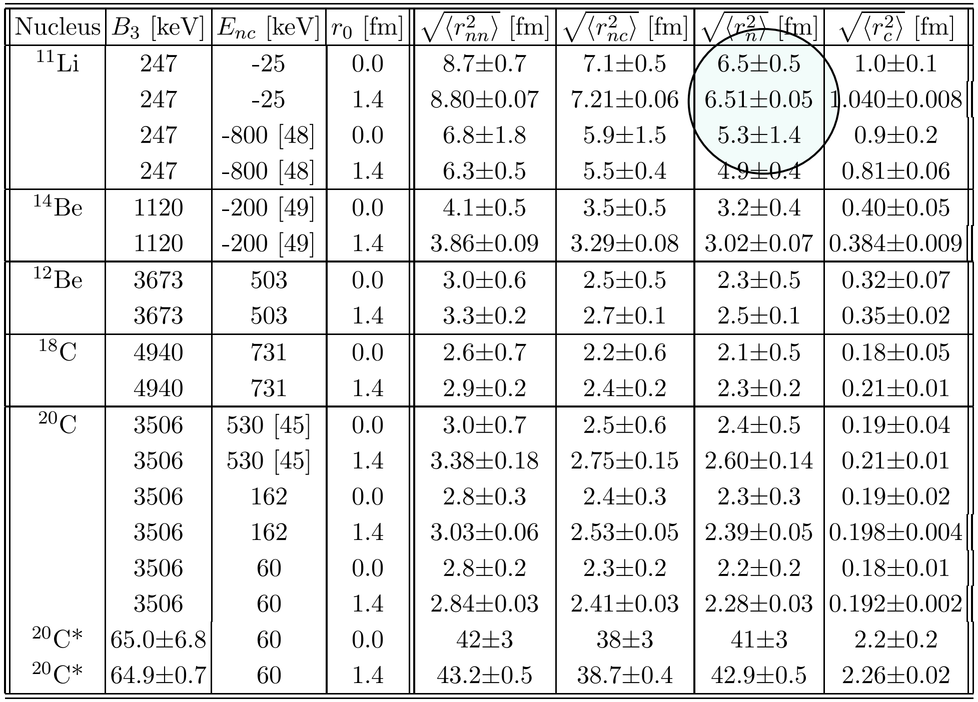


Core (A)	$-E_3$ (MeV)	$-E_{nA}$ (MeV)	$\sqrt{\langle r_{nn}^2\rangle}$ (fm)	$\sqrt{\langle r_{nn}^2\rangle}_{\mathrm{exp}}$ (fm)
^{4}He	0.973	0 (v)	5.1	
		0.3 (v)	4.6	5.9±1.2
		4.0 [23] (v)	3.6	
^{9}Li	0.32	0 (v)	9.2	6.6±1.5
		0.8 [24] (v)	5.9	
^{9}Li	0.29	0 (v)	9.7	
		0.05 [20,25,26] (v)	8.5	6.6±1.5
		0.8 [24] (v)	6.7	
^{12}Be	1.337	0 (v)	4.6	5.4±1.0
		0.2[27] (v)	4.2	
^{18}C	3.50	0.16 [3]	3.0	–
		0.53 [14]	4.4	–

Yamashita, Tomio and T. F.
NPA 735, 40 (2004)

^{11}Li

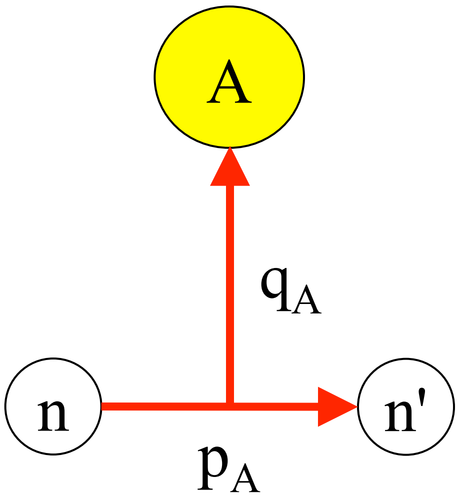
- Moriguchi et al. PRC88, 024610 (2013) - RIKEN reaction cross-section **r_n ~ 6.1 fm**
- **S_{2n}=369 keV** Smith et al. PRL101(2008)
- IMPROVE $E_v[^{10}\mathrm{Li}]$!



Nucleus	B_3 [keV]	E_{nc} [keV]	r_0 [fm]	$\sqrt{\langle r_{nn}^2\rangle}$ [fm]	$\sqrt{\langle r_{nc}^2\rangle}$ [fm]	$\sqrt{\langle r_n^2\rangle}$ [fm]	$\sqrt{\langle r_c^2\rangle}$ [fm]
^{11}Li	247	-25	0.0	8.7±0.7	7.1±0.5	6.5±0.5	1.0±0.1
	247	-25	1.4	8.80±0.07	7.21±0.06	6.51±0.05	1.040±0.008
	247	-800 [48]	0.0	6.8±1.8	5.9±1.5	5.3±1.4	0.9±0.2
	247	-800 [48]	1.4	6.3±0.5	5.5±0.4	4.9±0.4	0.81±0.06
^{14}Be	1120	-200 [49]	0.0	4.1±0.5	3.5±0.5	3.2±0.4	0.40±0.05
	1120	-200 [49]	1.4	3.86±0.09	3.29±0.08	3.02±0.07	0.384±0.009
^{12}Be	3673	503	0.0	3.0±0.6	2.5±0.5	2.3±0.5	0.32±0.07
	3673	503	1.4	3.3±0.2	2.7±0.1	2.5±0.1	0.35±0.02
^{18}C	4940	731	0.0	2.6±0.7	2.2±0.6	2.1±0.5	0.18±0.05
	4940	731	1.4	2.9±0.2	2.4±0.2	2.3±0.2	0.21±0.01
^{20}C	3506	530 [45]	0.0	3.0±0.7	2.5±0.6	2.4±0.5	0.19±0.04
	3506	530 [45]	1.4	3.38±0.18	2.75±0.15	2.60±0.14	0.21±0.01
	3506	162	0.0	2.8±0.3	2.4±0.3	2.3±0.3	0.19±0.02
	3506	162	1.4	3.03±0.06	2.53±0.05	2.39±0.05	0.198±0.004
	3506	60	0.0	2.8±0.2	2.3±0.2	2.2±0.2	0.18±0.01
	3506	60	1.4	2.84±0.03	2.41±0.03	2.28±0.03	0.192±0.002
^{20}C*	65.0±6.8	60	0.0	42±3	38±3	41±3	2.2±0.2
^{20}C*	64.9±0.7	60	1.4	43.2±0.5	38.7±0.4	42.9±0.5	2.26±0.02

Canham and Hammer
NPA 836 (2010) 275

Neutron-neutron correlation function



$$C_{nn}(\vec{p}_A) = \frac{\int d^3q_A \left|\Phi(\vec{q}_A, \vec{p}_A)\right|^2}{\int d^3q_A \rho(\vec{q}\,'_n)\rho(\vec{q}_n)}$$

$$\vec{q}_{n'} = \vec{p}_A - \frac{\vec{q}_A}{2} \qquad \vec{q}_n = -\vec{p}_A - \frac{\vec{q}_A}{2}$$

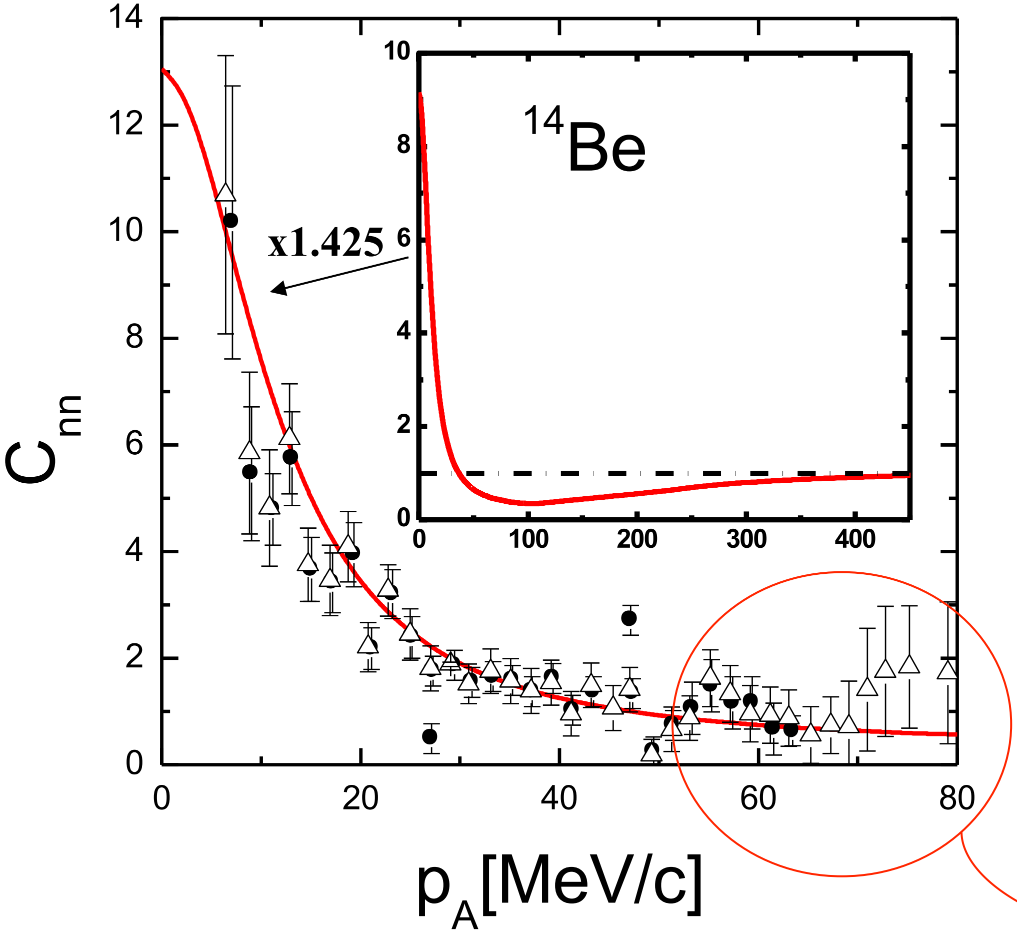
One-body density $$\rho(\vec{q}_{nA}) = \int d^3q_{n'A} \left|\Phi\left(-\vec{q}_{nA} - \vec{q}_{n'A}, \frac{\vec{q}_{nA} - \vec{q}_{n'A}}{2}\right)\right|^2$$

$\Phi \equiv \Phi(\vec{q}_A, \vec{p}_A)$ Breakup amplitude including the FSI between the neutrons

$$\Phi = \Psi(\vec{q}_A, \vec{p}_A) + \frac{1/(2\pi^2)}{\sqrt{E_{nn}} - ip_A}\int d^3p \frac{\Psi(\vec{q}_A, \vec{p})}{p_A^2 - p^2 + i\varepsilon}$$

Ψ is the three-body wave function

Neutron-neutron correlation function



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F. M. Marqués et al.
Phys. Rev. C **64**, 061301 (2001)

△

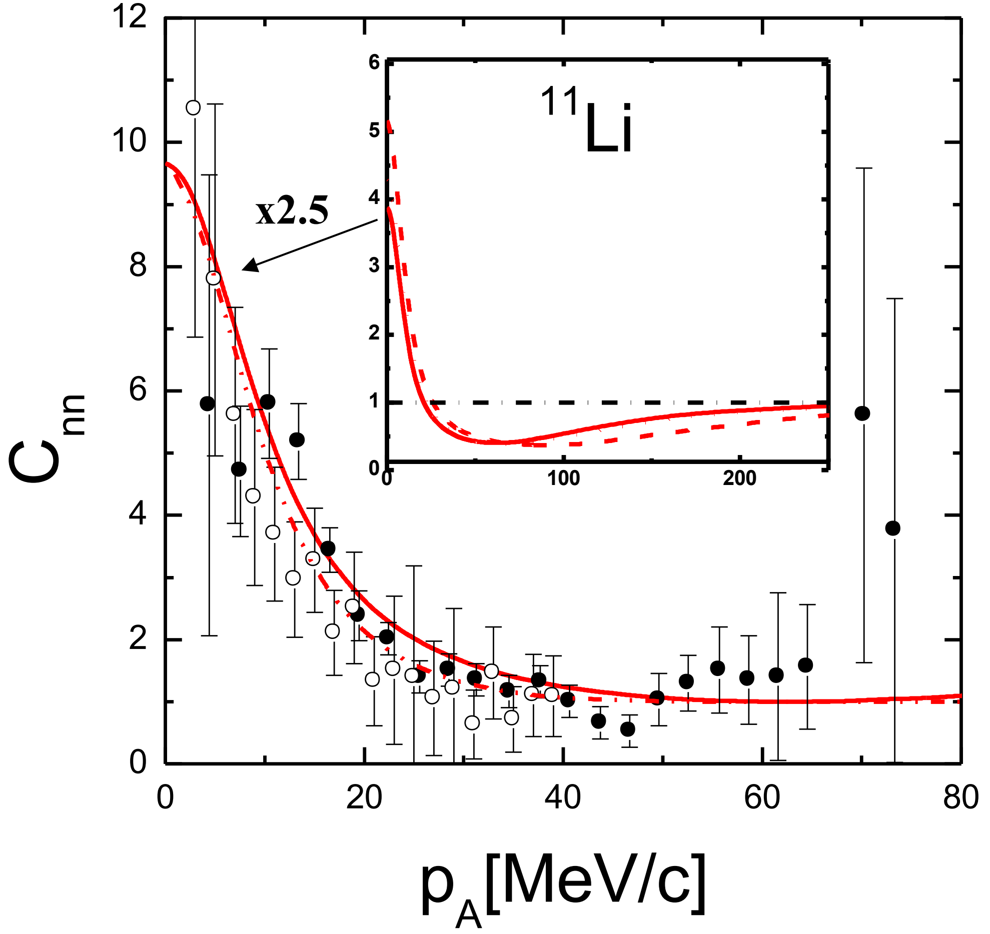
F. M. Marqués et al.
Phys. Lett. B **476**, 219 (2000)

— $E_3 = 1.337$ MeV
$E_{nA} = 0.2$ MeV
$E_{nn} = 0.143$ MeV

asymptotic region ?

Yamashita, TF, Tomio PRC 72, 011601(R) (2005)

Neutron-neutron correlation function



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F. M. Marqués et al.
Phys. Rev. C **64**, 061301 (2001)

○ — ·· —

M. Petrascu et al.
Nucl. Phys. A **738**, 503 (2004)

—— $E_3 = 0.29$ MeV
$E_{nA} = 0.05$ MeV

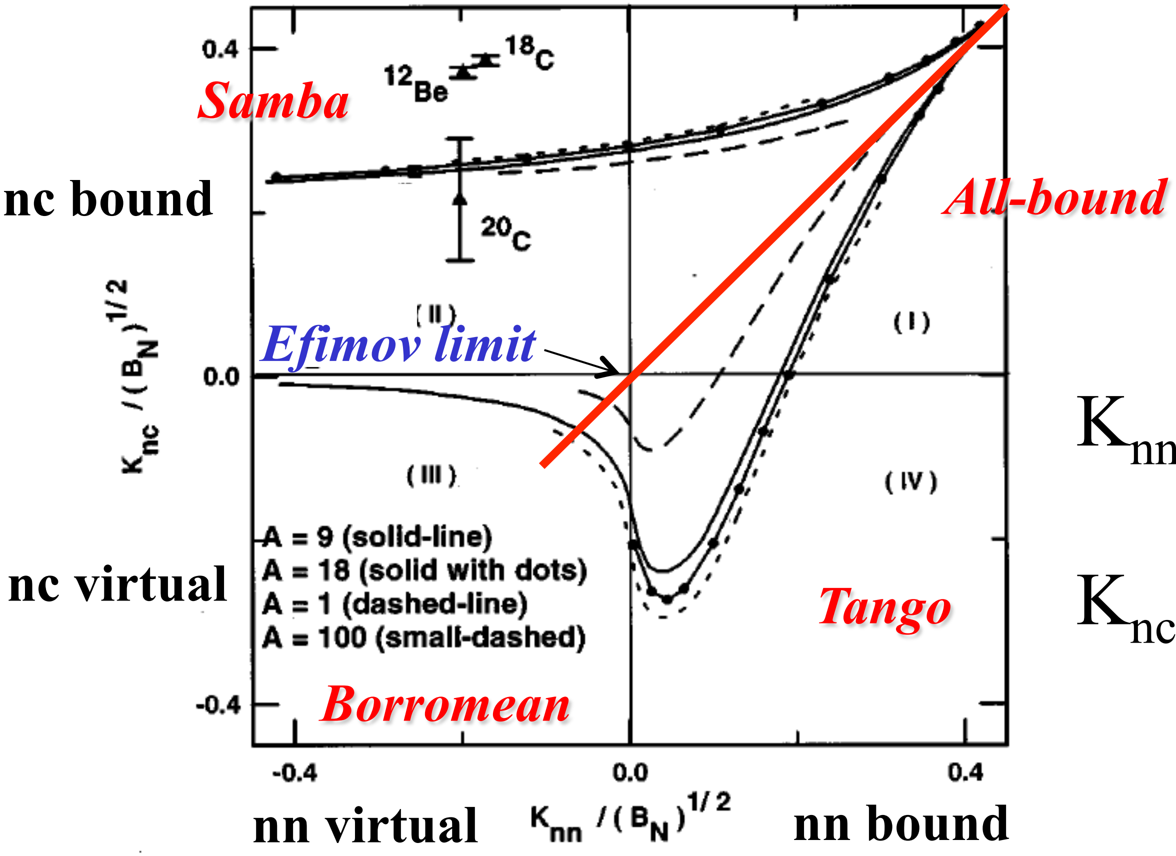
– – – $E_3 = 0.37$ MeV
$E_{nA} = 0.8$ MeV

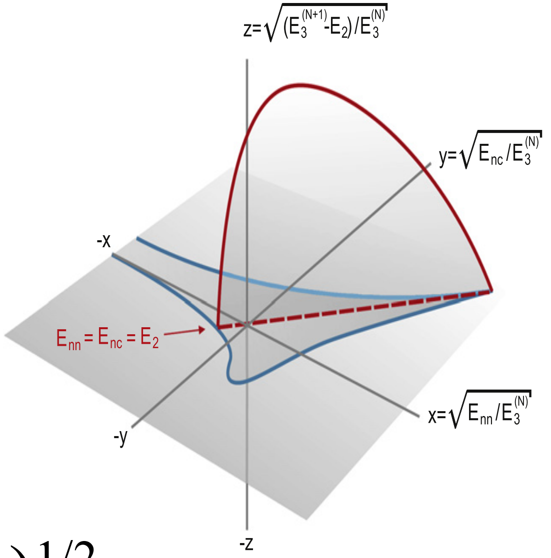
········ $E_3 = 0.37$ MeV
$E_{nA} = 0.05$ MeV

Enn = 0.143 MeV

Threshold for an excited Efimov state: Halo-nuclei

Critical condition for an excited (N+1)-th above the N-th state





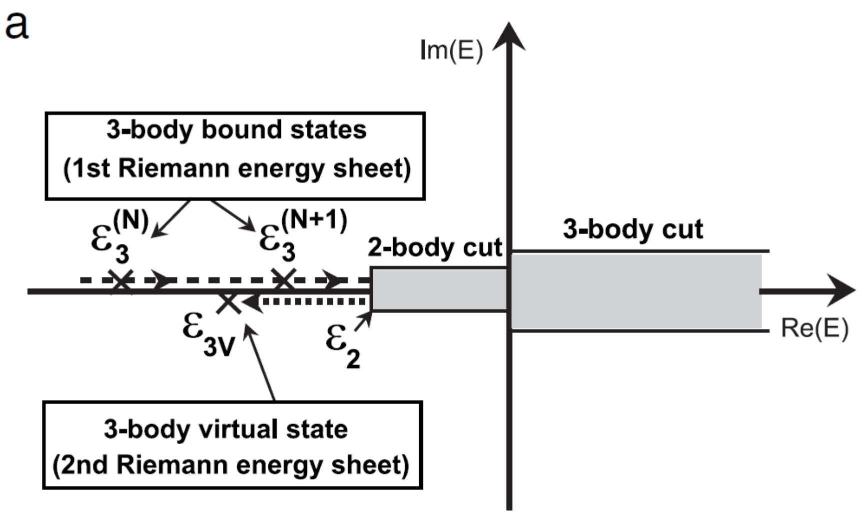
$K_{nn}=(B_{nn})^{1/2}$

$K_{nc}=(B_{nc})^{1/2}$

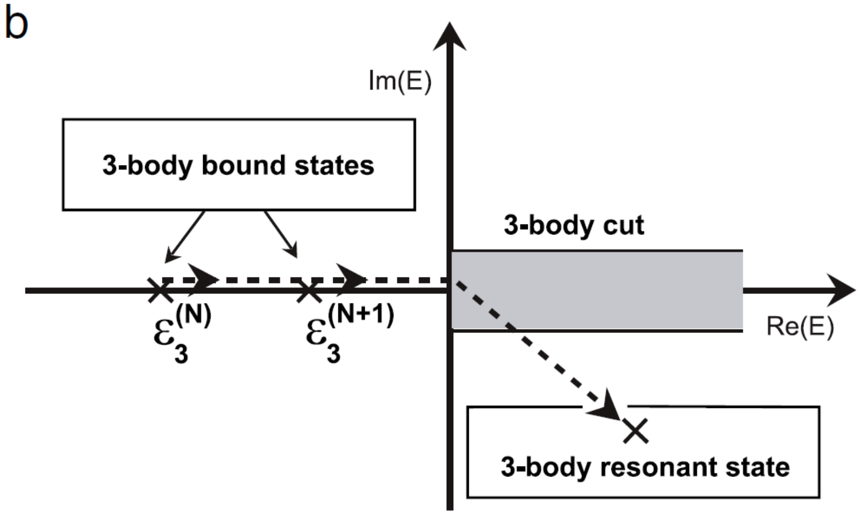
Amorim,TF,Tomio PRC56(1997)2378

Canham and Hammer EPJ A 37 (2008) 367; NPA 836 (2010) 275

analytic structure & Efimov state trajectory

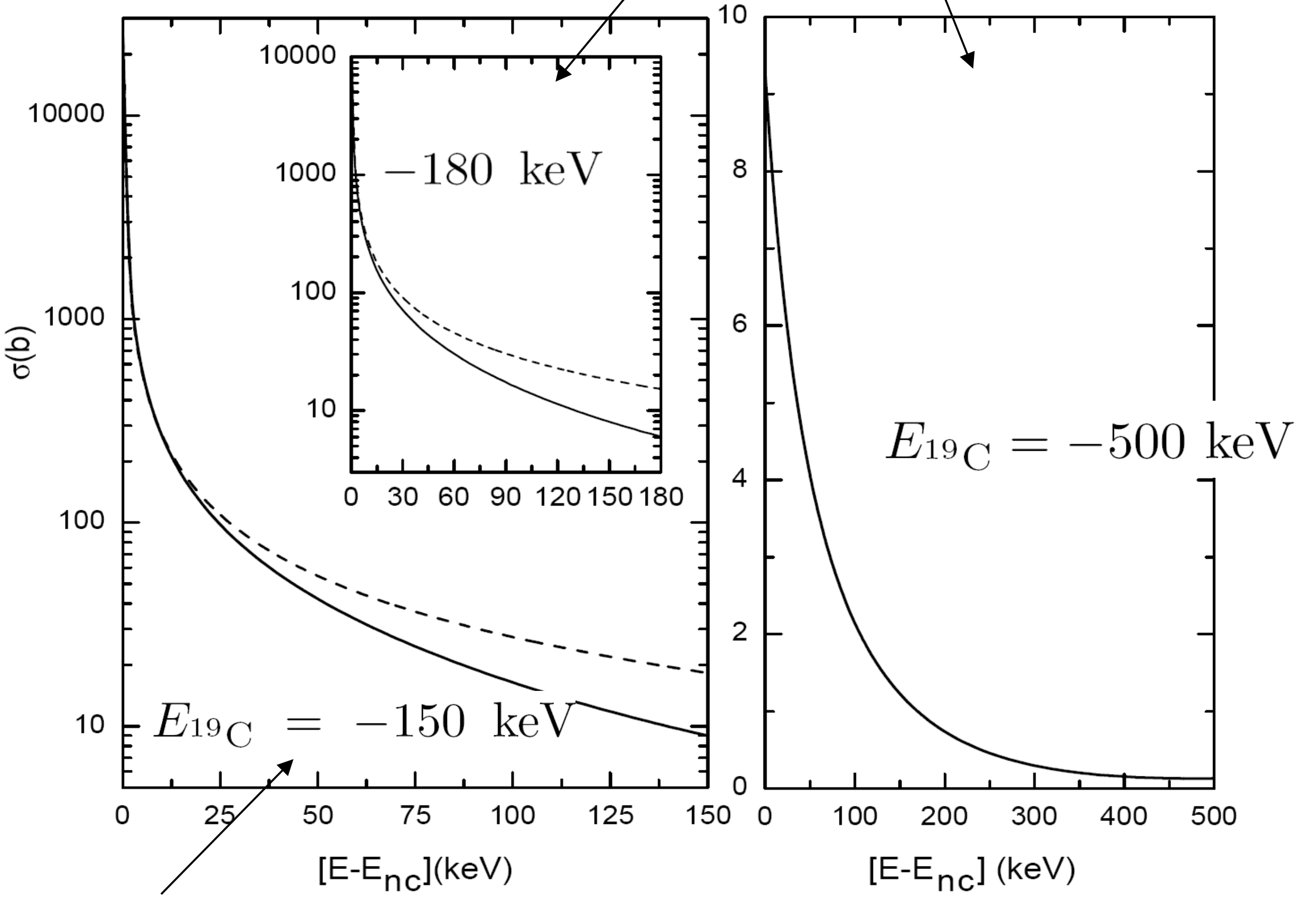


S.K. Adhikari and L. Tomio, Phys. Rev. C **26**, 83 (1982); S.K. Adhikari, A.C. Fonseca, and L. Tomio, *ibid.* **26**, 77 (1982).



F. Bringas, M.T. Yamashita and T. Frederico, Phys. Rev. A **69**, 040702(R) (2004).

n-^{19}C scattering and Efimov physics



^{20}C has a virtual Efimov state

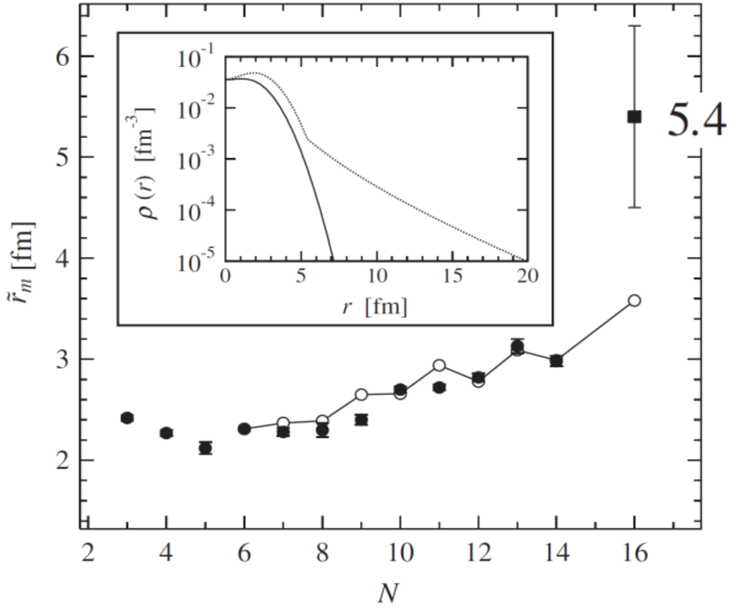
^{20}C has an excited bound Efimov state

Yamashita, TF,Tomio, PRL99 (2007)269201 & PLB660(2008)339

$^{22}C = n - n - {}^{20}C$

K. Tanaka *et al.*, Phys. Rev. Lett. **104** (2010) 062701

Reaction cross sections (σ_R) for ^{19}C, ^{20}C and the drip-line nucleus ^{22}C on a liquid hydrogen target have been measured at around 40*A* MeV by a transmission method. A large enhancement of σ_R for ^{22}C compared to those for neighboring C isotopes was observed. Using a finite-range Glauber calculation under an optical-limit approximation the rms matter radius of ^{22}C was deduced to be 5.4 ± 0.9 fm. It does not follow the systematic behavior of radii in carbon isotopes with $N \leq 14$, suggesting a neutron halo. It was found by an analysis based on a few-body Glauber calculation that the two-valence neutrons in ^{22}C preferentially occupy the $1s_{1/2}$ orbital.



5.4 ± 0.9 fm

$$\boxed{S_{2n} = 420 \pm 940 \text{ keV}}$$

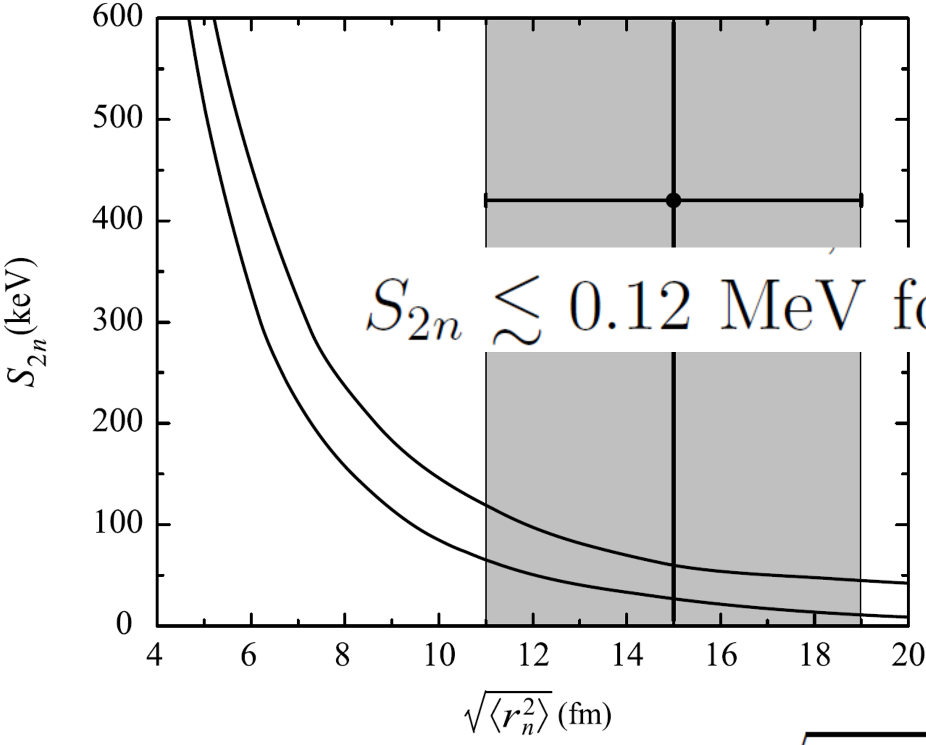
$$\tilde{r}_m^{22C} \equiv \langle (r_m^{22C})^2 \rangle^{1/2}$$

$$\tilde{r}_m^{20C} = 3\ fm$$

$$\boxed{\tilde{r}_n^{22C} = \sqrt{\frac{22}{2}}\sqrt{(\tilde{r}_m^{22C})^2 - \frac{20}{22}\left(\tilde{r}_m^{20C}\right)^2} \approx 15 \pm 3 fm}$$

$^{22}C = n - n - {}^{20}C$

^{21}C virtual state energy 0, -100 KeV. E_{nn}=-143KeV



$$S_{2n} = 420 \pm 940\ \text{keV}$$

$$S_{2n} \lesssim 0.12\ \text{MeV for } {}^{22}\text{C}$$

Yamashita, M de Carvalho, TF, Tomio,
PLB697(2011)90; A&E PLB715(2012)282

$$11\ \text{fm} \leqslant \sqrt{\langle r_n^2\rangle} \leqslant 19\ \text{fm}$$

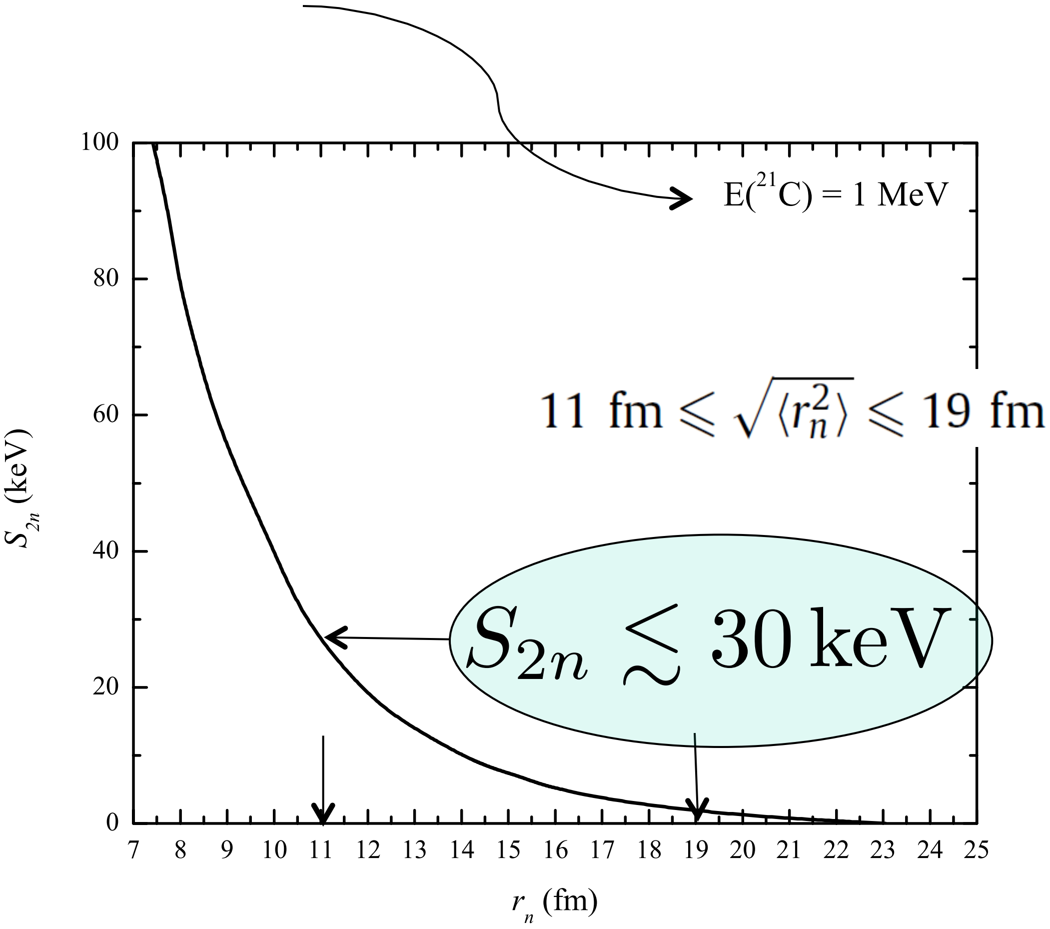
$$\sqrt{\langle r_{ch}^2({}^{22}\text{C})\rangle - \langle r_{ch}^2({}^{20}\text{C})\rangle} \gtrsim 0.9\ \text{fm}$$

H.T. Fortune, R. Sherr, Phys. Rev. C 85 (2012) 027303.

Acharya, Ji, Phillips PLB723(2013)19 [S_{2n} < 100 keV] (EFT)

$^{22}C = n - n - {}^{20}C$

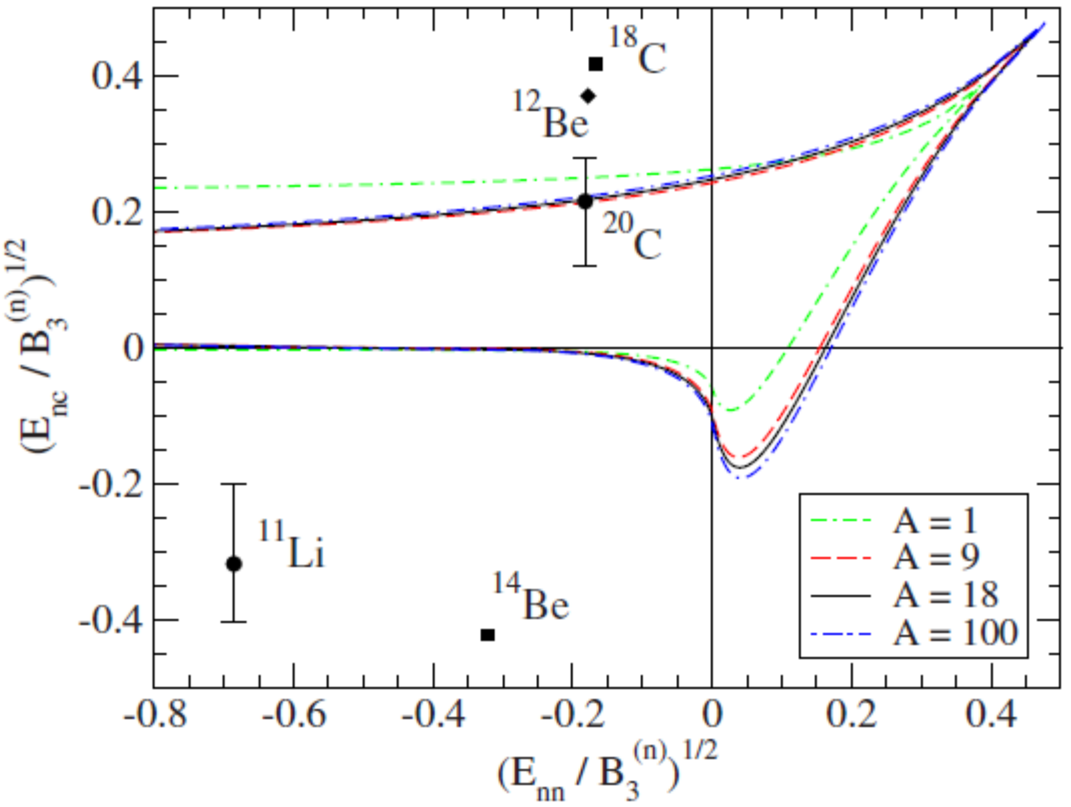
^{21}C Mosby et al. NPA 909, 69 (2013) – MSU - $|a_s| < 2.8$ fm (^{21}C virtual state)



$$11\ \text{fm} \leqslant \sqrt{\langle r_n^2 \rangle} \leqslant 19\ \text{fm}$$

$$S_{2n} \lesssim 30\,\text{keV}$$

$^{22}C = n - n - {}^{20}C$



^{22}C

^{21}C with a virtual state with energy 1 MeV

→ It is not possible an excited Efimov state/continuum resonance

If L_{total} is nonzero ?

- Virtual p-wave states of light non Borromean nn halo nuclei $E_{virtual}$~1.7 E_{nc}
- Delfino et al PRC61, 051301 (2000)

- Pigmy dipole 1^- resonance:
- M. Cubero et al, PRL 109, 262701 (2012) $^{11}Li+^{208}Pb$ close the Coulomb barrier → E_{res}=690 keV width=0.32 keV
- Fernandez-Garcıa et al PRL 110, 142701 (2013) $^{11}Li+^{208}Pb$ breakup around the Coulomb barrier

Determined by scattering lenghts only!

Summary

- Weakly bound & large systems: **few scales regime** in halo nuclei, molecules, trapped atoms
  CORRELATIONS BETWEEN OBSERVABLES→ CONSTRAINTS!

- Zero-range model n-n-c system:
  threshold conditions for excited states and resonances
  borromean configuration: **Efimov state→ resonance**
  at least one subsystem is bound: **Efimov state→ virtual state**

- Few-examples: ^{11}Li, ^{14}Be, ^{20}C, ^{22}C

- ^{20}C **Efimov state→ virtual state** **$E_{19C} > 165$ keV**

- ^{22}C large nn halo $S_{2n} \sim 30$ keV with ^{21}C virtual state 1 MeV (from $|a_s|<2.8$ fm)→

  No Efimov continuum resonance/excited state (range corrections?)

Outlook

- Neutron halo > 2n (no need of a 4-body scale)...
  - ^{12}Li = ^{10}Li+n+n+n , ^{21}C = ^{18}C+n+n+n
- Universality in scattering, breakup of halo nuclei & CDCC ...
- Pigmy resonances L_{total}=1,2, 3 ...
- Fix the tail of ab-initio calculations...

$$\mathcal{A}\left[\Psi({}^9Li)\times\Psi_{3B}({}^9Li-n-n)\right]$$

- Formation of neutron halo nuclei in neutron rich environment?
  How this affect neutron capture? ...

Collaborators:

Antonio Delfino (UFF/Brazil)
Filipe Bellotti (PhD/ITA/Aarhus)
Mohammadreza Hadizadeh (Ohio Univ)
Lauro Tomio (IFT/Brazil)
Marcelo Yamashita (IFT/Brazil)

THANK YOU!