Quark ensembles with infinite correlation length

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- Four-fermion interaction
- Thermodynamics of the quark ensemble. Fermi liquid.
- Transition layer of gas and liquid
  Quark droplets
- Infinite correlation length.
  Integrability (Thirring, Luttinger)

Four-fermion interaction

$$\mathcal{H} = -\bar{q}\,(i\gamma\boldsymbol{\nabla} + m)\,q - j^a_\mu \int d\boldsymbol{y}\; j'^b_\nu\; \langle A^a_\mu A'^b_\nu \rangle$$

$$j^a_\mu = \bar{q}\; t^a \gamma_\mu\; q$$

$$\langle A^a_\mu A'^b_\nu \rangle = G\;\delta(t-t')\;\delta^{ab}\;\delta_{\mu\nu}\;F(\boldsymbol{x}-\boldsymbol{y}) \qquad \text{white noice}$$

$F(\boldsymbol{x}) = \delta(\boldsymbol{x})$ (NJL), $F(\boldsymbol{p}) = \delta(\boldsymbol{p})$ (Keldysh model) $L \sim \Lambda^{-1}_{\rm QCD}$

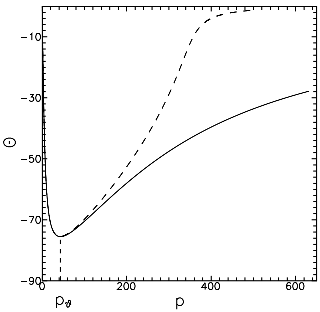
Bogolyubov transf. $|\sigma\rangle = T|0\rangle\;,\quad T = \prod_{p,s} \exp\left[\varphi_{\mathbf{p}}(a^+_{\mathbf{p},s} b^+_{-\mathbf{p},s} + a_{\mathbf{p},s} b_{-\mathbf{p},s})\right]$

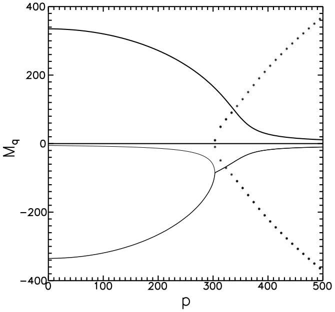
$$a\;|0\rangle = 0, \qquad b\;|0\rangle = 0$$

$A = T\;a\;T^\dagger\;,\quad$ preferenced ref. fr. $\quad$ fix. chiral phase

$$E = \min_{\varphi_{\mathbf{p}}} \langle \sigma | H | \sigma \rangle$$

Four-fermion interaction

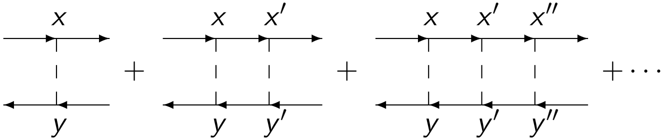




The most stable equilibrium angles $\theta = 2\varphi$ (in degrees) as function of momentum p in MeV. The solid line for NJL model, dashed one corresponds to the Keldysh model. Tuning $M_{\mathrm{Keld}}(0) = M_{\mathrm{NJL}}$

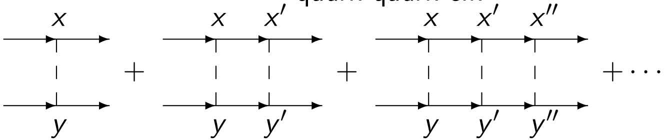
Three branches of solutions for dynamical quark mass (in MeV) for the Keld. model as a function of momentum (MeV). The imaginary parts of the solutions are shown by dots. Keld. model advantage– integrability

Keldysh model. $F(\mathbf{p}) = \delta(\mathbf{p})$ quark–anti-quark ch.

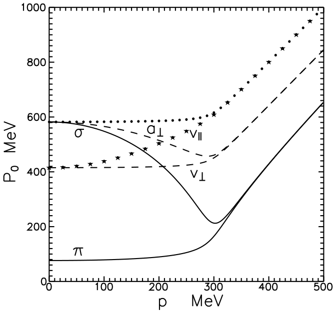


$$\varepsilon^2_{\pi,\sigma} = (E_p + E_q)^2 - 2G\,\frac{E_p + E_q}{E_p E_q}\,(E_p E_q \pm M_p M_q - \mathbf{pq})$$

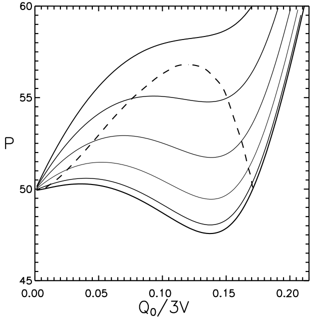
quark-quark ch.

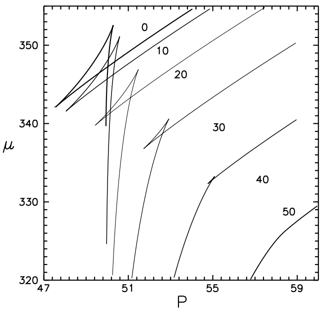


$$\varepsilon^2_{\pi,\sigma} = (E_p + E_q)^2 - \frac{2G}{N_c}\,\frac{E_p + E_q}{E_p E_q}\,(E_p E_q \mp M_p M_q - \mathbf{pq})$$



Energy of meson observables in Keldysh model
NJL–reasonable descr. of 4 meson nonets
Lorentz invar.





The ensemble press. P (MeV/fm^3) is shown as a funct. of charge density $\mathcal{Q}_0$ at temp. $T = 0$ MeV, ..., $T = 50$ MeV with spacing $T = 10$ MeV. The lowest curve corresponds to zero temp. The dashed curve shows the boundary of phase transition liquid–gas.

The fragments of the isotherms. The chemical potential μ (MeV) is plotted as a function of pressure P MeV/fm^3. The top curve corresponds to the zero isotherm and following down with spacing 10 MeV till the isotherm 50 MeV (the lowest curve). Landau theory of Fermi-liquid

Transition layer of gas and liquid (T=0) $\mathcal{Q}_0 \to \rho$

can two paheses coexist transition layer micr. grounds mean field

$M = 335$ MeV $\rho_g \sim 0$ $\overset{*}{M} \approx 70$ MeV $\rho_l = 3 \times 0.16$ ch/fm^3

$$\mathcal{L} = -\bar{q}\ (\hat{\partial} + M)\ q - \frac{1}{2}\ (\partial_\mu \sigma)^2 - U(\sigma) - \frac{1}{4}\ F_{\mu\nu} F_{\mu\nu} - \frac{m_v^2}{2}\ V_\mu V_\mu - $$
$$- g_\sigma\ \bar{q} q\ \sigma + i g_v\ \bar{q}\ \gamma_\mu\ q\ V_\mu\ \ldots,$$

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu\ , \quad U(\sigma) = m_\sigma^2\ \sigma^2/2 + b\ \sigma^3/3 + c\ \sigma^4/4 + \ldots$$

- Can param. of Lagr. be properly tuned $(g_\sigma\ ,\ g_v\ ,\ m_\sigma\ ,\ m_v\ ,\ b\ ,\ c)$, obtaining solutions approximating between two phases.

Transition layer of gas and liquid (T=0)

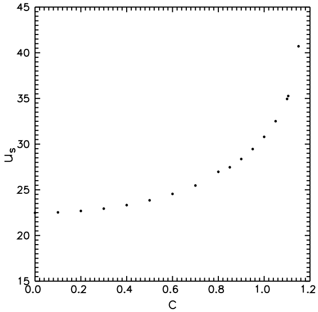
$$\Delta\ \sigma - m_\sigma^2\ \sigma = b\ \sigma^2 + c\ \sigma^3 + g_\sigma\ \rho_s\ ,$$
$$\Delta\ V - m_v^2\ V = -g_v\ \rho\ ,$$
$$(\hat{\nabla} + \overset{*}{M})\ q = (E - g_v\ V)\ q\ ,$$
$$\overset{*}{M} = M + g_\sigma \sigma, \qquad E = \left(\mathbf{p}^2 + \overset{*}{M}{}^2\right)^{1/2}\ ,$$

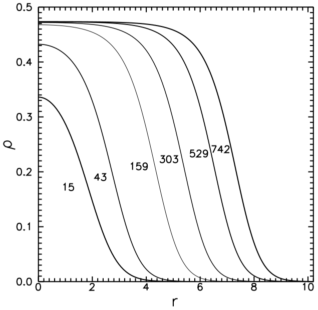
$$\xi(x) = \int^{P_F} d\widetilde{\mathbf{p}}\ q_{\mathbf{p}}(x)\bar{q}_{\mathbf{p}}(x)$$
$$\rho_s(x) = Tr\left\{\xi(x), 1\right\},\quad \rho(x) = Tr\left\{\xi(x), \gamma_4\right\}$$

Thomas-Fermi approximation, $P_F(x)$:

$$\rho = \gamma \int^{P_F} d\widetilde{\mathbf{p}} = \gamma\ P_F^3/6\pi^2\ ,\quad \rho_s = \gamma \int^{P_F} d\widetilde{\mathbf{p}}\ \ \overset{*}{M}/E$$

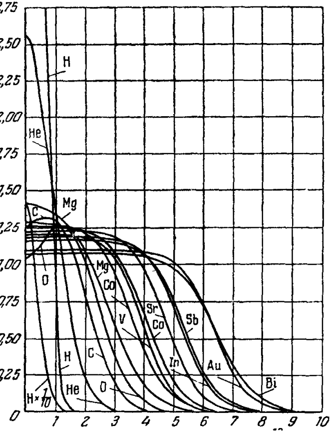
Transition layer of gas and liquid (T=0)





The surface tens. coef. u_s in MeV. $t \sim 1$—2 fm.

The quark density distributions ρ (ch/fm^3) as function of distance r (fm). Walecka model.



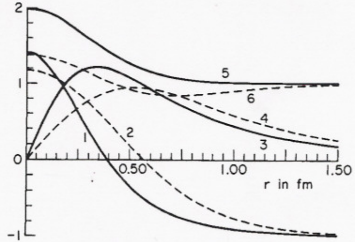


Fig. 1. The σ and π fields in units of F_π. Solid lines are for the present calculation, dashed lines for that of ref. [5]. Curves 1 and 2 show σ, 3 and 4 show π, 5 and 6 show $\sigma^2 + \pi^2$.

R.Hofstadter;   Thomas–Fermi appr. W. Broniowski and M. K. Banerjee, Phys.Lett.**B158**(1978)335. Apparently, the phase transition of chiral symm. (partial) restoration has already realized as the mixed phase of physical vacuum and baryonic matter. $m_\sigma \sim 1\ GeV$. BCS—4 meson non. + baryon

Exact (Thirring, Luttinger) integrability

- $1+1$ dimensions (Thirring (1958))

$$\begin{aligned}\mathcal{H} &= q_1^+ \; i\partial_x \; q_1 - q_2^+ \; i\partial_x \; q_2 + m \, (q_1^+ q_2 + q_2^+ q_1) + \\ &+ g \, (q_1^+ q_1 \; q_2'^+ q_2' + q_2^+ q_2 \; q_1'^+ q_1')\end{aligned}$$

$$L \sim \Lambda_{\rm QCD}^{-1} \quad const \quad \text{Peiod. bound. cond.} \;\; \widetilde{p} = n/L \quad t^a \to 1$$

Reference vacuum state $|0)$, $q_{1x}|0) = q_{2x}|0) = 0$
(Dried Dirac sea)

$a_{ik} = \int dx \; e^{-ikx} \; q_{ix}$, $\quad q_{ix} = \int d\widetilde{k} \; e^{ikx} \; a_{ik}$, $\quad \widetilde{k} = k/(2\pi)$
$(m = 0)$

$$\begin{aligned}\mathcal{H}_0 &= \sum_{-\infty<k<\infty} k \; a_{1k}^+ a_{1k} - \sum_{-\infty<k<\infty} k \; a_{2k}^+ a_{2k} \\ \mathcal{V} &= 2g \sum_{k,p} a_{1k+p}^+ a_{1k} \sum_{l} a_{2l}^+ a_{2l-p}\end{aligned}$$

Exact (Thirring, Luttinger) integrability

$H=\int dx\ \mathcal{H},\ H=H_0+V,\ H\ |0)=0,\ H_0\ |0)=0,\ V\ |0)=0$

System "Charge" $Q=\int dx\ \left(q_1^+ q_1+q_2^+ q_2\right)=\sum_k \left(a_{1k}^+ a_{1k}+a_{2k}^+ a_{2k}\right)$ commutes with the Hamiltonian $[H,Q]=0$.

$$|0\rangle=\prod_{k\geq -\Lambda}^{-P} a_{1k}^+ \prod_{l\geq P}^{\Lambda} a_{2l}^+\ |0)$$

$$\mathcal{H}_0\ |0\rangle=\left(\sum_{-\Lambda}^{-P} k-\sum_{P}^{\Lambda} k\right)|0\rangle\ ,\quad \mathcal{V}\ |0\rangle=2gL\ (\Lambda-P)^2|0\rangle$$

$$\mathcal{E}_{\rm D}=-\Lambda(\Lambda+\tilde{1})+P(P+\tilde{1})+2gL\ (\Lambda-P)^2\ ,\quad \tilde{1}=2\pi/L\ ,\quad 2gL>1$$

$$\Lambda-P_{\rm min}=\frac{1}{2}\frac{2\Lambda+\tilde{1}}{2gL+\tilde{1}}\ \text{(Dirac sea width)},\ \ \mathcal{E}_{\rm min}=-\frac{1}{4}\frac{2\Lambda+\tilde{1}}{2gL+\tilde{1}}\ \left(2\Lambda+\tilde{1}\right)$$

Degeneracy of the vacuum state. Vacuum excitation. Add. remov. part. with mom. $\sim\Lambda/2$ gives small increase of energy $\sim d\mathcal{E}/dP|_{P=P_{\rm min}}$

Exact (Thirring, Luttinger) integrability

Comp. Thirring model with point like form of interact. $F(x) = \delta(x)$.
Hamilt. can be diagon. Bethe (1931)

$$|k_1, \ldots, k_N\rangle = \int \prod_{i=1}^{N_1} dx_i e^{ik_i x_i} \int \prod_{j=1}^{N_2} dy_j e^{ik_{N_1+j} y_j} \times$$

$$\times \prod_{i,j} [1 + \lambda_{ij} \epsilon(x_i - y_j)] \prod_{i=1}^{N_1} q_1^+(x_i) \prod_{j=1}^{N_2} q_2^+(y_j) |0\rangle$$

$\epsilon(x)$—step-like funct. $\epsilon(x) = -1$, at $x < 1$, $\epsilon(x) = 1$, at $x > 1$,
k_i—momentum i-th part., phase factor $\lambda_{ij} = -g/2S_{ij}$,
$S_{ij} = (k_i E_j - k_j E_i)/(k_i k_j - E_i E_j - \varepsilon^2)$, ε—inf. small infrared. reg.,
E_i—paricle energy, (m=0, $E_i = |k_i|$).

$$H|k_1, \ldots, k_N\rangle = \sum_{i=1}^{N} E_i \; |k_1, \ldots, k_N\rangle \; , \qquad N = N_1 + N_2$$

Exact (Thirring, Luttinger) integrability

Period. bound. cond. lead to particle momenta

$$k_i = \frac{2\pi n_i}{L} + \frac{2}{L}\sum_{j\neq i}^{N} \arctan(gS_{ij}/2)\ ,\ n_i = 0, \pm 1, \ldots, \pm N_0, N_0 = (N-1)/2$$

"Symm." vac. state $k_0 = 0$, $(n_0 = 0)$ (T. Fujita)

$$k_i = \frac{2\pi n_i}{L} + \frac{2N_0}{L}\arctan(g/2)\ ,\quad (n_i = 1, 2, \ldots, N_0),$$

$$k_i = \frac{2\pi n_i}{L} - \frac{2N_0}{L}\arctan(g/2)\ ,\quad (n_i = -1, -2, \ldots, -N_0).$$

$$E_0^{sym} = -\Lambda\left[N_0 + 1 - \frac{2N_0}{\pi}\arctan(g/2)\right]\ ,\quad N_0 = \frac{L}{2\pi}\Lambda$$

"non symm." states comparision NJL and Keldysh, $m \neq 0$, M.W. th.
V. Mastropietro and D. C. Mattis, "Luttinger model. The First 50 Years and Some New Directions" , Ser. on Direct. in Cond. Mat. Phys.—V. 20, 2014

Exact (Thirring, Luttinger) integrability

$$3+1\ (D+1)$$

$$V = g\ v\ ,\quad v = \int d\mathbf{x} d\mathbf{y}\ j^a_\mu(\mathbf{x})\ j^a_\mu(\mathbf{y})\ ,\quad j^a_\mu(\mathbf{x}) = \bar{q}_\mathbf{x}\ \gamma_\mu t^a\ q_\mathbf{x}$$

$$Q = \int d\mathbf{x}\ \bar{q}_\mathbf{x} \gamma_0\ q_\mathbf{x}\ ,\quad [H, Q] = 0\ ,\quad q_\mathbf{x}|0) = 0$$

$$|N) = \int d\mathbf{z}_1 \dots \int d\mathbf{z}_N\ \ \bar{q}_{\mathbf{z}_1} \bar{q}_{\mathbf{z}_2} \dots \bar{q}_{\mathbf{z}_N}\ \chi_{\mathbf{z}_1 \mathbf{z}_2 \dots \mathbf{z}_N}|0)$$

$$[v, \bar{q}_1 \bar{q}_2 \dots \bar{q}_N]\chi = 2N\ \bar{q}_1 \bar{q}_2 \dots \bar{q}_N\ \gamma_\mu t^a \gamma^0\ j\ \chi +$$
$$+N(N-1)\bar{q}_1 \dots \bar{q}_{N-1} \gamma_\mu t^a \gamma^0 \bar{q}_N \gamma_\mu t^a \gamma^0 \chi + N\bar{q}_1 \dots \bar{q}_N \gamma_\mu t^a \gamma^0 \gamma_\mu t^a \gamma^0 \chi$$

$$\boldsymbol{\lambda}\otimes\boldsymbol{\lambda}=\frac{4}{3}\,\Lambda_s-\frac{8}{3}\,\Lambda_a\ ,\quad \Lambda_s+\Lambda_a=E_\Lambda$$

$$\boldsymbol{\gamma}\gamma^0=\left\|\begin{array}{cc}\boldsymbol{\sigma} & 0\\ 0 & -\boldsymbol{\sigma}\end{array}\right\|,\ \boldsymbol{\gamma}\gamma^0=\left\|\begin{array}{cc}0 & \boldsymbol{\sigma}\\ \boldsymbol{\sigma} & 0\end{array}\right\|$$

$$\boldsymbol{\sigma}\otimes\boldsymbol{\sigma}=\Sigma_s-3\Sigma_a\ ,\quad \Sigma_s+\Sigma_a=E_\Sigma$$

$$\gamma_\mu\gamma^0\otimes\gamma^\mu\gamma^0=\Sigma_s+\Sigma_a-(\Sigma_s-3\Sigma_a)=4\ \Sigma_a$$

$$\boldsymbol{\lambda}\otimes\boldsymbol{\lambda}\ \ \boldsymbol{\gamma}\gamma^0\otimes\boldsymbol{\gamma}\gamma^0=\frac{16}{3}\,\Lambda_s\ \Sigma_a$$

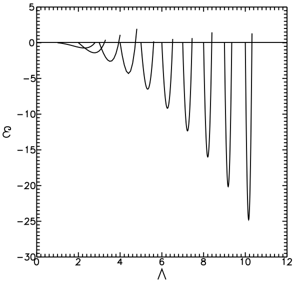
$$[v,\bar{q}_1\ldots\bar{q}_N]\,\chi\,|0)=\frac{4}{3}\,N\left[\Lambda_s\Sigma_a(N-3)-2\Lambda_a\Sigma_s\right]|N)\ ,\ \ (\Lambda_s\Sigma_a\approx\Lambda_a\Sigma_s)$$

$$\mathcal{E}_0 = -2N_c\ 4\pi \int_P^\Lambda \frac{k^2 dk\ k}{(2\pi)^3} = -\frac{2N_c}{2\pi^2}\ \frac{1}{4}\ (\Lambda^4 - P^4)$$

$$N = \mathcal{N}\ L^3\ ,\quad \mathcal{N} = 2N_c\ 4\pi \int_P^\Lambda \frac{k^2 dk}{(2\pi)^3} = \frac{2N_c}{2\pi^2}\ \frac{1}{3}\ (\Lambda^3 - P^3)$$

$$\mathcal{E}_{\mathrm{D}} = \mathcal{E}_0 + g\ \frac{4}{3}\mathcal{N}\ (N-1) - g\ \frac{16}{3}\mathcal{N}$$

param. of interact. $a = g\ L^3\ (a = g\ L^D)$ $\quad 1+1 \quad 2gL > 1$



Dirac sea energy as func. of Λ, at $a = 1.025$ for some bound moment. $P = 1, \ldots 10$.

$$\Lambda_1 = P \ , \quad \Lambda_2 = P + \Delta_\Lambda$$

$$\Delta_\Lambda \approx \frac{(D+1)\ P}{D[aDP^{D-1} - (D+1)/2]} \approx \frac{D+1}{aD^2} P^{2-D} \quad \text{(Dirac sea width)}$$

$$\mathcal{E}_{\rm D}(\Lambda') = -\frac{(D+1)^2 P^{D+1}}{4D\left[aDP^{D-1} - \frac{D+1}{2}\right]} \approx -\frac{(D+1)^2}{4aD^2} P^2, \ \Lambda' \approx (\Lambda_1 + \Lambda_2)/2$$

Exact (Thirring, Luttinger) integrability

- Dirac sea width squeezing
- $\Lambda_c \approx 2N_c/(2gL)\ 2\pi/L \qquad (3+1)$
- $L \sim \Lambda_{\rm QCD}^{-1}, \quad g \sim 300$ MeV
- Λ_c is not large
- Huge degeneracy $\qquad \sim$ surface area
- Jahn–Teller theorem. Pairing mechanism
- $E_{BCS} \sim -\Lambda^{D+1}$ in $D+1$
  $E_D \sim -\Lambda^2$

Droplets of quark liquid (T=0)

$$\rho_F(r) = \frac{\widetilde{\rho}_0}{1 + e^{(R_0 - r)/b}} \;, \qquad \widetilde{\rho}_0, R_0, b, t = 4\ln(3) b \qquad R_0 = r_0 N_q^{1/3}$$

N_q	$\widetilde{\rho}_0$ (ch/fm^3)	R_0 (fm)	t (fm)	r_0 (fm)	m_σ (MeV)	η
15	0.34	1.84	2.24	0.74	351	0.65
43	0.43	2.19	2.28	0.75	384	0.73
159	0.46	4.19	2.29	0.77	409	0.78
303	0.47	5.23	2.29	0.78	417	0.795
529	0.47	6.37	2.27	0.79	423	0.805
742	0.47	7.15	2.27	0.79	426	0.81

Four fermion interaction

$$\mathcal{H} = -\bar{q}\,(i\gamma\boldsymbol{\nabla} + im)\,q - \bar{q}\;t^a\gamma_\mu\;q\;\int d\boldsymbol{y}\;\bar{q}'\;t^b\gamma_\nu\;q'\;\langle A^a_\mu A'^b_\nu\rangle$$

$$\langle A^a_\mu A'^b_\nu\rangle = G\;\delta^{ab}\;\delta_{\mu\nu}\;F(\boldsymbol{x}-\boldsymbol{y})$$

$$F(\boldsymbol{x}) = \delta(\boldsymbol{x})\;\;\text{(NJL)},\qquad F(\boldsymbol{p}) = \delta(\boldsymbol{p})\;\;\text{(Keldysh)}$$

- Bogolyubov transformation
- (One-particle approximation) Mean field approximation
- Alternative? Complementarity? (applicability of one-particle approximation for the ensemble, force—correlations)
- Phase transitions. Algebra (dominating) correlators. Renormgroup. Universality
- Exact integrability, $1+1$ dimensions (Thirring, Lattinger, Lieb–Mattis ...)