A description of neutron rich nuclei within the Deformed QRPA

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http://ssanp.ssu.ac.kr

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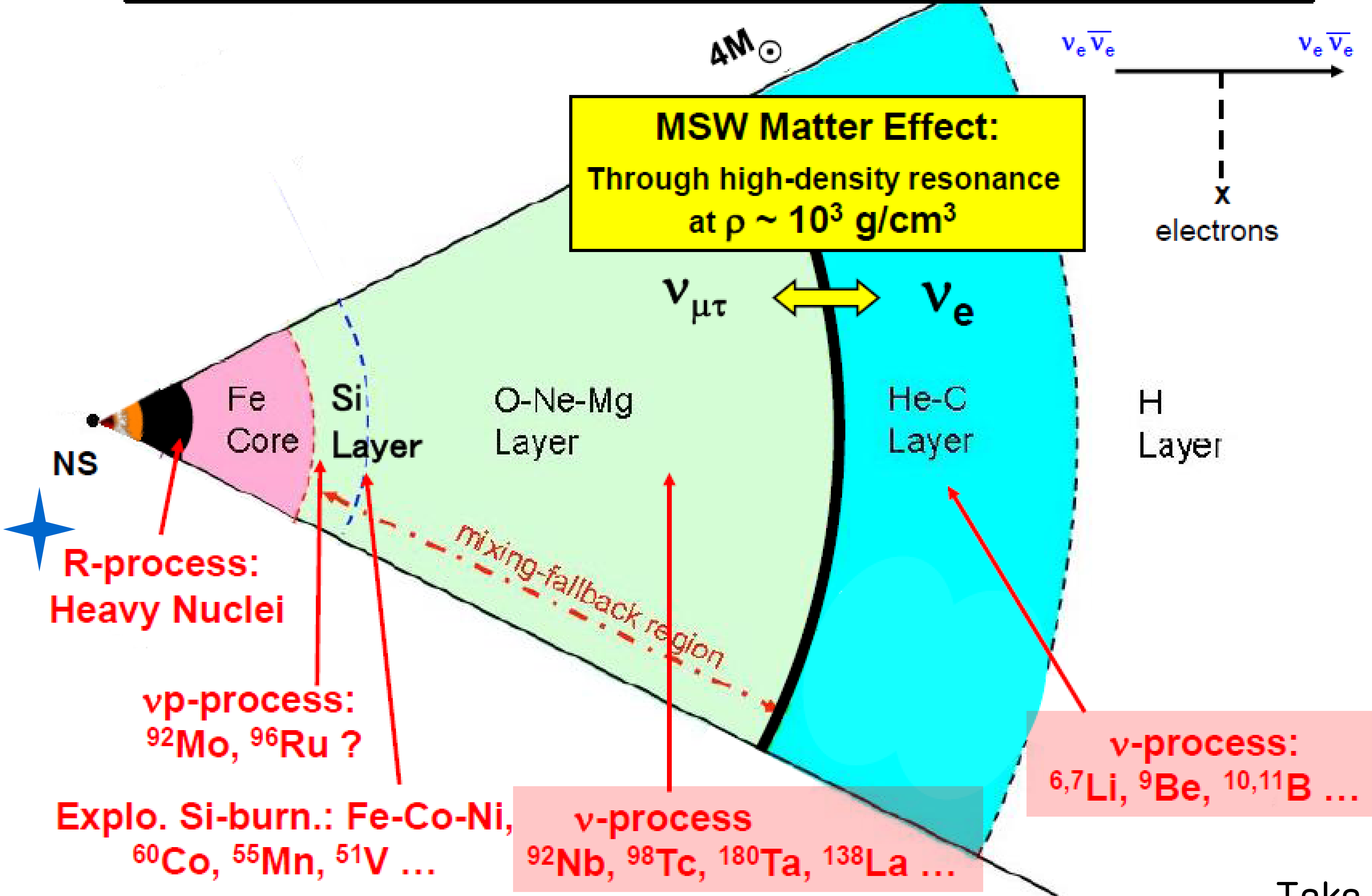
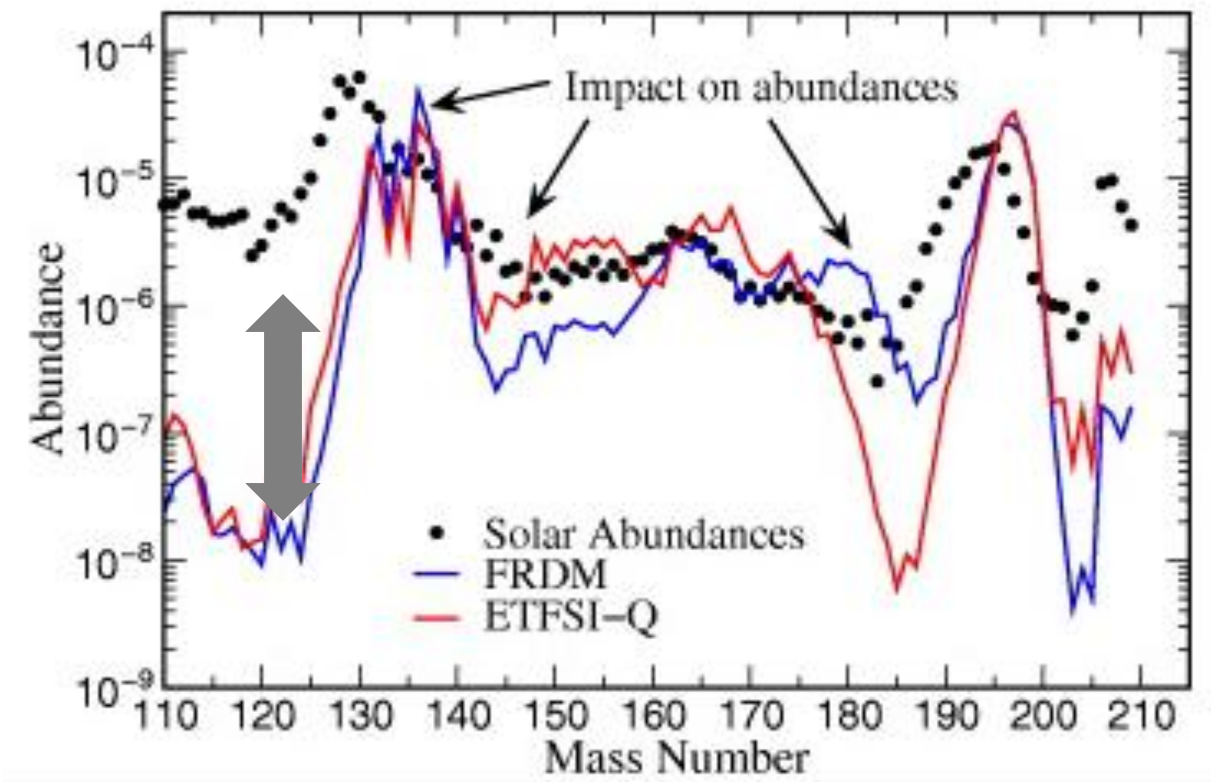
Neutrino Oscillation and SN-Nucleosynthesis



Taka Kajino

❖ The r-process abundances

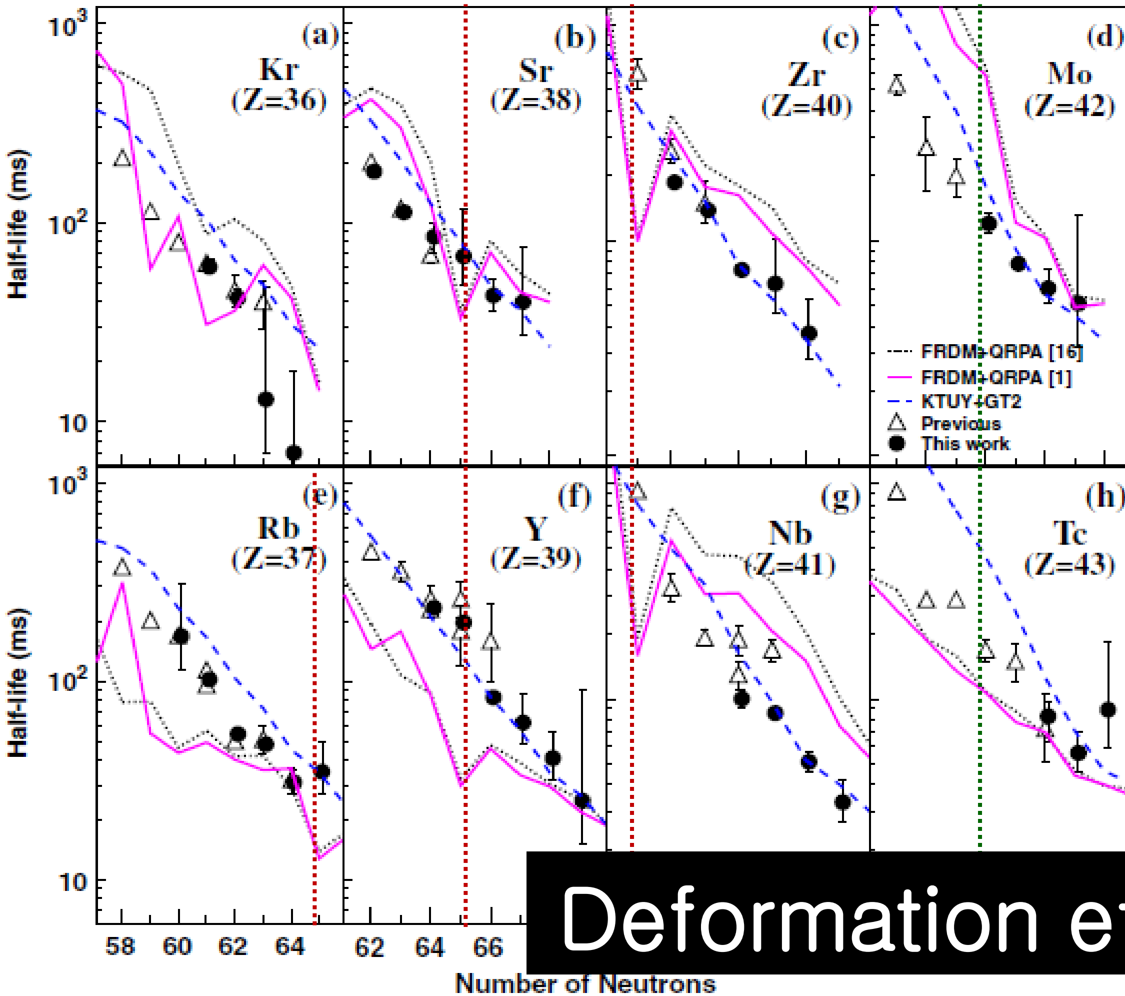
Pinedo 's talk
and Rebcecca's talk in ECT workshop !!!



- Two different mass models, FRDM(finite-range droplet mass) and ETFSI(extended Thomas Fermi Strutinsky integral), underestimate the abundances by an order of magnitude or more at A≈110 region.
- The main effect of the newly measured β-decay half-lives is an enhancement in the calculated abundance of isotope with A=110~120, relative to abundances calculated using β-decay half-lives estimated with the FRDM+QRPA. [N. Nishimura *et al*., PRC. 85, 048801(2012)]

❖ β-decay half-lives for Kr to Tc isotopes

[S. Nishimura *et al*., PRL 106, 052502(2011)]

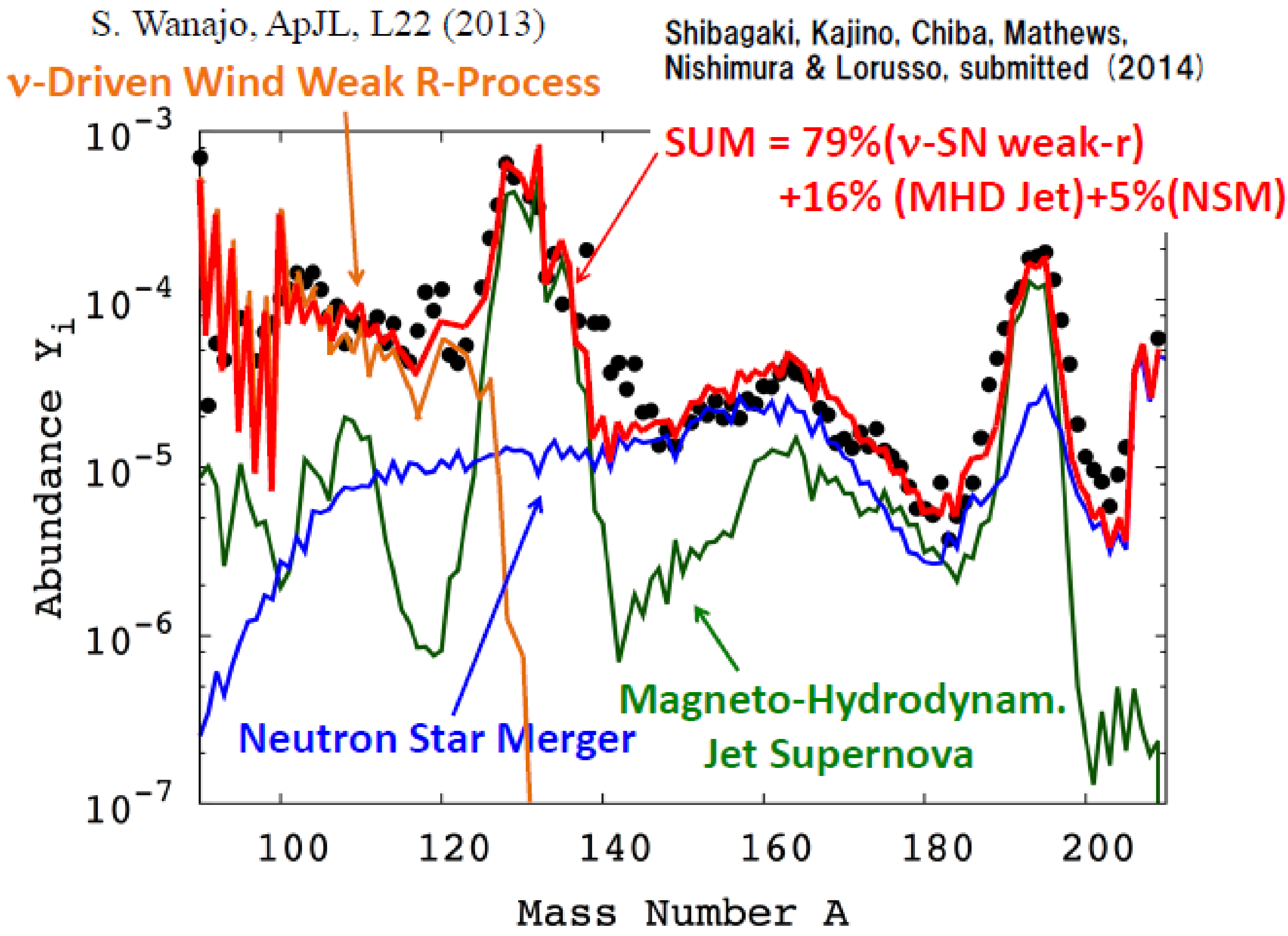


Giuseppe's talk !!!

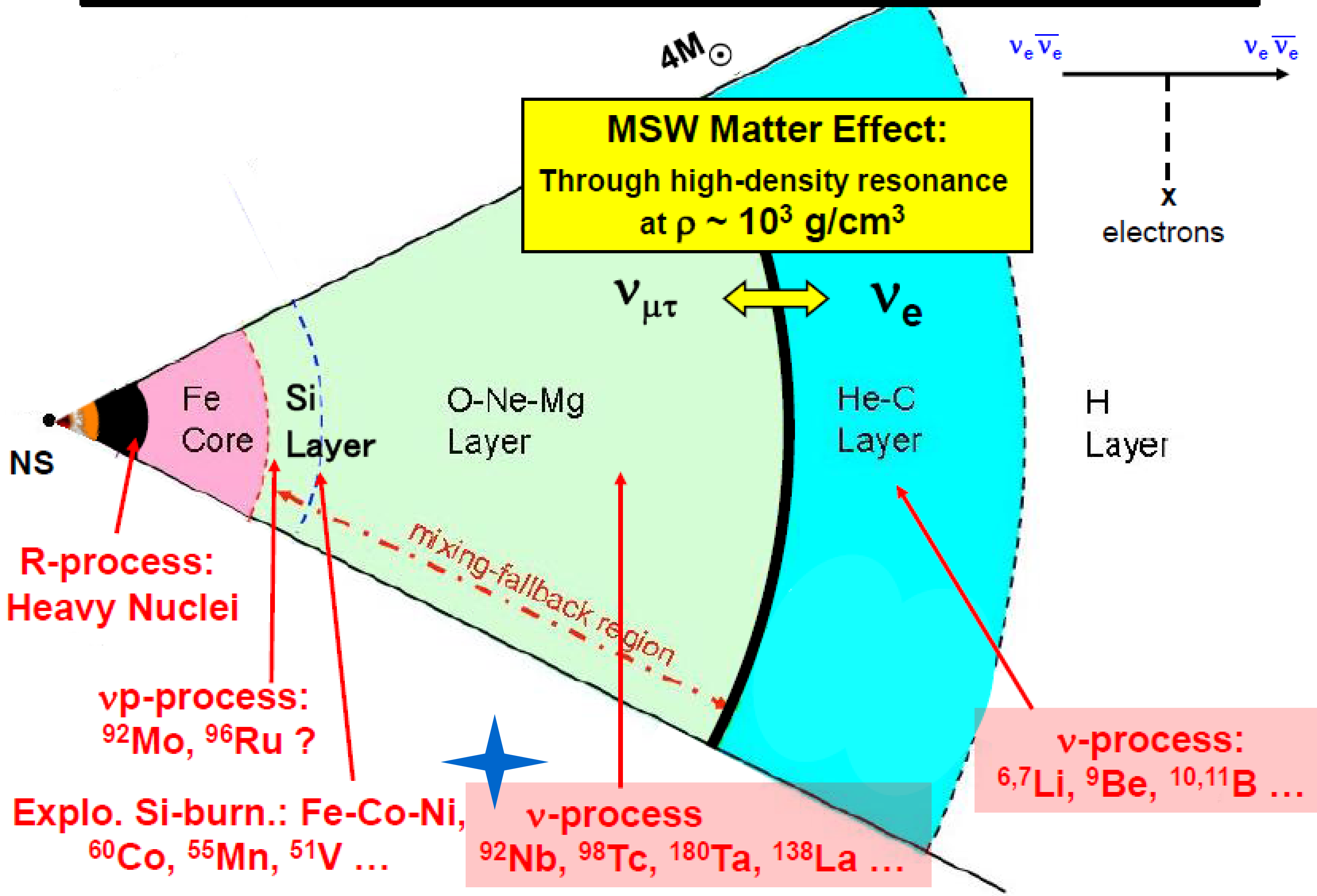
Deformation effects ???

- FRDM+QRPA calculation underpredicts the $T_{1/2}$ of the **N=65** isotones for Rb, Sr, Y, Zr, and Nb. [P. Möller *et al*., At. Data Nucl. Data Tables 66, 131(1997)]
- The KTUY+GT2 model overestimates the $T_{1/2}$ for Mo and Tc below **N=70**.

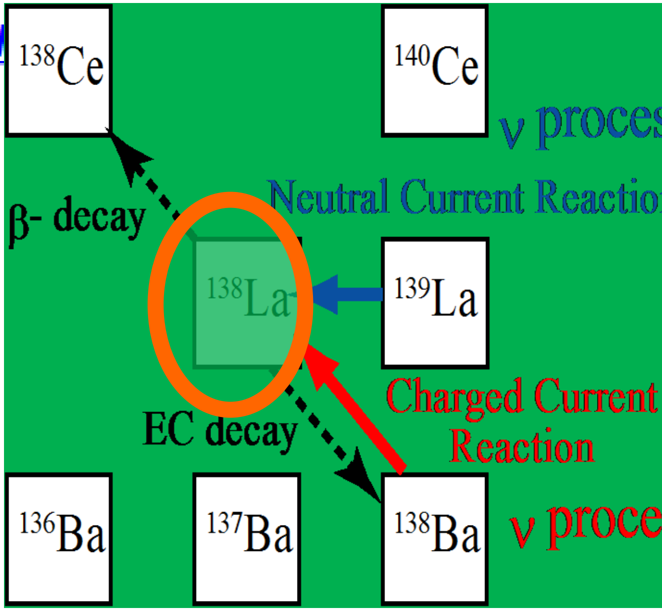
Recipe to reproduce solar r-elements



Neutrino Oscillation and SN-Nucleosynthesis



Theoretical Calculation for ν–Nucleus Cross Sections



New Shell Model cal. with NEW Hamiltonian: ν-^{12}C, ^{4}H

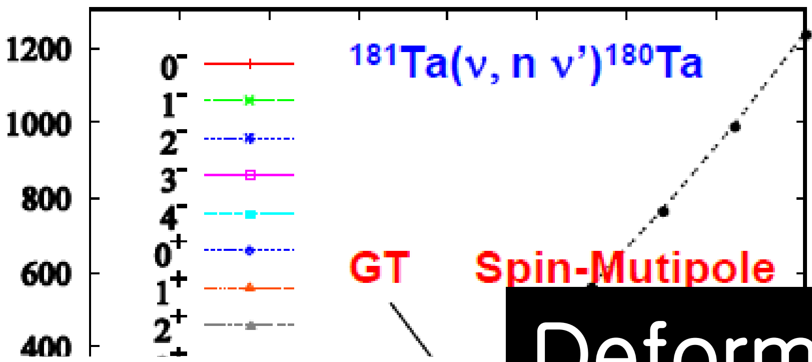
Suzuki, Chiba, Yoshida, Kajino & Otsuka, PR C74 (2006), 034307.
Suzuki, Fujimoto & Otsuka, PR C67, 044302 (2003)

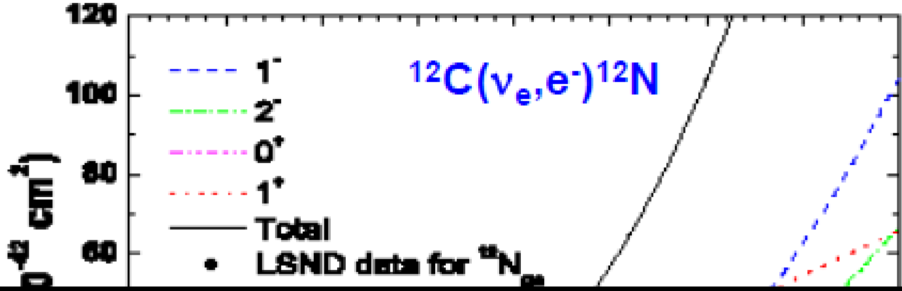
^{12}C: New Hamiltonian = Spin-isospin flip int. with tensor force to explain neutron-rich exotic nuclei.

- μ-moments of p-shell nuclei
- GT strength for ^{12}C→^{12}N, ^{14}C→^{14}N, etc. (GT)
- DAR (ν,ν'), (ν,e-) cross sections

QRPA cal.: ν-^{180}Ta, ^{138}La, ^{98}Tc, ^{92}Nb, ^{42}Ca, ^{12}C, ^{4}He…

Cheoun, et al., PRC81 (2010), 028501; PRC82 (2010), 035504;
J. Phys. G37 (2010), 055101; PRC 83 (2011), 028801





Deformation (effects) Nuclei ???

THE ASTROPHYSICAL JOURNAL LETTERS, 779:L9 (5p

SUPERNOVA NEUTRINO NUCLEOSYNTHESIS OF THE RADIOACTIVE ^{92}Nb OBSERVED IN PRIMITIVE METEORITES

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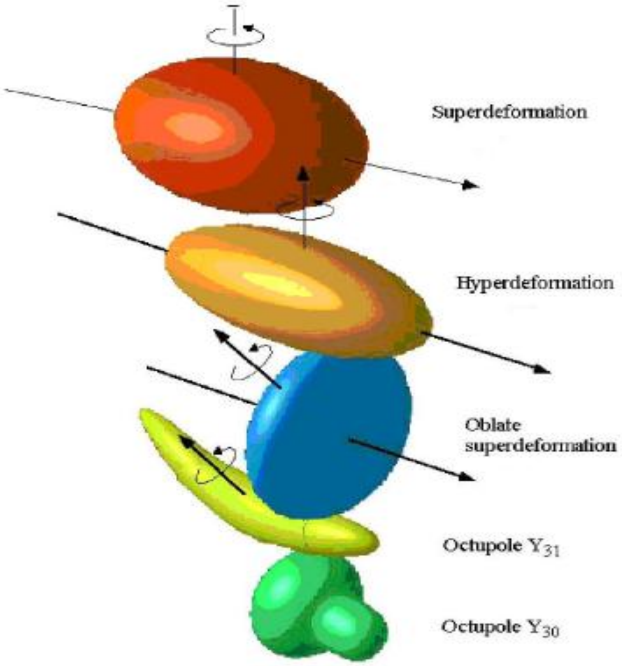
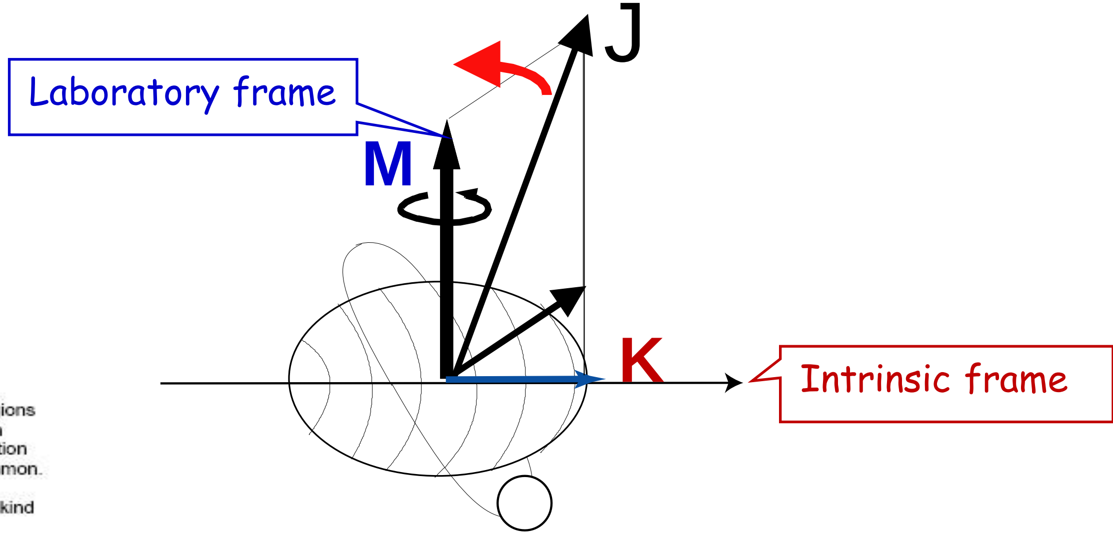
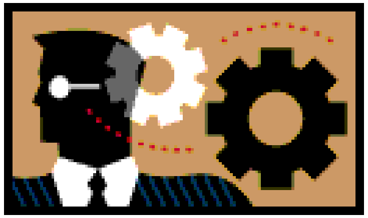


Figure 1.2. Various shapes observed or expected in nuclei. Exotic orbitals that appear in regions far from the stability line may provide some new types of deformation. The superdeformation (top) and pear shape (bottom) have been observed experimentally; the oblate superdeformation has been predicted but not observed—less deformed oblate shapes are, however, quite common. The hyperdeformation (second from the top) has been seen in certain nuclei. The octupole banana-type deformation has not been observed in such extreme form, but vibrations of this kind are well known.





❖ Total Hamiltonian of a many body system

In deformed basis Hamiltonian can be written as

$$H = H_0 + H_{\rm int} \ ,$$

$$H_0 = \sum_{\alpha\rho_\alpha\alpha'} \epsilon_{\alpha\rho_\alpha\alpha'} c^\dagger_{\alpha\rho_\alpha\alpha'} c_{\alpha\rho_\alpha\alpha'} \qquad (\alpha' = p, n) \ ,$$

$$H_{\rm int} = \sum_{\alpha\beta\gamma\delta\rho_\alpha\rho_\beta\rho_\gamma\rho_\delta,\ \alpha'\beta'\gamma'\delta'} V_{\alpha\rho_\alpha\alpha'\beta\rho_\beta\beta'\gamma\rho_\gamma\gamma'\delta\rho_\delta\delta'} c^\dagger_{\alpha\rho_\alpha\alpha'} c^\dagger_{\beta\rho_\beta\beta'} c_{\delta\rho_\delta\delta'} c_{\gamma\rho_\gamma\gamma'} ,$$

where, α : single particle state.

$\rho_\alpha (\pm 1)$: sign of the angular momentum projection .

Ω_α : projection of the total angular momentum on the nuclear symmetry axis.

$-\Omega_\alpha$: time reversal state.

❖Single particle states (SPSs)

The SPSs are calculated from the eigen-equation of the total Hamiltonian in a deformed (Nilsson) basis obtained by the deformed axially symmetric Woods-Saxon potential.

Obtained by the eigenvalue Eq. of the total Hamiltonian

Deformed harmonic oscillator wave function

$$|\alpha\rho_\alpha = +1 >= \sum_{Nn_z} [b^{(+)}_{Nn_z\Omega_\alpha} |N, n_z, \Lambda_\alpha, \Omega_\alpha = \Lambda_\alpha + 1/2 >$$
$$+ b^{(-)}_{Nn_z\Omega_\alpha} |N, n_z, \Lambda_\alpha + 1, \Omega_\alpha = \Lambda_\alpha + 1 - 1/2 >],$$

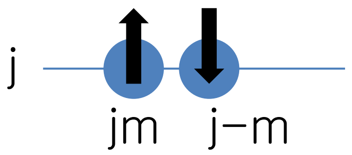
N : main quantum number in deformed basis
n_z : the numbers of node the basis function in z direction
Λ : the projection of the orbital angular momentum onto the z axis

The time –reversed state is

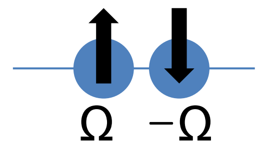
$$|\alpha\rho_\alpha = -1 >= \sum_{Nn_z} [b^{(+)}_{Nn_z\Omega_\alpha} |N, n_z, -\Lambda_\alpha, \Omega_\alpha = -\Lambda_\alpha - 1/2 >$$
$$- b^{(-)}_{Nn_z\Omega_\alpha} |N, n_z, -\Lambda_\alpha - 1, \Omega_\alpha = -\Lambda_\alpha - 1 + 1/2 >]$$

❖ Deformed BCS for the ground state

spherical basis



deformed basis



$\Omega = 1/2$

j	m
$\frac{1}{2}$	$\frac{1}{2}$
$\frac{3}{2}$	$\frac{1}{2}$
$\frac{5}{2}$	$\frac{1}{2}$
$\frac{7}{2}$	$\frac{1}{2}$
…	

Since the deformed s. p. states are expanded in terms of a spherical s. p. basis,
the s. p. states with different orbital and total angular momenta in the spherical basis states would be mixed.

❖ DQRPA eq with neutron-proton pairing correlations.

$$\begin{pmatrix} A^{1111}_{\alpha\beta\gamma\delta}(K) & A^{1122}_{\alpha\beta\gamma\delta}(K) & A^{1112}_{\alpha\beta\gamma\delta}(K) & B^{1111}_{\alpha\beta\gamma\delta}(K) & B^{1122}_{\alpha\beta\gamma\delta}(K) & B^{1112}_{\alpha\beta\gamma\delta}(K) \\ A^{2211}_{\alpha\beta\gamma\delta}(K) & A^{2222}_{\alpha\beta\gamma\delta}(K) & A^{2212}_{\alpha\beta\gamma\delta}(K) & B^{2211}_{\alpha\beta\gamma\delta}(K) & B^{2222}_{\alpha\beta\gamma\delta}(K) & B^{2212}_{\alpha\beta\gamma\delta}(K) \\ A^{1211}_{\alpha\beta\gamma\delta}(K) & A^{1222}_{\alpha\beta\gamma\delta}(K) & A^{1212}_{\alpha\beta\gamma\delta}(K) & B^{1211}_{\alpha\beta\gamma\delta}(K) & B^{1222}_{\alpha\beta\gamma\delta}(K) & B^{1212}_{\alpha\beta\gamma\delta}(K) \\ -B^{1111}_{\alpha\beta\gamma\delta}(K) & -B^{1122}_{\alpha\beta\gamma\delta}(K) & -B^{1112}_{\alpha\beta\gamma\delta}(K) & -A^{1111}_{\alpha\beta\gamma\delta}(K) & -A^{1122}_{\alpha\beta\gamma\delta}(K) & -A^{1112}_{\alpha\beta\gamma\delta}(K) \\ -B^{2211}_{\alpha\beta\gamma\delta}(K) & -B^{2222}_{\alpha\beta\gamma\delta}(K) & -B^{2212}_{\alpha\beta\gamma\delta}(K) & -A^{2211}_{\alpha\beta\gamma\delta}(K) & -A^{2222}_{\alpha\beta\gamma\delta}(K) & -A^{2212}_{\alpha\beta\gamma\delta}(K) \\ -B^{1211}_{\alpha\beta\gamma\delta}(K) & -B^{1222}_{\alpha\beta\gamma\delta}(K) & -B^{1212}_{\alpha\beta\gamma\delta}(K) & -A^{1211}_{\alpha\beta\gamma\delta}(K) & -A^{1222}_{\alpha\beta\gamma\delta}(K) & -A^{1212}_{\alpha\beta\gamma\delta}(K) \end{pmatrix} \begin{pmatrix} \tilde{X}^m_{(\gamma 1 \delta 1)K} \\ \tilde{X}^m_{(\gamma 2 \delta 2)K} \\ \tilde{X}^m_{(\gamma 1 \delta 2)K} \\ \tilde{Y}^m_{(\gamma 1 \delta 1)K} \\ \tilde{Y}^m_{(\gamma 2 \delta 2)K} \\ \tilde{Y}^m_{(\gamma 1 \delta 2)K} \end{pmatrix} = \hbar\Omega^m_K \begin{pmatrix} \tilde{X}^m_{(\alpha 1 \beta 1)K} \\ \tilde{X}^m_{(\alpha 2 \beta 2)K} \\ \tilde{X}^m_{(\alpha 1 \beta 2)K} \\ \tilde{Y}^m_{(\alpha 1 \beta 1)K} \\ \tilde{Y}^m_{(\alpha 2 \beta 2)K} \\ \tilde{Y}^m_{(\alpha 1 \beta 2)K} \end{pmatrix}$$

$$A^{\alpha''\beta'',\,\gamma''\delta''}_{\alpha\bar\beta,\,\gamma\bar\delta}(K) = (E_{\alpha\alpha''}+E_{\beta\beta''})\delta_{\alpha\gamma}\delta_{\alpha''\gamma''}\delta_{\bar\beta\bar\delta}\delta_{\beta''\delta''} - \sigma_{\alpha\alpha''\bar\beta\beta''}\sigma_{\gamma\gamma''\bar\delta\delta''}$$
$$\times\,[g_{pp}(u_{\alpha\alpha''}u_{\bar\beta\beta''}u_{\gamma\gamma''}u_{\bar\delta\delta''}+v_{\alpha\alpha''}v_{\bar\beta\beta''}v_{\gamma\gamma''}v_{\bar\delta\delta''})\,V^K_{\alpha\bar\beta,\,\gamma\bar\delta}$$
$$+\,g_{ph}(u_{\alpha\alpha''}v_{\bar\beta\beta''}u_{\gamma\gamma''}v_{\bar\delta\delta''}+v_{\alpha\alpha''}u_{\bar\beta\beta''}v_{\gamma\gamma''}u_{\bar\delta\delta''})\,V^K_{\alpha\delta,\,\gamma\beta}$$
$$+\,g_{ph}(u_{\alpha\alpha''}v_{\bar\beta\beta''}v_{\gamma\gamma''}u_{\bar\delta\delta''}+v_{\alpha\alpha''}u_{\bar\beta\beta''}u_{\gamma\gamma''}v_{\bar\delta\delta''})\,V^K_{\alpha\gamma,\,\delta\beta}],$$

$$B^{\alpha''\beta'',\,\gamma''\delta''}_{\alpha\bar\beta,\,\gamma\bar\delta}(K) = -\,\sigma_{\alpha\alpha''\bar\beta\beta''}\sigma_{\gamma\gamma''\bar\delta\delta''}$$
$$\times\,[-g_{pp}(u_{\alpha\alpha''}u_{\bar\beta\beta''}v_{\gamma\gamma''}v_{\bar\delta\delta''}+v_{\alpha\alpha''}v_{\bar\beta\beta''}u_{\gamma\gamma''}u_{\bar\delta\delta''})\,V^K_{\alpha\bar\beta,\,\gamma\bar\delta}$$
$$+\,g_{ph}(u_{\alpha\alpha''}v_{\bar\beta\beta''}v_{\gamma\gamma''}u_{\bar\delta\delta''}+v_{\alpha\alpha''}u_{\bar\beta\beta''}u_{\gamma\gamma''}v_{\bar\delta\delta''})\,V^K_{\alpha\delta,\,\gamma\beta}$$
$$+\,g_{ph}(u_{\alpha\alpha''}v_{\bar\beta\beta''}u_{\gamma\gamma''}v_{\bar\delta\delta''}+v_{\alpha\alpha''}u_{\bar\beta\beta''}v_{\gamma\gamma''}u_{\bar\delta\delta''})\,V^K_{\alpha\gamma,\,\delta\beta}],$$

$$V^K_{\alpha\bar\beta,\,\gamma\bar\delta} = \sum_J\sum_{abcd} F^{JK}_{\alpha a\bar\beta b}F^{JK}_{\gamma c\bar\delta d}\,G(abcd,\ J)\ ,$$
$$V^K_{\alpha\delta,\,\gamma\beta} = \sum_J\sum_{abcd} F^{JK'}_{\alpha a\bar\delta d}F^{JK'}_{\gamma c\bar\beta b}\,G(adcb,\ J)\ ,$$
$$V^K_{\alpha\gamma,\,\delta\beta} = \sum_J\sum_{abcd} F^{JK}_{\alpha a\gamma c}F^{JK}_{\bar\beta b\bar\delta d}\,G(acdb,\ J).$$

By Including deformation

where $F^{JK'}_{\alpha a\beta b} = B^{\alpha}_{a}\ B^{\beta}_{b}\ C^{JK'}_{j_a\Omega_\alpha j_b\Omega_\beta}$ with $K' = \Omega_\alpha + \Omega_\beta$.

Realistic two body interaction was taken by Brueckner G-matrix, which is a solution of the Bethe-Goldstone Eq., derived from the Bonn-CD one-boson exchange potential.

Expansion of the deformed state by a spherical basis

- To exploit G-matrix elements and matrix elements of transition operators, which are calculated on the spherical basis, the deformed basis is expanded in terms of a spherical basis.

$$|\alpha\Omega\rangle = \sum_{Nn_z} b_{Nn_z\Omega}\left(|N, n_z, \Lambda\rangle\right)$$

$$\sum_a |a\Omega\rangle \quad (a \equiv n\ell j)$$

Sph. harmonic oscillator w. f.

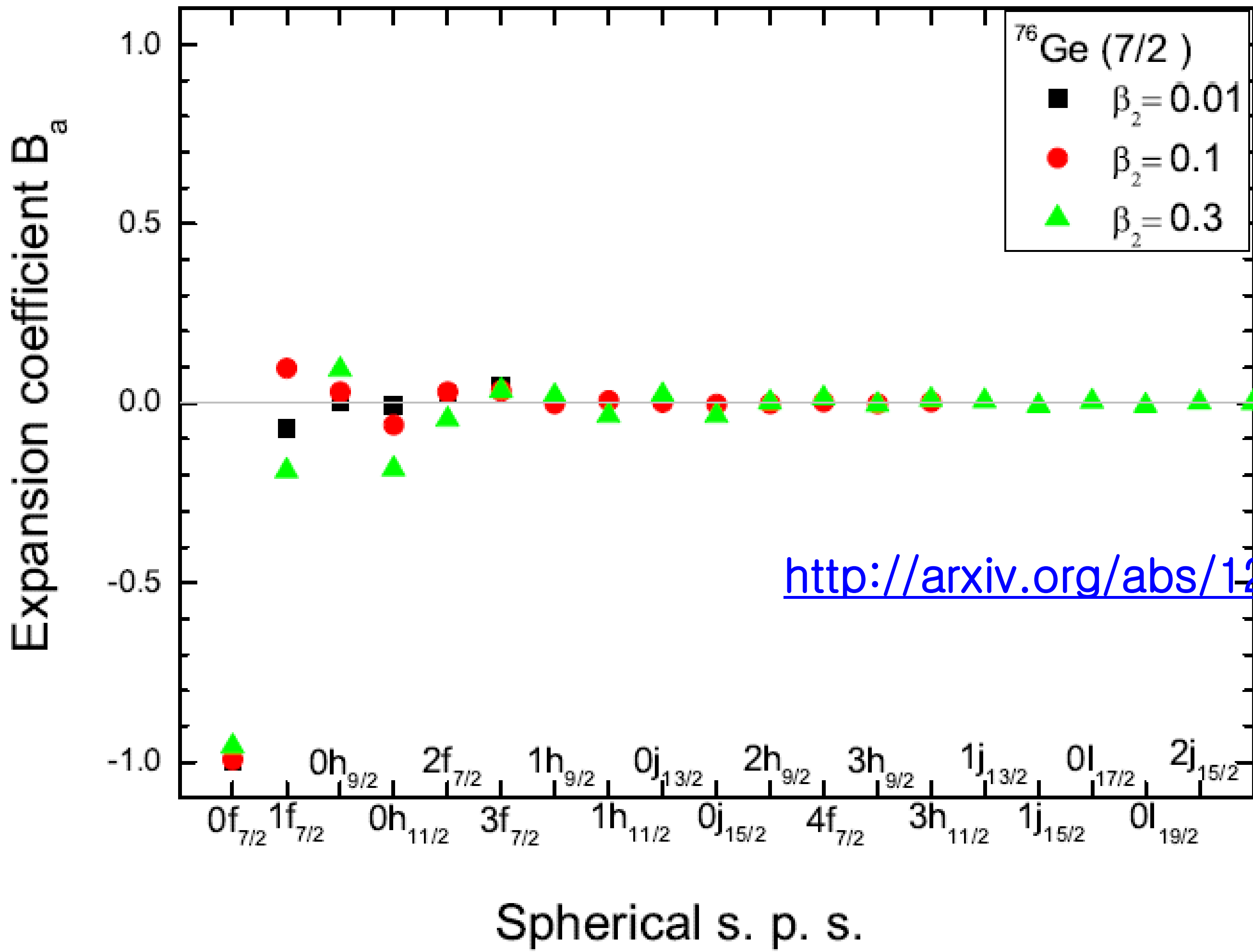
$$|\alpha\Omega_\alpha >= \sum_a B_a^\alpha \; |a\Omega_\alpha >,$$

The expansion coefficient B is

$$B_a^\alpha = \sum_{Nn_z\Sigma} C_{l\Lambda\frac{1}{2}\Sigma}^{j\Omega_\alpha} A_{Nn_z\Lambda}^{N_0 l} \; b_{Nn_z\Sigma}.$$

C-G coef. of orbital & spin | spatial overlap integral | eigenvalue eq. of the total Hamiltonian

❖ Expansion coefficient B with different β_2 values

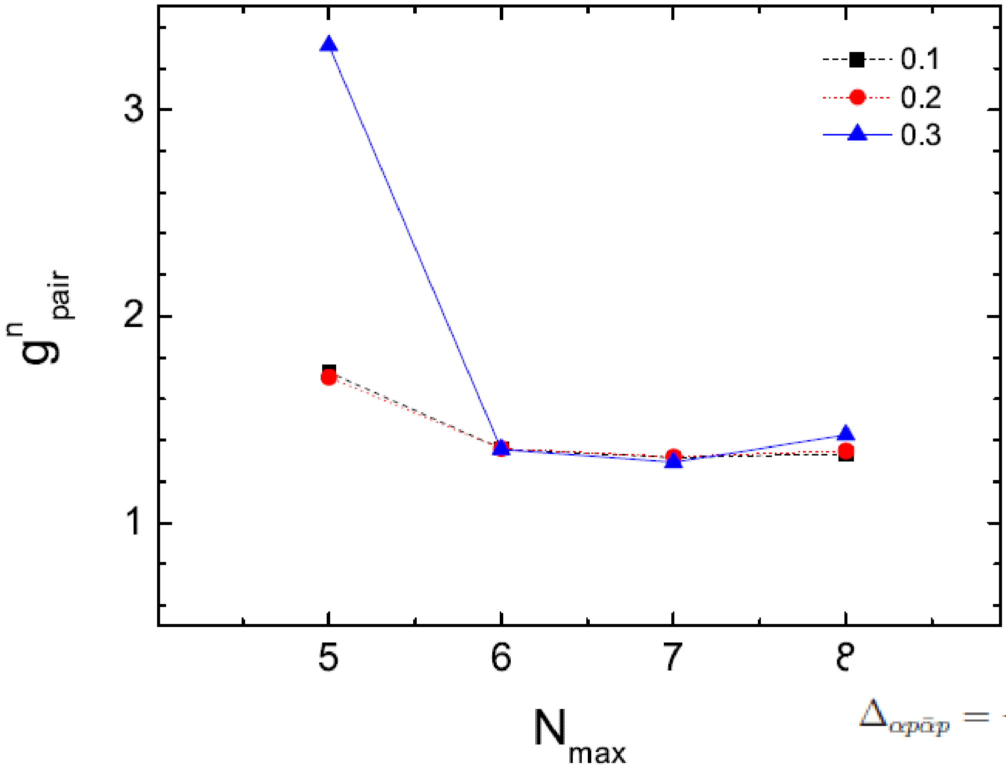


http://arxiv.org/abs/1205.4561 v4

➢ Number of the spherical s. p. basis increases as the β_2 value increases.

❖ Particle model space N_{max} & pairing strength g_{pair}

The particle model space 5hω is not enough to reproduce the empirical pairing gap. Therefore, the particle model space can be used beyond 6hω in G−matrix. In this calculation we use N_{max}=10hω in G−matrix. (5hω in deformed basis)



$$\Delta_{\alpha p \bar{\alpha} p} = -\frac{1}{2} \sum_{J,c} g^p_{\text{pair}} F^{J0}_{\alpha a \bar{\alpha} a} F^{J0}_{\gamma c \bar{\gamma} c} G(aacc, J)(u^*_{1p_c} v_{1p_c} + u^*_{2p_c} v_{2p_c}) \;,$$

FIG. 2: (Color online) Dependence of neutron pairing strength g^n_{pair} in Eq. (11) on particle model space N^{sph}_{max} in $G-$matrix. Black dashed, red dotted, and blue solid points are results for $\beta_2 = 0.1$, 0.2, and 0.3, respectively.

❖ Strength of particle-particle and particle-hole.

$$A^{\alpha''\beta'',\,\gamma''\delta''}_{\alpha\bar\beta,\,\gamma\bar\delta}(K) = (E_{\alpha\alpha''}+E_{\beta\beta''})\delta_{\alpha\gamma}\delta_{\alpha''\gamma''}\delta_{\bar\beta\bar\delta}\delta_{\beta''\delta''} - \sigma_{\alpha\alpha''\bar\beta\beta''}\sigma_{\gamma\gamma''\bar\delta\delta''}$$
$$\times\,[g_{pp}(u_{\alpha\alpha''}u_{\bar\beta\beta''}u_{\gamma\gamma''}u_{\bar\delta\delta''}+v_{\alpha\alpha''}v_{\bar\beta\beta''}v_{\gamma\gamma''}v_{\bar\delta\delta''})\,V^K_{\alpha\bar\beta,\,\gamma\bar\delta}$$
$$+\,g_{ph}(u_{\alpha\alpha''}v_{\bar\beta\beta''}u_{\gamma\gamma''}v_{\bar\delta\delta''}+v_{\alpha\alpha''}u_{\bar\beta\beta''}v_{\gamma\gamma''}u_{\bar\delta\delta''})\,V^K_{\alpha\delta,\,\gamma\beta}$$
$$+\,g_{ph}(u_{\alpha\alpha''}v_{\bar\beta\beta''}v_{\gamma\gamma''}u_{\bar\delta\delta''}+v_{\alpha\alpha''}u_{\bar\beta\beta''}u_{\gamma\gamma''}v_{\bar\delta\delta''})\,V^K_{\alpha\gamma,\,\delta\beta}],$$

$$B^{\alpha''\beta'',\,\gamma''\delta''}_{\alpha\bar\beta,\,\gamma\bar\delta}(K) = -\,\sigma_{\alpha\alpha''\bar\beta\beta''}\sigma_{\gamma\gamma''\bar\delta\delta''}$$
$$\times\,[-g_{pp}(u_{\alpha\alpha''}u_{\bar\beta\beta''}v_{\gamma\gamma''}v_{\bar\delta\delta''}+v_{\alpha\alpha''}v_{\bar\beta\beta''}u_{\gamma\gamma''}u_{\bar\delta\delta''})\,V^K_{\alpha\bar\beta,\,\gamma\bar\delta}$$
$$+\,g_{ph}(u_{\alpha\alpha''}v_{\bar\beta\beta''}v_{\gamma\gamma''}u_{\bar\delta\delta''}+v_{\alpha\alpha''}u_{\bar\beta\beta''}u_{\gamma\gamma''}v_{\bar\delta\delta''})\,V^K_{\alpha\delta,\,\gamma\beta}$$
$$+\,g_{ph}(u_{\alpha\alpha''}v_{\bar\beta\beta''}u_{\gamma\gamma''}v_{\bar\delta\delta''}+v_{\alpha\alpha''}u_{\bar\beta\beta''}v_{\gamma\gamma''}u_{\bar\delta\delta''})\,V^K_{\alpha\gamma,\,\delta\beta}],$$

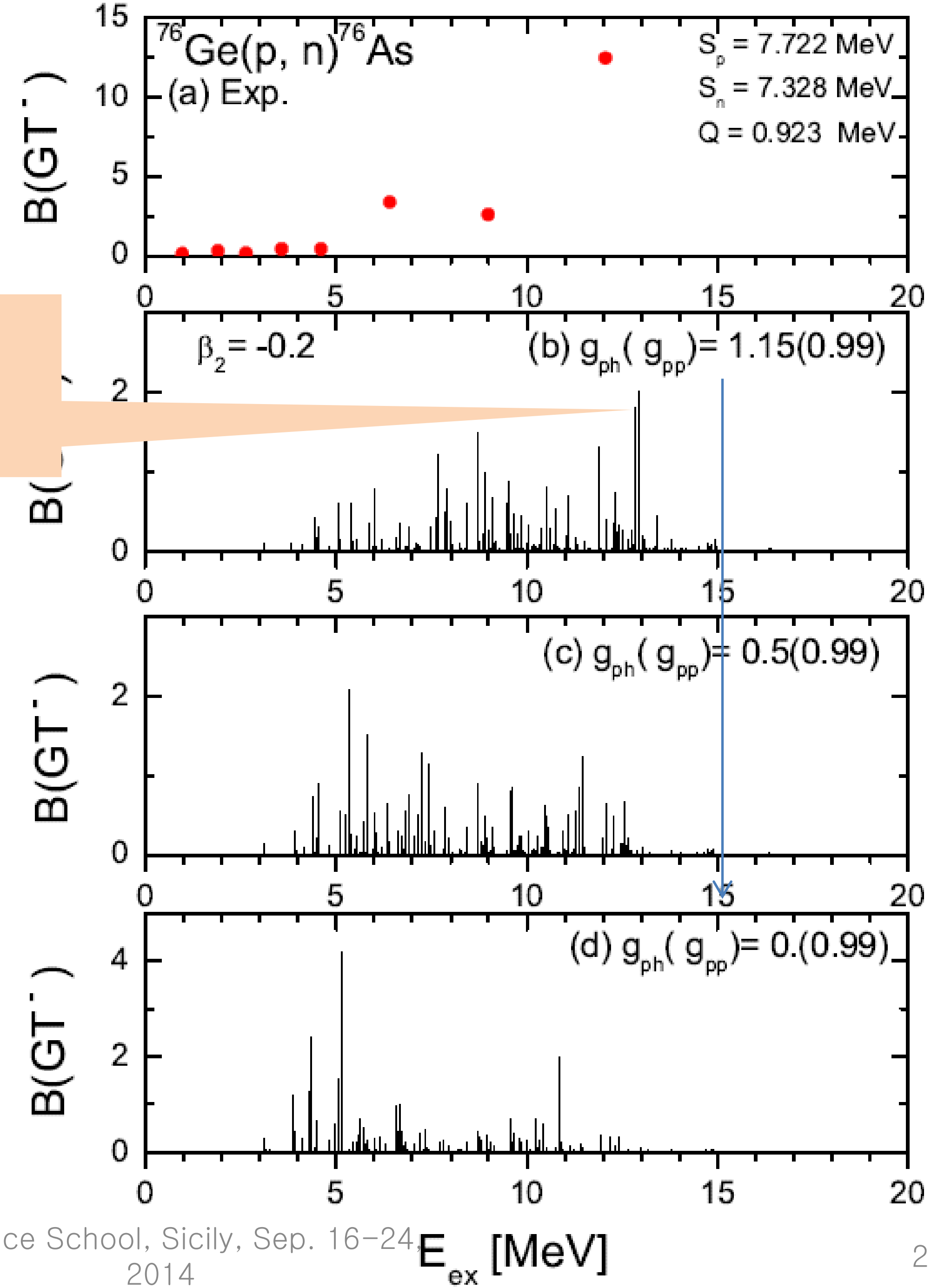
$$V^K_{\alpha\bar\beta,\,\gamma\bar\delta} = \sum_J\sum_{abcd} F^{JK}_{\alpha a\bar\beta b}F^{JK}_{\gamma c\bar\delta d}\,G(abcd,\ J)\ ,$$
$$V^K_{\alpha\delta,\,\gamma\beta} = \sum_J\sum_{abcd} F^{JK'}_{\alpha a\bar\delta d}F^{JK'}_{\gamma c\bar\beta b}\,G(adcb,\ J)\ ,$$
$$V^K_{\alpha\gamma,\,\delta\beta} = \sum_J\sum_{abcd} F^{JK'}_{\alpha a\gamma c}F^{JK'}_{\beta b\delta d}\,G(acdb,\ J).$$

g_{ph} is determined by adjusting the calculated positions of the GT giant resonances for ^{48}Ca, ^{90}Zr, and ^{208}Pb.
g_{pp} is determined by a fitting procedure to β-decay half-lives of nuclei with Z≤ 40.

where $F^{JK'}_{\alpha a\beta b} = B^{\alpha}_{a}\ B^{\beta}_{b}\ C^{JK'}_{j_a\Omega_\alpha j_b\Omega_\beta}$ with $K' = \Omega_\alpha + \Omega_\beta$.

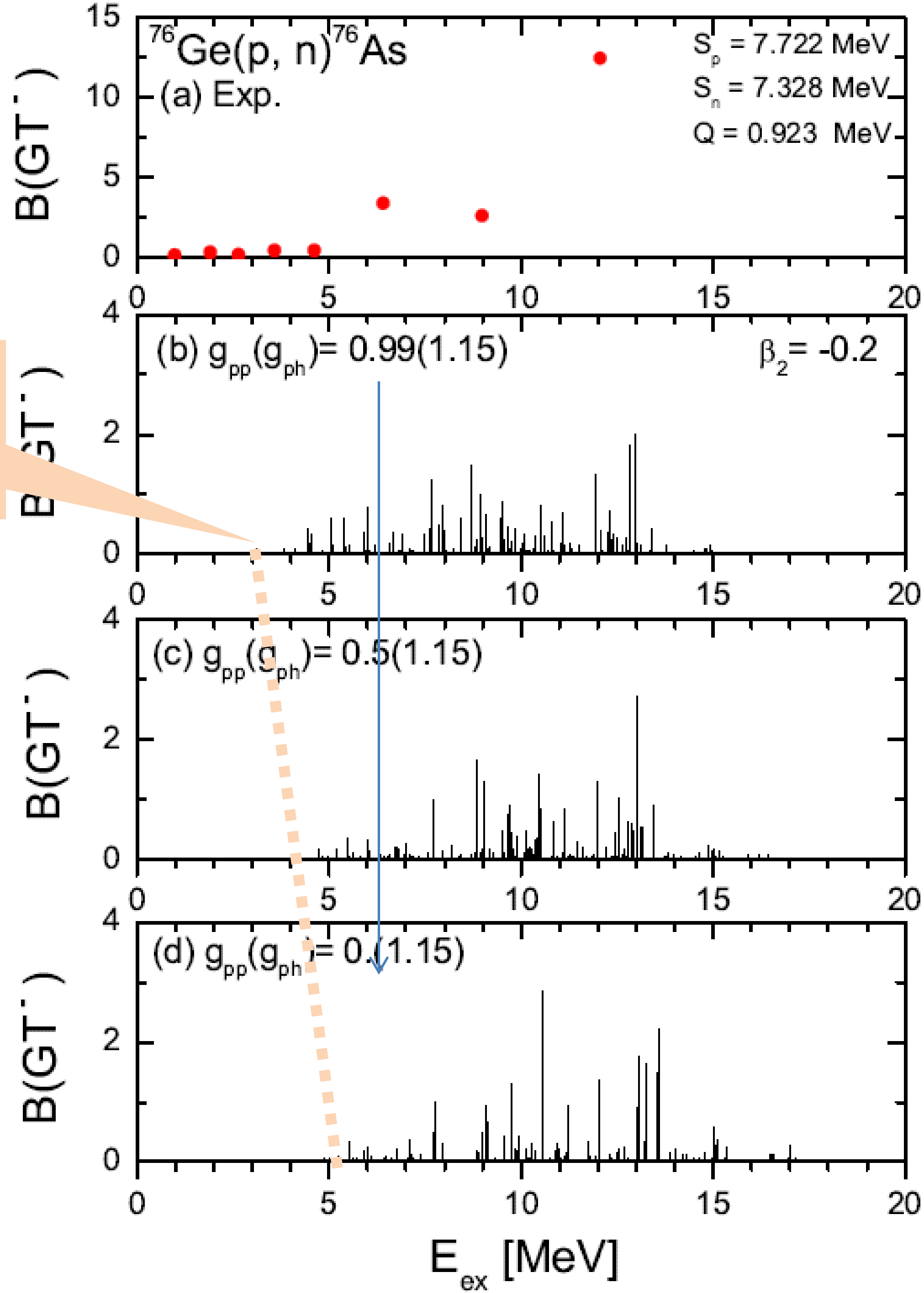
❖ Particle-hole strength g_{ph}

The energy of the GTGR is roughly reproduced.



❖ Particle-particle strength g_{pp}

All GT peaks get shifted to smaller energies as g_{pp} increase.



Test of the expansion method

$$< K^+, m|\hat{\beta}^-_K|\ QRPA > = \sum_{\alpha\rho_\alpha\beta\rho_\beta} < \alpha p\rho_\alpha|\tau^+\sigma_K|\beta n\rho_\beta > \ [u_{\alpha p}v_{\beta n}X^m_{(\alpha p\beta n)K} + v_{\alpha p}u_{\beta n}Y^m_{(\alpha p\beta n)K}]\ ,$$

In deformed bases [case I]

$$< \alpha_p\rho_\alpha|\tau^+\sigma_{K=0}|\beta_n\rho_\beta >= \delta_{\Omega_p\Omega_n}\rho_\alpha \sum_{Nn_z}[b^{(+)}_{Nn_z\Omega_p}b^{(+)}_{Nn_z\Omega_n} - b^{(-)}_{Nn_z\Omega_p}b^{(-)}_{Nn_z\Omega_n}],$$

$$\begin{aligned} < \alpha_p\rho_\alpha|\tau^+\sigma_{K=1}|\beta_n\rho_\beta > &= -\sqrt{2}\delta_{\Omega_p\Omega_n+1}\sum_{Nn_z} b^{(+)}_{Nn_z\Omega_p}b^{(-)}_{Nn_z\Omega_n} \quad (\rho_\alpha = \rho_\beta = +1) \\ &= +\sqrt{2}\delta_{\Omega_p\Omega_n+1}\sum_{Nn_z} b^{(-)}_{Nn_z\Omega_p}b^{(+)}_{Nn_z\Omega_n} \quad (\rho_\alpha = \rho_\beta = -1) \\ &= -\sqrt{2}\delta_{\Omega_p\frac{1}{2}}\delta_{\Omega_n-\frac{1}{2}}\sum_{Nn_z} b^{(+)}_{Nn_z\Omega_p}b^{(+)}_{Nn_z\Omega_n} \quad (\rho_\alpha = +1, \rho_\beta = -1), \end{aligned}$$

$$\begin{aligned} < \alpha_p\rho_\alpha|\tau^+\sigma_{K=-1}|\beta_n\rho_\beta > &= \sqrt{2}\delta_{\Omega_p\Omega_n-1}\sum_{Nn_z} b^{(-)}_{Nn_z\Omega_p}b^{(+)}_{Nn_z\Omega_n} \quad (\rho_\alpha = \rho_\beta = +1) \\ &= -\sqrt{2}\delta_{\Omega_p\Omega_n-1}\sum_{Nn_z} b^{(+)}_{Nn_z\Omega_p}b^{(-)}_{Nn_z\Omega_n} \quad (\rho_\alpha = \rho_\beta = -1) \\ &= +\sqrt{2}\delta_{\Omega_p-\frac{1}{2}}\delta_{\Omega_n\frac{1}{2}}\sum_{Nn_z} b^{(+)}_{Nn_z\Omega_p}b^{(+)}_{Nn_z\Omega_n} \quad (\rho_\alpha = +1, \rho_\beta = -1). \end{aligned}$$

In spherical bases [case II]

$$< \alpha_p\rho_\alpha|\tau^+\sigma_K|\beta_n\rho_\beta > = \sum_{ab} F^{1K}_{\alpha_p a\beta_n b}\frac{< a_p||\tau^+\sigma_K||b_n >}{\sqrt{3}},$$

$$< a_p||\tau^+\sigma_K||b_n > = \sqrt{6}\delta_{n_a n_b}\delta_{l_a l_b}\sqrt{2j_a+1}\sqrt{2j_b+1}(-1)^{l_a+j_a+\frac{3}{2}}\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ j_b & j_a & l_a \end{Bmatrix}.$$

❖ Test of the expansion method for the Gamow-Teller strength

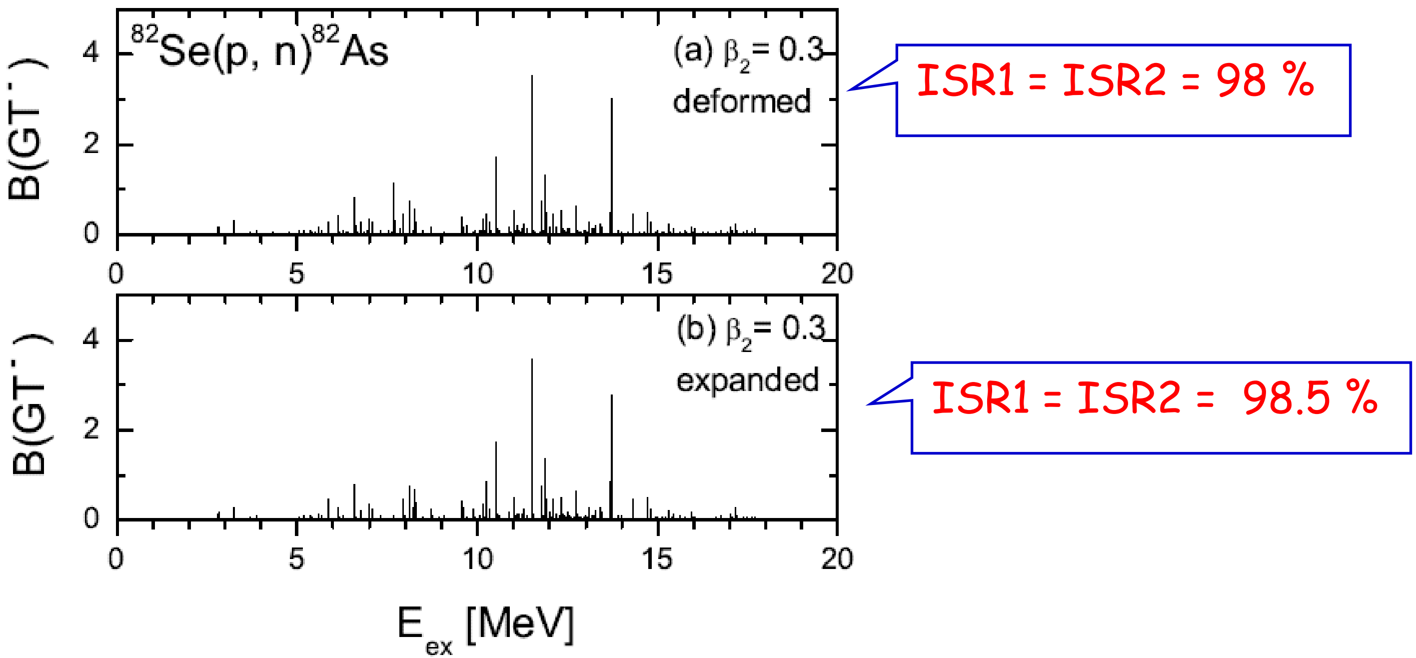


FIG. 5: (Color online) GT strength distributions for ^{82}Se at $\beta_2 = 0.3$. They are calculated in deformed basis by Eq. (30)~(32) (a) and in the spherical basis by Eq. (33) (b).

❖ Test of the DQRPA by the IKEDA sum rule w and w/o the closure relation

$$(S^-_{GT} - S^+_{GT})_{ISR\ II} = \sum_{K=0,\pm1} \sum_{m} [\,|<K^+, m|\hat{\beta}^-_K|\ QRPA>|^2 - |<K^+, m|\hat{\beta}^+_K|\ QRPA>|^2]$$

$$= \sum_{K=0,\pm1} \sum_{m} \sum_{\alpha\rho_\alpha\beta\rho_\beta} |<\alpha p\rho_\alpha|\tau^+\sigma_K|\beta n\rho_\beta>|^2\ (u^2_{\alpha p}v^2_{\beta n} - v^2_{\alpha p}u^2_{\beta n})[(X^m_{(\alpha p\beta n)K})^2 - (Y^m_{(\alpha p\beta n)K})^2]$$

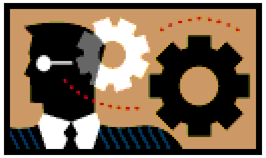
$$(S^-_{GT} - S^+_{GT})_{ISR\ I} = \sum_{K=0,\pm1} \sum_{\alpha\rho_\alpha\beta\rho_\beta} |<\alpha p\rho_p|\tau^+\sigma_K|\beta n\rho_n>|^2(v^2_n - v^2_p)\ .$$

Results of the Gamow-Teller strength distributions

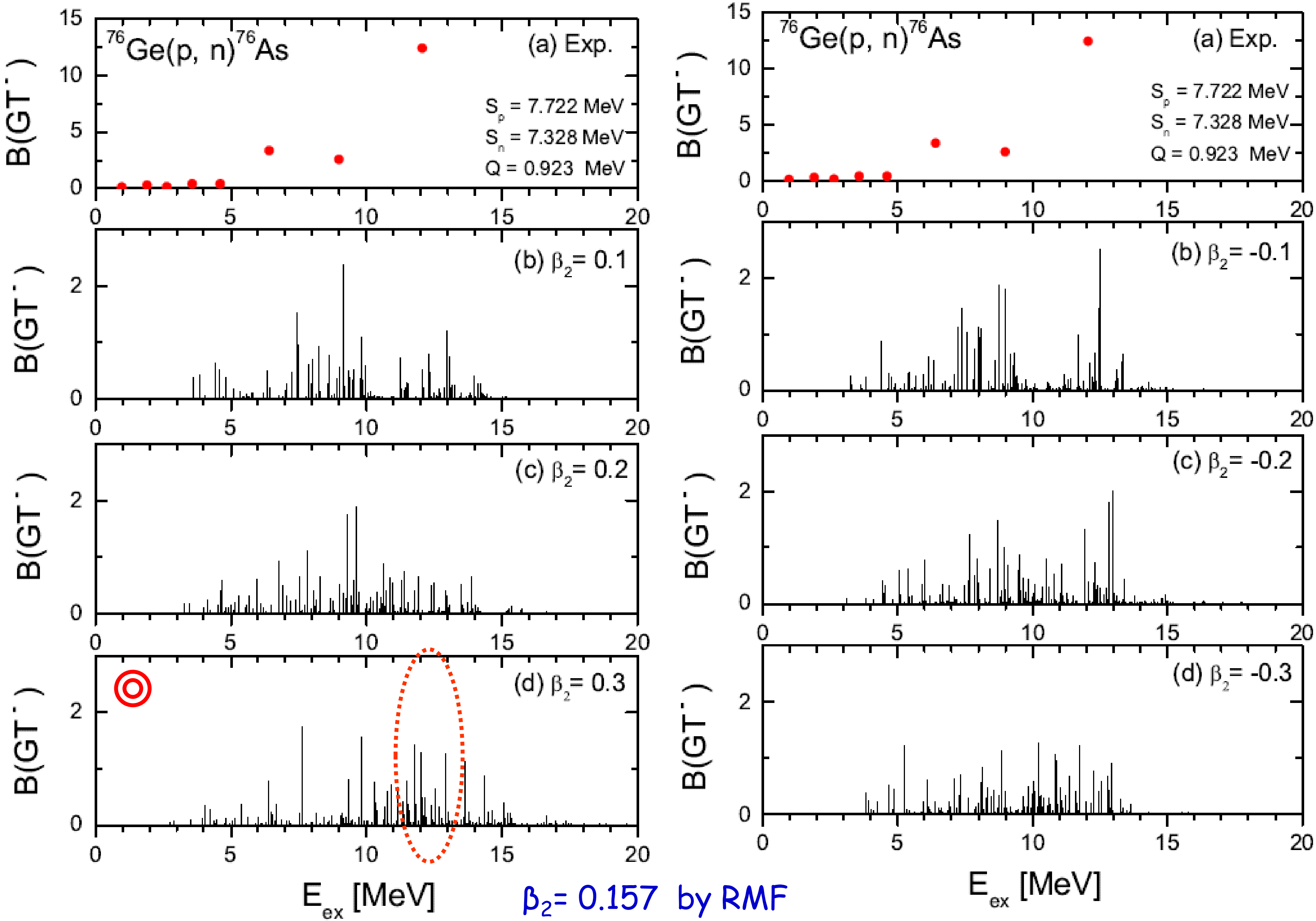
http://arxiv.org/abs/1205.4561 v4

[11] E. Ha and M-K. Cheoun, Phys. Rev. C **88**, 017603 (2013).

[12] E. Ha and M-K. Cheoun, Few Body Syst. **54** 1389-1392 (2013).

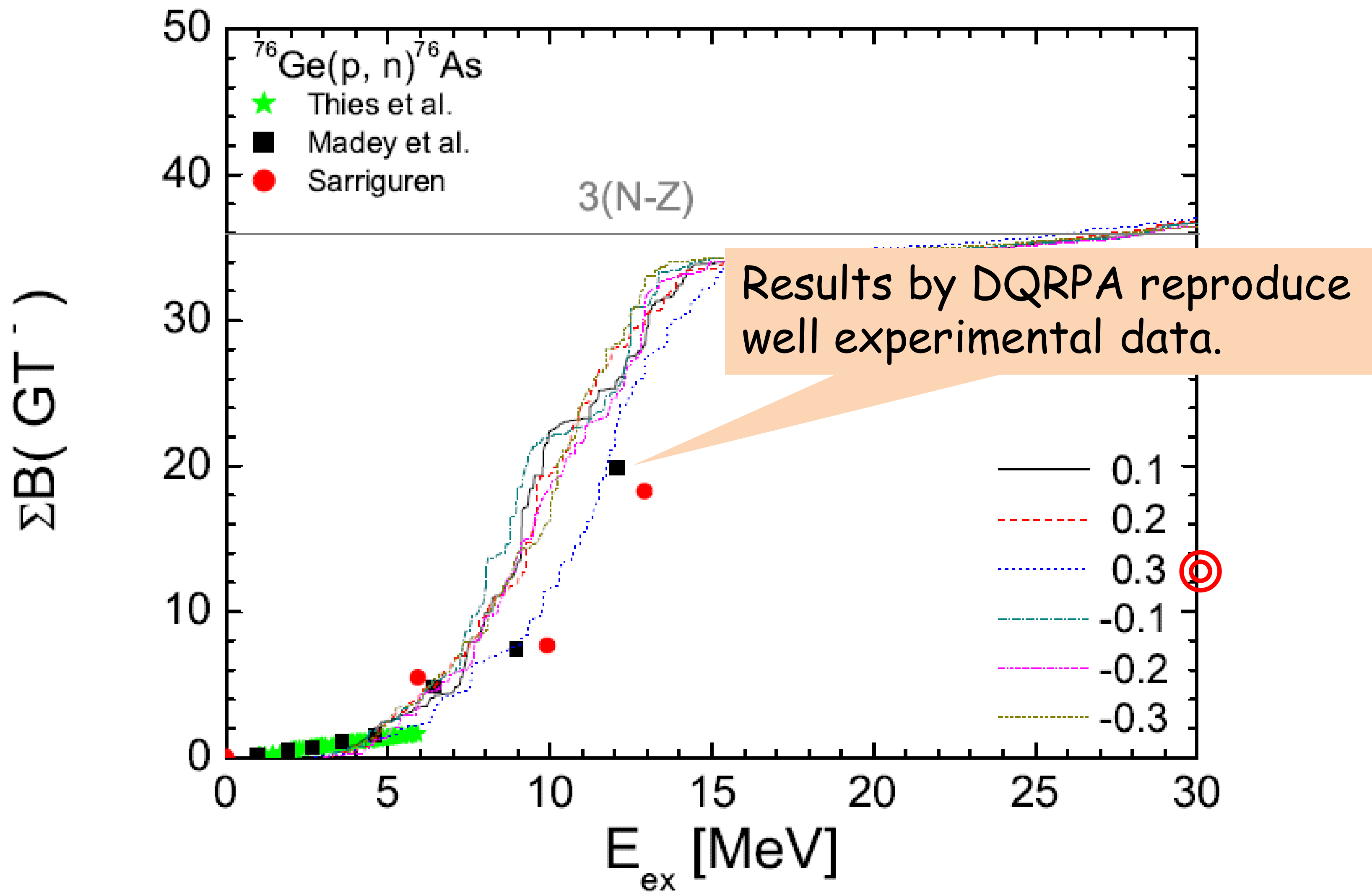


❖ GT(-) strength for ^{76}Ge with different β_2 value



β_2= 0.157 by RMF
0.262 from B(E2)

❖ Running sum of GT(-) strength for ^{76}Ge

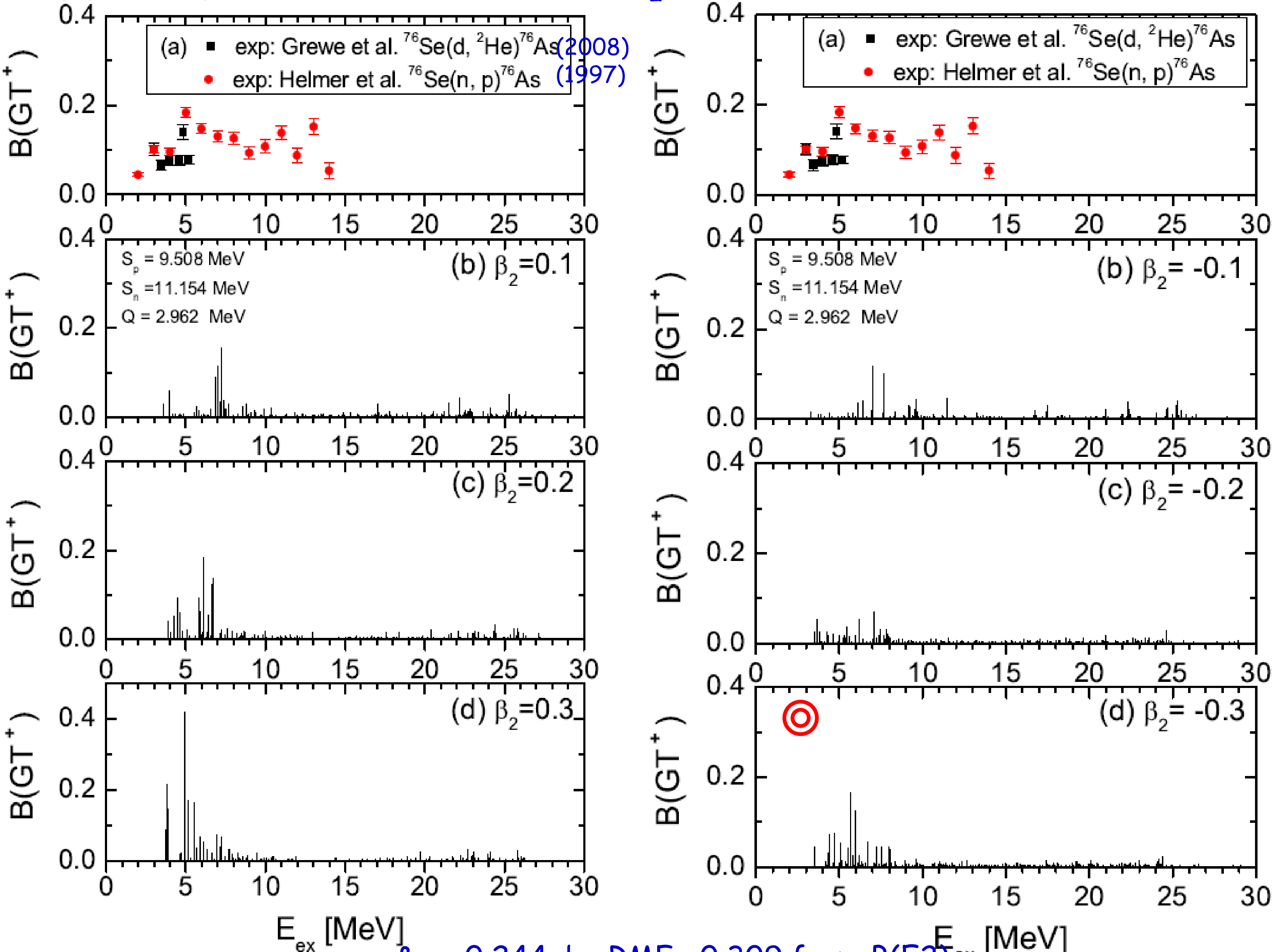


ISR_{exp} = 55% (up to 12MeV)

There may be a possibility of the high-lying GT state above 12.0 MeV.

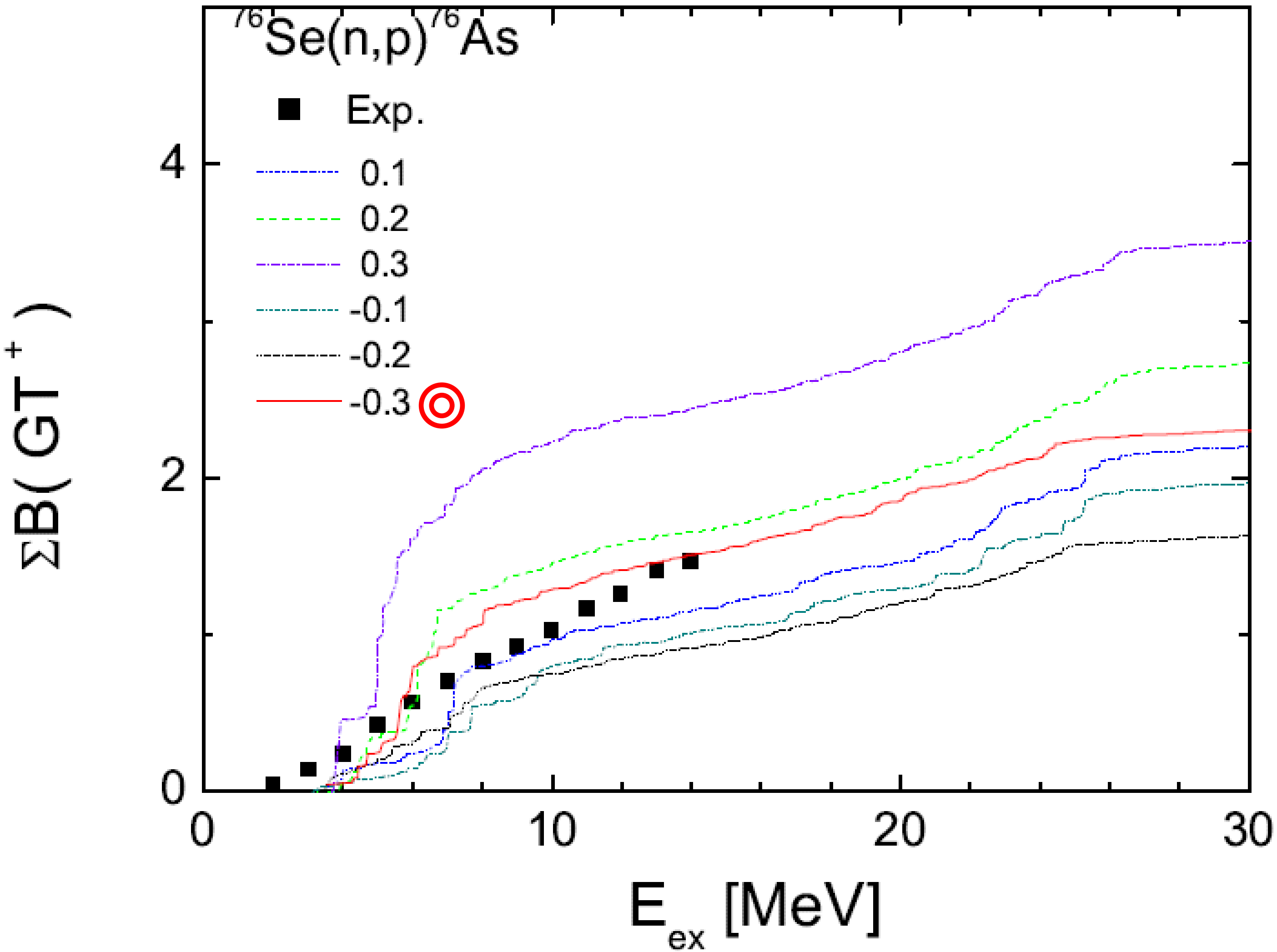
$ISR_{DQRPA} \approx 98$ %

❖ GT(+) strength for ^{76}Se with different β_2 value

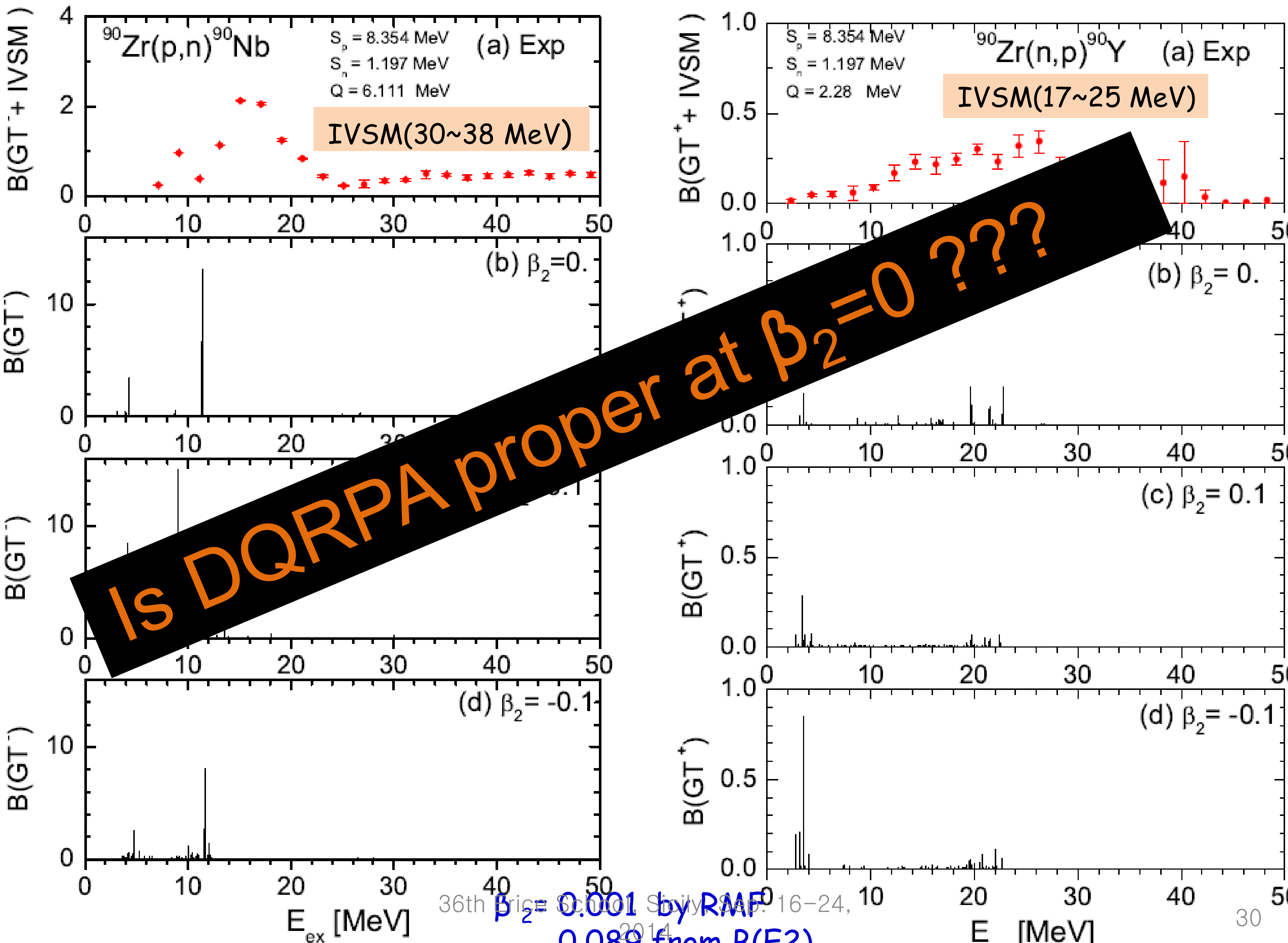


β_2= -0.244 by RMF, 0.309 from B(E2)

❖ Running sum of GT(+) strength for ^{76}Se



❖ GT(-,+) strength for ^{90}Zr with different β_2 value



β_2= 0.001 by RMF
0.089 from B(E2)

❖ Does DQRPA go back to QRPA at β_2=0 ??

If we take the limit, the deformed basis goes back to the spherical basis.
But, for $\beta_2 \neq 0$, the deformed basis has many components because the angular momentum projection Ω_j may have angular momenta higher j.

- Ω_j can be composed of different j values in the β_2 = 0.3.

$$|Nn_z\Lambda : 000> \; = \; |0s\frac{1}{2}> \quad ; \beta_2 = 0,$$

$$|Nn_z\Lambda : 000> \; = \; 0.98|0s\frac{1}{2}> +0.0005|1s\frac{1}{2}> +0.094|0d\frac{5}{2}> +0.0116|0d\frac{3}{2}> + \cdots \quad ; \beta_2 = 0.3.$$

- Single particle states by eigenequation of total Hamiltonian are linear combination of the deformed basis even if we take the β_2 = 0 limit.

$$|\frac{1}{2}> \; = \; 0.93|0s\frac{1}{2}> -0.36|1s\frac{1}{2}> +0.04|2s\frac{1}{2}> +0.01|3s\frac{1}{2}> + \cdots \quad ; \beta_2 = 0,$$

$$|\frac{1}{2}> \; = \; 0.92|0s\frac{1}{2}> -0.36|1s\frac{1}{2}> -0.03|0d\frac{3}{2}> +0.04|0d\frac{5}{2}> + \cdots \quad ; \beta_2 = 0.1.$$

- One may notice | Ω_j = 1/2 > state has other components |ns1/2 > although main component is |0s1/2 >.

- **Therefore, the extension to β_2=0 values may not be exact, but approximate treatment.**

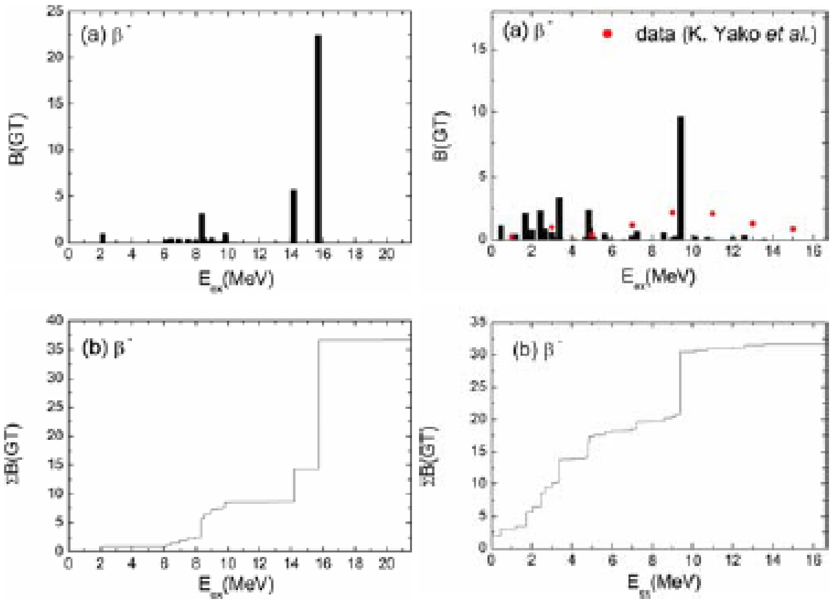


Fig. 1. GT(–) strength distributions for ^{90}Zr with *np* pairing (right) and without *np* pairing (left) with respect to the parent nucleus ^{90}Zr. The experimental Q value between ^{90}Zr and ^{90}Nb is 6.111 MeV. Experimental data are from Refs. 20 and 21

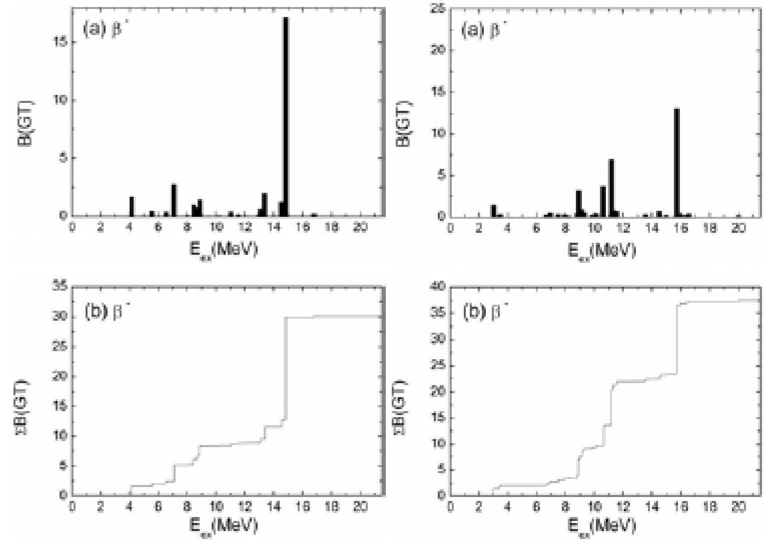


Fig. 2. GT(–) strength distributions for ^{92}Zr with *np* pairing (right) and without *np* pairing (left) and with respect to the parent nuclei ^{92}Zr. The experimental Q value between ^{92}Zr and ^{92}Nb is 2.005 MeV.

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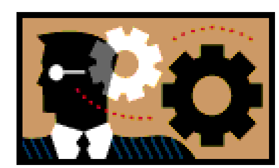
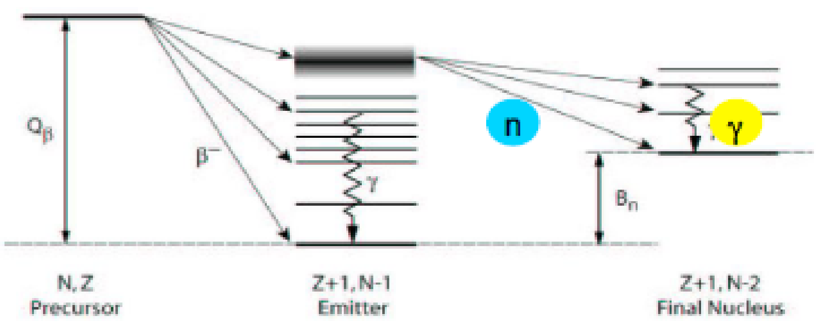
Results of the nuclear beta decays for Zr and Mo isotopes

Table II. $Q^{exp}_{\beta_-}(Q^{FRDM}_{\beta_-})$ used in this work with β_2 values.

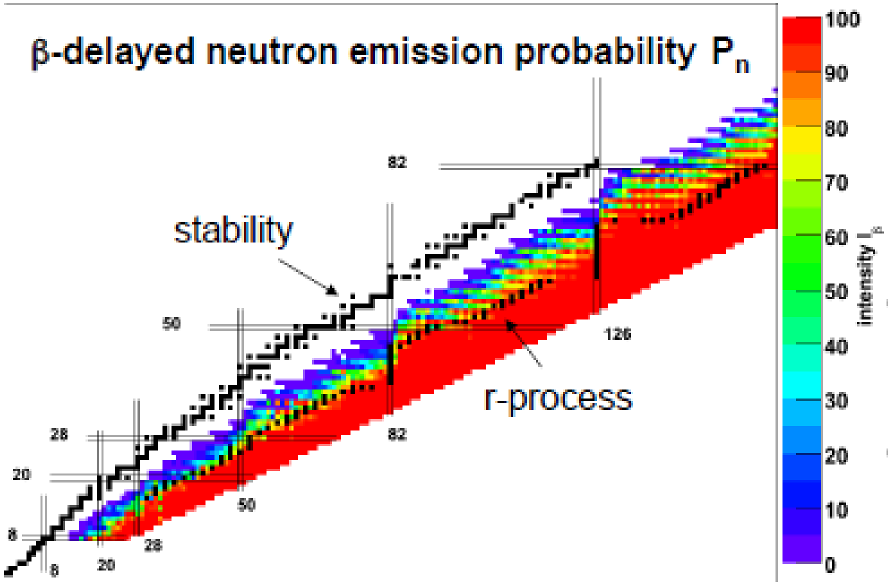
nuclide	$Q^{exp}_{\beta_-}(Q^{FRDM}_{\beta_-})$ [MeV]	$\beta^{E2}_2(\beta^{RMF}_2)$
^{102}Zr	4.61	0.427
^{104}Zr	5.9	0.371
^{106}Zr	7.2	0.375
^{108}Zr	8.6	0.381
^{110}Zr	9.3	0.401
^{112}Zr	(10.77)	0.421
^{104}Mo	2.16	0.362
^{106}Mo	3.52	0.354
^{108}Mo	4.65	(-0.27)
^{110}Mo	5.5	(-0.278)
^{112}Mo	7.2	(-0.264)
^{114}Mo	8.4	(-0.229)



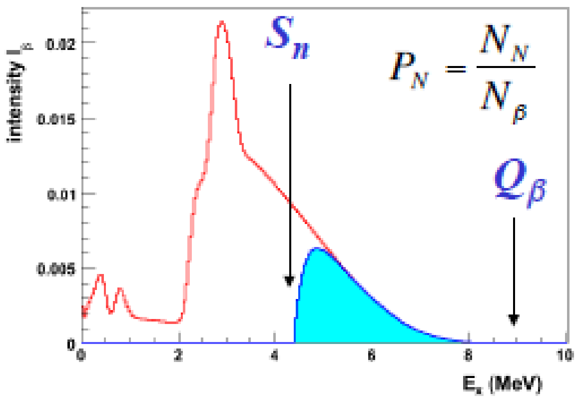
Beta decay in the neutron rich side



If $S_n < Q_\beta$
and the decay proceeds to states above S_n, neutron emission competes and can dominate over γ-ray de-excitation



The process will dominate far from stability on the n-rich side. To have a full picture of the strength …



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Beta Decay
Allowed decays

Fermi transitions

$$\langle J_f M_f T_f T_{0f} | T_{\mp} | J_i M_i T_i T_{0i} \rangle = \sqrt{T_i(T_i+1) - T_{0i}(T_{0i} \mp 1)}\,\delta_{J_i J_f}\delta_{M_i M_f}\delta_{T_i T_f}\delta_{T_{0i} \mp 1 T_{0f}}$$

In reality, isospin is violated by the electromagnetic force, but the violation is weak.

$J_f = J_i$	$(\Delta J = 0)$
$T_f = T_i \neq 0$	$(\Delta T = 0,$ but $T_i = 0 \to T_f = 0$ forbidden$)$
$T_{0f} = T_{0i} \mp 1$	$(\Delta T_0 = 1)$
$\Delta\pi = 0$	no parity change

T_+ has rank unity!

Gamow-Teller transitions

The matrix element strongly depends on the structure of the wave function!

$\Delta J = 0,1$	but $J_i = 0 \to J_f = 0$ forbidden
$\Delta T = 0,1$	but $T_i = 0 \to T_f = 0$ forbidden
$T_{0f} = T_{0i} \mp 1$	$(\Delta T_0 = 1)$
$\Delta\pi = 0$	no parity change

The absolute values of GT matrix elements are generally smaller than those for Fermi transitions.

$$fT = \frac{const}{\langle F \rangle^2 + g_A^2 \langle GT \rangle^2}$$

squared matrix elements

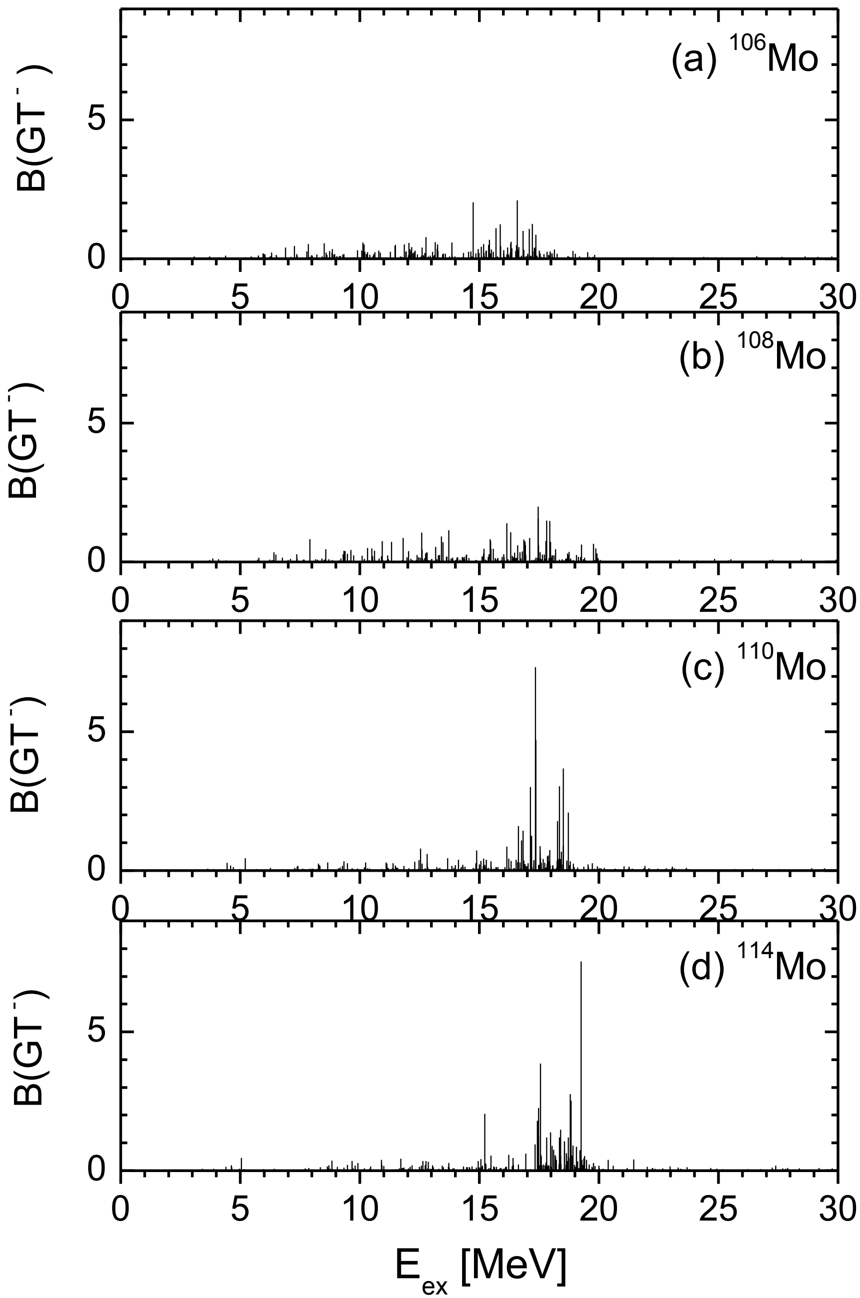
Beta Decay
Forbidden transitions

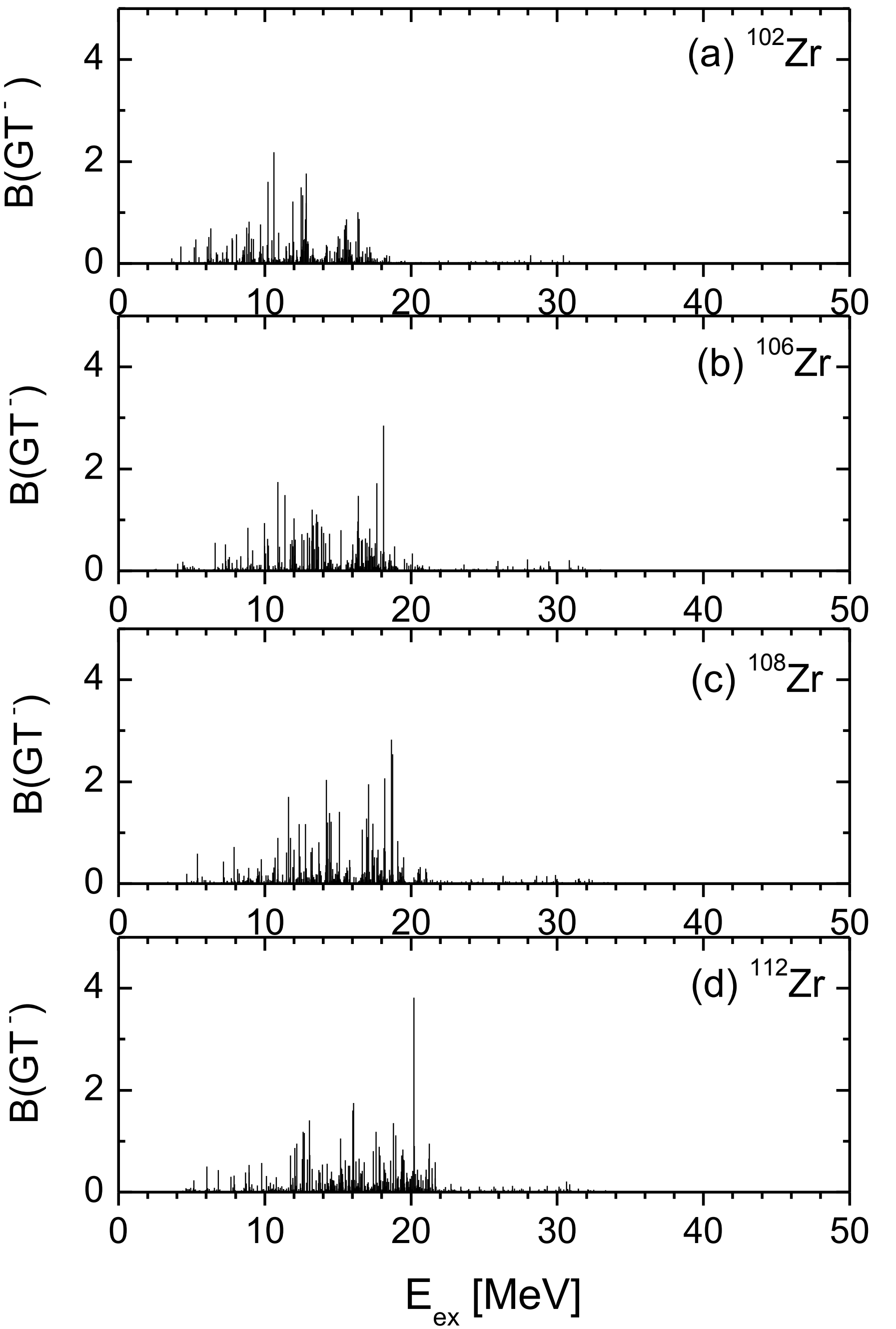
The angular momentum (L) of the systems (e + nu) can be non-zero (in the center-of-mass frame of the system).

$$\Delta J = L \text{ or } L \pm 1, \quad \Delta\pi = (-1)^L$$

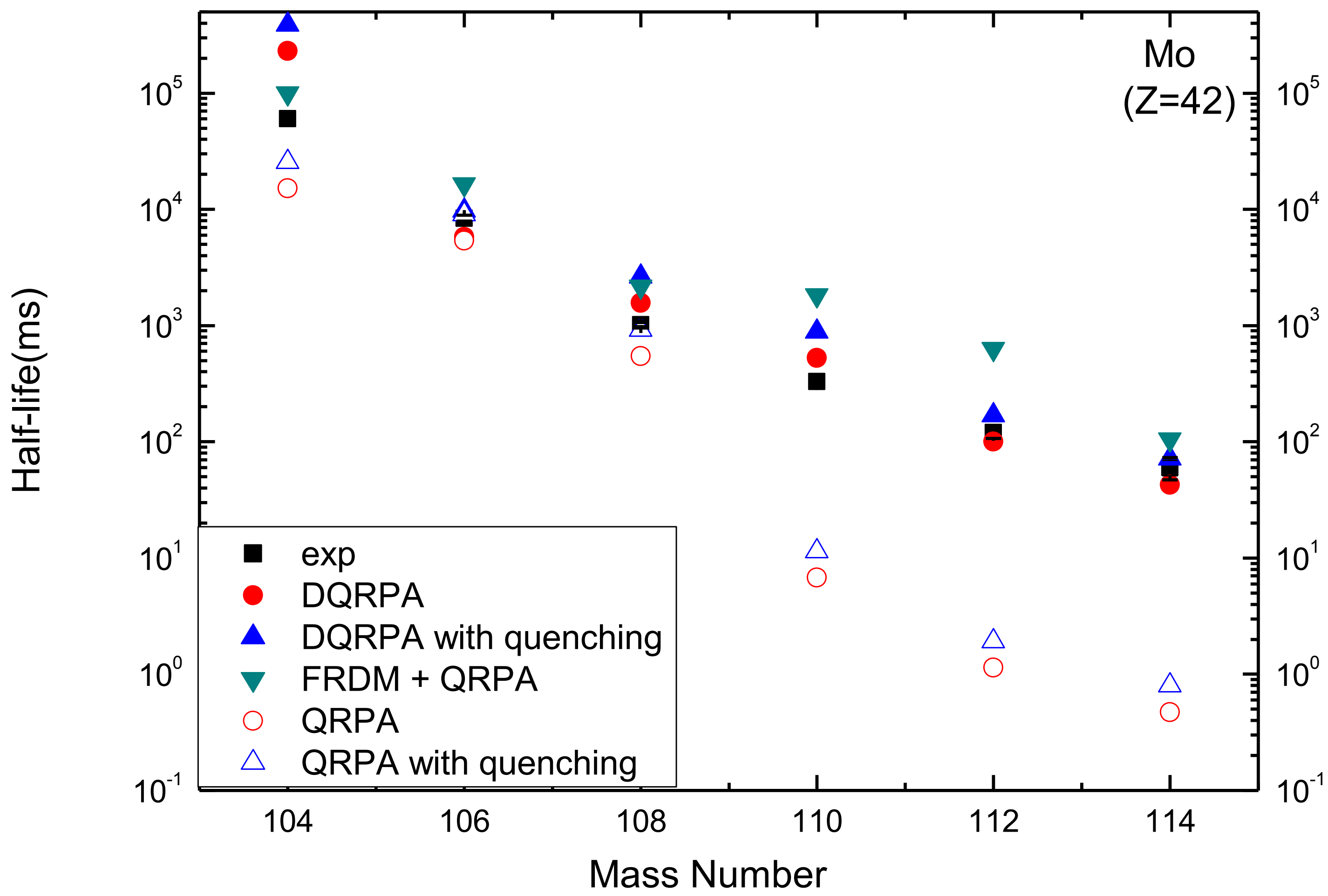
Decay type	ΔJ	ΔT	$\Delta\pi$	$\log_{10} ft_{1/2}$
Superallowed	$0^+ \to 0^+$	0	no	3.1–3.6
Allowed	0, 1	0, 1	no	2.9–10
First forbidden	0, 1, 2	0, 1	yes	5–19
Second forbidden	1, 2, 3	0, 1	no	10–18
Third forbidden	2, 3, 4	0, 1	yes	17–22
Fourth forbidden	3, 4, 5	0, 1	no	22–24

❖ GT(-) strength for $^{106\sim114}Mo$





❖ β-decay half-life of $^{104\sim114}Mo$ with QRPA



Summary

0. R- and nu- processes in a view point of nuclear models, QRPA.

1. We used the deformed WS potential and then performed the deformed BCS and deformed QRPA with a realistic two-body interaction calculated by **Brueckner G-matrix based on Bonn potential**.

2. Results of the Gamow-Teller strength, B(GT±), for ^{76}Ge, ^{76,82}Se, and ^{92}Zr show that the deformation effect leads to a fragmentation of the GT strength into **high-lying GT excited states.**

3. Our results show that **the running sum** of the GT strength distributions for ^{76}Ge and ^{76,82}Se reproduce well experimental data .

4. Preliminary results of the beta decay of neutron rich nuclei **show the importance of the deformation.**

5. We are preparing to calculate data relevant to heavy nucleus, A>110, by using a super computer.

6. Future plan 1 : apply to the M1 and E2 transitions of even-odd or odd-odd nuclei, and consider the continuum states in DQRPA.

7. **Future plan 2 : New Mass Model**

Thanks for your attention !!