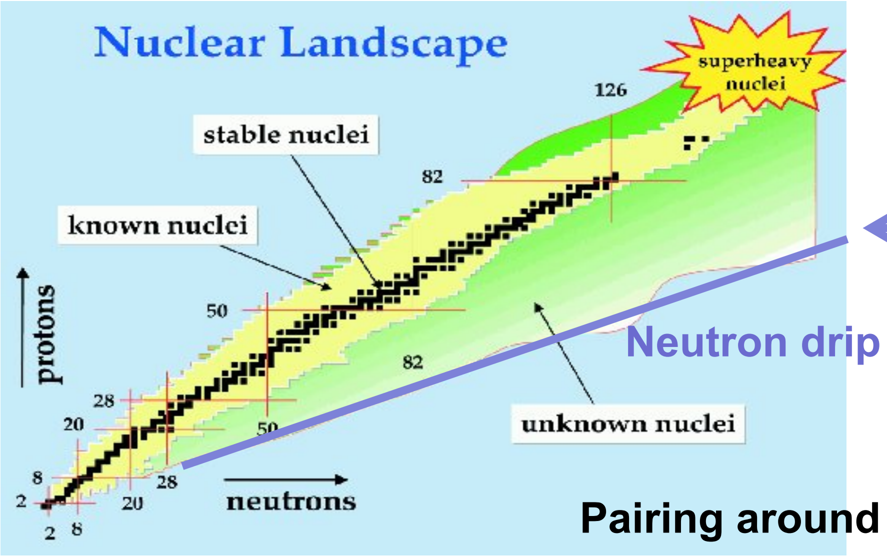
International School of Nuclear Physics: *nuclei in the laboratory and in the cosmos*, ERICE, 2014

ETTORE MAJORANA FOUNDATION AND
CENTRE FOR SCIENTIFIC CULTURE
TO PAY A PERMANENT TRIBUTE TO GALILEO GALILEI, FOUNDER OF MODERN SCIENCE AND TO ENRICO FERMI, "THE ITALIAN NAVIGATOR", FATHER OF THE WEAK FORCES

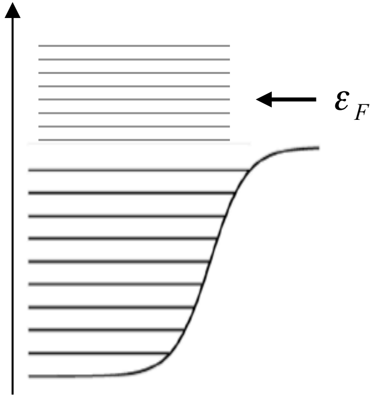
Superfluidity and thermal properties of the crust of Neutron Stars

Jérôme Margueron, IPN-Lyon, France.

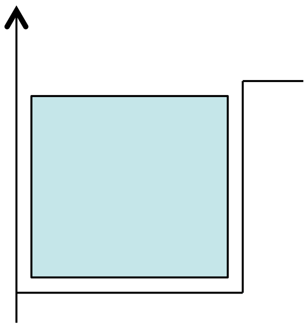
Superfluidity in non-uniform systems,
Cooling and crust thermalisation of neutron stars
Surprising phenomenons in non-uniform systems



Transition Outer/inner crust



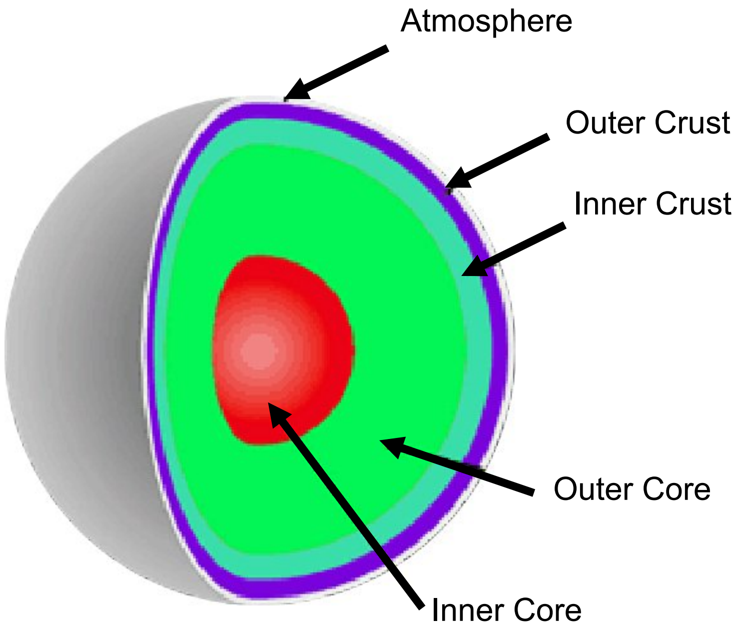
Equivalent piece of nuclear matter



Pairing around and beyond the drip line

Global picture of a Neutron Star

Remnant of a core-collapse supernova (Baade & Zwickly, Phys. Rev. 1934)



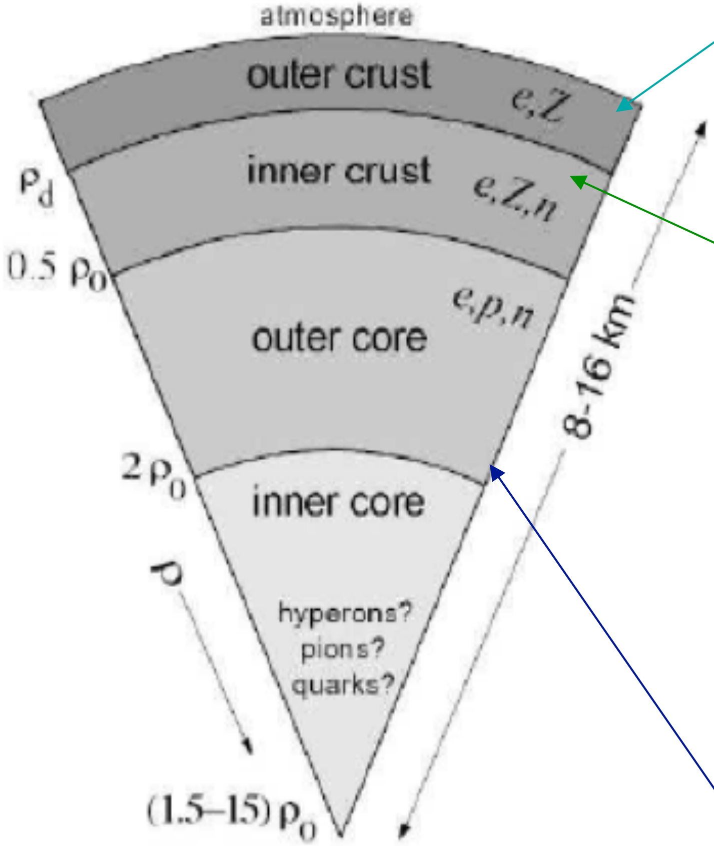
R ~ 10 - 12 km

M ~ 1.2 - 2 M_{sun}

P ~ 1 ms - 10 s

B ~10^{12} - 10^{16} G

More detailed anatomy of a neutron star

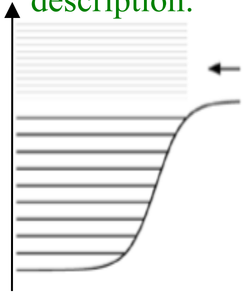


Composition of the crust

$\rho < 10^3$ g/cm^3 : ^{56}Fe bcc lattice+cloud of electrons
$\rho > 10^3$ g/cm^3 : electrons fully ionized
$\rho > 10^6$ g/cm^3 : electrons are relativistic
$\rho > 10^7$ g/cm^3 : $^{56}\text{Fe} \rightarrow {}^{84}\text{Se} \rightarrow \ldots$
$\rho > 10^9$ g/cm^3 : e capture:
n rich exotic nuclei $^{82}\text{Ge} \rightarrow \ldots \rightarrow {}^{118}\text{Kr}$

$\rho > 4.3\ 10^{11}$ g/cm^3 : neutron drip
→ nuclear clusters (lattice)
surrounded by a neutron gas

microscopic description:



Pasta phase

$\rho > 2.0\ 10^{14}$ g/cm^3 : nuclei dissolve
→ homogeneous nuclear matter

We need a theoretical modeling able to bridge the transition from nuclei to uniform matter.

Self-consistent nuclear mean field models

Going towards very N rich nuclei

Self-consistent mean-field theories and density functional theory

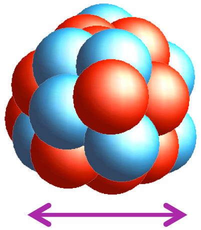
Properties of finite systems:
masses, radii, pairing, evolution of the shells, deformation, collective modes, molecular states, …

↔

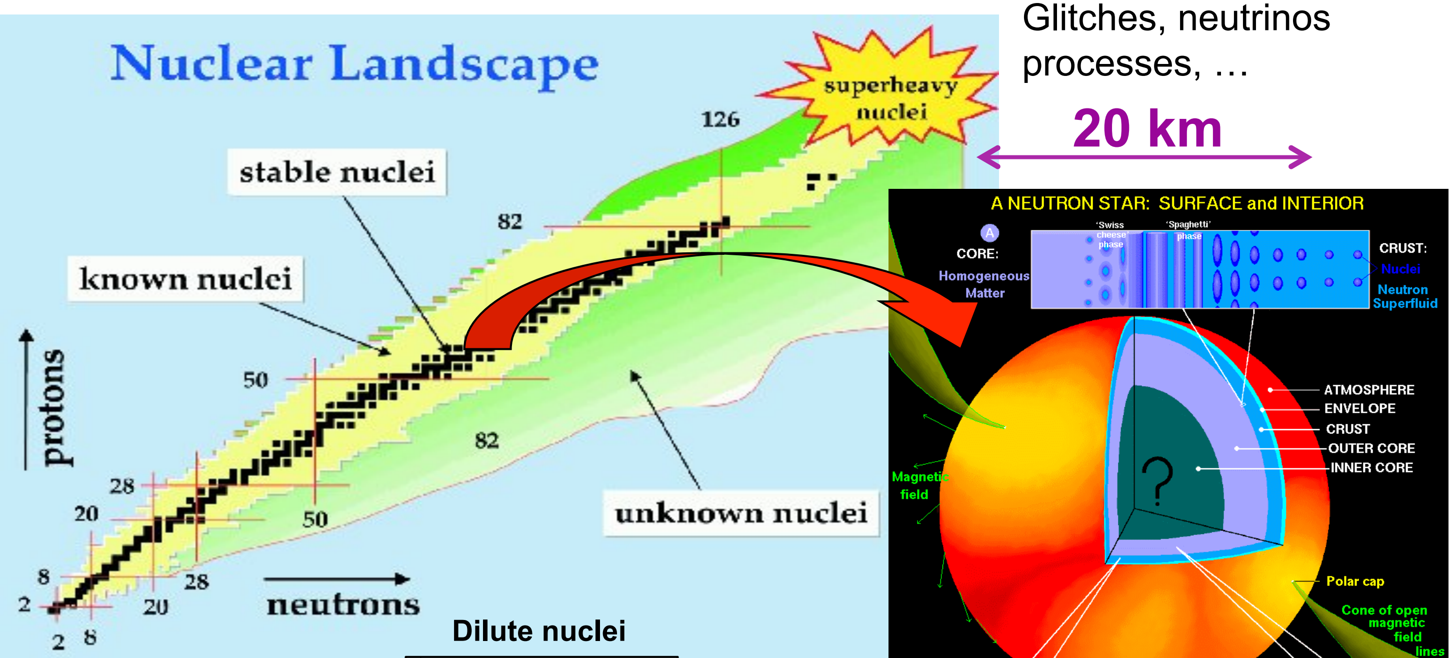
General properties of matter:
incompressibility, symmetry energy equation of state, …

↔

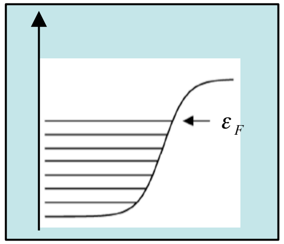
Application to neutron stars and supernovae:
Masses, radii, cooling, Glitches, neutrinos processes, …



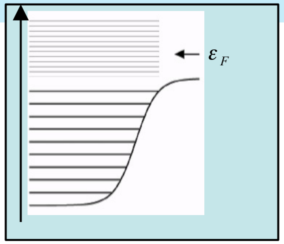
10 fm



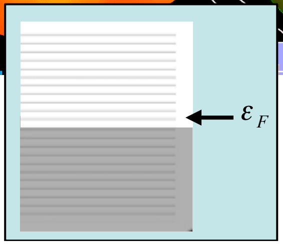
Exotic nuclei



Dilute nuclei

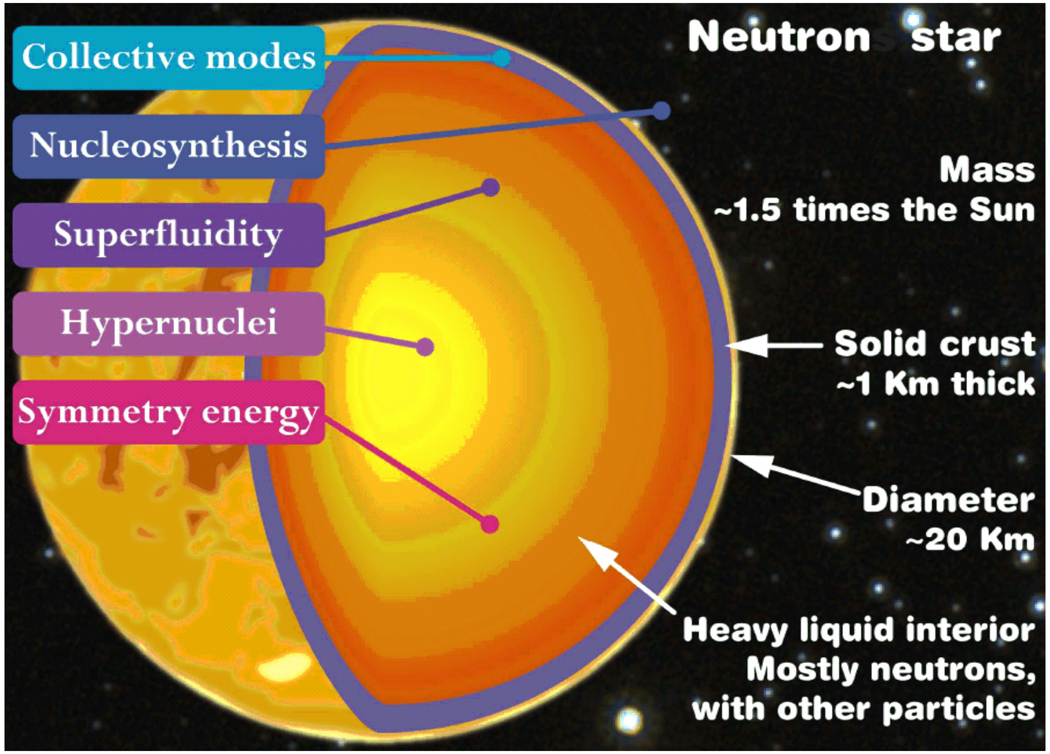


Finite temperature,
Beyond drip line.



A laboratory for modern physics !

Neutron star is a laboratory to study matter under *extreme conditions* (density, temperature, ...)



***Interdisciplinary* field:**

Quantum Liquids:
E_F/T~100-1000
→ Fermi-Dirac statistic

Special Relativity:
v_F/c~0.2-0.4

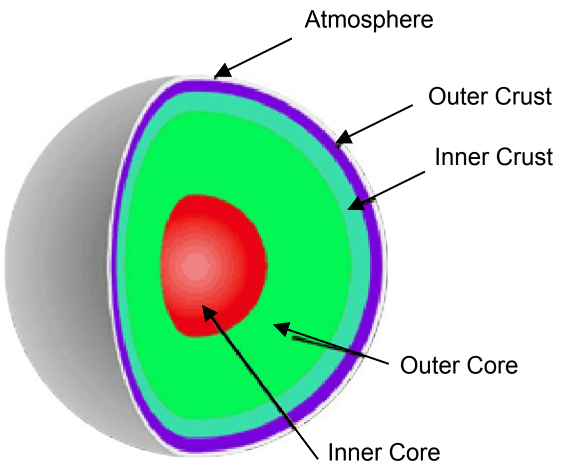
General Relativity:
R_{sh}/R~0.1-0.2

Magnetic fields:
$B\sim10^{12}-10^{16}$ G
→ origin, MHD ?

Deconfined Quarks matter:
$\rho > 10^{15}$ g/cm^3

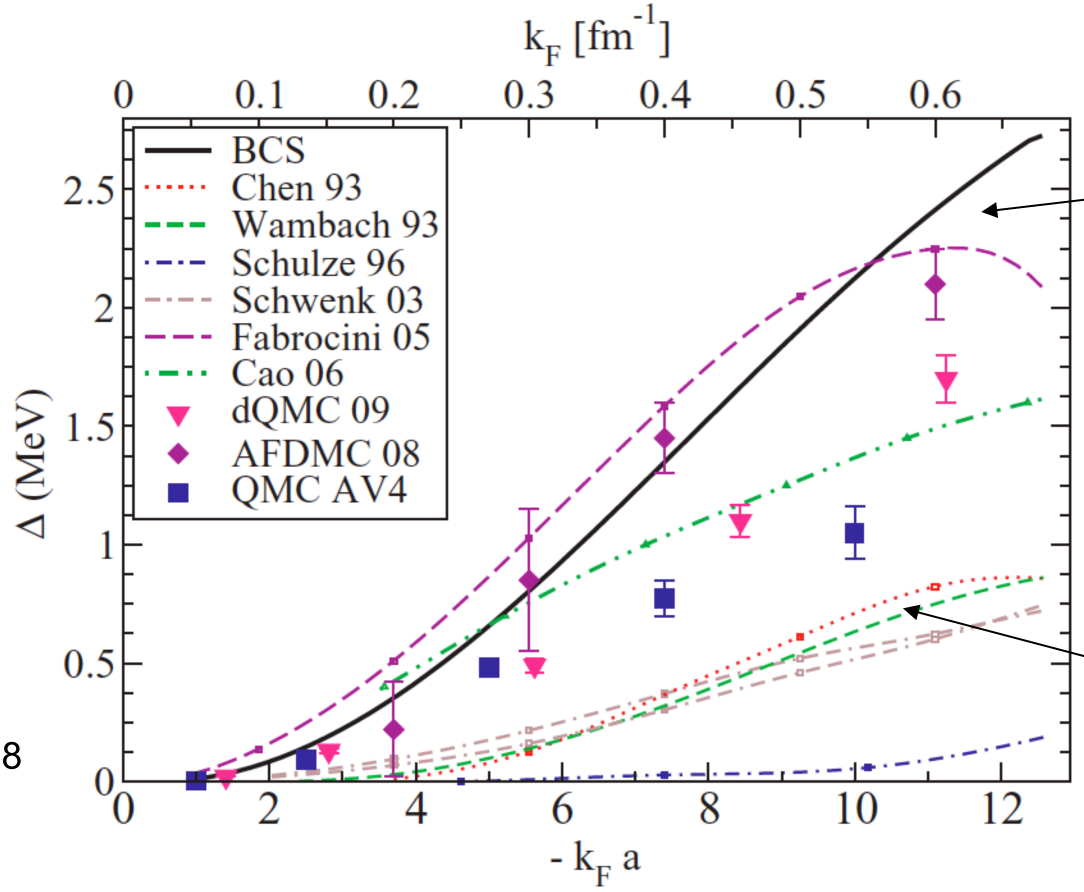
Superfluid, rotation, cold atomic gas, BEC/BCS crossover...

Superfluidity in uniform and non-uniform systems



Pairing gap in neutron matter: comparisons of different approaches

BCS, BCS+polarisation, QMC, AFDMC, ...



BCS

Strong
(max at 3 MeV),

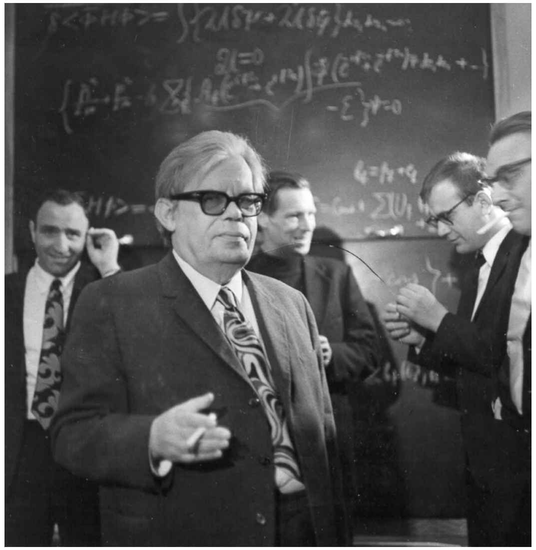
Weak
(max at 1 MeV).

Gezerlis, Carlson,
Phys. Rev. C 81
(2010)

Lombardo, Schulze,
Lect. Notes Phys. 578
(2001)

What is the effect of strong/weak pairing on the cooling of neutron stars ?

Superfluidity in non-uniform systems



In non-uniform systems, Bogoliubov introduced field operators: $\hat{\psi}(r)$

Generalized Bogoliubov-Valatin transformation:

$$\left\{ \begin{aligned} \psi_\uparrow(r) &= \sum_n \mathrm{u}_n(r) b_{n\uparrow} + \mathrm{v}_n(r) b^+_{n\downarrow} \\ \psi_\downarrow(r) &= \sum_n \mathrm{u}_n(r) b_{n\downarrow} - \mathrm{v}_n(r) b^+_{n\uparrow} \end{aligned} \right.$$

With a little bit of algebra:

$$\begin{bmatrix} h-\mu & \Delta \\ \Delta^* & -h+\mu \end{bmatrix} \begin{pmatrix} \mathrm{u}_n(r) \\ \mathrm{v}_n(r) \end{pmatrix} = E_n \begin{pmatrix} \mathrm{u}_n(r) \\ \mathrm{v}_n(r) \end{pmatrix}$$

Hartree-Fock-Bogoliubov equation

Mean field $h[\rho(r)], \Delta[\rho(r)]$

Quasi-particle wave function

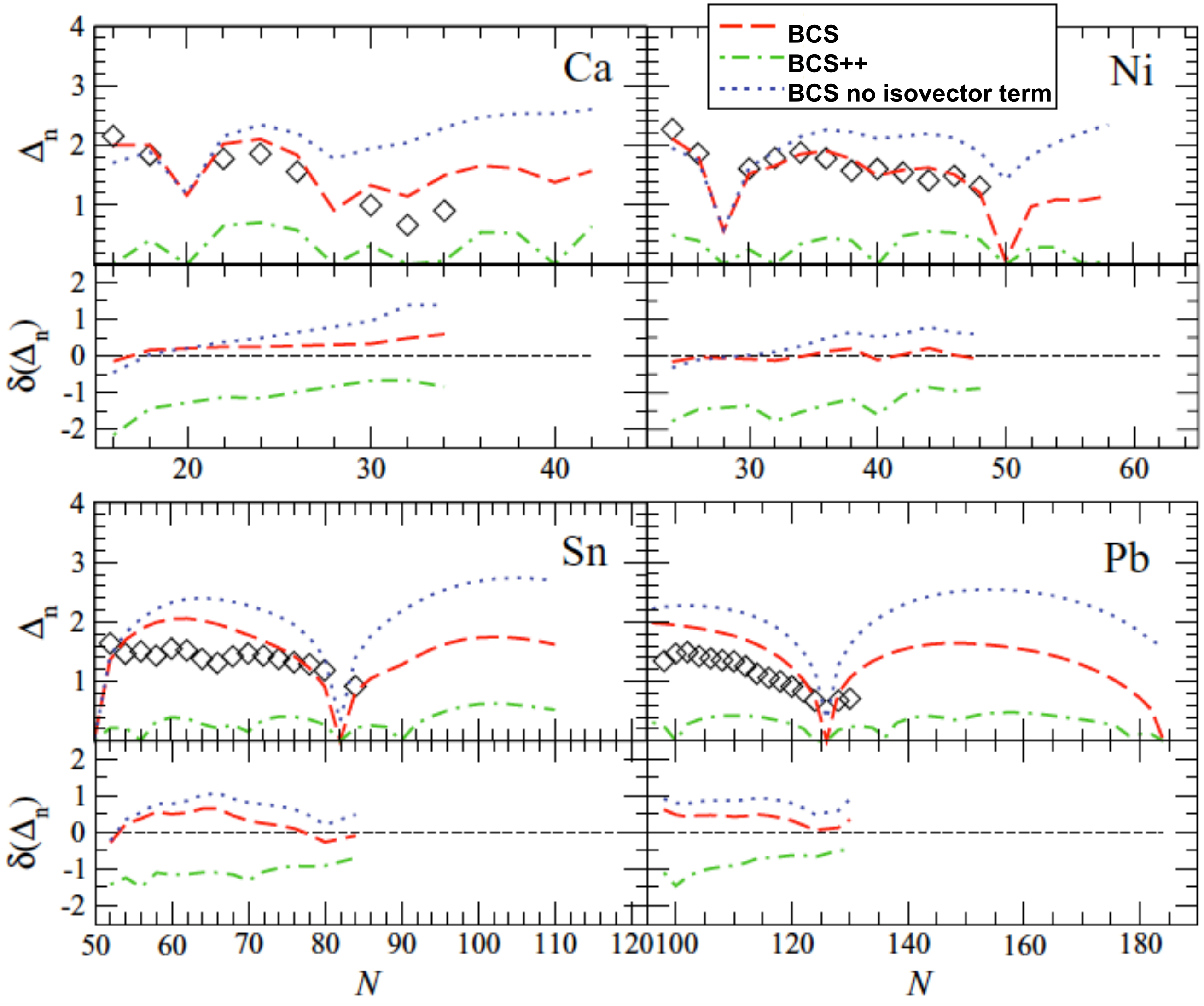
Densities

$$\rho(r) = \sum_a (2J_a+1)\mathrm{v}_a^2(r)$$

$$\kappa(r) = \sum_a (2J_a+1)\mathrm{u}_a(r)\mathrm{v}_a(r)$$

→ *Self-consistent mean-field*

Application to semi-magic isotopes

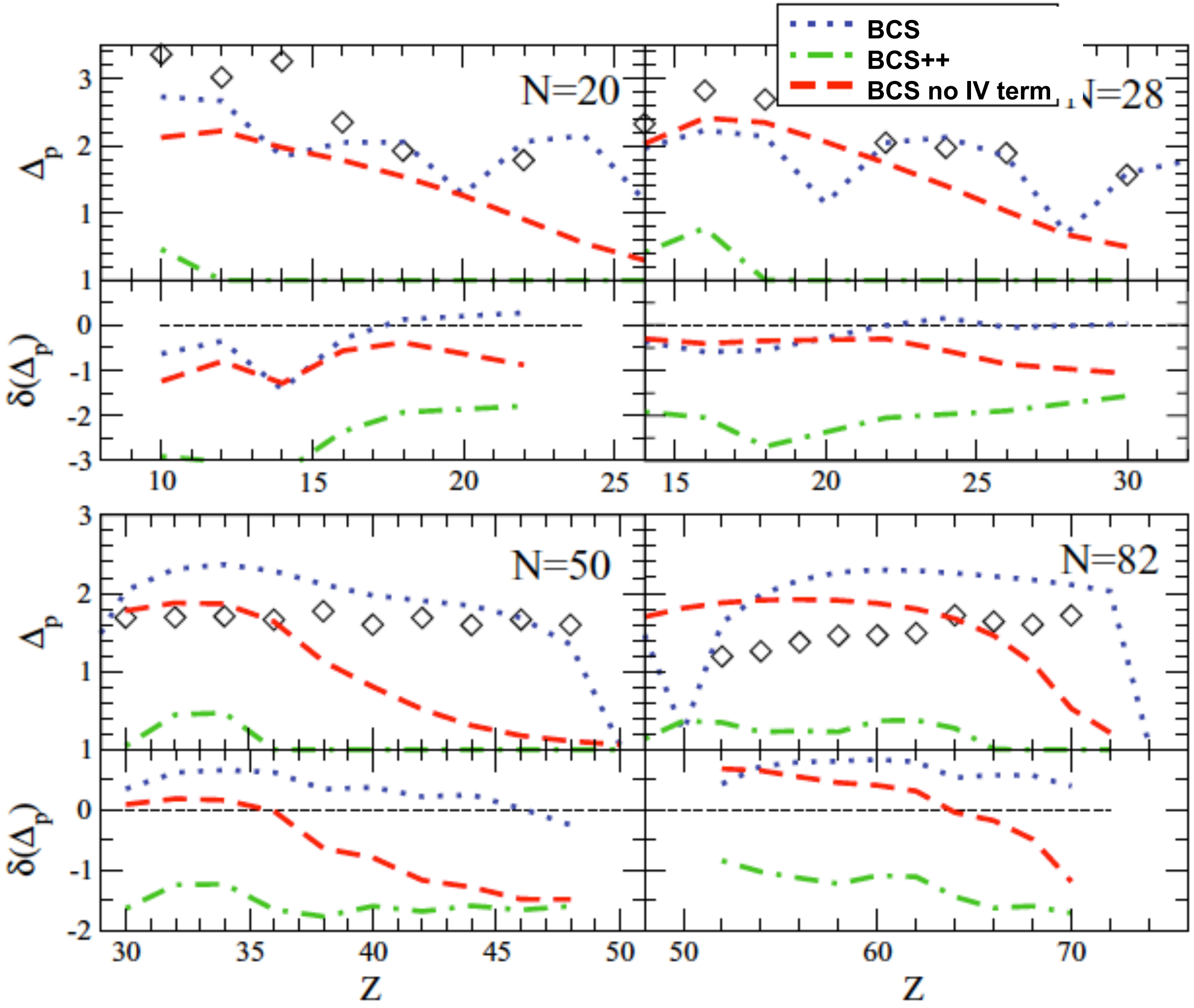


BCS with isovector term reproduce better the isotopic trend.

BCS++ is too weak.

J.M., Sagawa, Hagino, PRC 77 (2008)

Application to semi-magic isotones



Coulomb interaction disregarded.

BCS with isovector term reproduce better the isotonic trend.

BCS++ is too weak.

J.M., Sagawa, Hagino, PRC 77 (2008)

Why BCS++ is so poor in nuclei ?

- Polarisation is a surface effect : the phonons in uniform matter and in nuclei are very different.
- In uniform matter, the effective mass is peaked at the Fermi energy, while in Skyrme mean field, there is no **surface peaked effective mass**.

➔ A surface peaked effective mass in the mean field model could increase the level density and enhance the pairing gap at the Fermi level.

$$\Delta_F \approx 2\epsilon_F \exp[2/(N_0 v_{pair})]$$

More in Duguet et al., PRC (2008)

Mean field with surface peaked effective mass

Inspired from Ma & Wambach, NPA 1983

Our approach : introduce a correction to the EDF such as to get :
→ a surface peaked effective mass (energy-independent)
→ a moderate effect on the mean field

Standard Skyrme energy density :

$$\mathcal{H}(\mathbf{r}) = \mathcal{K}(\mathbf{r}) + \sum_{T=0,1} \mathcal{H}_T(\mathbf{r})$$

Bender *et al.*,
Rev. Mod. Phys. **75** (2003)

$$\begin{aligned}
\mathcal{K}(\mathbf{r}) &= \frac{\hbar^2}{2m}\tau(\mathbf{r}) \, , \\
\mathcal{H}_T(\mathbf{r}) &= C_T^{\rho}\rho_T^2(\mathbf{r}) + C_T^{\Delta\rho}\rho_T(\mathbf{r})\Delta\rho_T(\mathbf{r}) \\
&+ C_T^{\tau}\rho_T(\mathbf{r})\tau_T(\mathbf{r}) \\
&+ C_T^{J}\mathbb{J}_T^2(\mathbf{r}) + C_T^{\nabla J}\rho_T(\mathbf{r})\nabla\cdot\mathbf{J}_T(\mathbf{r})
\end{aligned}$$

Correction term : (isoscalar)

Fantina et al.,
J. Phys. G 38 (2011)

$$\boxed{\begin{aligned}
\mathcal{H}_0^{\mathrm{corr}}(\mathbf{r}) &= C_0^{\tau(\nabla\rho)^2}\tau(\mathbf{r})\left(\nabla\rho(\mathbf{r})\right)^2 \\
&+ C_0^{\rho^2(\nabla\rho)^2}\rho(\mathbf{r})^2\left(\nabla\rho(\mathbf{r})\right)^2
\end{aligned}}$$

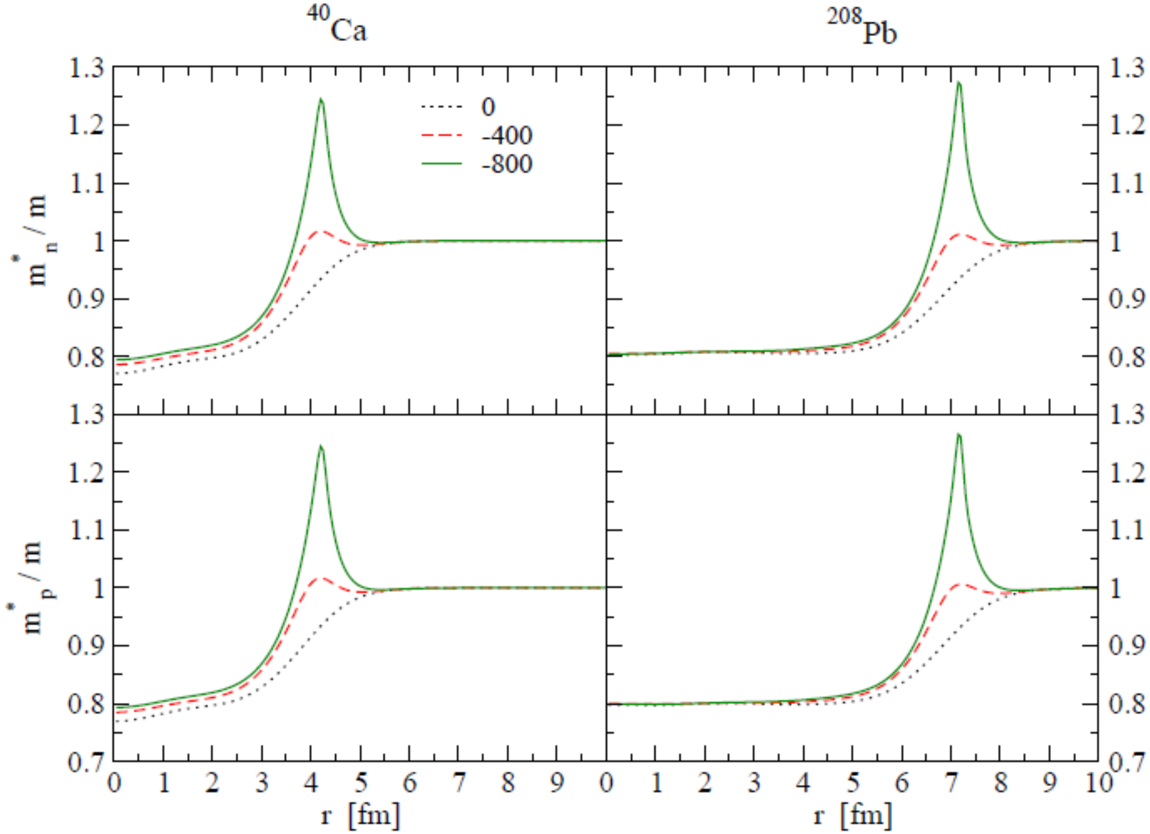
surface peaked effective mass

Zalewski, Olbratowski, Satula, PRC81 (2010)

mean field compensation

Effective mass profile in ^{40}Ca & ^{208}Pb

Effective mass : $$\frac{\hbar^2}{2m_q^*(\mathbf{r})} \equiv \frac{\delta H}{\delta \tau_q} = \underbrace{\frac{\hbar^2}{2m} + C_q^\tau \rho_q(\mathbf{r})}_{\text{standard Skyrme } k\text{-mass}} + C_0^{\tau(\nabla\rho)^2}(\nabla\rho(\mathbf{r}))^2$$



Fantina et al., J. Phys. G 38 (2011)

Level density

Density of states:

$$g(E) \equiv \frac{\mathrm{d}N(E)}{\mathrm{d}E} = \sum_{\substack{\lambda_1<F\\ \lambda_2>F}} (2j_{\lambda_2}+1)\ \delta(E-(\epsilon_{\lambda_2}-\epsilon_{\lambda_1}))$$

Number of states:

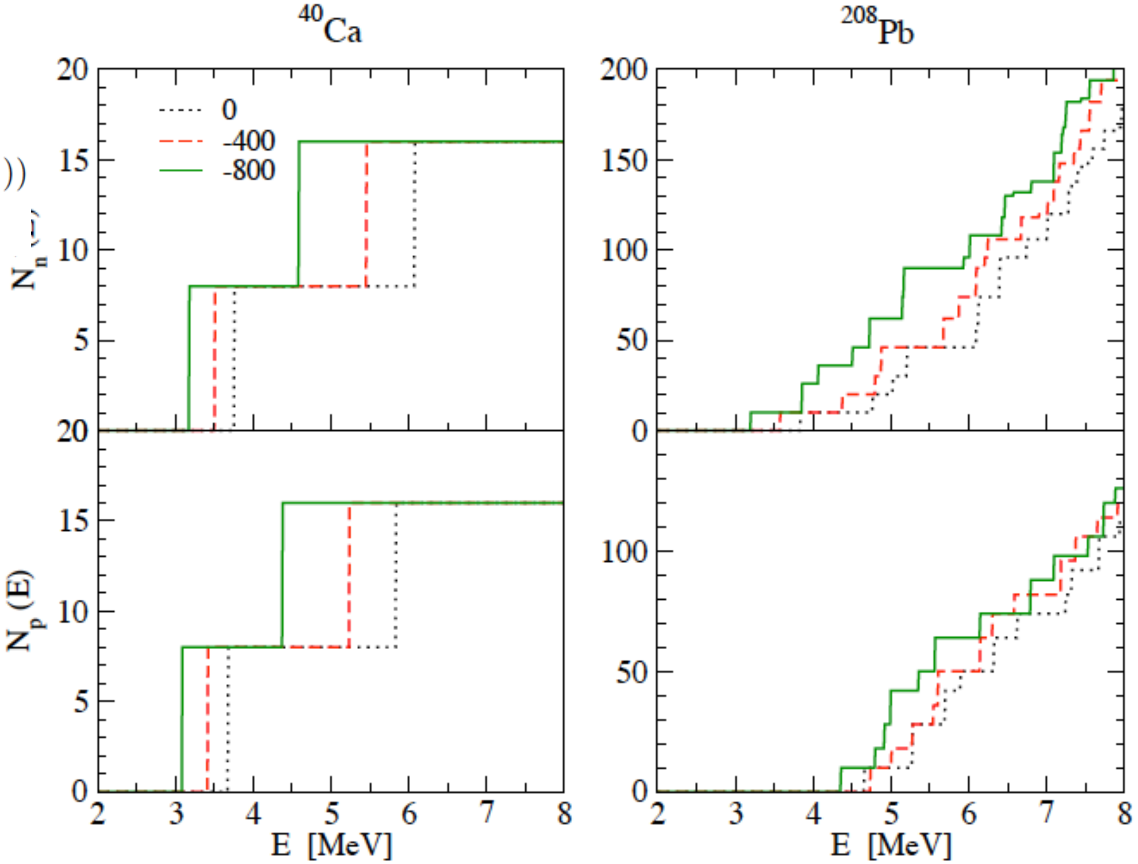
$$N(E) = \int dE\ g(E)$$

pairing properties

Pairing gaps

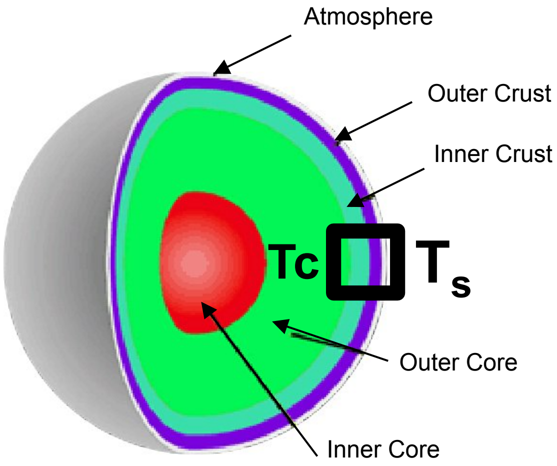
$C_0^{\tau(\nabla\rho)^2}$	η		
(MeV fm^{10})	0	0.5	1
0	1.30	1.30	1.30
−200	1.33	1.36	1.47
−400	1.37	1.43	1.62
−600	1.42	1.52	1.79
−800	1.49	1.60	1.96

Pairing interaction: volume mixed surface

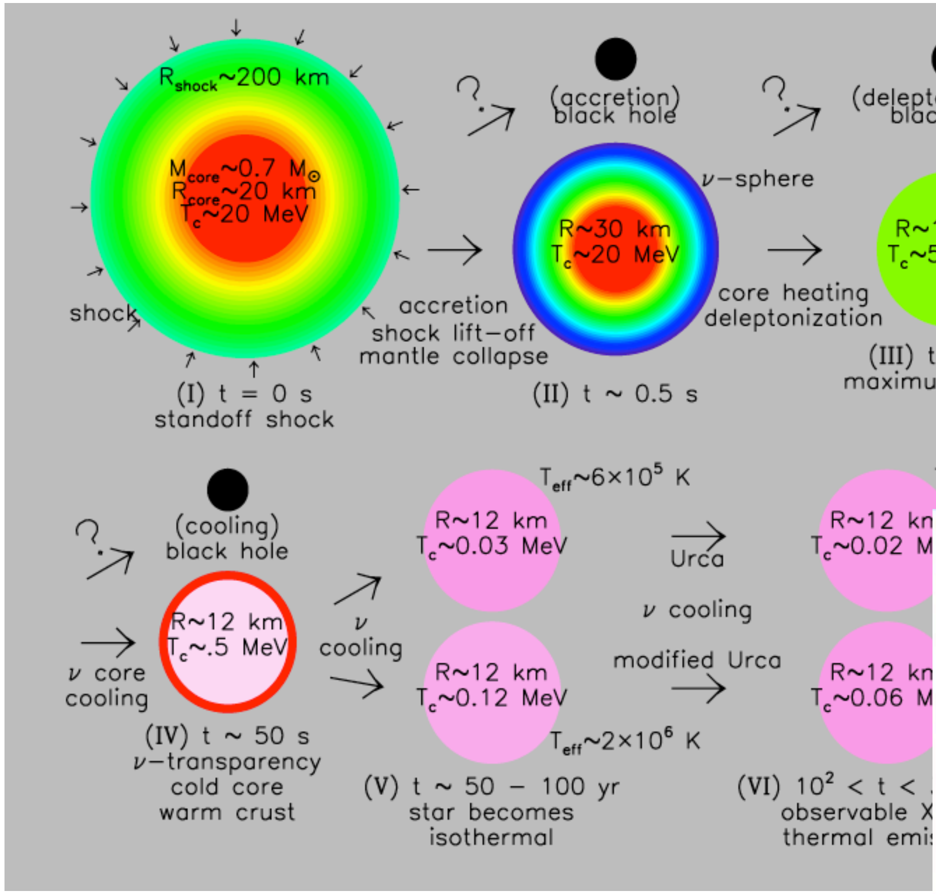


Fantina et al., J. Phys. G 38 (2011)

Thermal relaxation of the neutron star crust

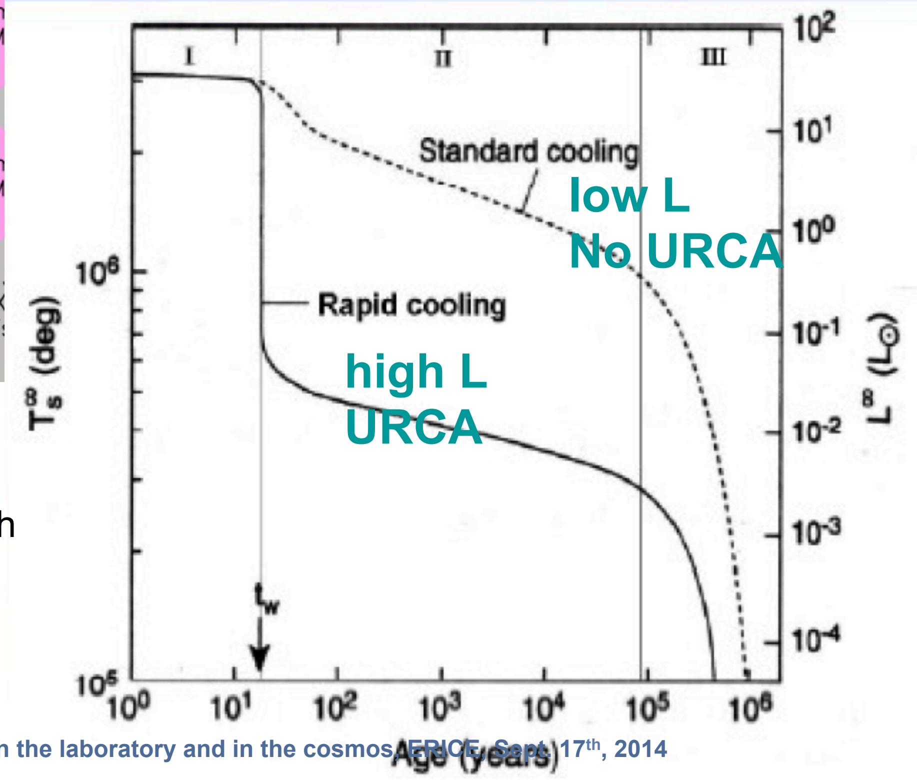


Cooling of Neutron stars



Lattimer & Prakash, Phys. Rep. 442 (2007)

Several phases of cooling:	*Time-scales:*
• *Convective*	*s*
• *Conductive*	*1-100 y*
• *Radiative*	*>10^5 y*

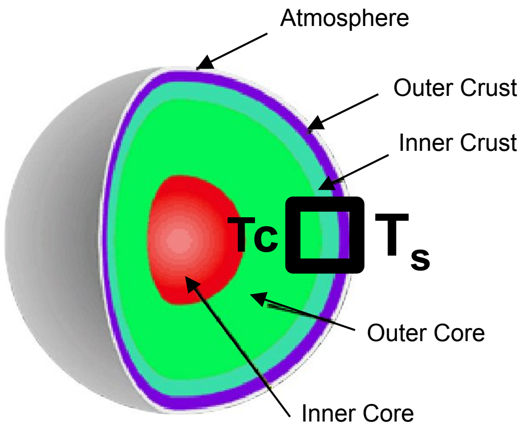


I: **Crust thermalization** epoch
II: ν cooling epoch
III: Photon epoch

Crust thermalization

Fast cooling of the core (induced by URCA process) leave the crust at a temperature around 10^9-10^{10} K after few days.

- → after ~1 year: Tcore << Tcrust ~0.5 MeV,
- → next ~10-100 years: **thermalization** of the crust.

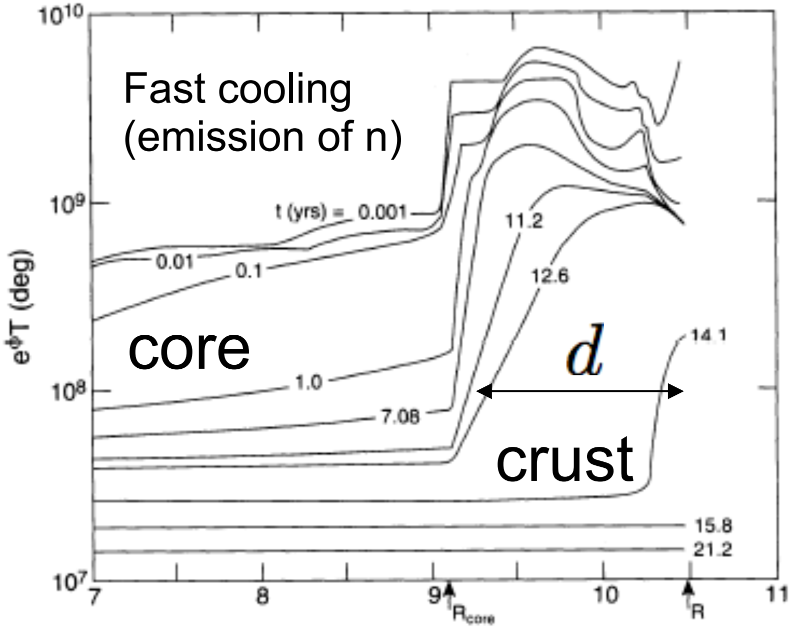


Thermalization time :

$$\tau \propto \frac{d^2}{D}$$

with $D = \frac{K}{\sum_i C_{v,i} \approx C_{v,n}}$

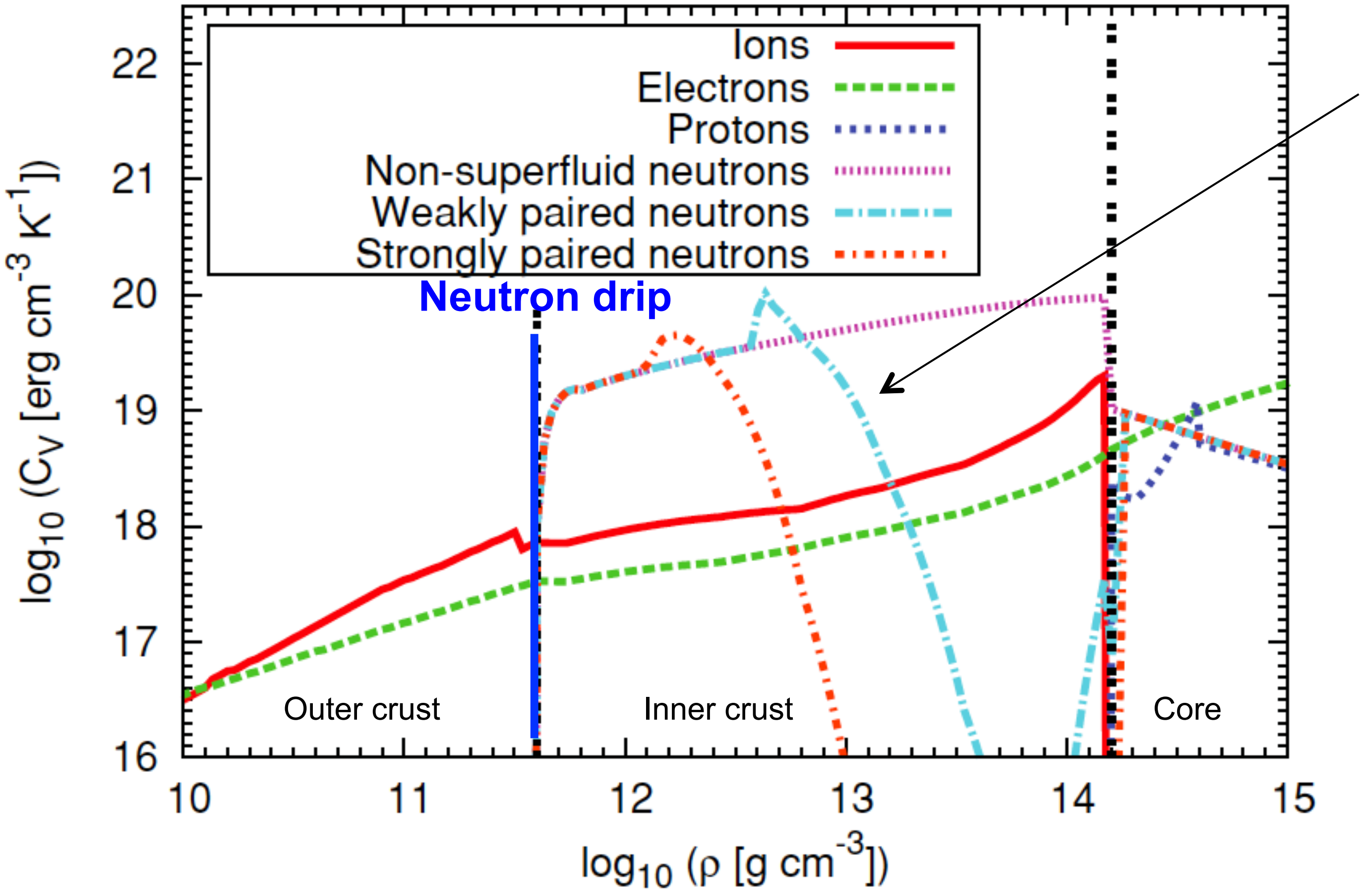
Lattimer et al, ApJ425 (1994) 802



1- Superfluidity and cooling of neutron stars

Fortin et al., PRC 88 (2010)

Log T = 9



Suppression of Cv in the superfluid phase

↓

Increase the diffusivity in the crust (D=K/Cv)

↓

Reduces the thermal relaxation time of the crust (τ=R^2/D)

↔ **Relaxation time of LMXRT**

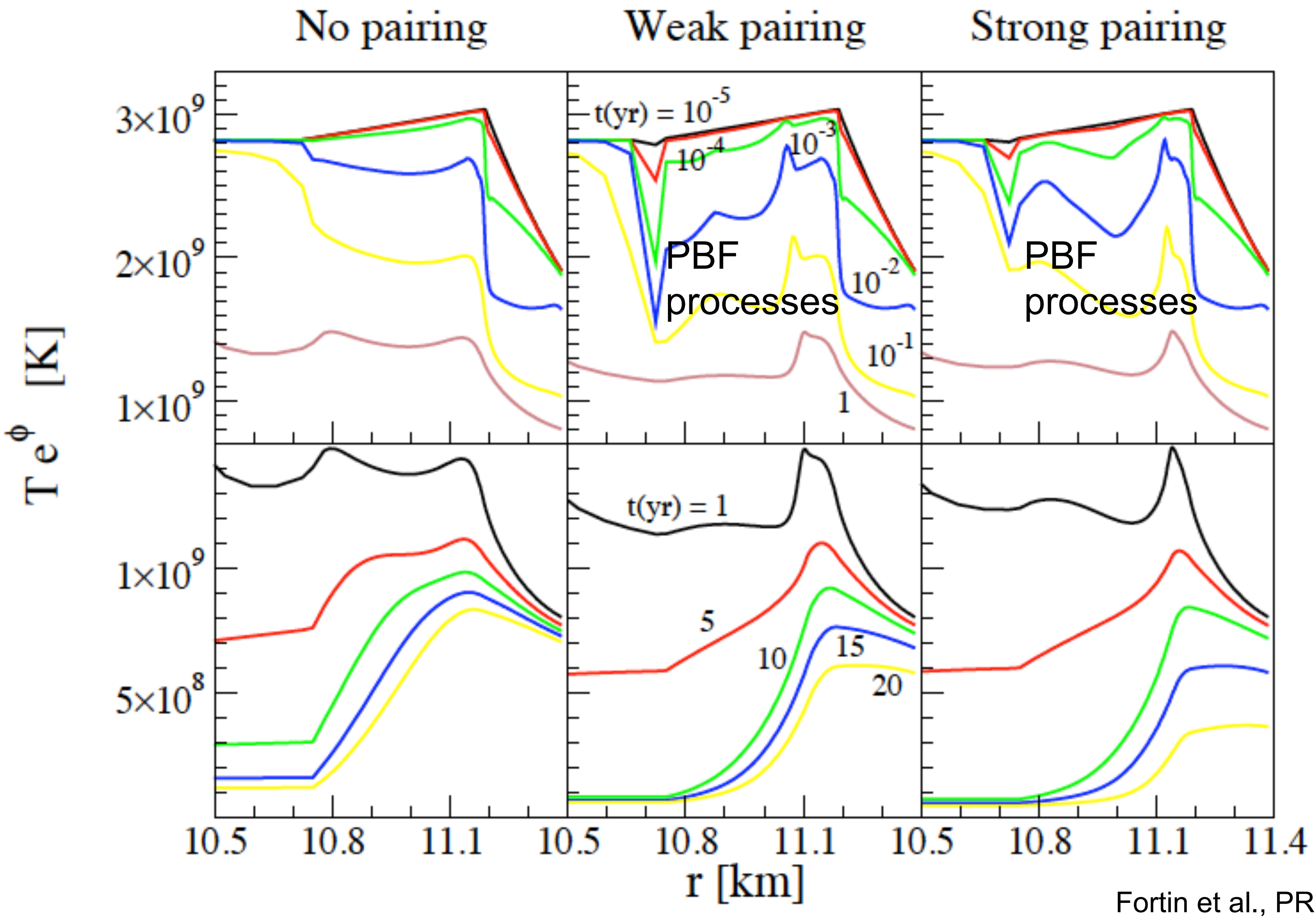
↔ **cooling of young neutron stars**

Temperature profiles in the crust

Heat transport equation (Thorne 1977):

$$\frac{\partial}{\partial r}\left[\frac{K r^2}{\Gamma(r)} e^{\phi} \frac{\partial}{\partial r}(e^{\phi} T)\right] = r^2 \Gamma(r) e^{\phi} \left(C_V \frac{\partial T}{\partial t} + e^{\phi} Q_\nu\right)$$

Cooling code of D. Page.



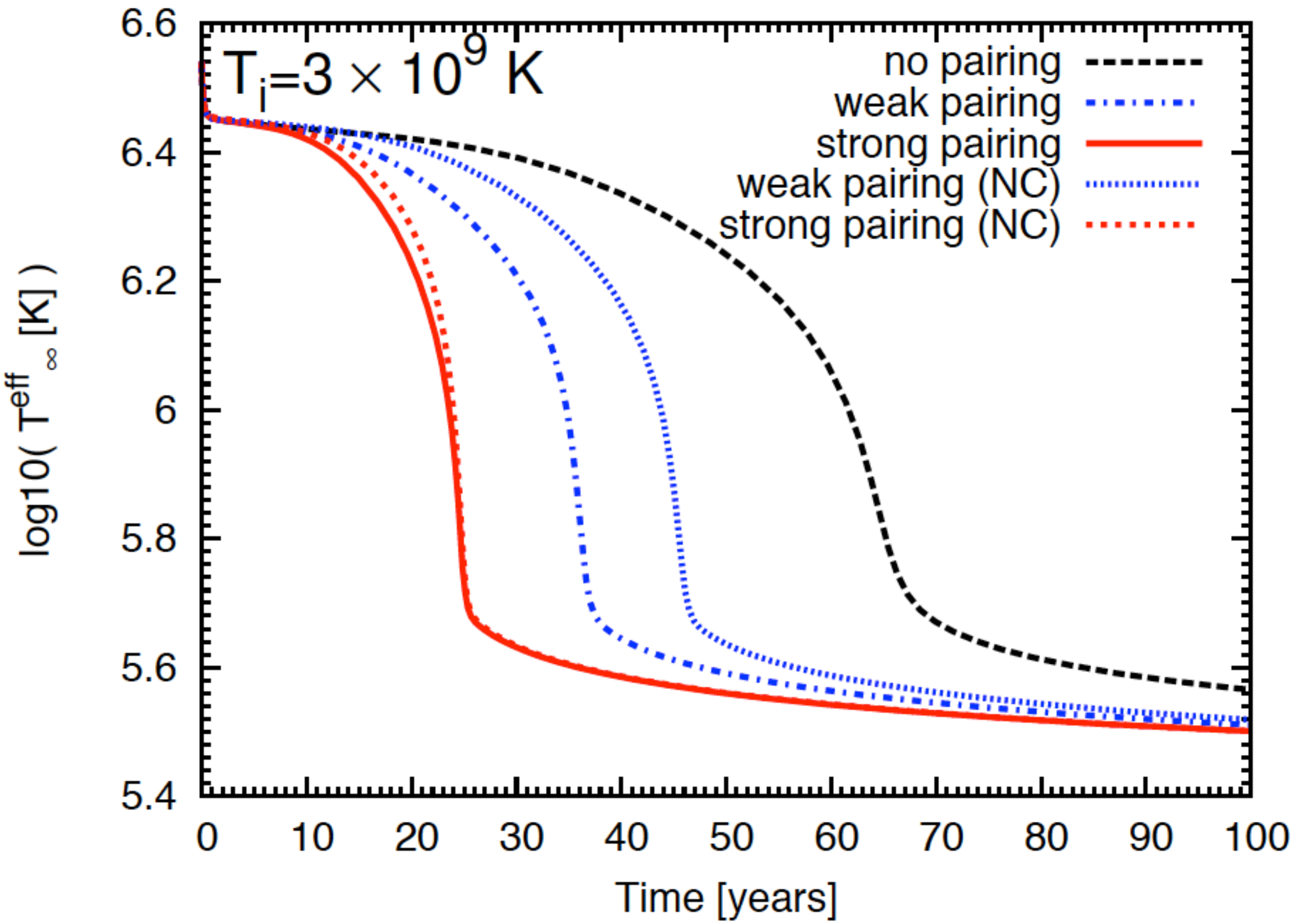
1.6 M_o
Fast cooling

1st year

next 20 yrs

Fortin et al., PRC 88 065804 (2010)

Surface temperature



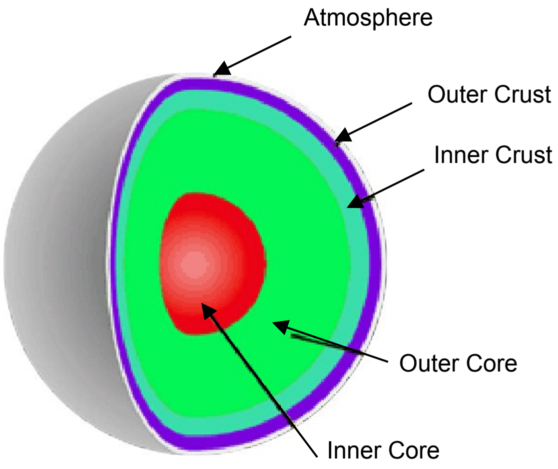
1.6 M_o
Fast cooling

NC=No clusters

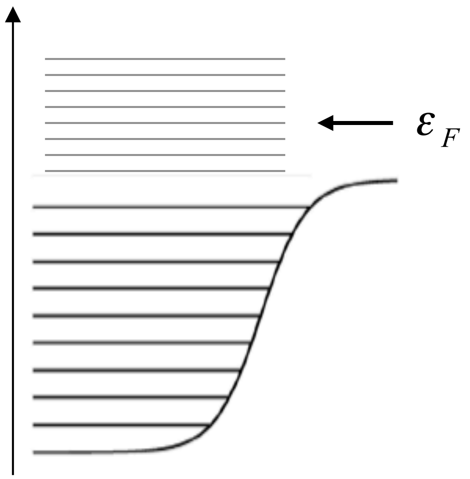
Hierarchy in the cooling time depending on the cooling scenario.
For weak pairing: the presence of non-uniform matter makes the cooling faster.
For strong pairing: almost no effect of non-uniform matter.

Fortin et al., PRC 88 065804 (2010)

Surprising features of superfluidity

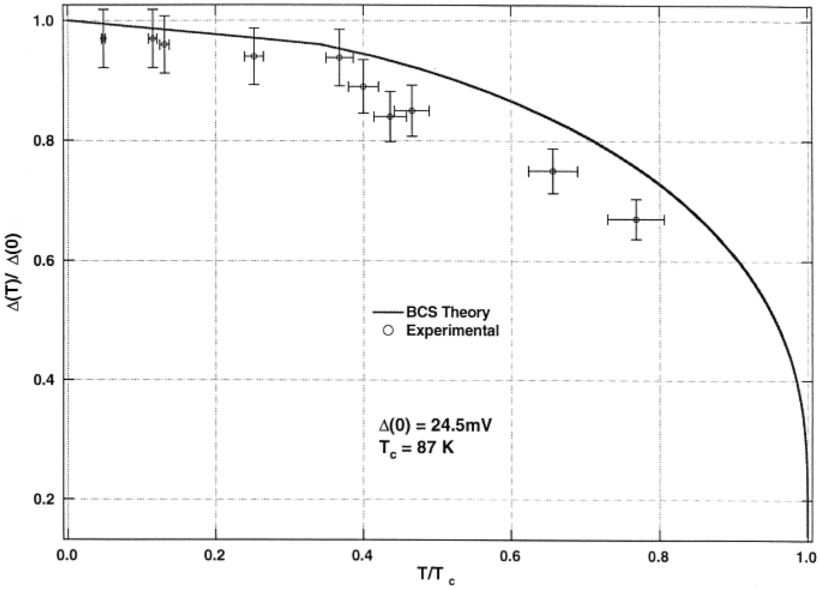


Transition outer / inner crust

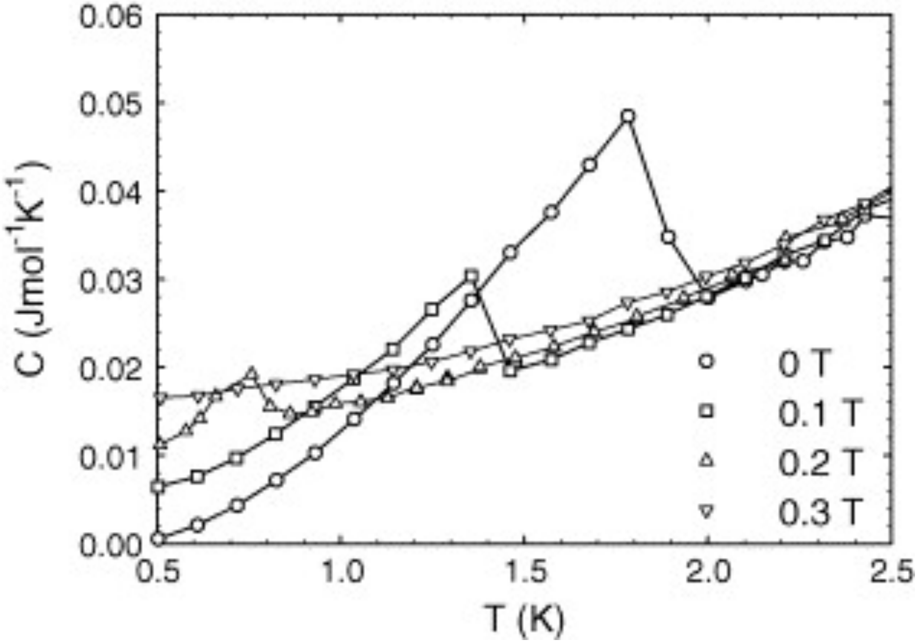


BCS at finite temperature

pairing gap



Specific heat



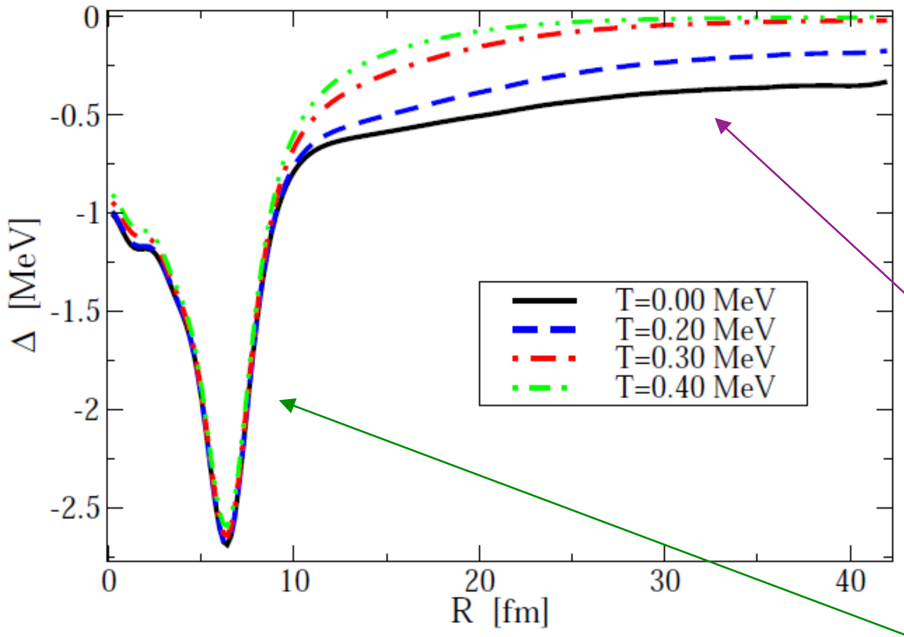
T/T_c

critical temperature

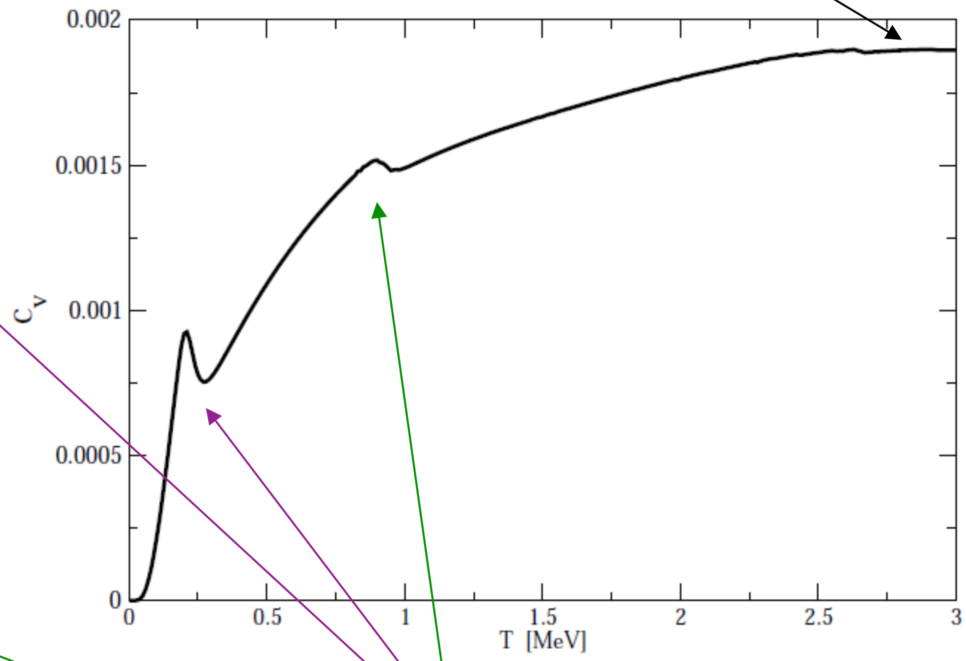
Neutrons specific heat in ^{500}Zr

N=460, Z=40

Pairing field profile
at various temperatures:



Neutron specific heat:



Classical regime

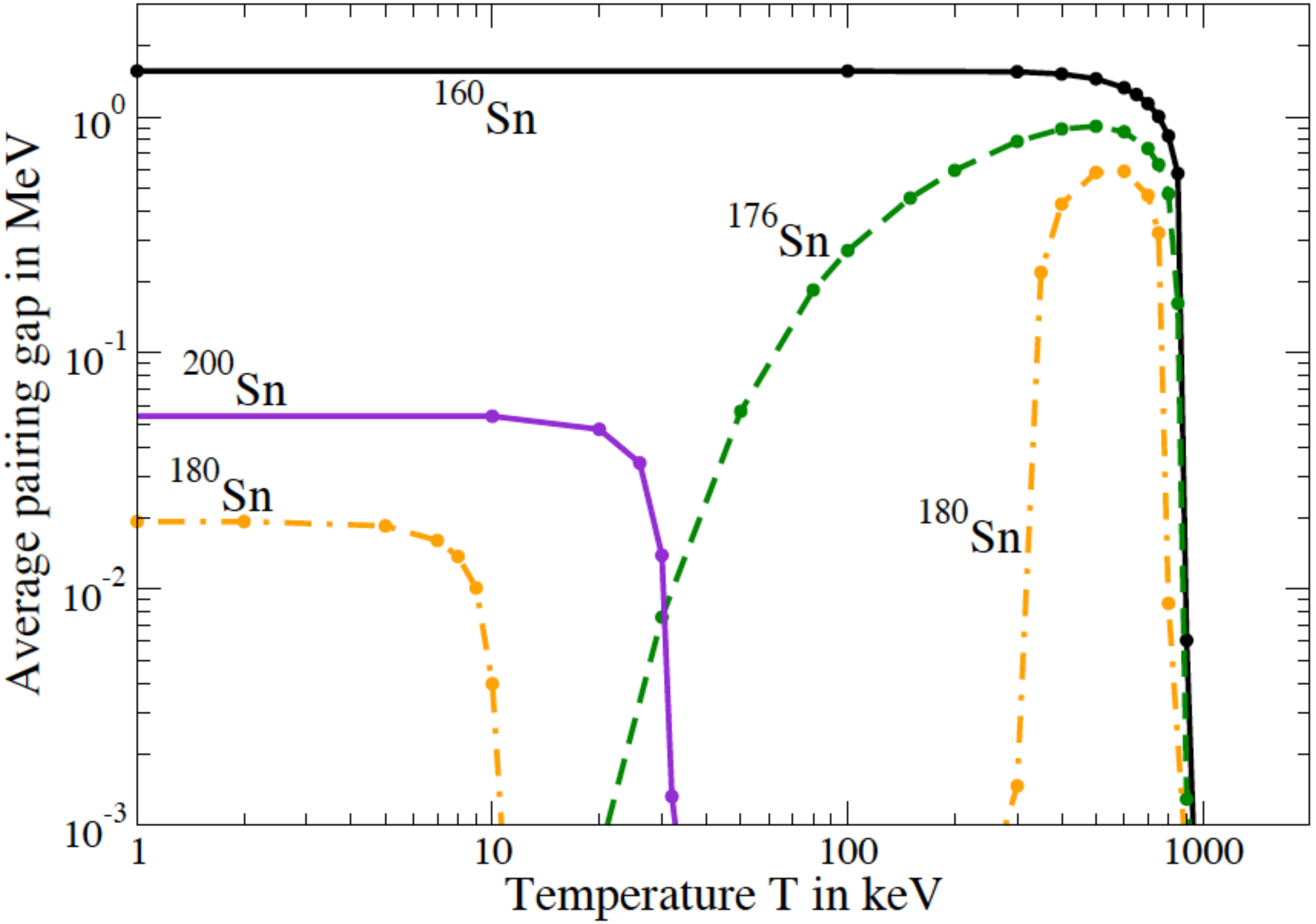
Disappearance of superfluidity

in the neutron gas

in the cluster

Fortin et al., PRC 88 065804 (2010)

Pairing reentrance phenomenon in Sn at the drip



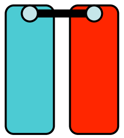
Temperature populates excited states:
1- kinetic energy cost induces a quenching of pairing,
2- in some cases, pairing occurs among thermally occupied excited states.

J.M. & Khan, PRC 2012

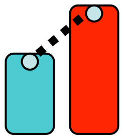
Pairing reentrance phenomenon

Superfluidity is destroyed by increasing the temperature…
But a bit of temperature sometimes helps in restoring superfluidity !

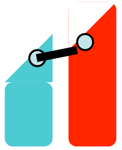
Pairing reentrance in asymmetric systems:



Pairing in symmetric systems



Asymmetry detroys pairing



Temperature in asymmetric systems restore superfluidity

In nuclear matter: pairing in the T=0 (deuteron) channel

Sedrakian, Alm, Lombardo, PRC 55, R582 (1997)

In spin-asymmetric cold atom gas

Castorina, Grasso, Oertel, Urban, Zappala, PRA 72, 025601 (2005)
Chien, Chen, He, Levin, PRL 97, 090402 (2006)

In higly polarized Liquid ^{3}He, ^{4}He

Frossati, Bedell, Wiegers, Vermeulen, PRL 57 (1986)

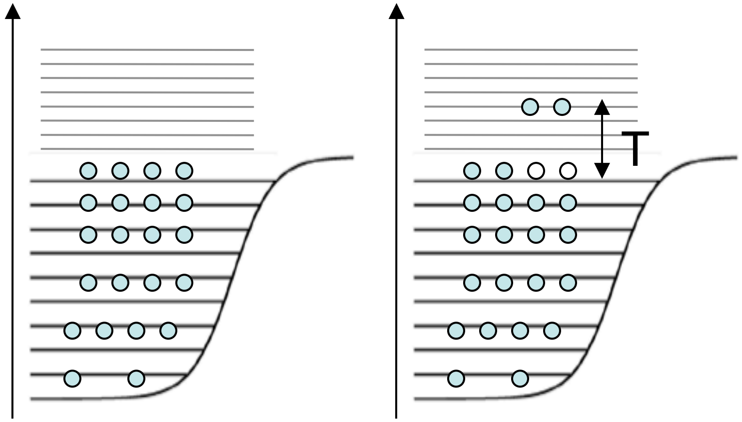
Pairing in heated rotating nuclei

Dean, Langanke, Nam, and Nazarewicz, PRL105, 212504 (2010).

Pairing reentrance in finite systems:

In magic nuclei, the presence of low-energy resonances, populated at low temperature, can help superfluidity to appear.

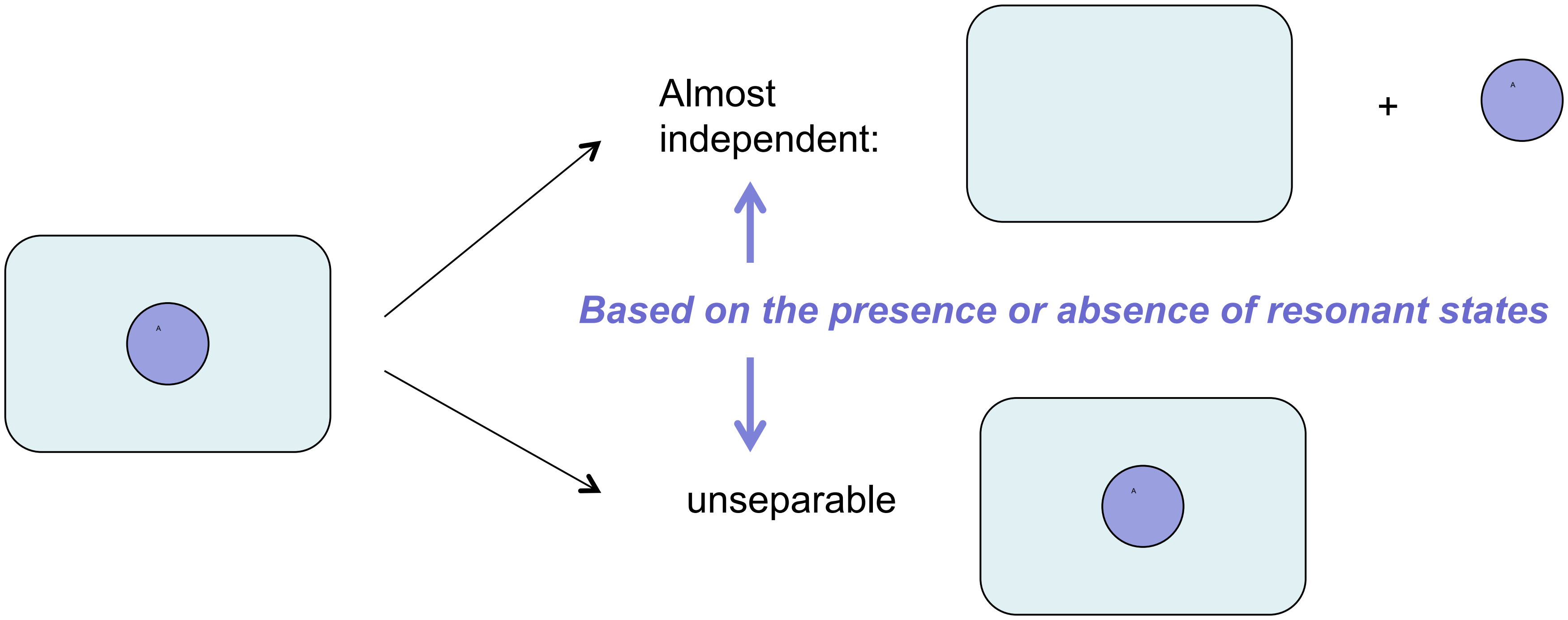
J.M., Khan, PRC 2012



A simple picture beyond the drip?

What is the interplay between the gas and the nuclei ?

Depending on the shell structure of drip nucleus:



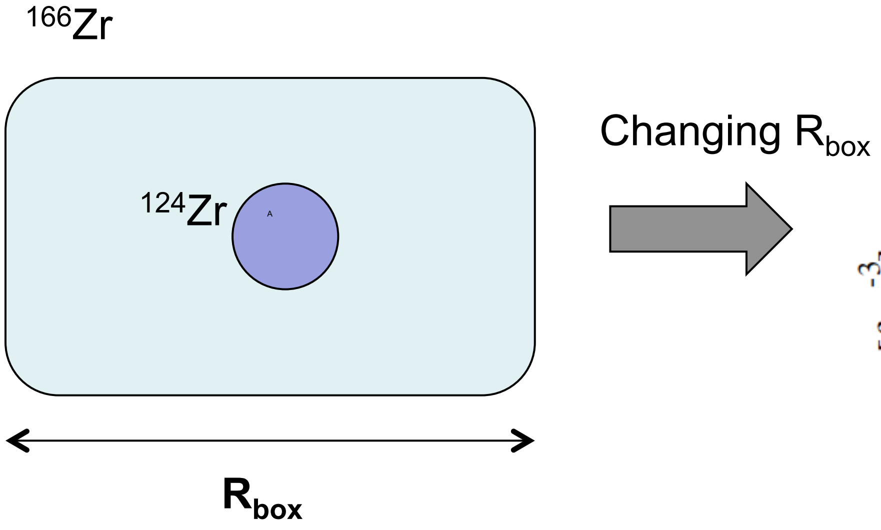
Almost independent:

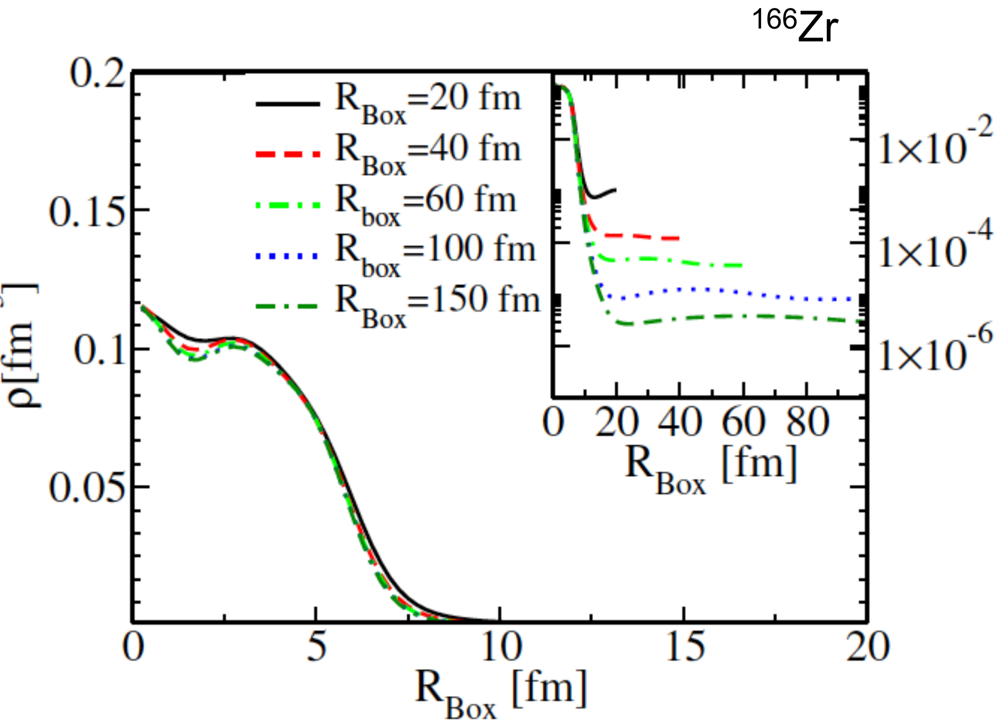
Based on the presence or absence of resonant states

unseparable

The limit of very dilute clusters

Interaction of a shallow gas with a nucleus

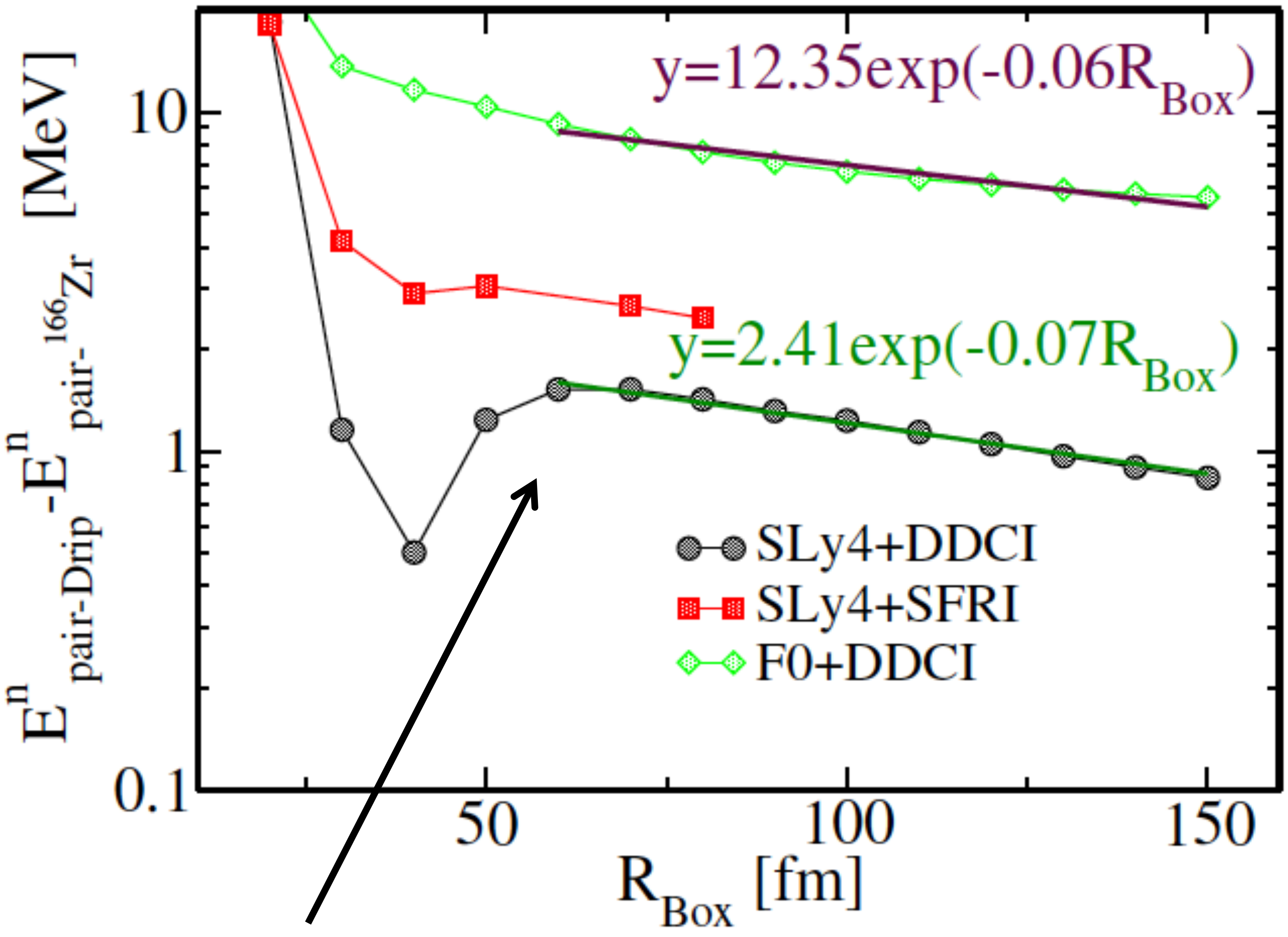




At which density ρ_{gas} the gas can be neglected?

Pastore, JM, Schuck, Vinas, PRC 2013

Decreasing the gas density (by increasing the volume)

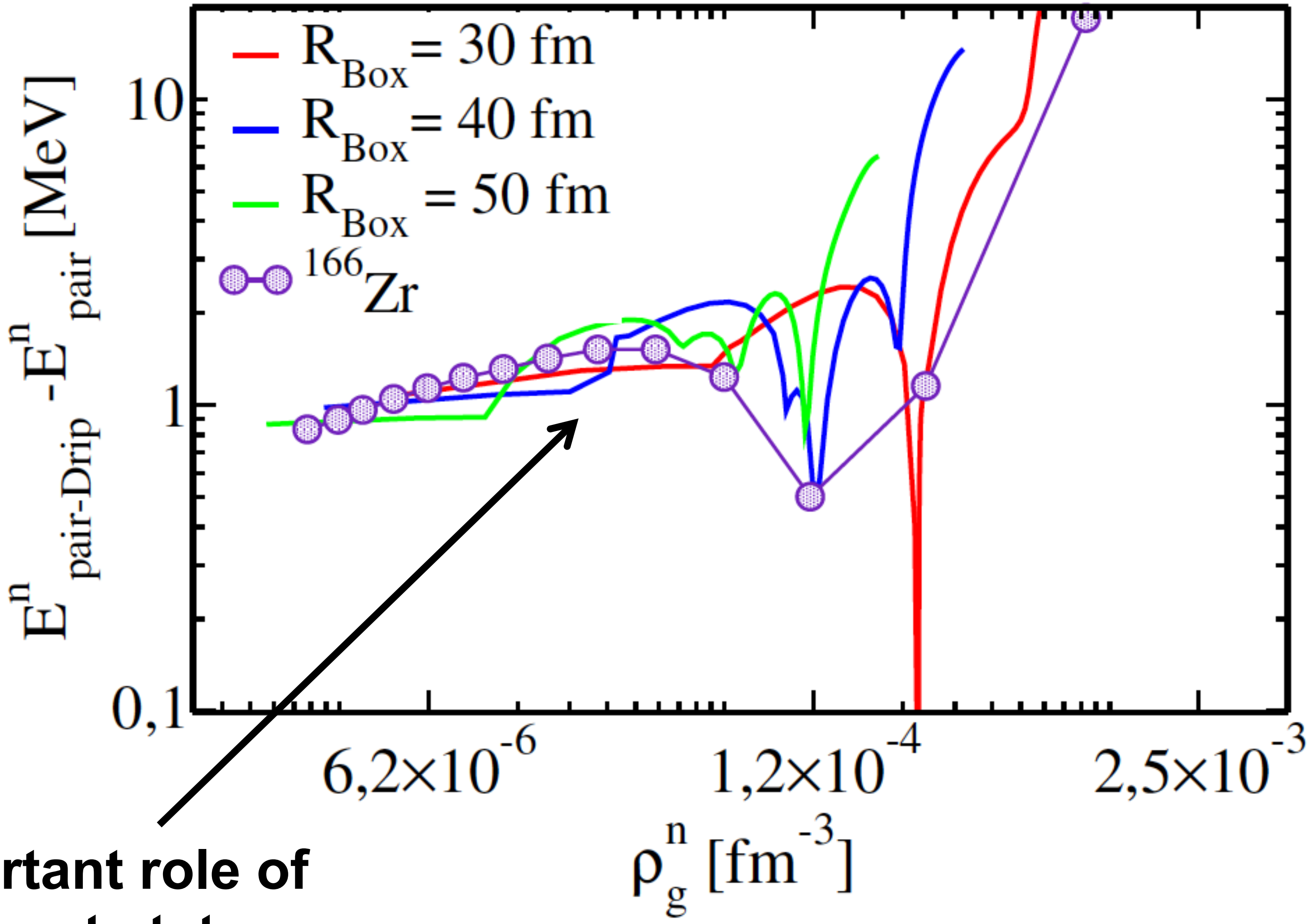


Important role of resonant states

Pastore, JM, Schuck, Vinas, PRC 2013

Decreasing the gas density (by decreasing N)

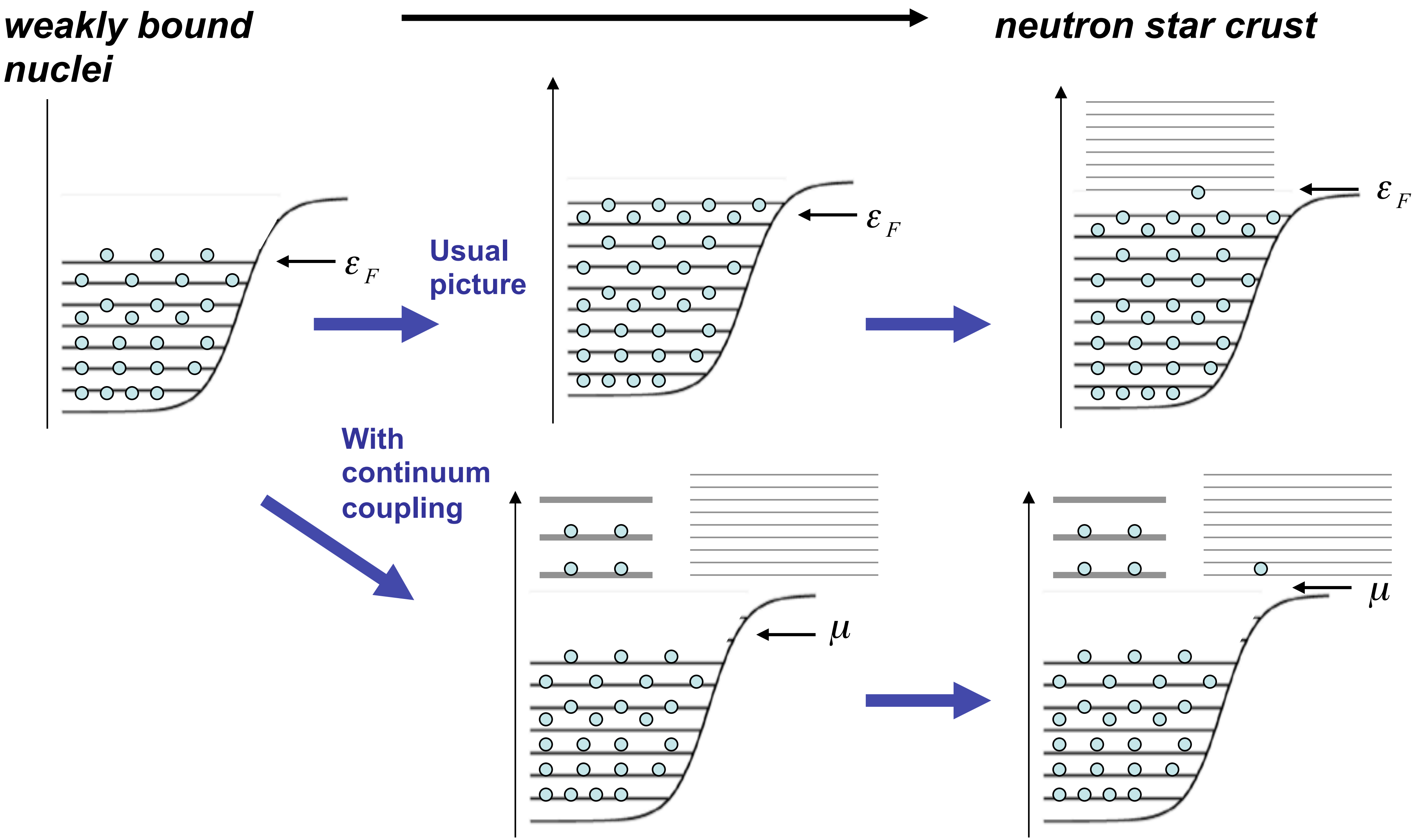
Fix Rbox, and decrease the total number of neutrons



Important role of resonant states

Pastore, JM, Schuck, Vinas, PRC 2013

Towards a better understanding of the neutron drip (line and -ing)



weakly bound nuclei

neutron star crust

Usual picture

With continuum coupling

Conclusions

- The transition between the **outer / inner crust** offers a fascinating playground to apply and test pairing theories.
- Since two superfluids overlap (gas+nucleus), surprising features occurs, mostly due to the **resonant states**.

These non-trivial features of superfluidity are interesting for:

- Neutron stars: Models for the crust including pairing shall be revised taking into account finite temperature in non-uniform nuclear clusters.
- Could resonant states be responsible for the **entrainment** phenomenon?
- For **nuclear experiments**: role of continuum coupling going towards the drip line for 20<Z<40.

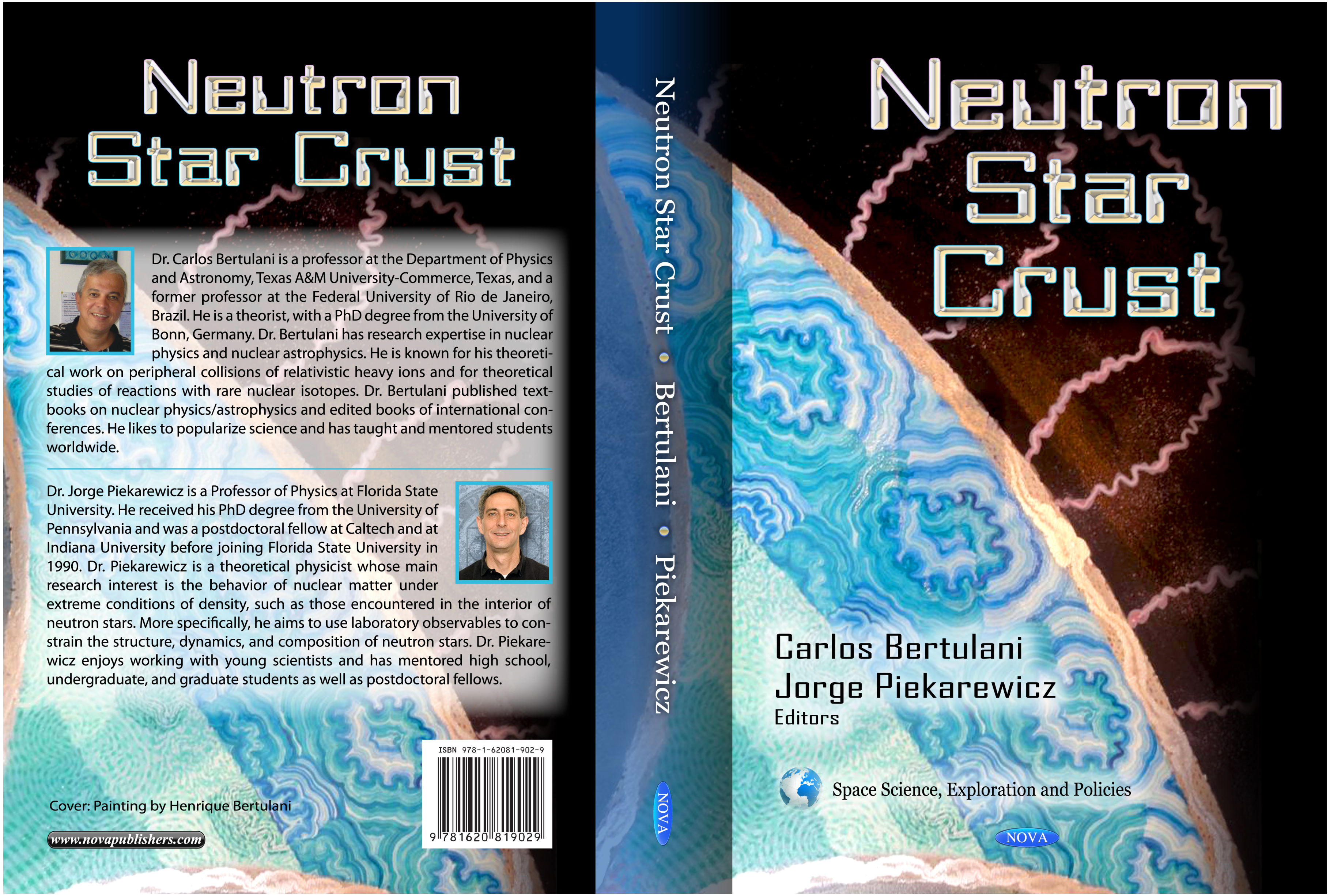
Outlooks

Pairing reentrance in other nuclei? Under investigation,

Similar effects in cold-atomic gas (Pastore, ArXiv 2014)

Correlations beyond HFB (QRPA, etc…)

Collaborators: A. Fantina (ULB Bruxelles), A. Pastore (ULB Bruxelles), N. Sandulescu (NIPNE Romania), N. Van Giai (IPNO), E. Khan (IPNO), P. Schuck (IPNO), X. Vinas (Barcelona), D. Page (UNAM Mexico), P. Pizzochero (Milan).



Neutron Star Crust

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Dr. Jorge Piekarewicz is a Professor of Physics at Florida State University. He received his PhD degree from the University of Pennsylvania and was a postdoctoral fellow at Caltech and at Indiana University before joining Florida State University in 1990. Dr. Piekarewicz is a theoretical physicist whose main research interest is the behavior of nuclear matter under extreme conditions of density, such as those encountered in the interior of neutron stars. More specifically, he aims to use laboratory observables to constrain the structure, dynamics, and composition of neutron stars. Dr. Piekarewicz enjoys working with young scientists and has mentored high school, undergraduate, and graduate students as well as postdoctoral fellows.

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Carlos Bertulani
Jorge Piekarewicz
Editors

Space Science, Exploration and Policies

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The crust of neutron stars

Negele & Vautherin NPA 207 (1973)

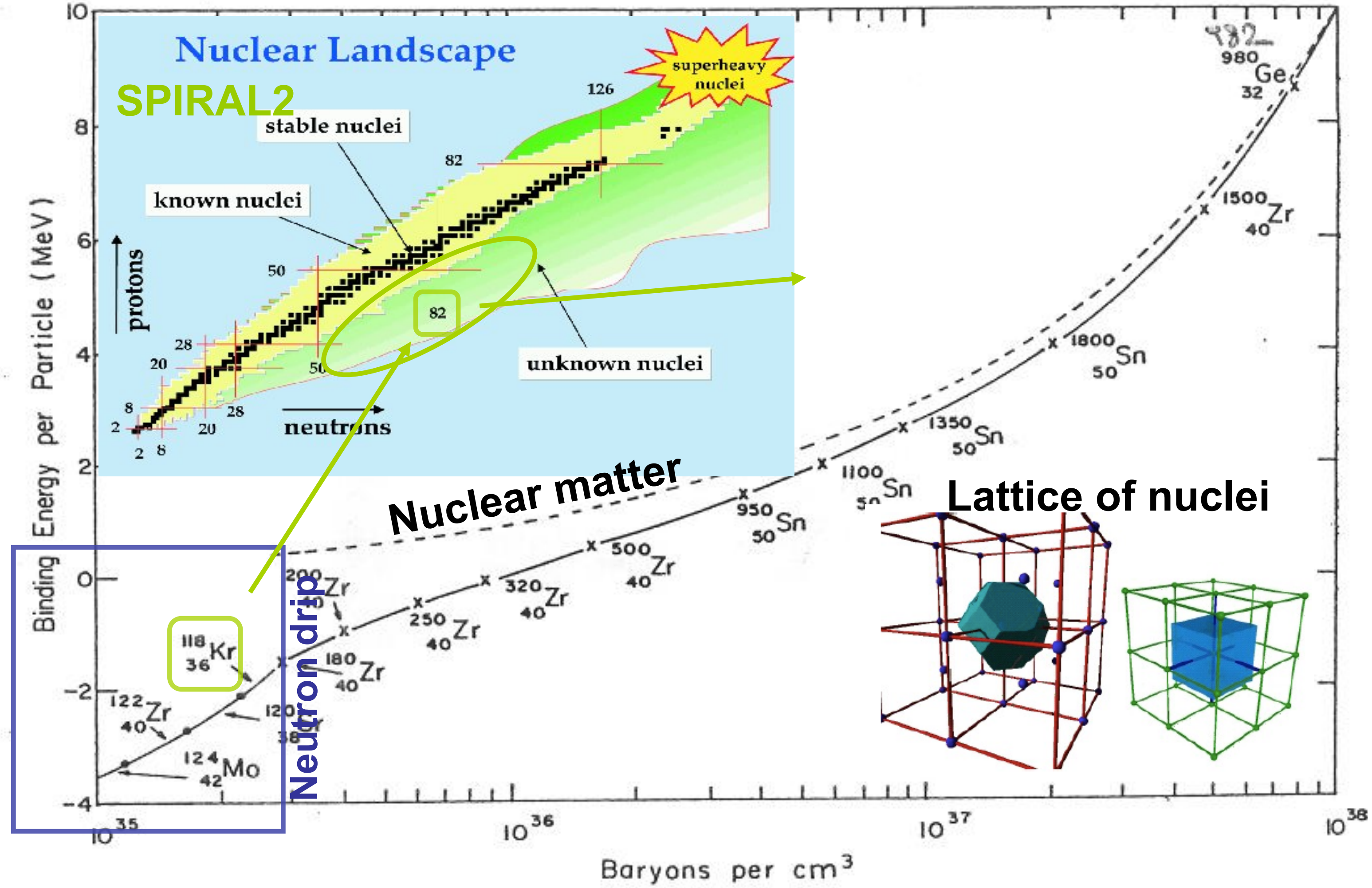
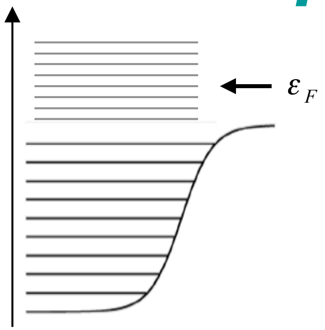


Fig. 2. Energy per particle versus baryon density.

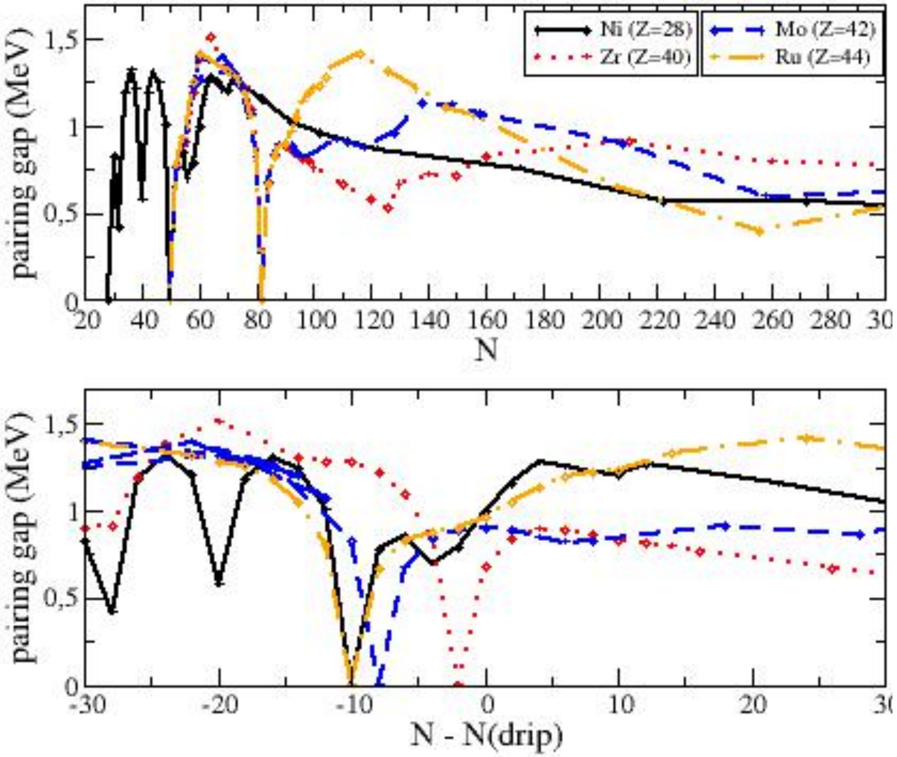
A systematic study based on 8 isotopes with 28<Z<50

Nucleus	Z	N_{drip}	group	N_{res}
Ni	28	60	$\mathcal{A}_1$	3.0
Kr	36	82	$\mathcal{A}_2$	0.0
Sr	38	82	$\mathcal{A}_2$	0.0
Zr	40	84	$\mathcal{A}_1$	2.2
Mo	42	90	$\mathcal{A}_1$	8.0
Ru	44	92	$\mathcal{A}_1$	3.0
Sn	50	126	$\mathcal{A}_2$	0.0
Te	52	126	$\mathcal{A}_2$	0.0

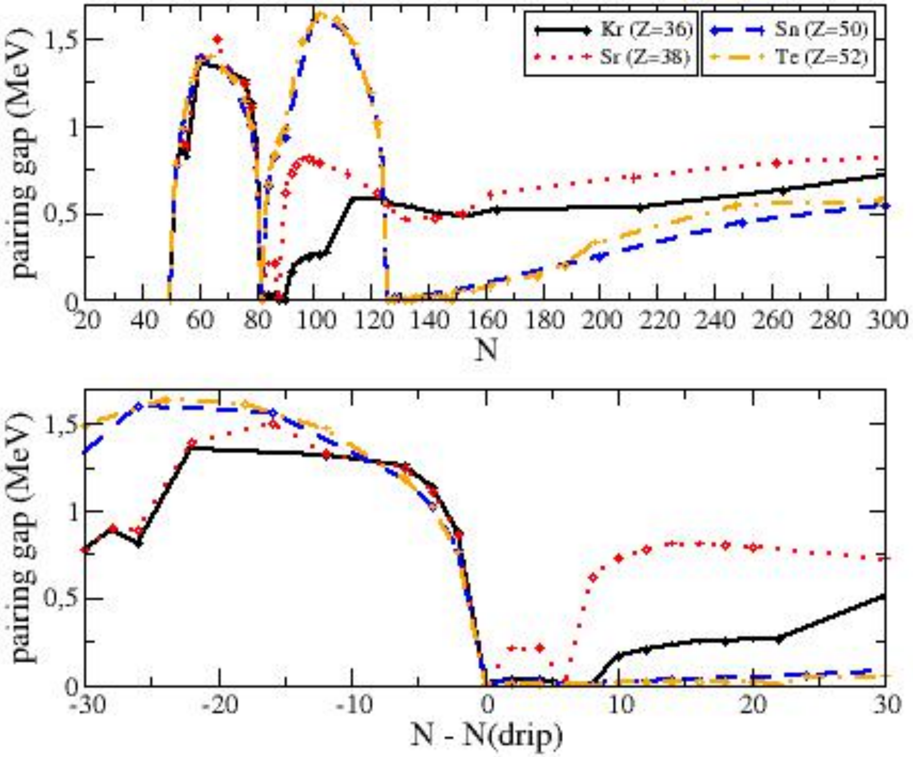


Suppression or persistence of pairing upon overflowing neutrons.

group $\mathcal{A}_1$



group $\mathcal{A}_2$

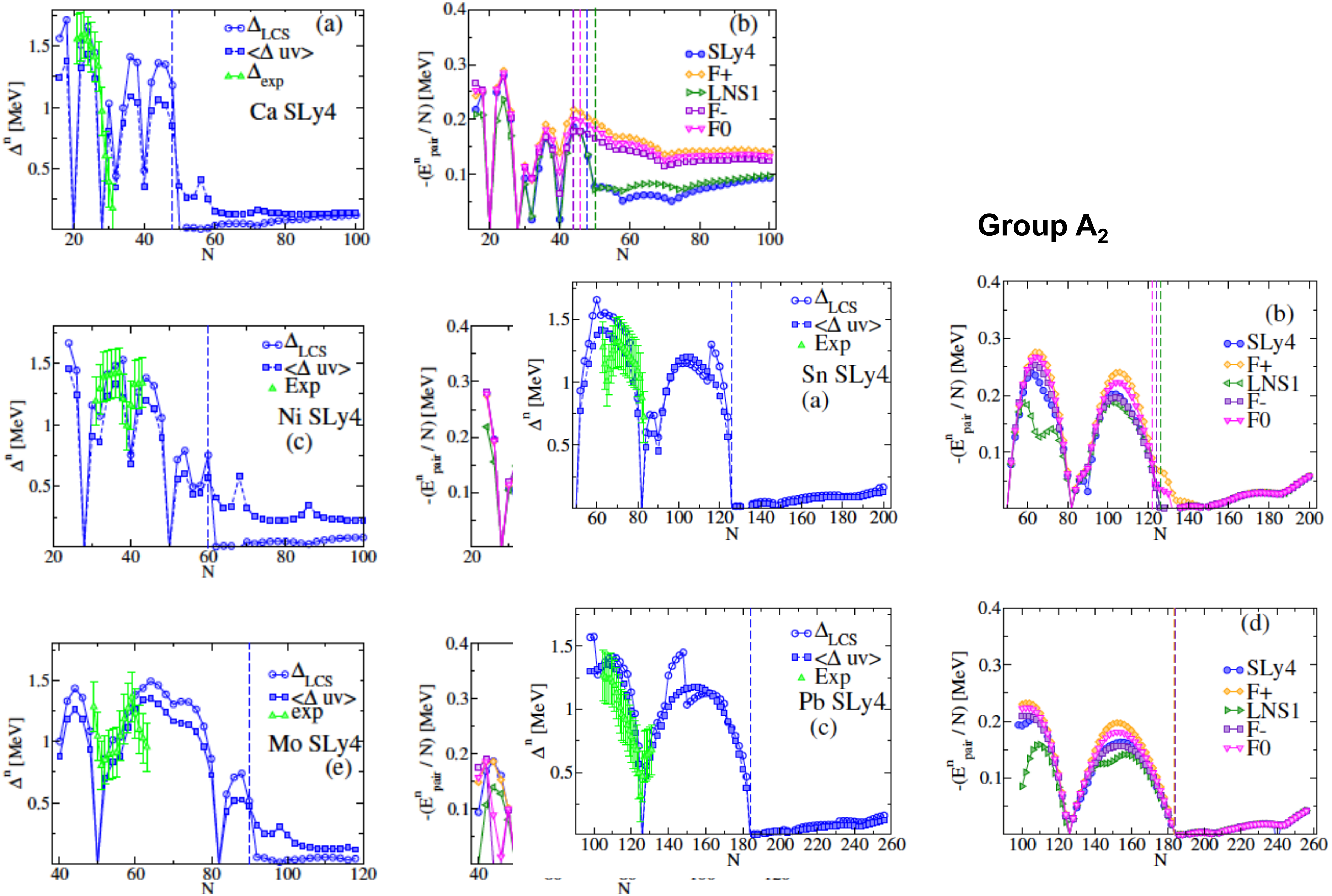


J.M. & Khan, PRC 2012

Weak dependence on the model

Group A_1

Group A_2



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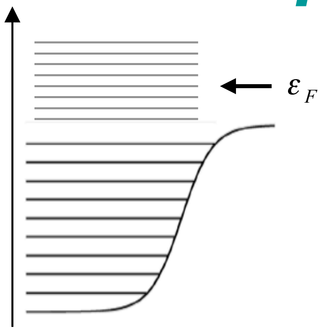
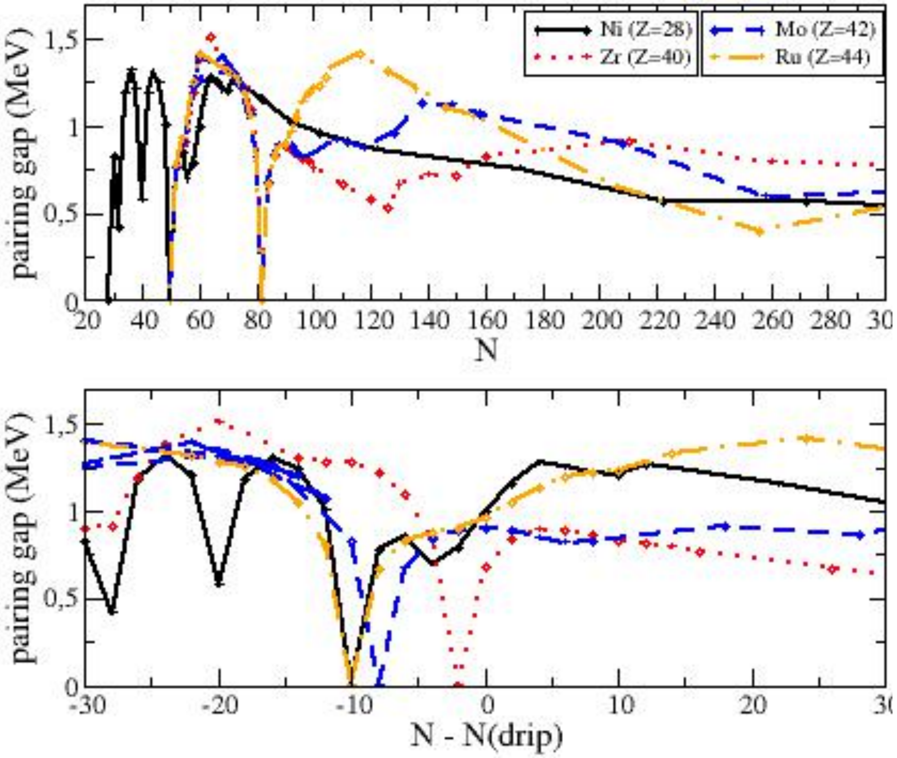
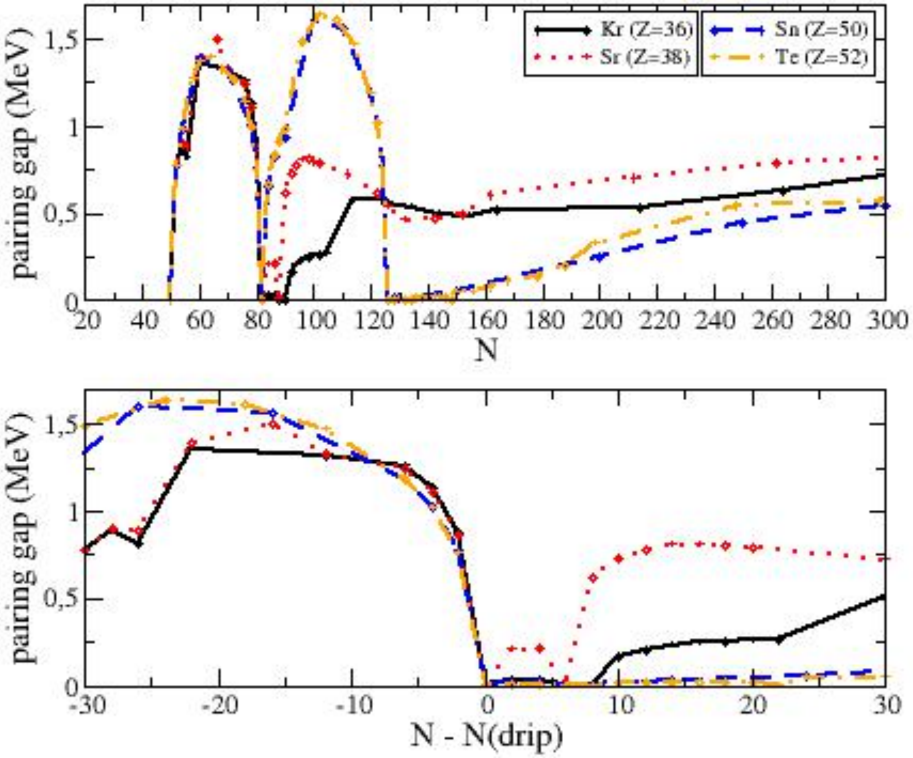


Pairing is suppressed in the absence of occupied resonant states at the drip-line.

group $\mathcal{A}_1$



group $\mathcal{A}_2$



J.M. & Khan, PRC 2012

Spontaneous symmetry breaking

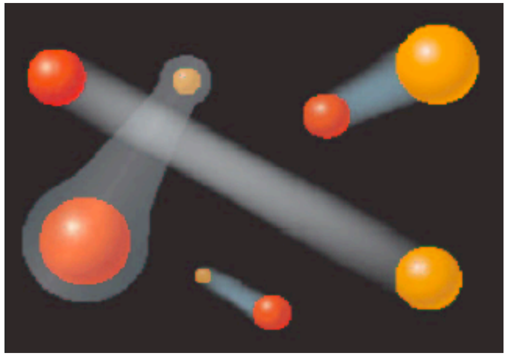
2 recent Nobel prices:

2008 **Yoichiro Nambu**
"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"
Makoto Kobayashi and **Toshihide Maskawa**
"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"

2013 **François Englert** and **Peter W. Higgs**
"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"

Uniform matter

Superfluid matter



	Uniform matter	Superfluid matter
Ground state:	$\left\| HF \right\rangle$	$\left\| BCS \right\rangle$
invariant under rotation: Symmetry U(1)	$\left\| HF \right\rangle = \exp(-i\varphi)\left\| HF \right\rangle$	$\left\| BCS \right\rangle \neq \exp(-i\varphi)\left\| BCS \right\rangle$
Non-vanishing operator:	$\left\langle HF \right\| c_{k\uparrow} c_{-k\downarrow} \left\| HF \right\rangle = 0$	$\left\langle HF \right\| c_{k\uparrow} c_{-k\downarrow} \left\| HF \right\rangle = \Delta_k$

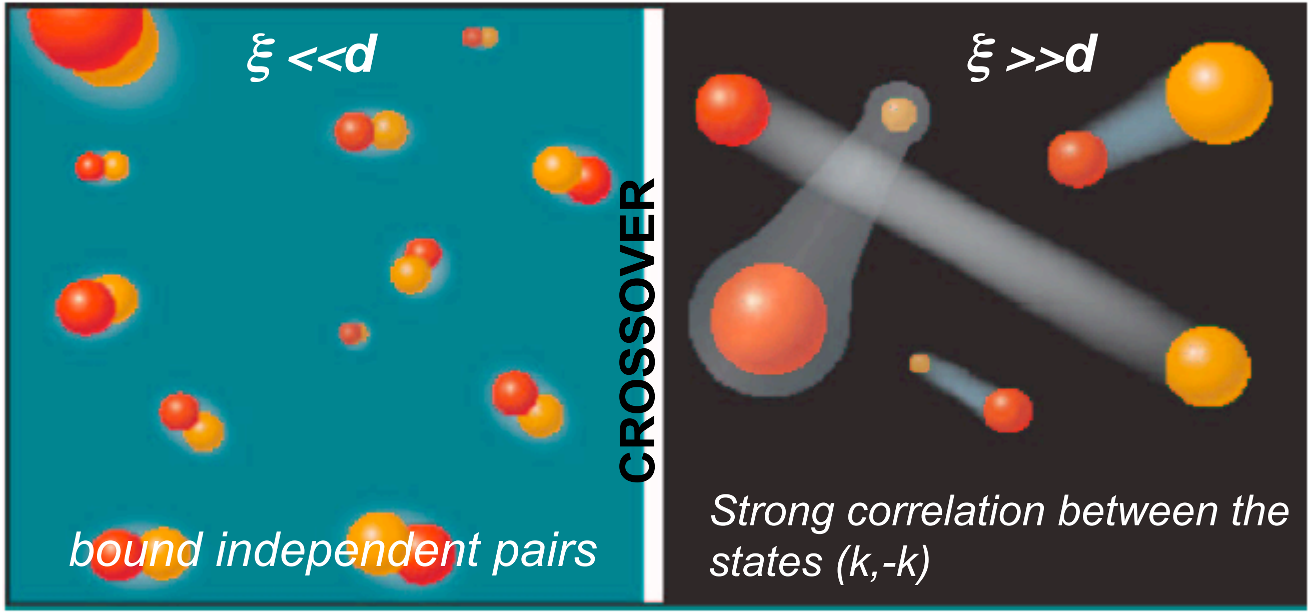
BCS/BEC condensate of Cooper pairs

(ultra-cold atomic gases)

(^{6}Li, ^{40}K)
Fermi gas (2 components)
Interaction in the s-wave

N of atoms ~10^4-10^6
Densities ~10^{13}-10^{14} atoms/cm^3
Temperatures~10-100 nK

BEC (molecules) BCS (weak limit)



a: scattering length,
d: average distance.

a>0 *(Feshbach resonance)* a<0

$$\delta\varepsilon_k \approx \Delta \Rightarrow \delta k \approx \frac{m\Delta}{\hbar^2 k_F} \Rightarrow \delta \mathrm{r} \approx \frac{\hbar^2 k_F}{m\Delta} = \xi$$

For neutrons:
a ~ -18.5 fm

→ **Low density neutron matter is close to the unitary limit (crossover)**