Hadrons in the PNJL model for interacting quarks

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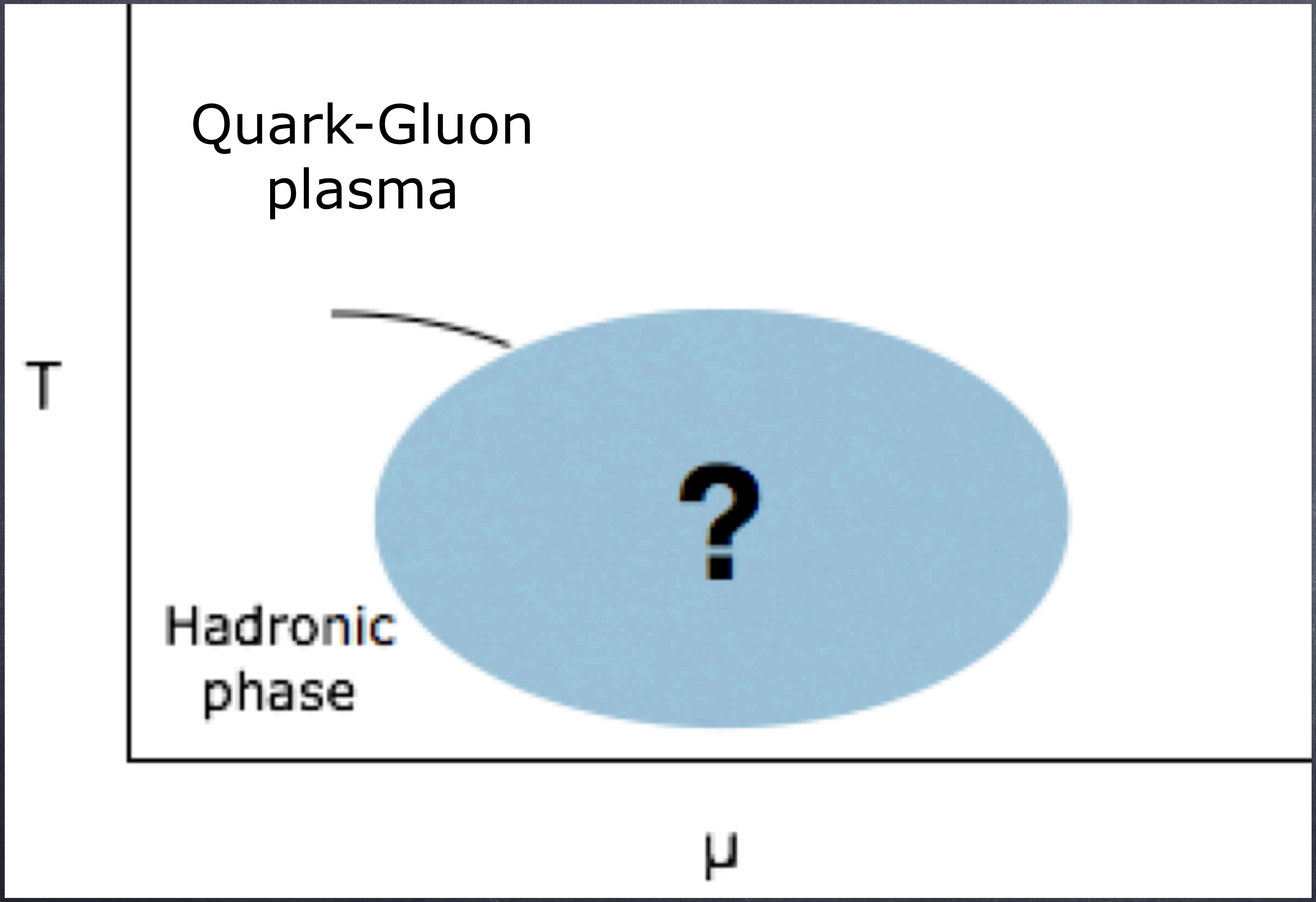
in collaboration with **Tetsuo Matsui** (the Open University of Japan)
and **Gordon Baym** (university of Illinois)

- KY and T. Matsui, Nucl. Phys. A913 (2013) 19.
- KY and T. Matsui, Nucl. Phys. A922 (2014) 237.
- KY, T. Matsui, Gordon Baym, Nucl. Phys. A933 (2015) 245.

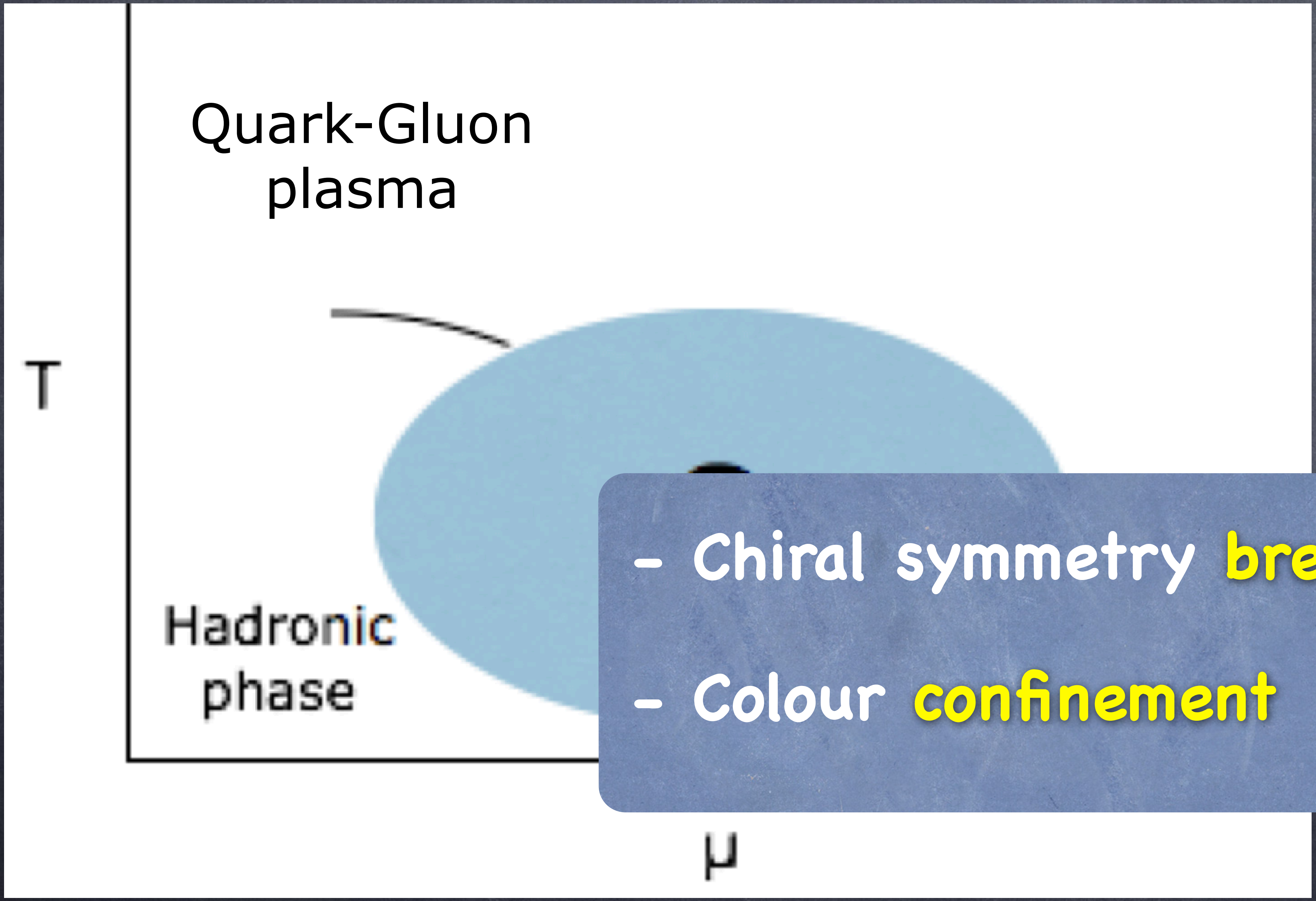
Outline

- Introduction
    - motivation of this work
    - how to approach
- Mesons in the PNJL model
    - equation of state
    - collective modes
- Baryons in the PNJL model
- Summary

Quark-Hadron Phase Transition

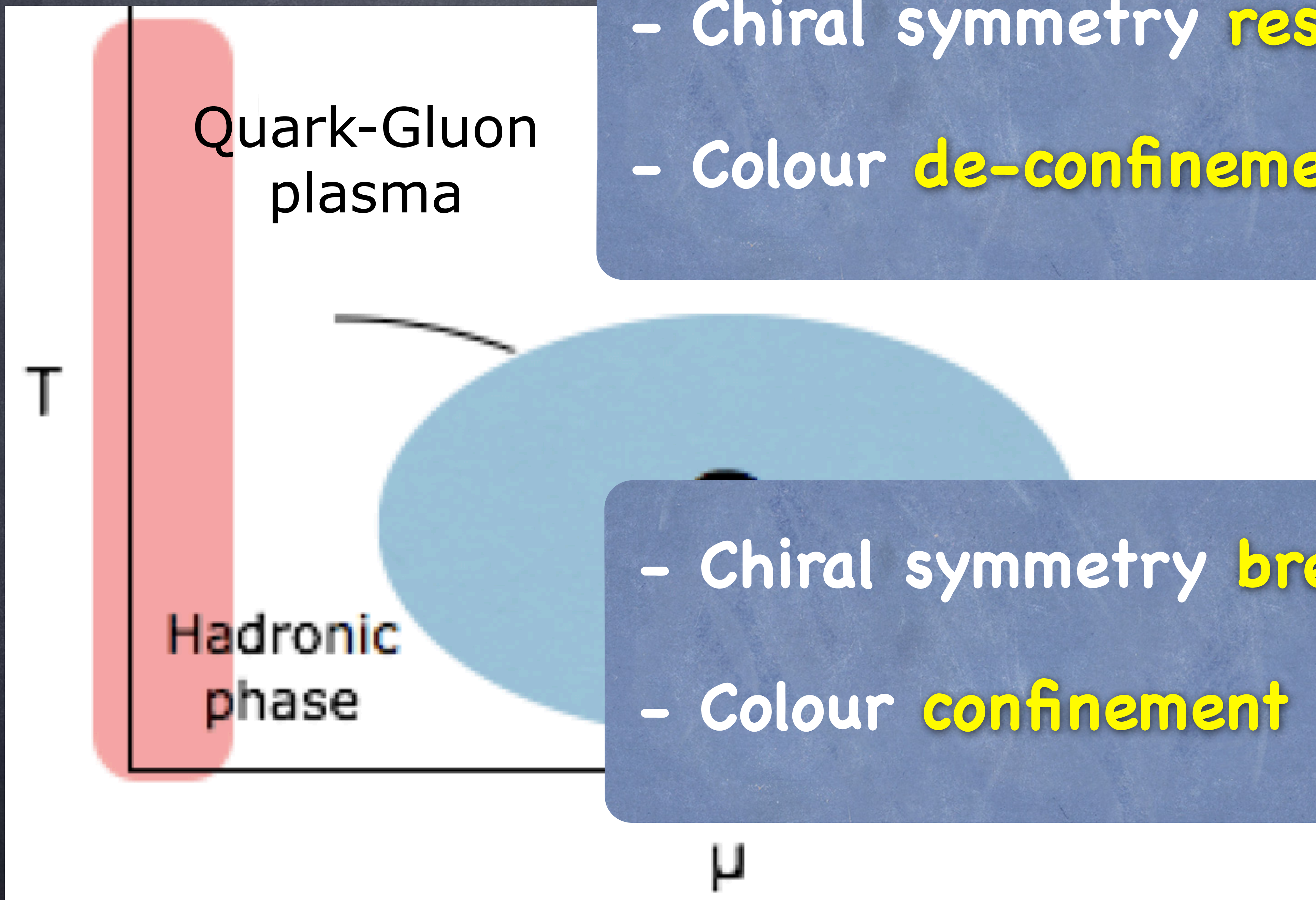


Quark-Hadron Phase Transition



- Chiral symmetry **breaking**
- Colour **confinement**

Quark-Hadron Phase Transition



- Chiral symmetry **restoration**
- Colour **de-confinement**

- Chiral symmetry **breaking**
- Colour **confinement**

Transitions

chiral transition

- spontaneous chiral symmetry breaking generates quark mass
  in the vacuum : $M \sim 300$ MeV
- as T increases, chiral symmetry restores : $m_0 \sim 3\text{-}7$ MeV
- order parameter : chiral condensate $\langle \bar{q}q \rangle$

Transitions

chiral transition

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de-confinement transition

- characterised by expectation value of Polyakov loop

$$L(x) = \mathcal{P}\exp\Big[-ig\int_0^\beta dx_4 A_4(\boldsymbol{x}, x_4)\Big], \quad \mathcal{P}: \text{path ordering}$$

$$l = \frac{1}{N_c}\mathrm{tr}L\ , \qquad \Phi(\boldsymbol{x}) = \langle l(\boldsymbol{x})\rangle$$

- relation between P-loop and quark free energy : $\Phi = e^{-\beta f_q}$

confinement : $f_q = \infty$ ⟹ $\Phi = 0$

de-confinement : $f_q < \infty$ ⟹ $\Phi \neq 0$

Order parameters

	chiral condensate	Expectation value of Polyakov loop
low temperature	$\langle \bar{q}q \rangle \neq 0$	$\Phi = 0$
high temperature	$\langle \bar{q}q \rangle = 0$	$\Phi \neq 0$

Purpose

describe the hadrons at low temperatures,
and study how they melt as the temperature increases

For this, we use an effective model which describes

- chiral transition and de-confinement transition simultaneously
- (quarks : fundamental fields
  hadrons : composed by quarks

➡ **Nambu-Jona-Lasinio model with Polyakov loop (PNJL model)** K. Fukushima, 2004

PNJL model

- chiral phase transition

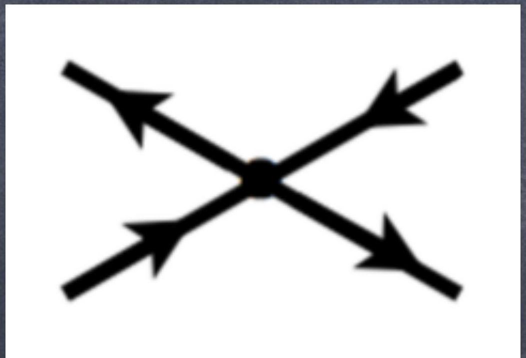
  ➡ Nambu-Jona-Lasinio (NJL)

  Y. Nambu, G. Jona-Lasinio, 1961

  T. Hatsuda, T. Kunihiro, 1985

  V. Bernard, U. G. Meissner, I. Zahed, 1987

interaction :



- de-confinement transition

  ➡ Polyakov loop

  A. M. Polyakov, 1978

  L. Susskind, 1979

PNJL model

Mesons
in the PNJL model

2 flavour PNJL model

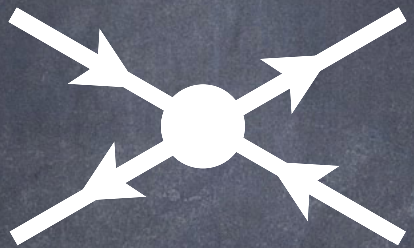
K. Fukushima, 2004

Partition function

$$Z = \int [dq][d\bar{q}] \exp\Big[\int_0^\beta d\tau \int d^3x \; \mathcal{L}(q,\bar{q})\Big]$$

$$\mathcal{L} = \bar{q}(i\gamma_\mu D^\mu - m_0)q + \mathcal{L}_4 - \mathcal{U}[\bar{\Phi}, \Phi, T]$$

$$D_\mu = \partial_\mu + g\boxed{A_0}\delta_{\mu,0} \quad , \quad A_4 = i A_0$$



m_0 : bare quark mass

breaks chiral symmetry explicitly

2 flavour PNJL model

K. Fukushima, 2004

Partition function

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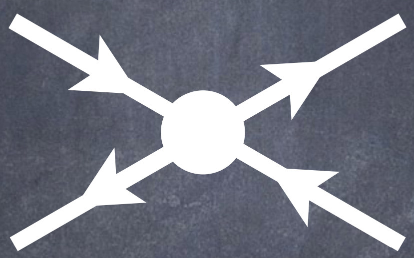


Effective potential (as a function of the Polyakov) :

represent dynamics of gluon phenomenologically

Method

- Calculate the partition function which is given in terms of **the integral over the quark fields** in the path integral method.

- Meson fields are introduced as **auxiliary fields**, then the partition function contains the integrals over the **quark fields** and **meson fields**.

- The simplest way to calculate the partition function is the **mean field approximation**

mean field approximation is NOT good

- Meson fields are treated as **uniform back ground** under the **mean field approximation**

- Even at low temperatures, **mesonic excitations cannot be described**

- However, mesonic excitations must dominate equation of state at low temperatures

- Taking fluctuations into account by expanding the effective action up to the second order
- **MFA + mesonic correlations**

Thermodynamic potential

- partition function after inserting boson fields :

$$Z(T, A_4) = \int [d\bar{q}][dq][d\phi]\mathrm{e}^{-I(\bar{q},q,\phi,A_4)}$$

- integrating over **fermion fields**

$$\Rightarrow \quad Z(T, A_4) = \int [d\phi]e^{-I(\phi',A_4)}$$

- expand the effective action up to second order in the fluctuations around the stationary point, in order to take mesonic correlations into account

Thermodynamic potential

- partition function

$$Z(T, A_4) \simeq e^{-I_0} \int [d\varphi] \exp\left[-\frac{1}{2} \frac{\delta^2 I}{\delta\phi'_i \delta\phi'_j} \bigg|_{\phi'=\phi'_0} \varphi_i \varphi_j \right]$$

- integrating over **boson fields**

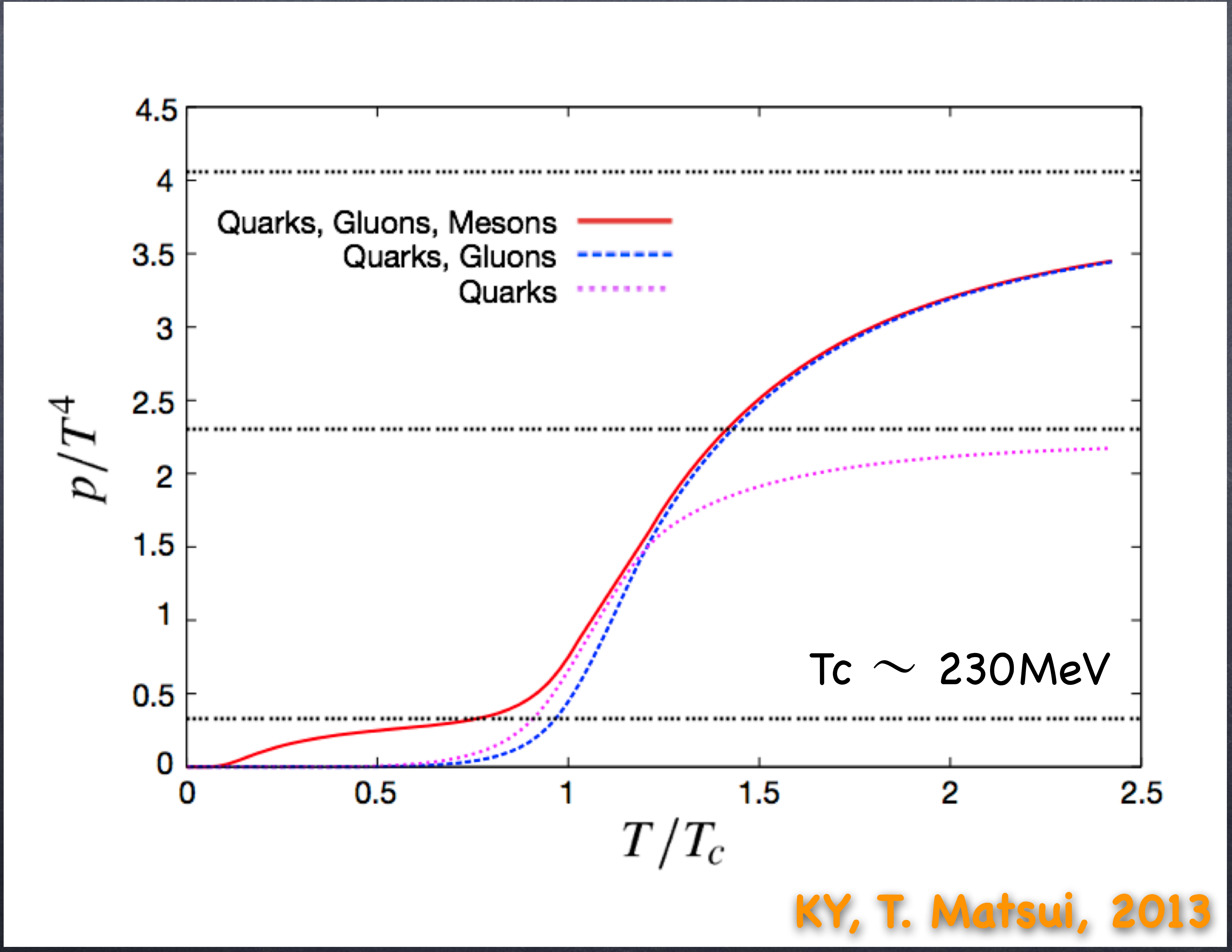
$$\Rightarrow \quad \Omega(T, A_4) = T \left(\underline{I_0} + \underline{\frac{1}{2} \mathrm{Tr_M} \ln \frac{\delta^2 I}{\delta\phi_i \delta\phi_j}} \right)$$

contribution from **mean field**

contribution from **mesonic excitations**

Pressure

$$p(T) = -\frac{1}{V}\Omega(T)$$



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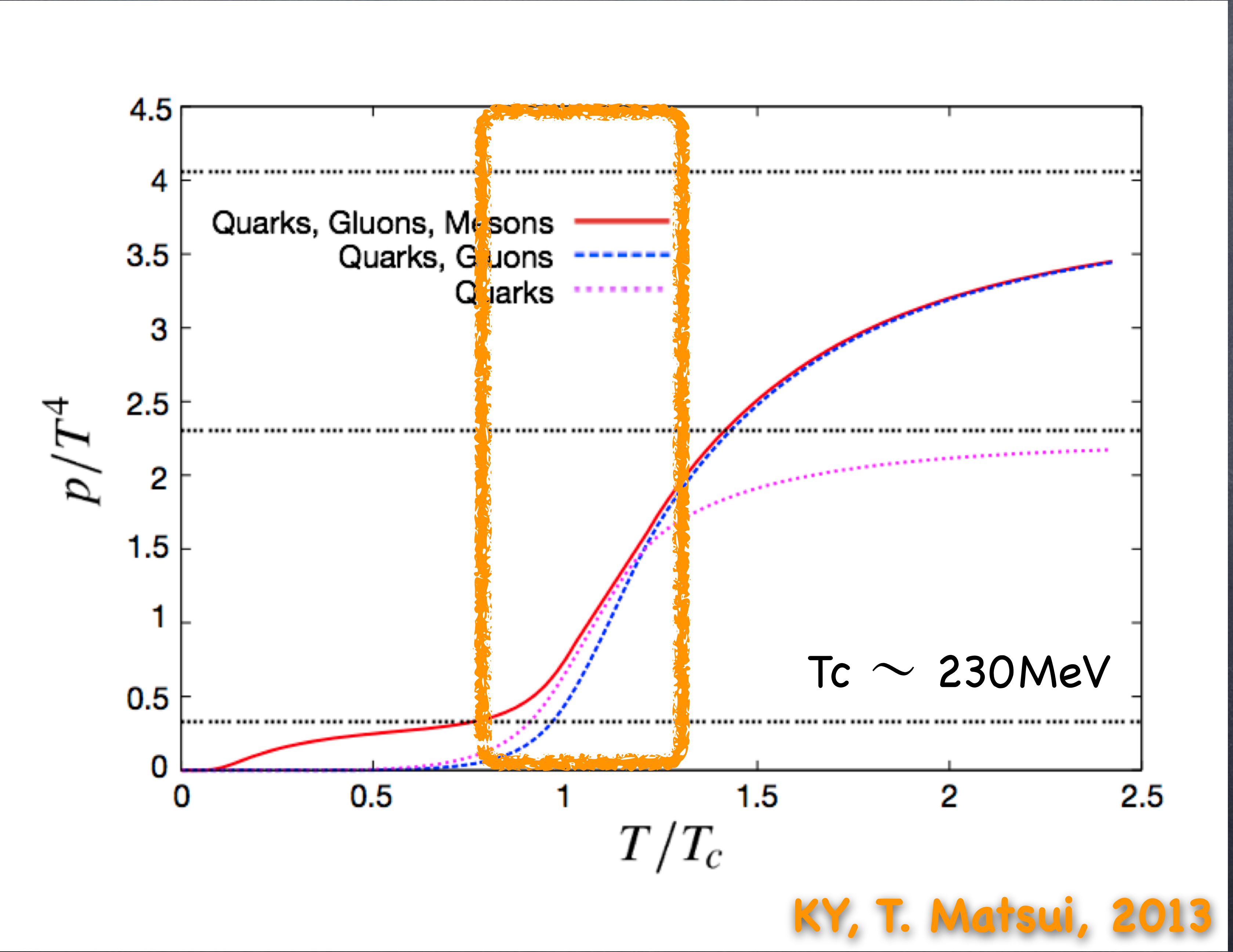
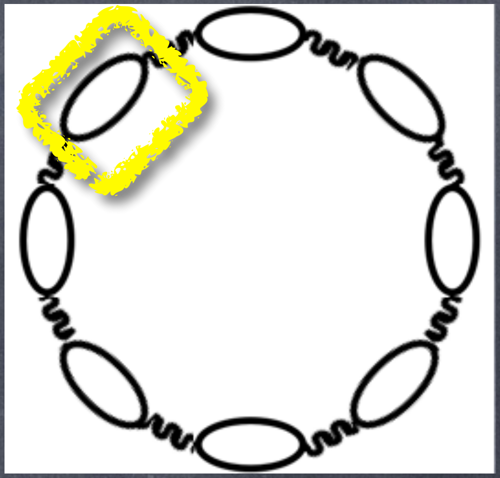


Mesonic Correlations

- Contribution of mesonic correlations to pressure

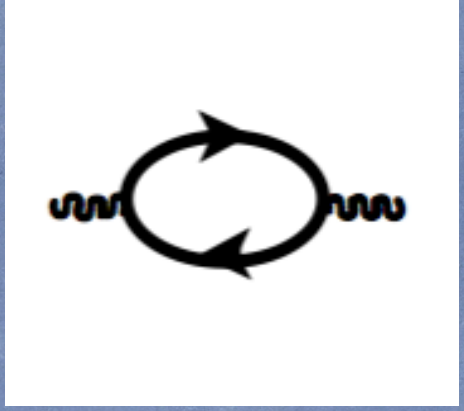
$$p_M = -\frac{T}{2V}\mathrm{Tr}_M \ln \left. \frac{\delta^2 I}{\delta\phi_i \delta\phi_j}\right|_{\phi=\phi_0}$$

$$p_M(T) = -\frac{1}{2}\int \frac{d^3q}{(2\pi)^3}\frac{1}{2\pi i}\int_0^\infty d\omega \left[1+\frac{2}{e^{\beta\omega}-1}\right] \times \left\{3\ \ln\left[\frac{\mathcal{M}_\pi(\omega+i\epsilon,q)}{\mathcal{M}_\pi(\omega-i\epsilon,q)}\right] + \ln\left[\frac{\mathcal{M}_\sigma(\omega+i\epsilon,q)}{\mathcal{M}_\sigma(\omega-i\epsilon,q)}\right]\right\}$$



M(ω, q) : $\mathcal{M}_\pi(\omega\ \ ,q) = 1 - 2G\Pi_\pi(\omega\ \ ,q)$

Π(ω, q) :



Mesonic Correlations

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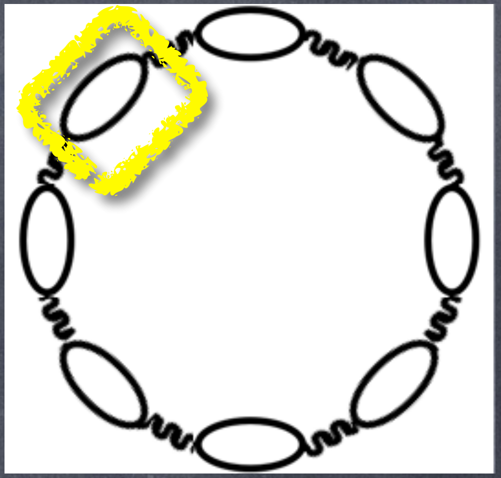
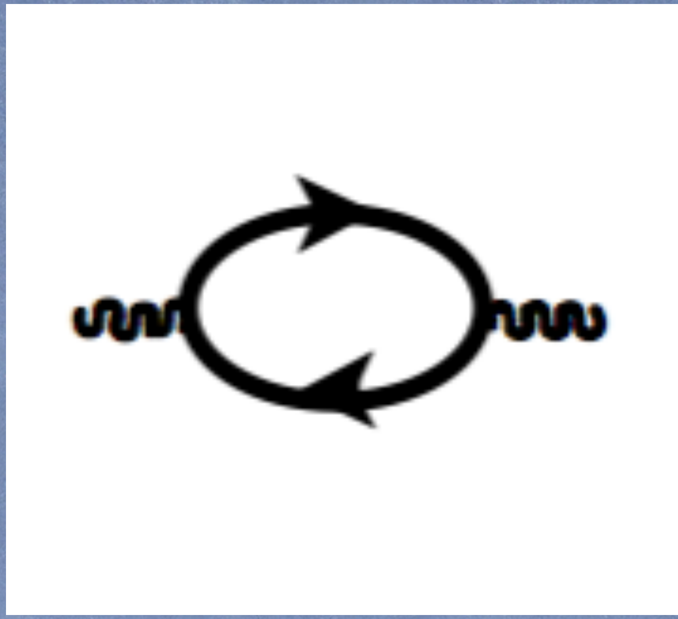
$$p_M = -\frac{T}{2V}\mathrm{Tr}_M \ln \left.\frac{\delta^2 I}{\delta\phi_i \delta\phi_j}\right|_{\phi=\phi_0}$$

$$\begin{aligned} p_M(T) \quad &= \quad -\frac{1}{2}\int \frac{d^3q}{(2\pi)^3}\frac{1}{2\pi i}\int_0^\infty d\omega \left[1+\frac{2}{e^{\beta\omega}-1}\right] \\ &\quad \times \left\{3\ \ln\left[\frac{\mathcal{M}_\pi(\omega+i\epsilon,q)}{\mathcal{M}_\pi(\omega-i\epsilon,q)}\right] + \ln\left[\frac{\mathcal{M}_\sigma(\omega+i\epsilon,q)}{\mathcal{M}_\sigma(\omega-i\epsilon,q)}\right]\right\}\end{aligned}$$

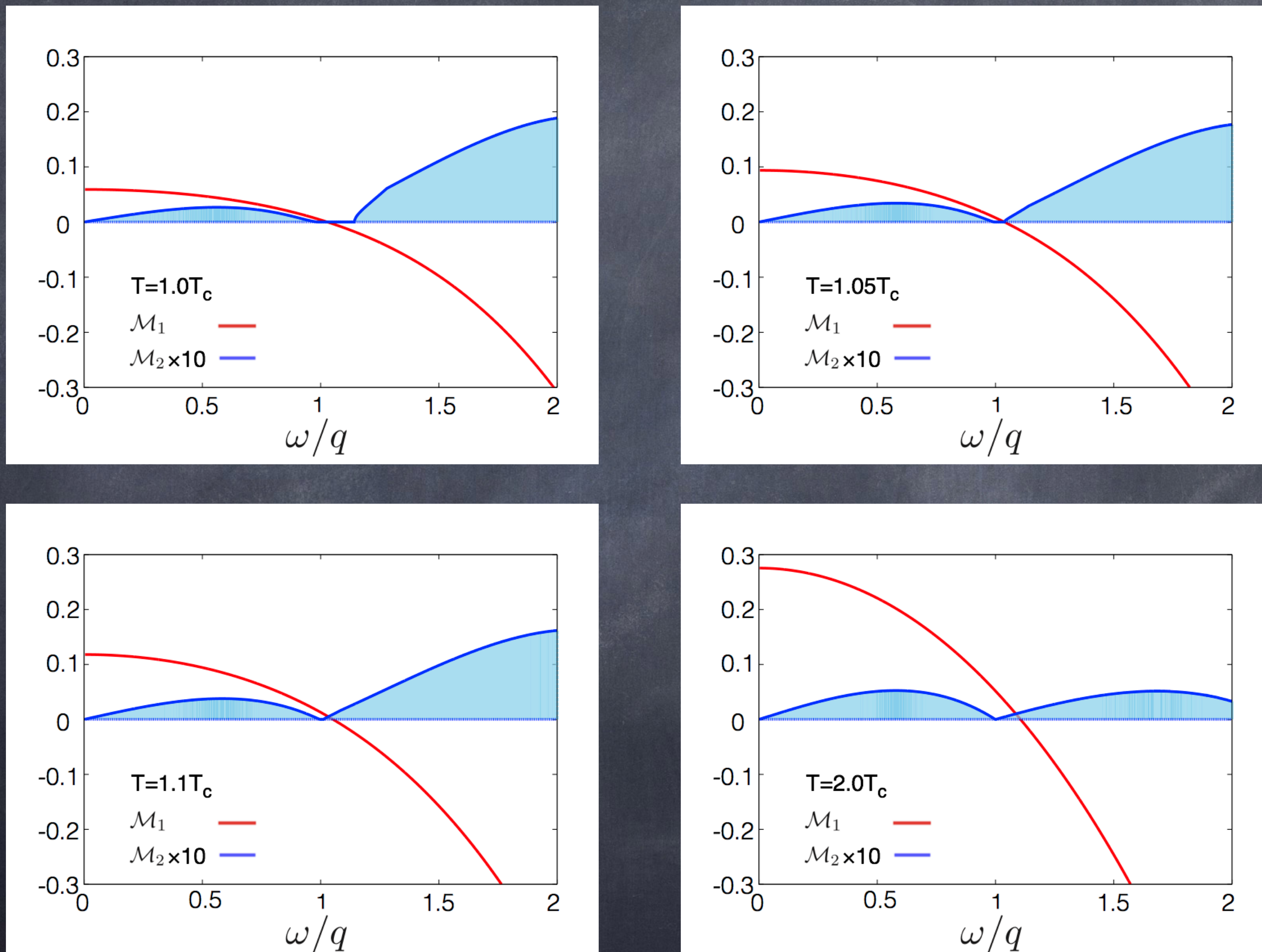




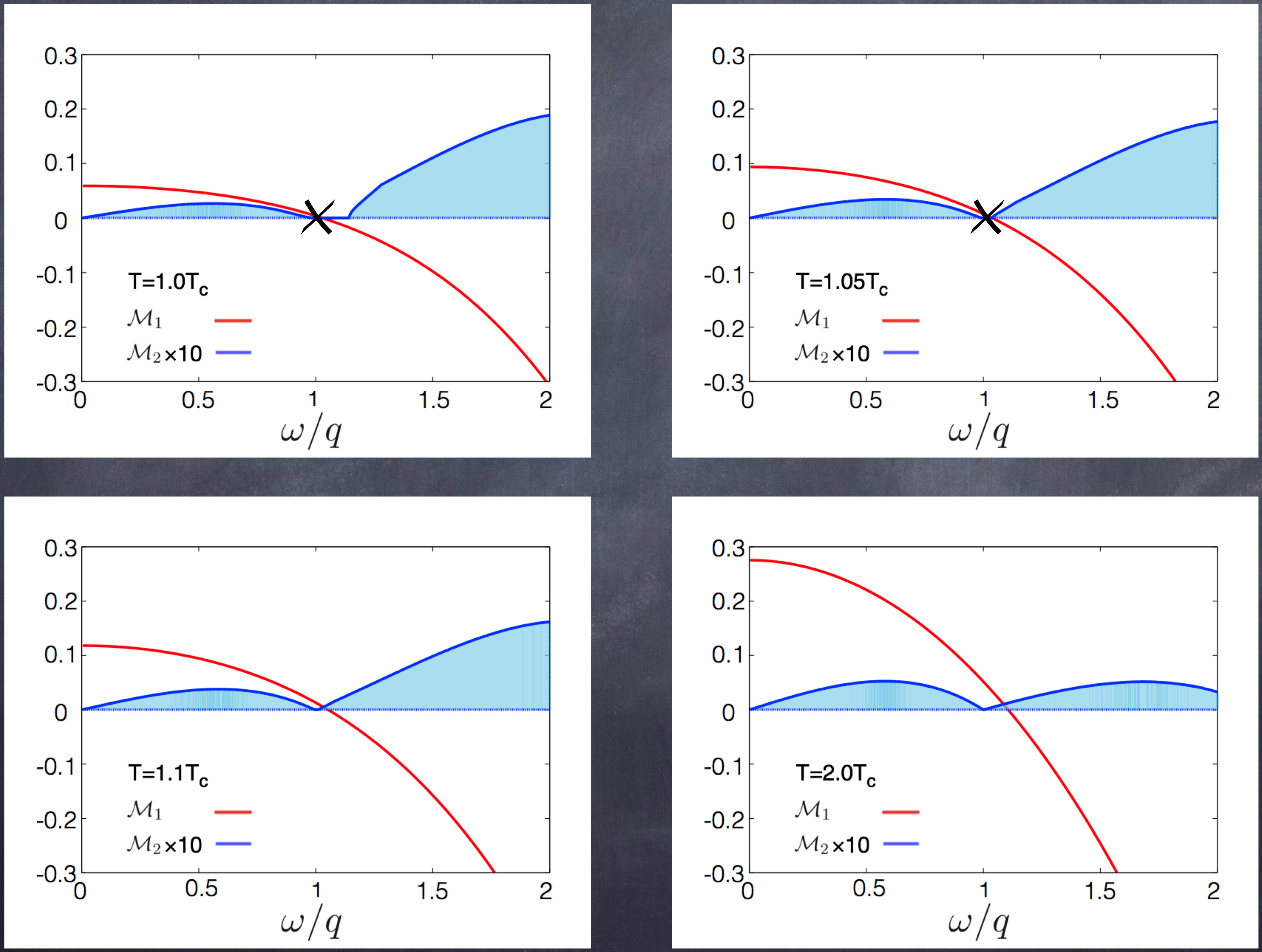
$$\mathrm{Re}[1-2G\Pi_{\pi/\sigma}]$$

$$\mathrm{Im}[1-2G\Pi_{\pi/\sigma}]$$

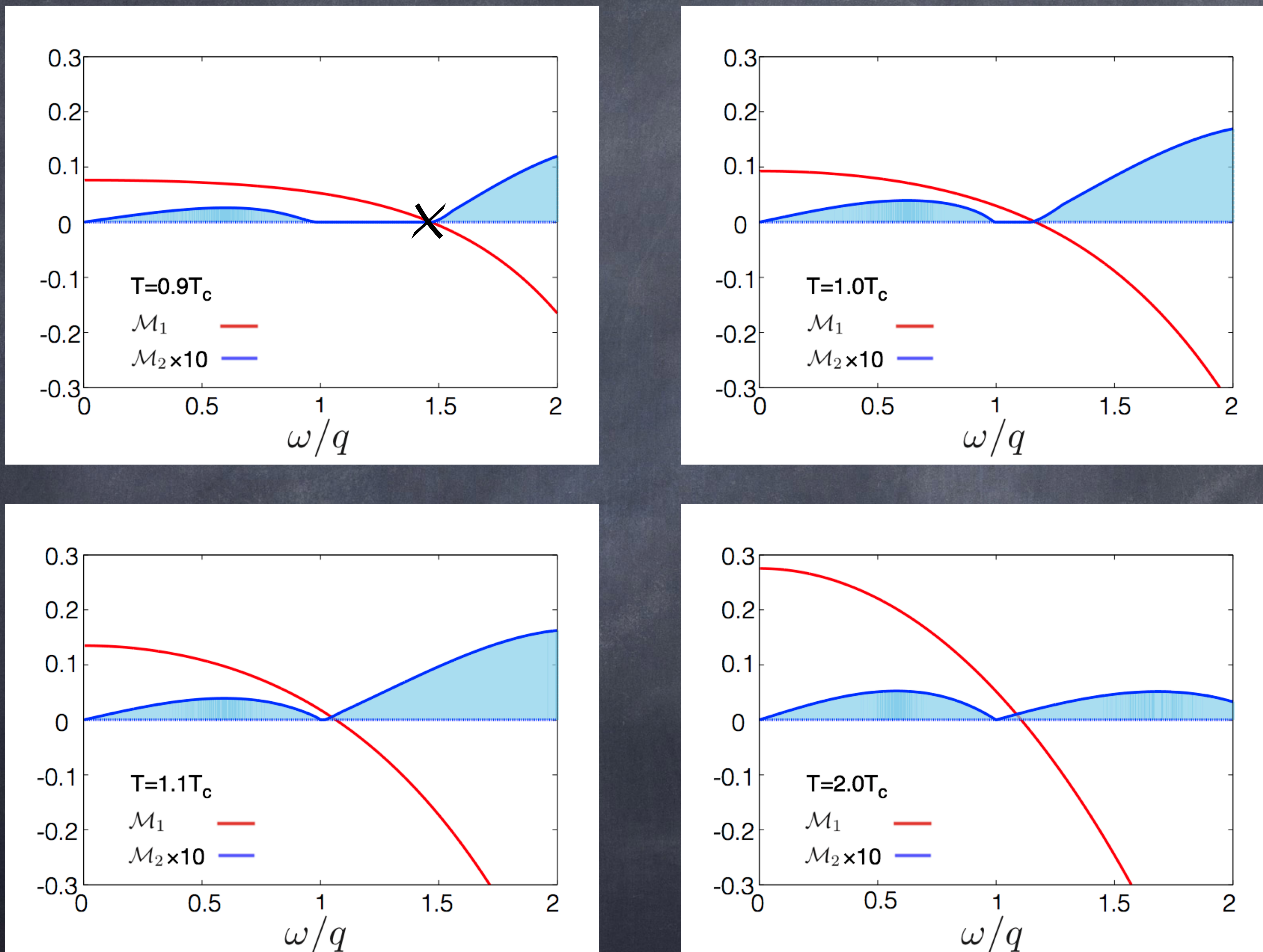
Collective mode of pion



Collective mode of pion



Collective mode of sigma meson



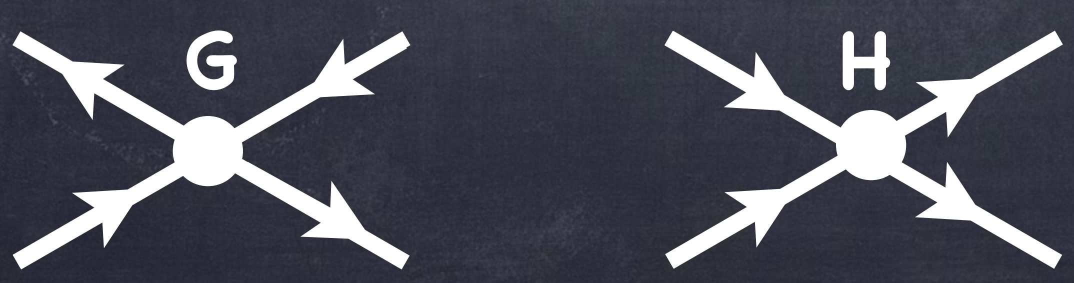
Baryons
in the PNJL model

The way to describe baryons

idea

- rewrite quark fields to auxiliary fields, like mesons' case
- assume that baryons are constructed by **quarks** and **diquarks**
- need **diquark interactions** in Lagrangian

$$\mathcal{L} = \bar{q}(i\not{D} - m)q + \mathcal{L}_{\bar{q}q} + \mathcal{L}_{qq} - \mathcal{U}[\bar{\Phi}, \Phi, T]$$



Insert auxiliary fields

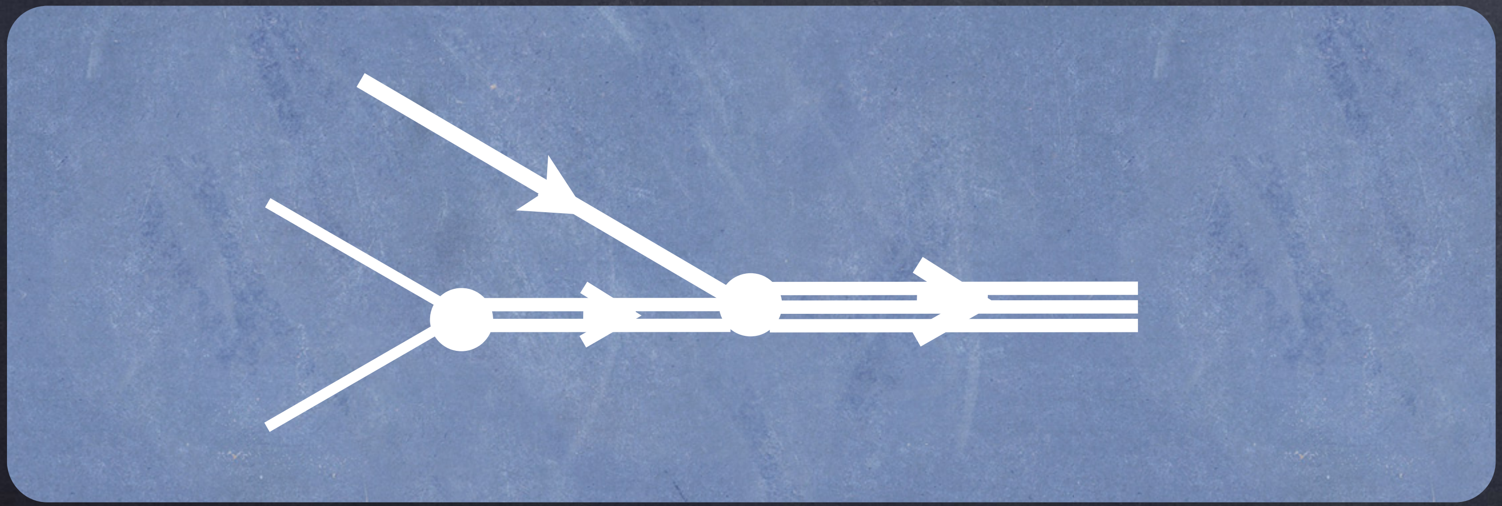
- mesons

$$G\bar{q}q \to \phi$$

- diquarks

$$Hqq \to \Delta$$

- baryons



$$\boxed{\lambda} q\Delta \to B$$

Partition function

- effective action is a function of quarks, mesons, diquarks and baryons

- perform first quark integrals

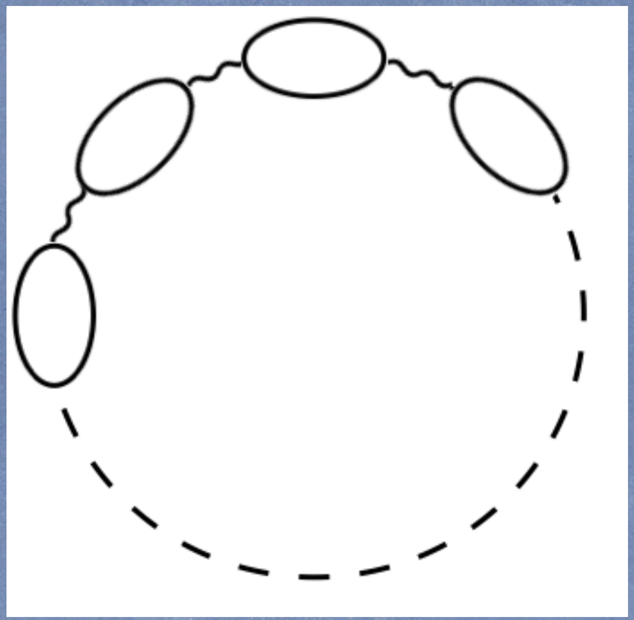
$$Z = \int [d\phi][d\bar{\Delta}][d\Delta][d\bar{B}][dB]e^{-\boxed{I_{eff}(\phi,\bar{\Delta},\Delta,\bar{B},B)}}$$

- approximation to the auxiliary fields in the same way as mesons

- obtain the equation of state

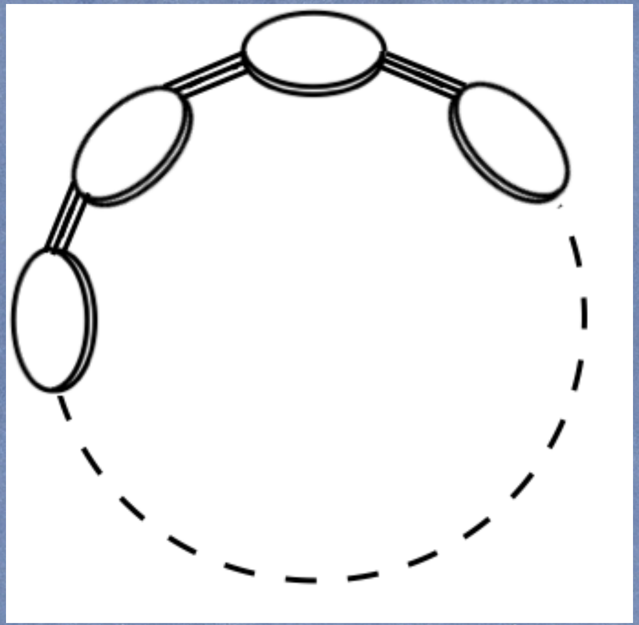
Pressure

- **mesons**



$$p_M(T) = -\frac{1}{2}\int \frac{d^3q}{(2\pi)^3}\int \frac{d\omega}{2\pi i}\left[1+2f_B(\omega)\right] \times\left\{3\ln\left[\frac{\mathcal{M}_\pi(\omega+i\epsilon,q)}{\mathcal{M}_\pi(\omega-i\epsilon,q)}\right]+\ln\left[\frac{\mathcal{M}_\sigma(\omega+i\epsilon,q)}{\mathcal{M}_\sigma(\omega-i\epsilon,q)}\right]\right\}$$

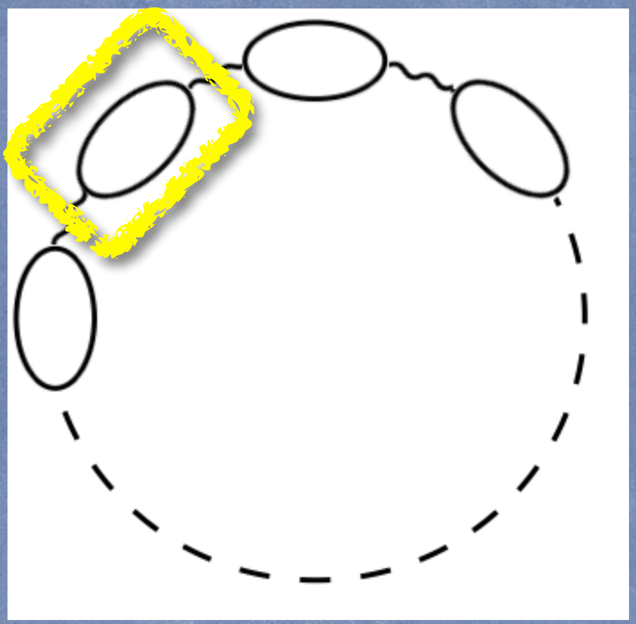
- **baryons**



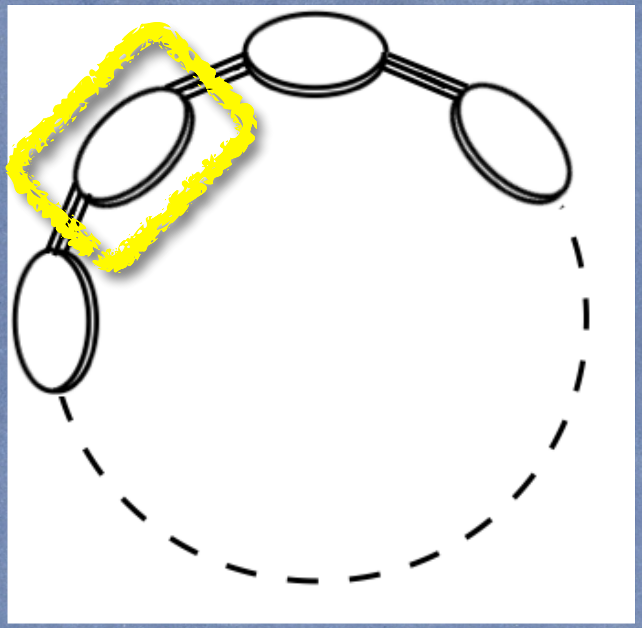
$$p_B = \frac{1}{2}\int \frac{d^3P}{(2\pi)^3}\int \frac{dE}{2\pi i}\Big[f(E-3\mu)+f(E+3\mu)-1\Big] \times\ln\left[\frac{\mathcal{M}_B(E+i\delta,P)}{\mathcal{M}_B(E-i\delta,P)}\right]$$

Collective modes

- mesons

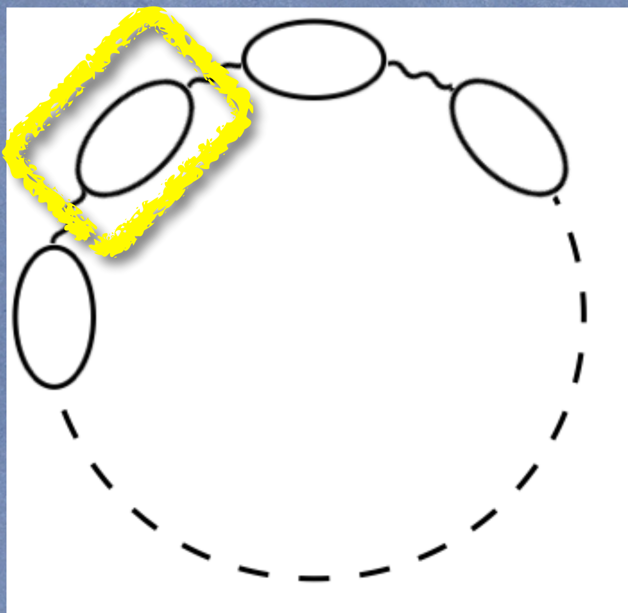


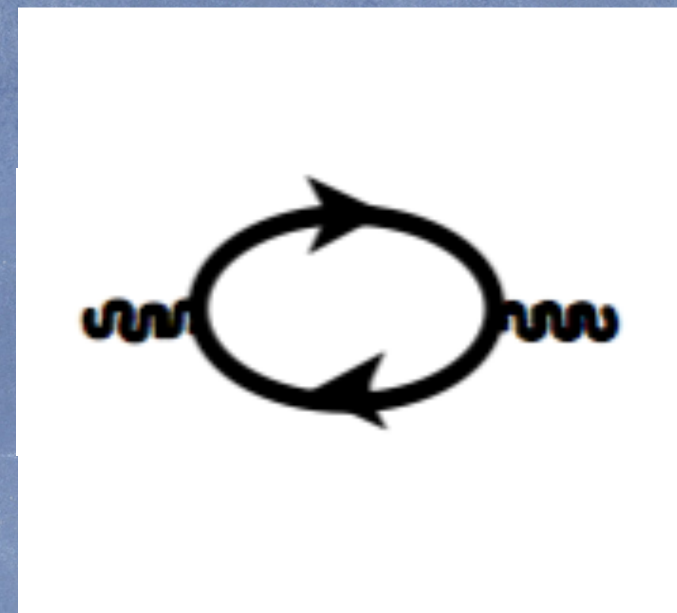
- baryons



Collective modes

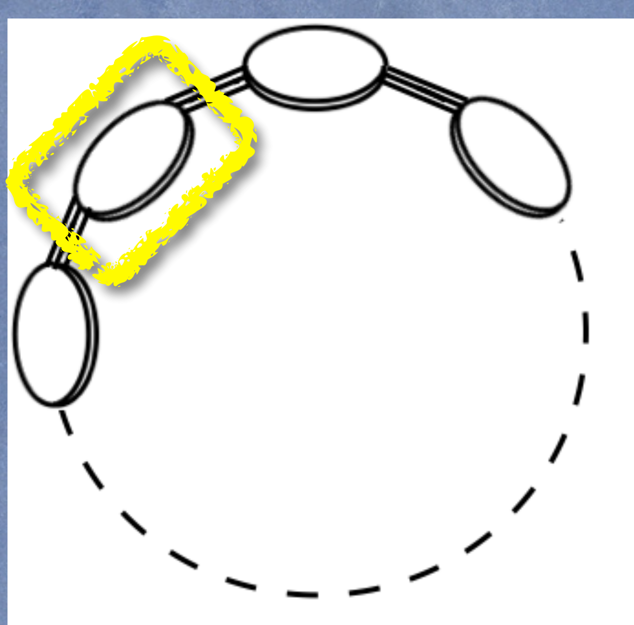
- mesons





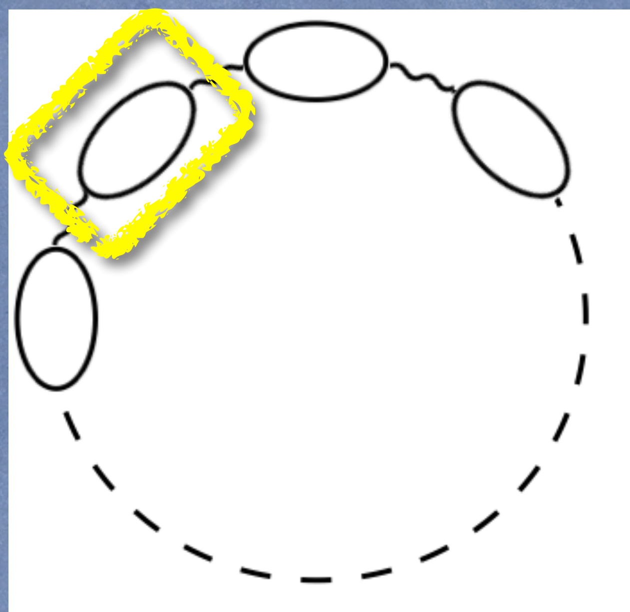
$$\mathrm{Re}[1 - 2G\Pi_{\pi/\sigma}]$$
$$\mathrm{Im}[1 - 2G\Pi_{\pi/\sigma}]$$

- baryons



Collective modes

- mesons



$$\mathrm{Re}[1 - 2G\Pi_{\pi/\sigma}]$$
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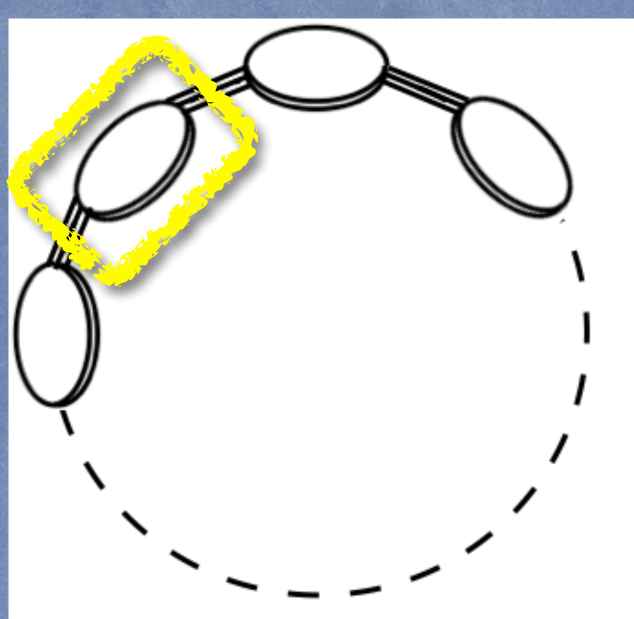
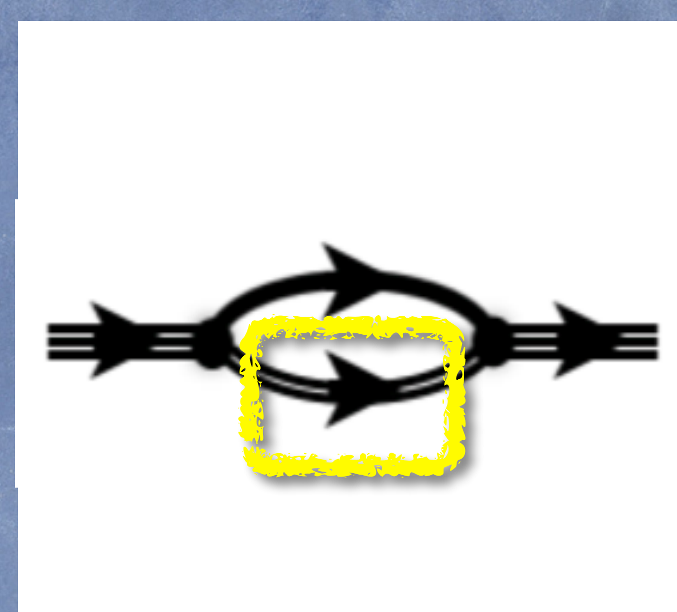
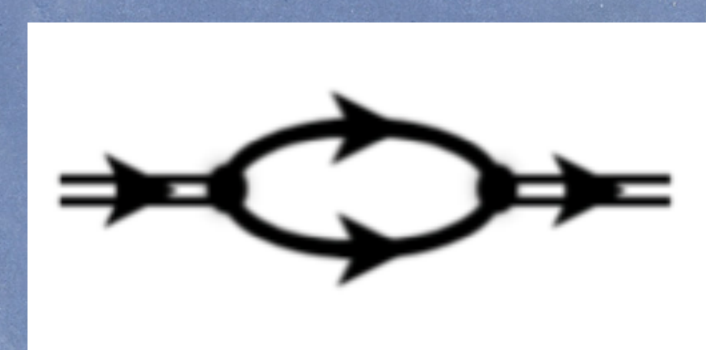
- baryons

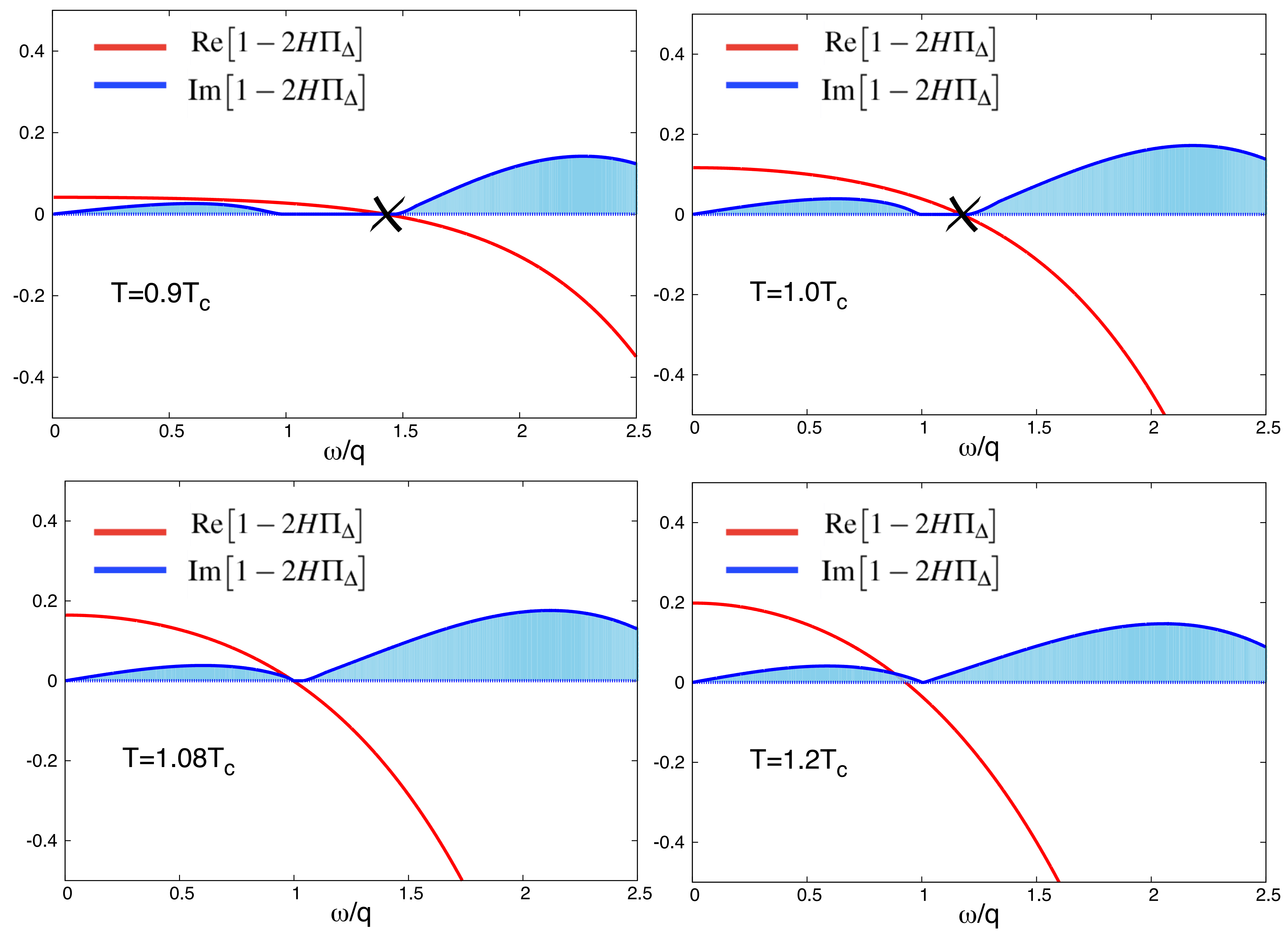




$$\mathrm{Re}[1 - 2G\Pi_{\Delta}]$$
$$\mathrm{Im}[1 - 2G\Pi_{\Delta}]$$



Collective mode of diquark



Summary

Summary

- We have calculated the melting temperatures of the pion and sigma meson.

- The collective modes of mesons at low T melt into the quarks and antiquarks in the intermediate T.

- We have described baryons composed of quarks and diquarks.

- Diquark correlation in baryons disappears as the temperature increases.

Back Up

Effective potential $U(\Phi,\bar{\Phi},T)$

C. Ratti, M. A. Thaler, W. Weise, 2006

principle

- $U(\Phi,\bar{\Phi},T)$ satisfies SU(3) center symmetry
  like pure gauge QCD Lagrangian
- $U(\Phi,\bar{\Phi},T)$ has a single minimum at $\Phi=0$ at T<<Tc
- Φ gets close to 1 at T>>Tc

the simplest form of effective potential

$$\frac{\mathcal{U}(\Phi,\bar{\Phi},T)}{T^4} = -\frac{b_2}{2}\bar{\Phi}\Phi - \frac{b_3}{6}(\Phi^3+\bar{\Phi}^3) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2$$

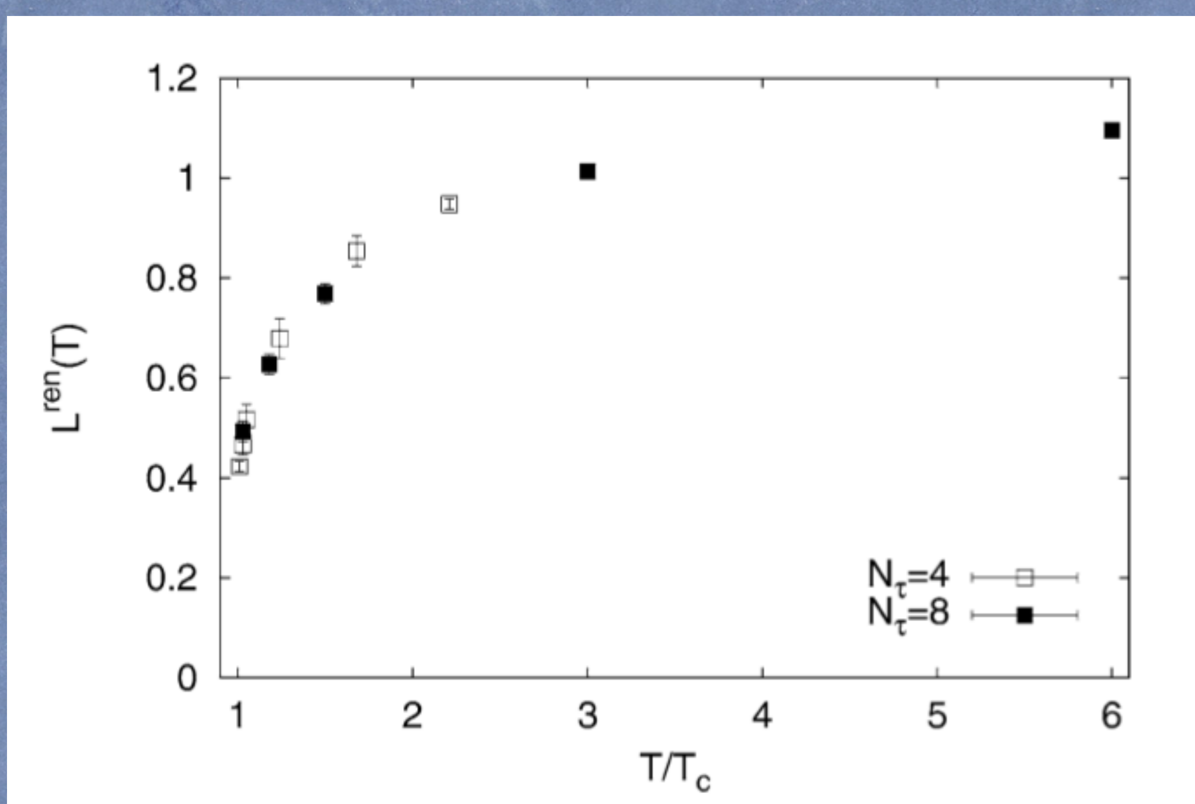
$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$$

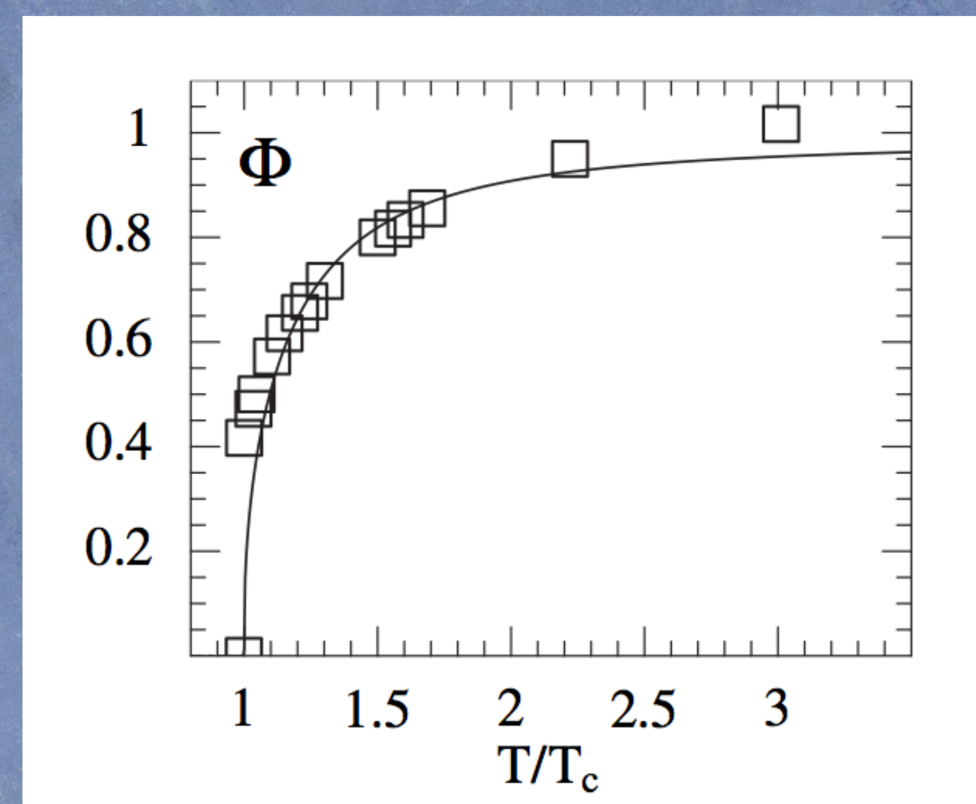
Effective potential $U(\Phi,\bar{\Phi},T)$

C. Ratti, M. A. Thaler, W. Weise, 2006

parameters in $U(\Phi,\bar{\Phi},T)$

- $U(\Phi,\bar{\Phi},T)$ contains 7 parameters
- They are fixed to reproduce lattice datas in pure gauge sector





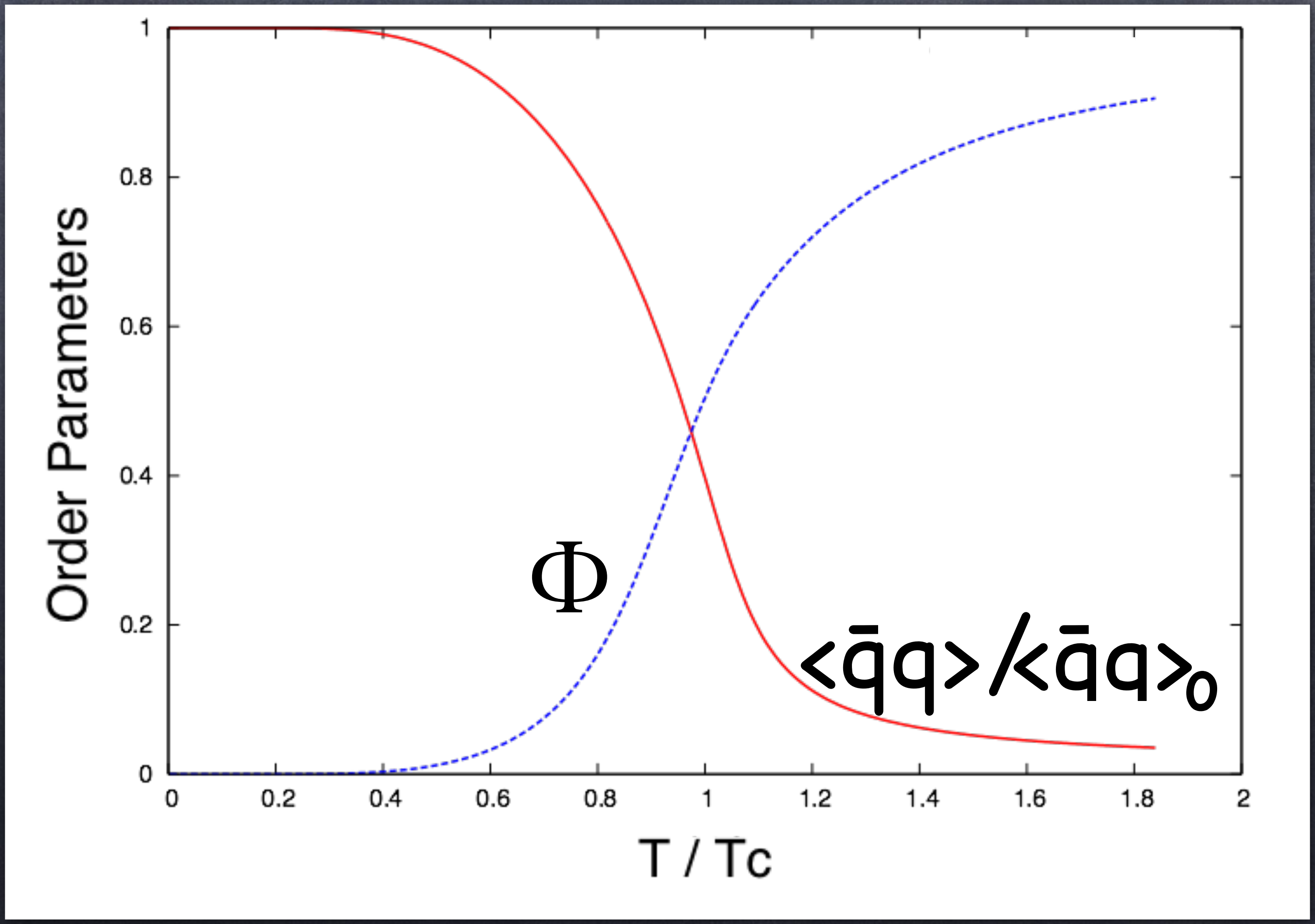
Kaczmarek, Karsch, Petreczky, Zantow, 2002

Order parameters (2f)

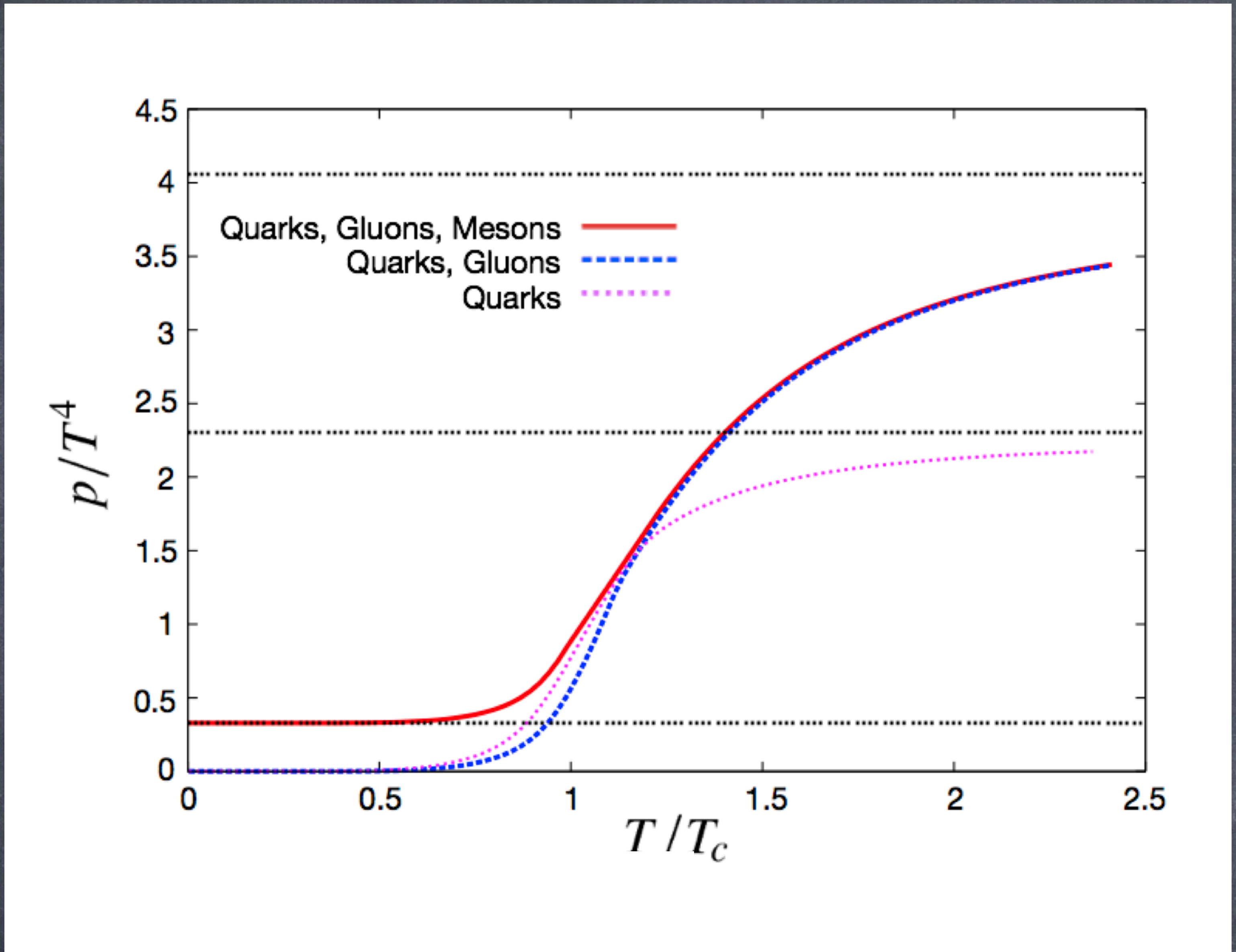
- quark mass M and expectation value of Polyakov loop are determined by stationary conditions

$$\left.\frac{\delta I}{\delta \phi_i}\right|_{\phi_0 = M} = 0 \quad , \qquad \frac{\delta I}{\delta \Phi} = 0$$

$$M - m_0 = -2G\langle \bar{q}q \rangle$$

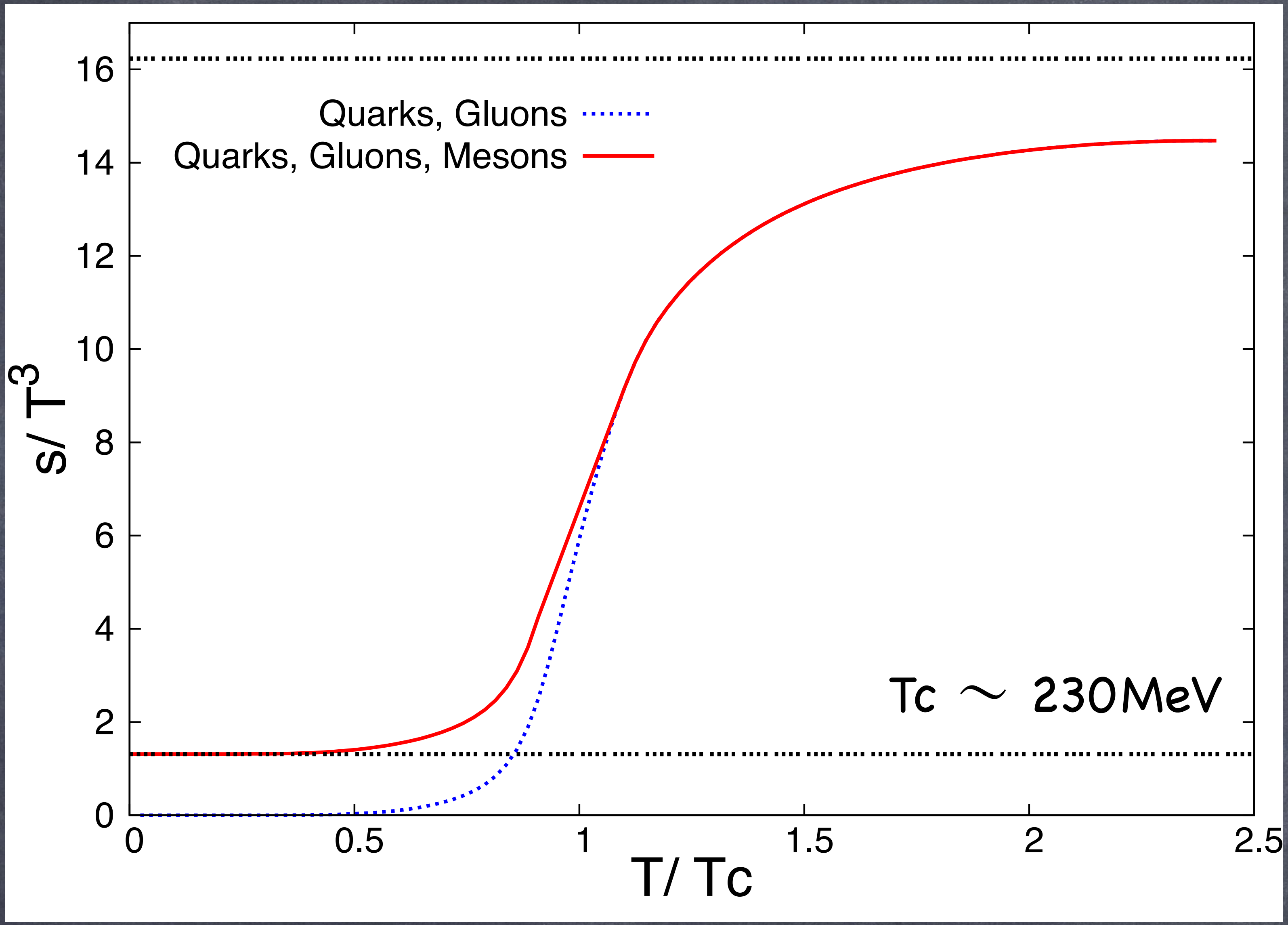


Pressure (2f) m_0=0



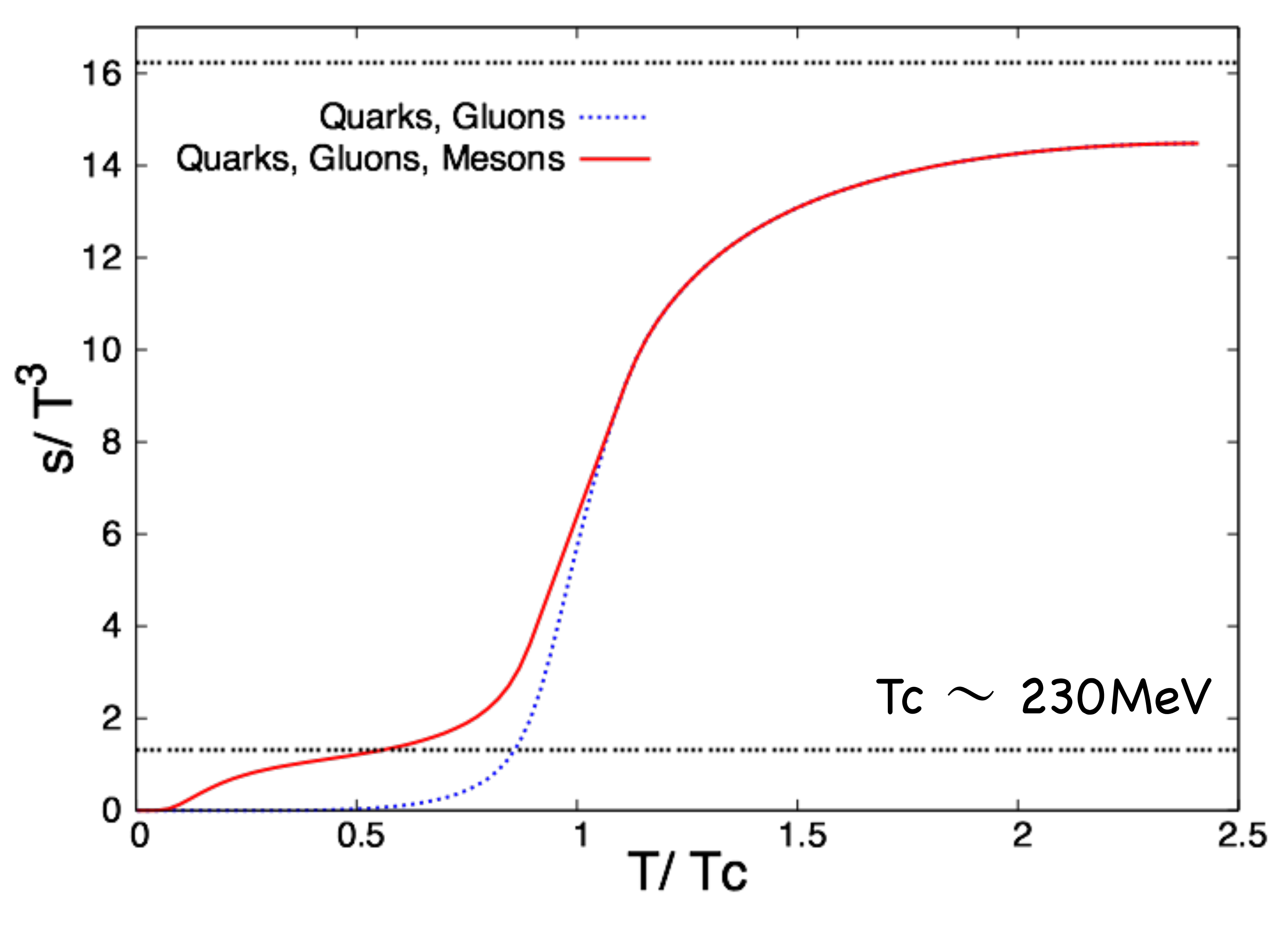
KY, T. Matsui, 2013

Entropy (2f) m_0=0



- **Collective excitations** carry entropy at low T.
- **Free quarks and gluons** carry entropy at high T.

Entropy (2 flavour)



KY, G. Baym, T. Matsui, 2015

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Quark distribution function

Two extreme cases

- De-confining phase

$$\Phi = \bar{\Phi} = 1 \quad \Rightarrow \quad f_\Phi(E_p)|_{\Phi=1} = \frac{1}{e^{\beta E_p}+1}$$

quark distribution function

- Confining phase

$$\Phi = \bar{\Phi} = 0 \quad \Rightarrow \quad f_\Phi(E_p)|_{\Phi=0} = \frac{1}{e^{3\beta E_p}+1}$$

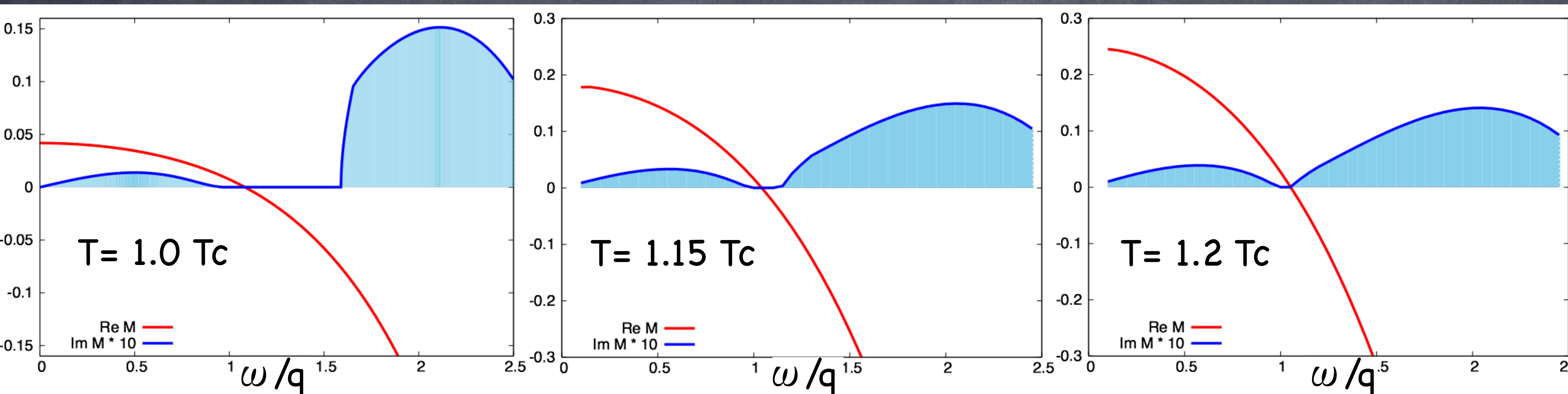
triad quark distribution function

$$E_p = \sqrt{p^2 + M_0^2}$$

Mo is determined by Gap equation

Collective modes

Pion



Kaon