# EM STRUCTURE OF THE NONET OF PSEUDOSCALAR MESONS AND CONTRIBUTIONS OF $e^+e^- \to M\bar{M}$ AND $e^+e^- \to \gamma M$ PROCESSES TO MUON g-2 ANOMALY and $\alpha(M_Z^2)$

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School/Workshop on Nuclear Physics'15, Erice, 16.-24. Sept.

# Outline

# INTRODUCTION

The **anomalous magnetic moment of the muon**

$$a_\mu = \frac{g-2}{2},$$

to be measured experimentally and evaluated theoretically, provides an **extremely clean test of the Standard Model (SM)** of elementary particle physics.
Therefore - it is important to achieve in its evaluation the inequality

$$(a_\mu^{exp} - a_\mu^{th}) < \Delta(a_\mu^{exp} - a_\mu^{th}).$$

# INTRODUCTION

Another quantity - the **running QED fine structure coupling constant**

$\alpha(M_Z^2)$,

is also very important to be known with very high precision - as **almost all SM predictions of observable depend on its value**.

# INTRODUCTION

In both quantities,

$a_\mu$ and $\alpha(M_Z^2)$,

**dominant sources of the total uncertainties in theoretical predictions** are **hadronic contributions**, which can be reduced to the calculation of dispersion integrals through

$\sigma_{tot}(e^+e^- \to hadrons)$.

Almost all evaluations of these integrals have been carried out **by the integration through existing experimental data points** on $\sigma_{tot}(e^+e^- \to hadrons)$, joining them by straight lines, i.e. **by using the so-called "trapezoidal rule"**.

# INTRODUCTION

In this contribution we would like to demonstrate, **how the errors of the evaluated integrals** through two-body total cross-sections $\sigma(e^+e^- \to M\bar{M})$ and $\sigma(e^+e^- \to \gamma M)$ **can be reduced** by exploiting the Unitary and Analytic (U& A) model of EM structure of the nonet of pseudoscalar mesons

$$\pi^+, \pi^0, \pi^-, K^+, K^0, \bar{K}^0, K^-, \eta, \eta'$$

**in comparison with evaluation of the same integrals** and at the same energy intervals **by the numerical integration through experimental points**.

# MUON g-2 ANOMALY

All **charged leptons**, $e^-, \mu^-, \tau^-$ (and also their antiparticles) are **described by the Dirac equation**.
The **magnetic moments** of these particles are **related to the spin** by means of the expression

$$\vec{\mu} = g\left(\frac{e}{2m_l}\right)\vec{s} \tag{1}$$

where the value of **gyromagnetic ratio** $g$ is predicted theoretically

*I.J.R.Aitchison and A.J.G.Hey: Gauge theories in particle physics, Bristol and Philadelphia, 2003*

to be $g = 2$.

# MUON g-2 ANOMALY

However, interactions existing in nature **modify $g$ to be exceeding the value "2"** because of the emission and absorption of

- virtual photons (EM effects)
- intermediate vector and Higgs bosons (weak interaction effects)
- vacuum polarization into virtual hadronic states (strong interaction effects)

**Note:**
This is true for all three charged leptons, $e^-, \mu^-, \tau^-$.

# MUON g-2 ANOMALY

In order to describe this modification of $g$ theoretically, the **magnetic anomaly** has been introduced by the relation

$$a_l = \frac{g-2}{2} = a_l^{(1)}\left(\frac{\alpha}{\pi}\right) + \left(a_l^{(2)QED} + a_l^{(2)had}\right)\left(\frac{\alpha}{\pi}\right)^2 + a_l^{(2)weak} + O\left(\frac{\alpha}{\pi}\right)^3 \quad (2)$$

where $\alpha$ is the **fine structure constant of QED** to be $\alpha(0) = \frac{1}{137{,}036}$.

# MUON g-2 ANOMALY

While **contribution of** $a_l^{(2)weak}$ to the lepton $g-2$ anomaly $a_l$ in comparison with $a_l^{(2)QED}$ and $a_l^{(2)had}$ is **tiny** for all charged leptons, the **hadronic contribution** $a_l^{(2)had}$ i.e. $\frac{a_l^{(2)had}}{a_l^{(2)QED}}$ is **increasing with an increase of the lepton mass**.

So, if one would like to look for new physics beyond the SM (**investigating hadronic contributions**) the most suitable object could be the **anomalous magnetic moment of the** $\tau^-$**-lepton** $a_\tau$.
But its **experimental measurement is difficult to be carried out** due to the instability of the $\tau^-$ particle.

# MUON g-2 ANOMALY

On the other hand, the **electron anomalous magnetic moment**
$a_e^{exp} = 1159652180,73(0,28) \times 10^{-12}$
is theoretically **almost completely described by QED**

*Aoyama, Hayakawa, Kinoshita, Nio (2014)*

i.e. it is **not sensitive to hadronic contributions**.

Its **theoretical error is dominated only by the uncertainty in the input value of the QED fine structure coupling constant** $\alpha$.

# MUON g-2 ANOMALY

The **muon anomalous magnetic moment** $a_\mu$ - the **most suitable object for theoretical investigations**:

- **one of the best measured quantities in physics**
  $a_\mu^{(BNL)} = 116592080(63) \times 10^{-11}$
- though its **present accurate theoretical evaluation**
  $a_\mu^{SM} = 116591802(49) \times 10^{-11}$
  is still lower than $a_\mu^{exp}$
  $a_\mu^{(BNL)} - a_\mu^{SM} = 278(80);\ 3.6\sigma$,
  but **further theoretical and experimental improvements may lead to a revelation of a new physics beyond SM.**
- moreover, a **new measurement of it is expected in 2017**
  *G.Venanzoni: The FERMILAB muon g-2 experiment E989, Contr. to EPS HEP'15 in Vienna*

# MUON g-2 ANOMALY

The hadronic contributions $a_\mu^{had}$ - represented by

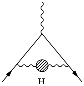


Fig.1: The lowest-order hadronic vacuum-polarization contributions.

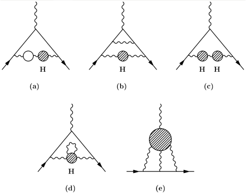


Fig.2:The third-order hadronic vacuum-polarization contributions.

# MUON g-2 ANOMALY

Some time ago **we have evaluated a contribution of the light-by-light** (LBL) diagram (the last one in Fig. 2)

*E.Bartos, A.Z.Dubnickova, S.Dubnicka, E.A.Kuraev, E.Zemlyanaya: Nucl. Phys. B632 (2002) 330* to be

$$a_\mu^{LBL} = (111.20 \pm 16.81) \times 10^{-11}$$

however **the most accurate value** has been obtained recently by

*A.Dorokhov et al: Contr. to the HS'15 Conf. in Horny Smokovec, Slovakia*

$$a_\mu^{LBL} = (168.0 \pm 12.5) \times 10^{-11}$$

# MUON g-2 ANOMALY

We do not see **any possibility of an improvement in evaluation** of all the rest of the **third-order hadronic vacuum-polarization diagrams in Fig. 2**, in comparison with the recent precise evaluation carried out by

*A.Kurz et al: Phys. Lett B (2014)*

However, we see **substantial improvement in evaluation** of the **lowest-order hadronic vacuum-polarization diagram in Fig.1**, which can be represented

*M.Gourdin, E. de Rafael: Nucl. Phys. B10 (1969) 667*

# MUON g-2 ANOMALY

by the dispersion integral

$$a_\mu^{(2)had} = \frac{1}{3}\left(\frac{\alpha(0)}{\pi}\right)^2 \left( \int_{4m_\pi^2}^{s_{(cut)}} \frac{ds}{s} R^{data}(s)K(s) + \int_{s_{(cut)}}^{\infty} \frac{ds}{s} R^{pQCD}(s)K(s) \right) \quad (3)$$

with

$$R(s) = \sigma_{tot}(e^+e^- \to had)/\frac{4\pi\alpha(0)^2}{3s}$$

and

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2+(1-x)\frac{s}{m_\mu^2}}.$$

# MUON g-2 ANOMALY

Just in the first integral of (3) **instead of integration through data** on $\sigma_{tot}(e^+e^- \to had)$ we evaluate contributions separately of

$$\sigma_{tot}(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha(0)^2}{3s}(1-4m_\pi^2/s)^{\frac{3}{2}} \mid F_{\pi^\pm}(s) \mid^2$$

$$\sigma_{tot}(e^+e^- \to K^+K^-) = \frac{\pi\alpha(0)^2}{3s}(1-4m_{K^\pm}^2/s)^{\frac{3}{2}} \mid F_{K^\pm}(s) \mid^2$$

$$\sigma_{tot}(e^+e^- \to K^0\bar{K}^0) = \frac{\pi\alpha(0)^2}{3s}(1-4m_{K^0}^2/s)^{\frac{3}{2}} \mid F_{K^0}(s) \mid^2$$

$$\sigma_{tot}(e^+e^- \to \pi^0\gamma) = \frac{\pi\alpha(0)^2}{6}(1-m_{\pi^0}^2/s)^3 \mid F_{\pi^0\gamma}(s) \mid^2$$

$$\sigma_{tot}(e^+e^- \to \eta\gamma) = \frac{\pi\alpha(0)^2}{6}(1-m_{\eta}^2/s)^3 \mid F_{\eta\gamma}(s) \mid^2$$

$$\sigma_{tot}(e^+e^- \to \eta'\gamma) = \frac{\pi\alpha(0)^2}{6}(1-m_{\eta'}^2/s)^3 \mid F_{\eta'\gamma}(s) \mid^2$$

# FINE STRUCTURE CONSTANT OF QED $\alpha(s)$ at $M_Z^2$

The **running fine structure coupling constant of QED** $\alpha(s)$ can be expressed as

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)}; \quad \alpha(0) = 1/137,036 \tag{4}$$

whereby $\Delta\alpha(s)$ is governed by the **renormalized vacuum polarization function** $\Pi_\gamma(s)$.

# FINE STRUCTURE CONSTANT OF QED $\alpha(s)$ at $M_Z^2$

$\Pi_\gamma(s)$ is defined by the Fourier transform of the time-ordered product of the **EM currents** $J^\mu_{em}(s)$ in the vacuum

$$(q^\mu q^\nu - q^2 g^{\mu\nu})\Pi_\gamma(q^2) = i\int d^4x e^{iqx} < 0 \mid T[j^\mu_{em}(x) j^\nu_{em}(0)] \mid 0 > \quad (5)$$

as

$$\Delta\alpha(s) = -4\pi\alpha(0) Re[\Pi_\gamma(s) - \Pi_\gamma(0)]. \quad (6)$$

**NOTE**: The $\Delta\alpha(s)$, for instance at $s = M_Z^2$ is large - due to the large change in scale, going from $s \to 0$ (Thomson limit) to the mass of $Z$ resonance.

# FINE STRUCTURE CONSTANT OF QED $\alpha(s)$ at $M_Z^2$

In perturbation theory, the **leading order contributions** represented by the free fermion loops (see Fig. 3)

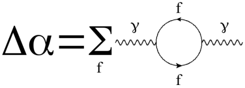


Fig.3: The leading light fermion ($m_f \ll M_Z$) contributions.

give

$$\Delta\alpha(s) = \frac{\alpha(0)}{3\pi} \sum_f Q^2 N_{cf} (ln \frac{s}{m_f^2} - \frac{5}{3}) \qquad (7)$$

where $Q$... the fermion charge; $N_{cf}$...the color factor - 1 for **leptons** and 3 for **quarks**.

# FINE STRUCTURE CONSTANT OF QED $\alpha(s)$ at $M_Z^2$

One distinguishes contributions in $\Delta\alpha(s)$

- from leptons $(e, \mu, \tau)$
- from 5 light quarks $u, d, c, s, b \quad (mass < 5GeV)$
- from "top" - quark $t$ $(mass \approx 175GeV)$

Then

$$\Delta\alpha(s) = \Delta\alpha_l(s) + \Delta^{(5)}_{had}(s) + \Delta\alpha_{top}(s). \qquad (8)$$

# FINE STRUCTURE CONSTANT OF QED $\alpha(s)$ at $M_Z^2$

The **leptonic contributions** are calculable in perturbation theory, where at leading order the free leptons yield

$$\Delta\alpha_l(s) = \frac{\alpha(0)}{3\pi} \sum_{f=e,\mu,\tau} [ln\frac{s}{m_l^2} - \frac{5}{3}] \tag{9}$$

Then numerically $\Delta\alpha_l(M_Z^2) \approx 0.031498$.

# FINE STRUCTURE CONSTANT OF QED $\alpha(s)$ at $M_Z^2$

**Since the** $t$**-quark is heavy** $(m_t \gg M_Z \approx 91 GeV)$, one can not use the light fermion approximation for it and **it decouples like**

$$\Delta\alpha_{top}(s) \approx -\frac{\alpha(0)}{3\pi}\frac{4}{15}\frac{M_Z^2}{m_t} \to 0. \tag{10}$$

# FINE STRUCTURE CONSTANT OF QED $\alpha(s)$ at $M_Z^2$

A serious problem is the 5 light quarks, $u, d, s, c, b$ contribution $\Delta\alpha_{had}^{(5)}(s)$, **due to the light masses of these quarks it can not be calculated** in the framework of the "perturbative" QCD (pQCD).

Fortunately - **one can evaluate it from** $e^+e^- \to hadrons$ **data**, like in muon g-2 anomaly, by **exploiting dispersion relation**

$$Re\Pi_\gamma(s) - \Pi_\gamma(0) = \frac{s}{\pi} Re \int_{s_0}^{\infty} \frac{Im\Pi_\gamma(s')}{s'(s'-s-i\varepsilon)} ds' \qquad (11)$$

**and the optical theorem**

$$Im\Pi_\gamma(s) = \frac{s}{e^2}\sigma_{tot}(e^+e^- \to had). \qquad (12)$$

# FINE STRUCTURE CONSTANT OF QED $\alpha(s)$ at $M_Z^2$

In terms of the **total cross-section ratio**

$$R(s) = \frac{\sigma_{tot}(e^+e^- \to had)}{\sigma_{tot}(e^+e^- \to \mu^+\mu^-)}, \tag{13}$$

where

$$\sigma_{tot}(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2(0)}{3s} \tag{14}$$

one finally obtains

$$\Delta\alpha_{had}^{(5)}(M_Z^2) = -\frac{\alpha(0)M_Z^2}{3\pi} Re \int_{4m_\pi^2}^{\infty} \frac{R(s')}{s'(s'-M_Z^2-i\varepsilon)} ds'. \tag{15}$$

# $U\&A$ EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

In construction of **the** $U\&A$ **model**, EM FFs in $\sigma(e^+e^- \to M\bar{M})$ and $\sigma(e^+e^- \to \gamma M)$ are splitted into **isoscalar** and **isovector** parts

$$F_{\pi^\pm}(s) = F_\pi^{I=1}[W(s)]$$

$$F_{K^\pm}(s) = F_K^{I=0}[V(s)] + F_K^{I=1}[W(s)]$$

$$F_{K^0}(s) = F_K^{I=0}[V(s)] - F_K^{I=1}[W(s)]$$

$$F_{\pi^0\gamma}(s) = F_{\pi^0\gamma}^{I=0}[V(s)] + F_{\pi^0\gamma}^{I=1}[W(s)]$$

$$F_{\eta\gamma}(s) = F_{\eta\gamma}^{I=0}[V(s)] + F_{\eta\gamma}^{I=1}[W(s)]$$

$$F_{\eta'\gamma}(s) = F_{\eta'\gamma}^{I=0}[V(s)] + F_{\eta'\gamma}^{I=1}[W(s)].$$

# *U&A* EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

The model takes into account **all known properties of FFs**:

- normalization of FFs
- asymtotic behaviour as predicted by the quark model
- analytic properties of FFs
- unitarity conditions of FFs
- reality conditions of FFs
- experimental fact of a creation of vector mesons in $e^+e^- \to$ *had* process
- then $F^{I=1}(s)$ are **saturated** by $\rho, \rho', \rho'', etc$ and $F^{I=0}(s)$ by $\omega, \phi, \omega', \phi', etc.$

# *U&A* EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

Finally, every $F^{I=1}[W(s)]$ and $F^{I=0}[V(s)]$ **represent one analytic function** in the whole complex $s$-plane **besides two cuts on the positive real axis** to be defined on the four-sheeted Riemann surface and **depend on only physically interpretable parameters**.
To every considered vector meson, complex conjugate pair of poles on unphysical sheets appear.

Their **predictions** for the **nonet of pseudoscalar mesons** are presented in the following Figs.

# *U&A* EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

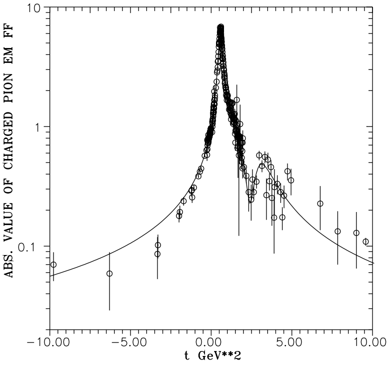


Figure: Prediction of pion EM FF behavior by *U&A* model.

# *U&A* EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

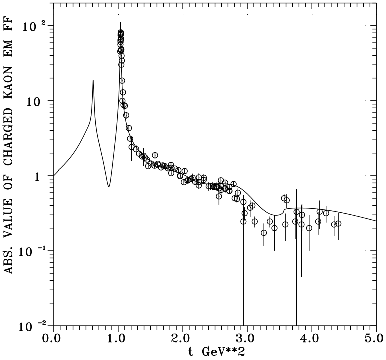


Figure: Prediction of charge kaon EM FF behavior by *U&A* model.

# $U\&A$ EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

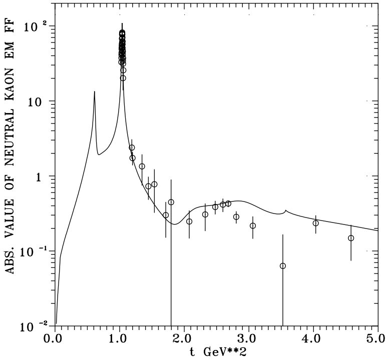


Figure: Prediction of neutral kaon EM FF behavior by $U\&A$ model.

# *U&A* EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

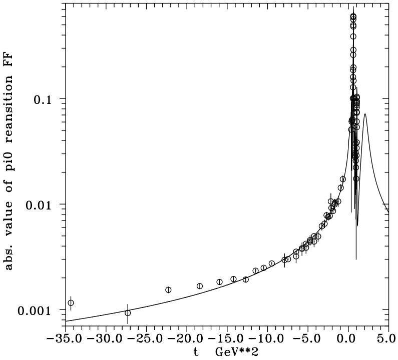


Figure: Prediction of $\pi^0\gamma$ transition EM FF behavior by *U&A* model.

# *U&A* EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

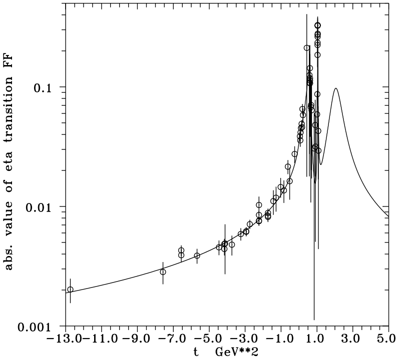


Figure: Prediction of $\eta\gamma$ transition EM FF behavior by *U&A* model.

# *U&A* EM STRUCTURE MODEL OF NONET OF PSEUDOSCALAR MESONS

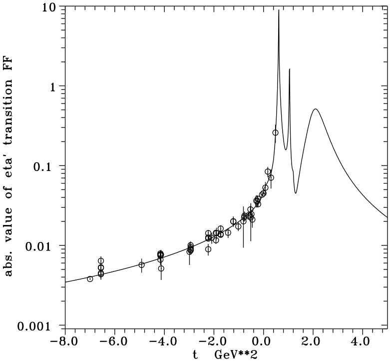


Figure: Prediction of $\eta'\gamma$ transition EM FF behavior by *U&A* model.

# EVALUATION OF CONTRIBUTIONS TO MUON g-2 AND $\alpha(M_Z^2)$

**Substituting** $U\&A$ **models** of $F^{I=1}[W(s)]$ and $F^{I=0}[V(s)]$ with corresponding numerical values of parameters **into two-body total cross-sections** $\sigma(e^+e^- \to M\bar{M})$ **and** $\sigma(e^+e^- \to \gamma M)$, one is ready to evaluate LO contributions to muon $g-2$ and $\alpha(M_Z^2)$. In order to have an opportunity to compare our results with other authors, **we shall carry out** both, **direct data integration and also integration by exploiting the** $U\&A$ **model** of the corresponding FFs, **at the interval** of energies $s_0 < s < 2.0449 GeV^2$.

# EVALUATION OF CONTRIBUTIONS TO MUON g-2 AND $\alpha(M_Z^2)$

**Results for the muon $g-2$ anomaly**:

$a_\mu^{had,LO}(e^+e^- \to \pi^+\pi^-) \times 10^{-11}$ for $4m_\pi^2 < s < 2.0449 GeV^2$

$U\&A$ model integration....$5128, 22^{+0,73}_{-0,67}$

direct data integration.....$5031, 22^{+28,94}_{-16,43}$

K.Hagiwara et al (2007).....$5008, 2 \pm 28, 70$

# EVALUATION OF CONTRIBUTIONS TO MUON g-2 AND $\alpha(M_Z^2)$

$a_\mu^{had,LO}(e^+e^- \to K^+K^-) \times 10^{-11}$ for $4m_K^2 < s < 2.0449 GeV^2$

$U\&A$ model integration....$224,67^{+1,23}_{-1,28}$
direct data integration.....$235,76^{+9,07}_{-5,00}$
K.Hagiwara et al (2004).....$216,2 \pm 7,6$

# EVALUATION OF CONTRIBUTIONS TO MUON g-2 AND $\alpha(M_Z^2)$

$a_\mu^{had,LO}(e^+e^- \to K^0\bar{K}^0) \times 10^{-11}$ for $4m_K^2 < s < 2.0449\,GeV^2$

$U\&A$ model integration....$128,38^{+0,76}_{-0,76}$
direct data integration.....$135,40^{+1,66}_{-0,96}$
K.Hagiwara et al (2004).....$131,6 \pm 3,1$

# EVALUATION OF CONTRIBUTIONS TO MUON g-2 AND $\alpha(M_Z^2)$

$a_\mu^{had,LO}(e^+e^- \to \pi^0\gamma) \times 10^{-11}$ for $m_{\pi^0}^2 < s < 2.0449\, GeV^2$

$U\&A$ model integration....$53,72 \pm 0,36$
M.Davier et al (2011).......$44,2 \pm 1,94$

# EVALUATION OF CONTRIBUTIONS TO MUON g-2 AND $\alpha(M_Z^2)$

$a_\mu^{had,LO}(e^+e^- \to \eta\gamma) \times 10^{-11}$ for $m_{\pi^0}^2 < s < 2.0449\, GeV^2$

$U\&A$ model integration....$11,55 \pm 0.08$
M.Davier et al (2011).......$6,40 \pm 0,24$

# EVALUATION OF CONTRIBUTIONS TO MUON g-2 AND $\alpha(M_Z^2)$

$a_\mu^{had,LO}(e^+e^- \to \eta'\gamma) \times 10^{-11}$ for $m_{\pi^0}^2 < s < 2.0449\,GeV^2$

$U\&A$ model integration....$20,69 \pm 9,65$

M.Davier et al (2011)....... ?

# EVALUATION OF CONTRIBUTIONS TO MUON g-2 AND $\alpha(M_Z^2)$

The best present evaluation of the hadronic contribution to the **RUNNING FINE STRUCTURE CONSTANT OF QED** $\alpha(s)$ is

$$\Delta\alpha^{(5)}_{had}(M_Z^2) = 0.027896 \pm 0.000395$$

If we add this result to the calculated value of
$\Delta_{leptons}(M_Z^2) = 0.031498$,
then for $\alpha(s)$ at the mass of the $Z$-boson one obtains

$$\alpha(m_Z^2) = \frac{\alpha(0)}{1-0.059394} = 1/128,897.$$

However, by exploiting our *U&A* model of the EM structure of pseudoscalar mesons **one can still improve this result substantially**.

# Conclusions

What are the **most important sources contributing to error reduction** in the muon $g-2$ anomaly and in the running fine structure constant of QED?

# Conclusions

- The $U\&A$ model **takes into account all known theoretical properties** of the considered EM FFs and **always describes the experimental data by a single analytic function in the space-like and time-like region simultaneously**.
- The $U\&A$ **model depends on only physically interpretable parameters** to be determined in a fitting of the data also outside the integration region, which leads to a decrease of the errors
- The $U\&A$ model predictions are mainly given by data with small errors and to large extent neglect the data points with huge uncertainties. **Just the latter contribute importantly to the result error in the direct integration through data points!**

# Thank you for your attention.