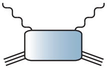
Nucleon structure properties with nonperturbative quarks and gluons

Gernot Eichmann
University of Giessen, Germany

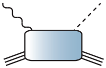
37th International School of Nuclear Physics
Erice, Sicily
September 20, 2015

Outline

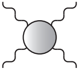
"Probing QCD with the electromagnetic interaction":



Compton scattering



Pion electro-production



Hadronic light-by-light

- typically studied with hadronic approaches
  (ChPT, dispersion relations, coupled-channel equations, models, . . .)

  ⇒ **quark-level description?**
  In terms of **QCD's Green functions:**
  Dyson-Schwinger, Bethe-Salpeter, Faddeev equations

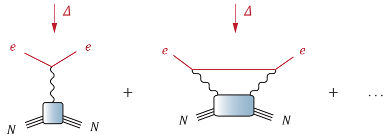
  . . . see talk by Reinhard Alkofer

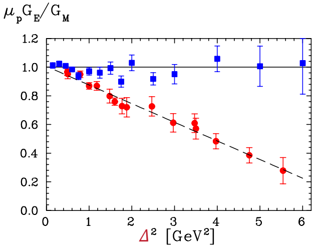
- **four-point functions** ⇒ complicated momentum and tensor structure

- involve **photons** ⇒ electromagnetic gauge invariance important

Compton scattering

- **Two-photon corrections to form factors:**
  can explain difference between Rosenbluth and polarization transfer measurements
  Guichon, Vanderhaeghen, PRL 91 (2003)



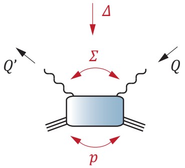


Arrington, Blunden, Melnitchouk
Prog. Part. Nucl. Phys. 66 (2011)

- **Proton radius puzzle:**
  can 2γ corrections explain difference between electron and muon measurements?
  So far: probably not, but . . .
  Carlson, Vanderhaeghen, 2011
  Birse, McGovern, EPJ A 48 (2012)

. . . see talk by Oleksandr Tomalak

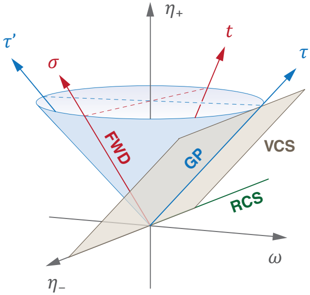
Compton scattering



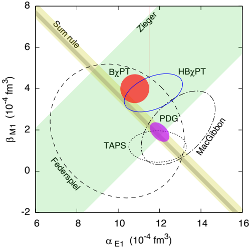
Four independent variables:

$$\eta_+ = \frac{Q^2 + Q'^2}{2m^2}, \quad \eta_- = \frac{Q \cdot Q'}{m^2},$$

$$\omega = \frac{Q^2 - Q'^2}{2m^2}, \quad \lambda = \frac{p \cdot \Sigma}{m^2}$$



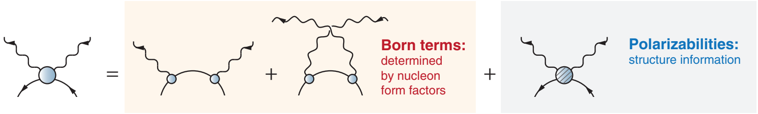
- **RCS:** nucleon polarizabilities
  Krupina & Pascalutsa, PRL 110 (2013)

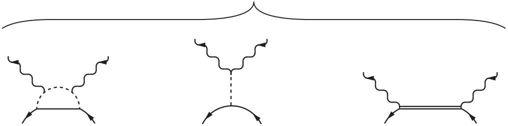


- **VCS:** generalized polarizabilities
- **DVCS:** handbag dominance, GPDs
- **Forward limit:** structure functions in DIS
- **Timelike region:** $p\bar{p}$ annhihilation at PANDA
- **Spacelike region:** two-photon corrections to nucleon form factors, proton radius puzzle?

Compton scattering

Compton amplitude = sum of **Born terms + 1PI structure part:**

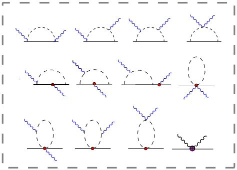




„Pion cloud“ (ChPT)

t-channel meson exchange ($\pi, \sigma, a_1, \dots$)

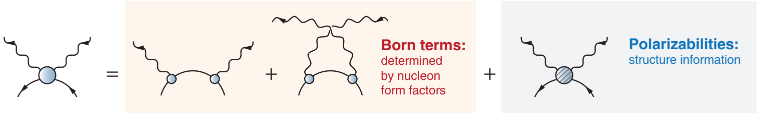
s/u-channel nucleon resonances ($\Delta, N^*, \dots$)

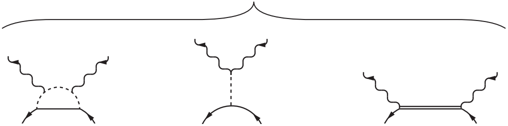


Griesshammer, McGovern, Phillips, Feldman, Prog. Part. Nucl. Phys. 67 (2012)

Compton scattering

Compton amplitude = sum of **Born terms + 1PI structure part:**

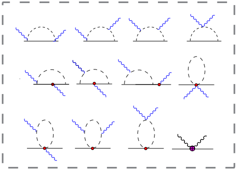




„Pion cloud“
(ChPT)

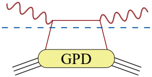
t-channel
meson exchange
$(\pi, \sigma, a_1, \dots)$

s/u-channel
nucleon resonances
$(\Delta, N^*, \dots)$



Griesshammer, McGovern, Phillips, Feldman, Prog. Part. Nucl. Phys. 67 (2012)

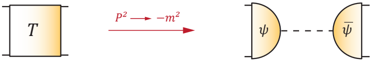
but also:



⇒ is there a common underlying **quark-level description?**

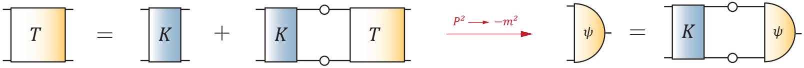
Bethe-Salpeter equations

- Extract hadron properties from **poles** in $q\bar{q}$, qqq **scattering matrices:**

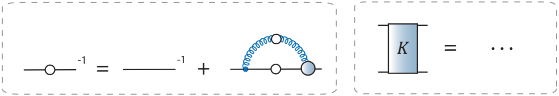


- Use **scattering equation** (inhomogeneous BSE) to obtain T in the first place: $T = K + K\,G_0\,T$

Homogeneous BSE for **BS amplitude:**

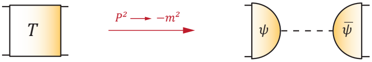


- Kernel is connected to **quark Dyson-Schwinger equation:**



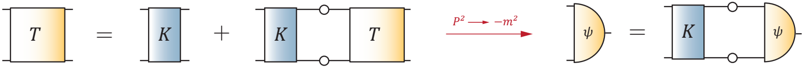
Bethe-Salpeter equations

- Extract hadron properties from **poles** in $q\bar{q}$, qqq **scattering matrices:**

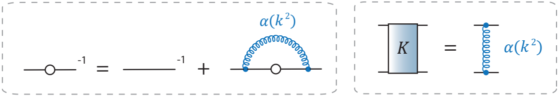


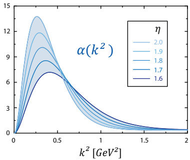
- Use **scattering equation** (inhomogeneous BSE) to obtain T in the first place: $T = K + K\,G_0\,T$

Homogeneous BSE for **BS amplitude:**



- Kernel is connected to **quark Dyson-Schwinger equation:**





Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

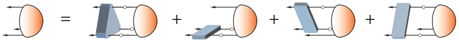
Rainbow-ladder: $\alpha(k^2) = \alpha_{\text{IR}}\left(k^2/\Lambda^2, \eta\right) + \alpha_{\text{UV}}(k^2)$

adjust scale Λ to observable, keep width η as parameter

Baryons

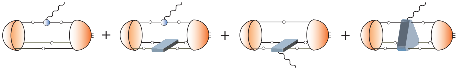
- Covariant Faddeev equation for **baryons:** keep 2-body interactions & rainbow-ladder, but no further approximations: $M_N = 0.94\,\text{GeV}$

  GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010), GE, PRD 84 (2011), Sanchis-Alepuz, Fischer, PRD 90 (2014), Sanchis-Alepuz, Fischer, Kubrak, PLB 733 (2014)



- **Baryon form factors:** nucleon and Δ FFs, $N \to \Delta\gamma$ transition

  GE, PRD 84 (2011), Sanchis-Alepuz, Williams, Alkofer, PRD 87 (2013), Alkofer, GE, Sanchis-Alepuz, Williams, Hyp. Int. 234 (2015)



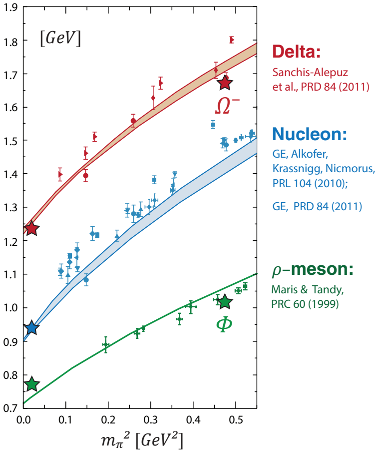
  Good overall description:

  em. gauge invariance √

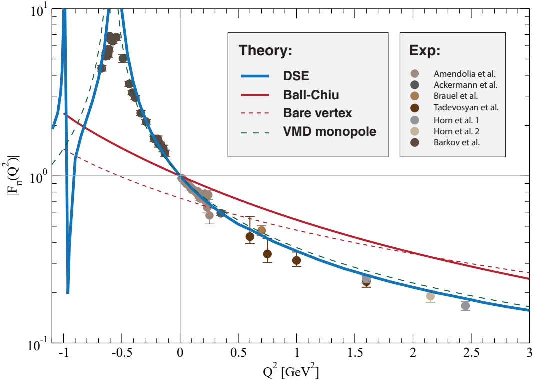
  vector-meson poles √

  but missing pion effects at low Q^2

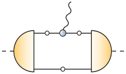
  . . . see talk by Reinhard Alkofer



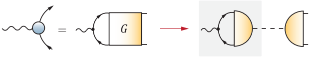
Pion form factor



A. Krassnigg (Schladming 2010),
Maris & Tandy, Nucl. Phys. Proc. Suppl. 161 (2006)

- Form factor from 

- **Timelike vector meson poles** automatically generated by quark-photon vertex BSE!

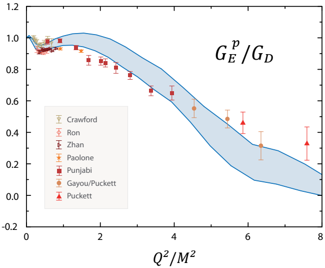


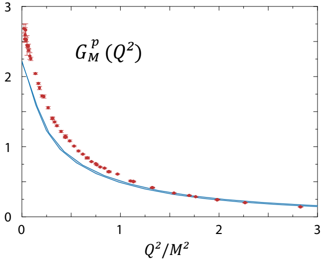
$\Rightarrow\ \Gamma^\mu =$ **Ball-Chiu** (em. gauge invariance)

$+$ **Transverse part** (vm. poles & dominance)

- Include **pion cloud** effects:
Kubrak, GE, Fischer, Williams, in preparation

Nucleon em. form factors



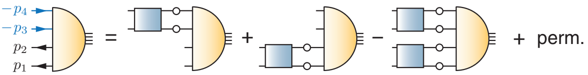


- same input, all ingredients calculated, model dependence shown by bands
  GE, PRD 84 (2011)
- **electric proton form factor:** consistent with data, possible zero crossing
- **magnetic form factors:** missing pion effects at low Q^2, $\kappa^s = -0.12$ reproduced (pion effects cancel!)
- **charge radii & magnetic moments** agree with lattice at larger quark masses, flat, no chiral divergences for radii
- Similar for axial & pseudoscalar FFs, Δ and $N \to \Delta\gamma$ transition form factors
  GE, Fischer, EPJ A 48 (2012), Sanchis-Alepuz et al., PRD 87 (2013), Alkofer et al., Hyperf. Int. 234 (2015)

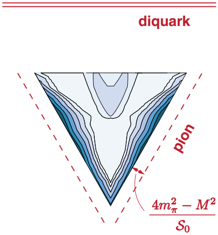
⇒ **"quark core without pion-cloud effects"**

Tetraquarks

- Solution of four-body equation (same input) reproduces mass pattern for **light scalar mesons:** σ, κ, a_0, f_0
  GE, Fischer, Heupel, 1508.07178 [hep-ph]

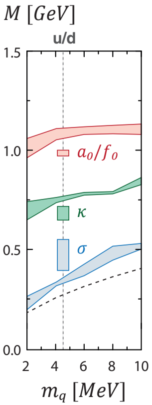


- BSE dynamically generates **pion poles** in wave function, drive σ mass from 1.5 GeV to ~350 MeV



Four quarks rearrange to **"meson molecule", diquarks irrelevant**

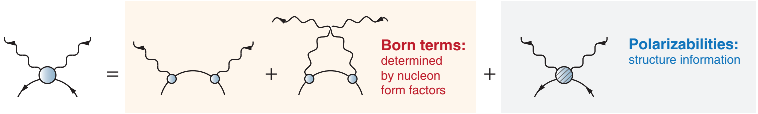
Tetraquark is at the same time dynamically generated resonance!

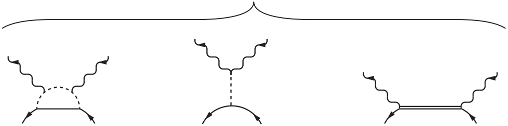


. . . see talk by Christian Fischer

Compton scattering ...

Compton amplitude = sum of **Born terms + 1PI structure part:**

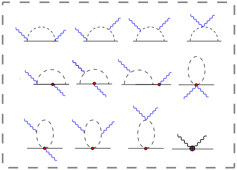




„Pion cloud“
(ChPT)

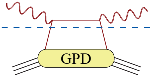
t-channel
meson exchange
$(\pi, \sigma, a_1, \ldots)$

s/u-channel
nucleon resonances
$(\Delta, N^*, \ldots)$



Griesshammer, McGovern, Phillips, Feldman, Prog. Part. Nucl. Phys. 67 (2012)

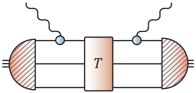
but also:



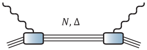
⇒ is there a common underlying **quark-level description?**

... at the quark level

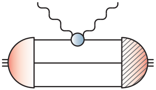
Derived closed expression for Compton amplitude at quark level
(here: rainbow-ladder, modulo crossing & permutation) GE, Fischer, PRD 85 (2012) & PRD 87 (2013)



s- and u-channel
nucleon resonances:

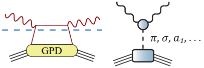


approximate this
(include all resonances)

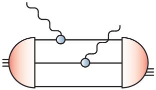


"Impulse approximation":

handbag diagrams +
t-channel **meson poles**



calculate this



cat's ears diagrams

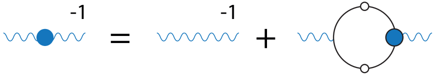
neglect this
(for the moment)

- √ crossing symmetry
- √ em. gauge invariance
- √ perturbative processes included
- √ s, t, u channel poles generated in QCD

But only **sum** is **gauge invariant**, not individual diagrams ⇒ problem!

Gauge invariance

Simplest example: photon vacuum polarization



$$\Pi^{\mu\nu}(Q) = a(Q^2)\,\delta^{\mu\nu} + b(Q^2)\,Q^\mu Q^\nu$$

- **Analyticity** ⇒ $a,\, b$ cannot have poles at $Q^2 = 0$ (intermediate massless particle, but $\Pi^{\mu\nu}$ = 1PI)
- **Transversality** ⇒ Ward identity: $Q^\mu \Pi^{\mu\nu}(Q) = 0 \quad \Rightarrow \quad a = -b\,Q^2$ (not $b = -a/Q^2$!!!)

In total:

$$\Pi^{\mu\nu}(Q) = \Pi(Q^2)\left(Q^2\,\delta^{\mu\nu} - Q^\mu Q^\nu\right) = \Pi(Q^2)\,t^{\mu\nu}_{QQ} \quad \sim \quad Q^2 \qquad\qquad t^{\mu\nu}_{AB} = A\cdot B\,\delta^{\mu\nu} - B^\mu A^\nu$$

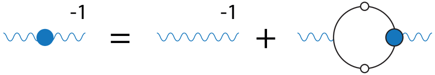
Or generally:

$$\Pi^{\mu\nu}(Q) = \underbrace{\Pi(Q^2)\,t^{\mu\nu}_{QQ}}_{\text{transverse part}} + \underbrace{\widetilde{\Pi}(Q^2)\,\delta^{\mu\nu}}_{\text{„gauge part“}}$$

- 1-loop in dim. reg: $\widetilde{\Pi}(Q^2) = 0$ . . . ok
- 1-loop with cutoff: $\widetilde{\Pi}(Q^2) \sim \Lambda^2 \neq 0$ . . . quadratic divergence!

Gauge invariance

Simplest example: photon vacuum polarization



$$\Pi^{\mu\nu}(Q) = a(Q^2)\,\delta^{\mu\nu} + b(Q^2)\,Q^\mu Q^\nu$$

- **Analyticity** ⇒ $a,\,b$ cannot have poles at $Q^2 = 0$ (intermediate massless particle, but $\Pi^{\mu\nu}$ = 1PI)
- **Transversality** ⇒ Ward identity: $Q^\mu \Pi^{\mu\nu}(Q) = 0 \quad \Rightarrow \quad a = -b\,Q^2$ (not $b = -a/Q^2$!!!)

In total:

$$\Pi^{\mu\nu}(Q) = \Pi(Q^2)\left(Q^2\,\delta^{\mu\nu} - Q^\mu Q^\nu\right) = \Pi(Q^2)\,t^{\mu\nu}_{QQ} \quad \sim \quad Q^2 \qquad\qquad t^{\mu\nu}_{AB} = A\cdot B\,\delta^{\mu\nu} - B^\mu A^\nu$$

Or generally:

$$\Pi^{\mu\nu}(Q) = \underbrace{\Pi(Q^2)\,t^{\mu\nu}_{QQ}}_{\text{transverse part}} + \underbrace{\widetilde{\Pi}(Q^2)\,\delta^{\mu\nu}}_{\text{„gauge part“}}$$

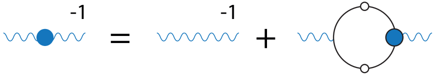
What if calculation **breaks** gauge invariance by **more** than cutoff? Transverse projection?

$$T^{\mu\alpha}_Q\,\Pi^{\alpha\beta}\,T^{\beta\nu}_Q = \left[\Pi(Q^2) + \frac{\widetilde{\Pi}(Q^2)}{Q^2}\right] t^{\mu\nu}_{QQ}$$

⇒ bad: kinematic singularities

Gauge invariance

Simplest example: photon vacuum polarization



$$\Pi^{\mu\nu}(Q) = a(Q^2)\,\delta^{\mu\nu} + b(Q^2)\,Q^\mu Q^\nu$$

- **Analyticity** ⇒ a, b cannot have poles at $Q^2 = 0$ (intermediate massless particle, but $\Pi^{\mu\nu}$ = 1PI)
- **Transversality** ⇒ Ward identity: $Q^\mu \Pi^{\mu\nu}(Q) = 0 \quad \Rightarrow \quad a = -b\,Q^2$ (not $b = -a/Q^2$!!!)

In total:

$$\Pi^{\mu\nu}(Q) = \Pi(Q^2)\left(Q^2\,\delta^{\mu\nu} - Q^\mu Q^\nu\right) = \Pi(Q^2)\,t^{\mu\nu}_{QQ} \quad \sim \quad Q^2 \qquad\qquad t^{\mu\nu}_{AB} = A\cdot B\,\delta^{\mu\nu} - B^\mu A^\nu$$

Or generally:

$$\Pi^{\mu\nu}(Q) = \underbrace{\Pi(Q^2)\,t^{\mu\nu}_{QQ}}_{\text{transverse part}} + \underbrace{\widetilde{\Pi}(Q^2)\,\delta^{\mu\nu}}_{\text{„gauge part“}}$$

- In general: need to project onto full **transverse + gauge basis**, subtract gauge part.
- **Compton amplitude:**
  32 tensors (18 transverse + 14 gauge).
  Transverse basis derived by Tarrach
  Tarrach, Nuovo Cim. 28 (1975)

Nucleon resonances I

- Calculate all s- & u-channel nucleon resonance contributions ($J = \frac{1}{2}^{\pm}, \frac{3}{2}^{\pm}$)

(+ crossed term)

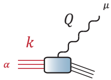
- Needs **offshell $N, N^*, \Delta, \ldots$ transition vertices.**
  Δ vertices must satisfy spin-3/2 gauge invariance, otherwise offshell spin-1/2 background
  Pascalutsa, Timmermanns, PRC 60 (1999); Shklyar, Lenske, PRC 80 (2009)

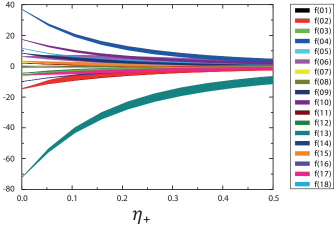
- General form of offshell $J = \frac{1}{2}^{\pm}$ (→8) and $\frac{3}{2}^{\pm}$ (→12) transition currents:
  GE, Ramalho, in preparation

$$\varepsilon^{\alpha\mu}_{kQ} = \gamma_5\, \varepsilon^{\alpha\mu\rho\sigma} k^\rho Q^\sigma \qquad \Leftrightarrow \qquad g_M\, (\partial^\mu \bar{\psi}^\alpha)\, \widetilde{F}^{\alpha\mu}\psi \quad \sim g_M\, \bar{\psi}^\alpha\, \gamma_5\, \varepsilon^{\alpha\mu}_{kQ}\, A^\mu \psi$$

$$t^{\alpha\mu}_{kQ} = k\cdot Q\, \delta^{\alpha\mu} - Q^\alpha k^\mu \qquad\qquad g_E\, (\partial^\mu \bar{\psi}^\alpha)\, \gamma_5\, F^{\alpha\mu}\psi \quad \sim g_E\, \bar{\psi}^\alpha\, \gamma_5\, t^{\alpha\mu}_{kQ}\, A^\mu \psi$$

$$\vdots \qquad\qquad\qquad\qquad \vdots$$

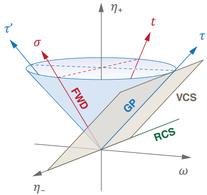




Residue of Δ exchange:
18 Compton FFs

Transverse basis works!

depends almost only on η_+
⇒ **same in all kinematic limits!**



Nucleon resonances II

- What about nucleon Born term?
- **offshell nucleon-photon vertex** depends on 12 tensor structures (8 transverse, 4 gauge)
- Must use **Dirac current**, otherwise Born term not gauge invariant

  (can be restored by adding terms in Compton amplitude ⇒ but then no longer just Born)
  GE, Fischer, PRD 87 (2013)

  ⇒ careful with **offshell form factors!**

(+ crossed term)

transverse:

$t^{\mu\nu}_{QQ}\,\gamma^\nu$ $t^{\mu\nu}_{QQ}\,ik^\nu$

$t^{\mu\nu}_{QQ}\,k\cdot Q\,\frac{i}{2}[\gamma^\nu,\not k]$ $t^{\mu\nu}_{QQ}\,k^\nu\not k$

$\frac{i}{2}[\gamma^\mu,\not Q]$ $t^{\mu\nu}_{Qk}\,k\cdot Q\,\gamma^\nu$

$\frac{1}{6}[\gamma^\mu,\not k,\not Q]$ $t^{\mu\nu}_{Qk}\,\frac{i}{2}[\gamma^\nu,\not k]$

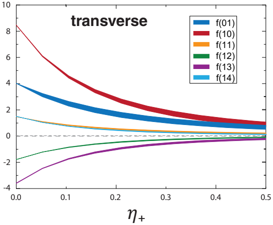
gauge:

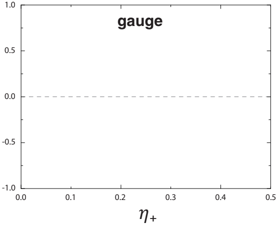
γ^μ

ik^μ

$k^\mu\not k$

$k\cdot Q\,\frac{i}{2}[\gamma^\mu,\not k]$





Compton FFs:
gauge part is zero

Nucleon resonances II

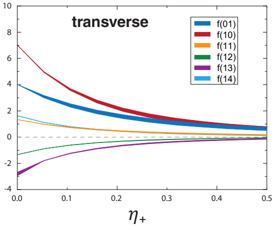
- What about nucleon Born term?
- **offshell nucleon-photon vertex** depends on 12 tensor structures (8 transverse, 4 gauge)
- Must use **Dirac current**, otherwise Born term not gauge invariant

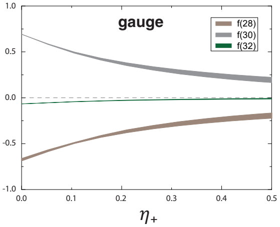
  (can be restored by adding terms in Compton amplitude ⇒ but then no longer just Born)
  GE, Fischer, PRD 87 (2013)

  ⇒ careful with **offshell form factors!**

(+ crossed term)

transverse		gauge
$t^{\mu\nu}_{QQ}\,\gamma^\nu$	$t^{\mu\nu}_{QQ}\,ik^\nu$	γ^μ
$t^{\mu\nu}_{QQ}\,k\cdot Q\,\frac{i}{2}[\gamma^\nu,\not k]$	$t^{\mu\nu}_{QQ}\,k^\nu\not k$	ik^μ
$\frac{i}{2}\,[\gamma^\mu,\not Q]$	$t^{\mu\nu}_{Qk}\,k\cdot Q\,\gamma^\nu$	$k^\mu\not k$
$\frac{1}{6}\,[\gamma^\mu,\not k,\not Q]$	$t^{\mu\nu}_{Qk}\,\frac{i}{2}\,[\gamma^\nu,\not k]$	$k\cdot Q\,\frac{i}{2}\,[\gamma^\mu,\not k]$



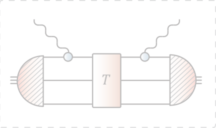


Compton FFs:
gauge part is zero

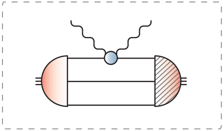
Add **non-Dirac current:** gauge part is nonzero, but transverse almost same!

⇒ **transverse + gauge basis works!**

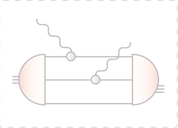
Compton amplitude



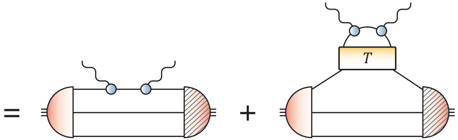
Nucleon resonances



"Impulse approximation"



cat's ears diagrams



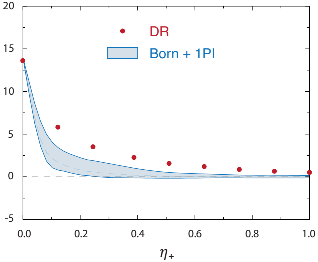
Born: reproduces perturbative handbag

1PI: reproduces t-channel meson poles

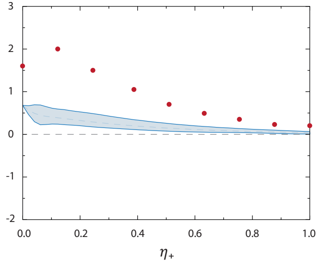
- Offshell quark Compton vertex: 6 kinematic invariants, 128 tensors (72 + 56), ⇒ difficult!
- Derived inhomogeneous BSE, solved in rainbow-ladder
  GE, Fischer, PRD 87 (2013)
- Calculate Compton amplitude, project on **transverse** + gauge ⇒ extract **polarizabilities!**

Proton polarizabilities

$\alpha_E + \beta_M$
$[10^{-4}\ \mathrm{fm}^3]$

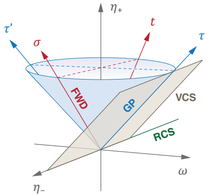


β_M
$[10^{-4}\ \mathrm{fm}^3]$

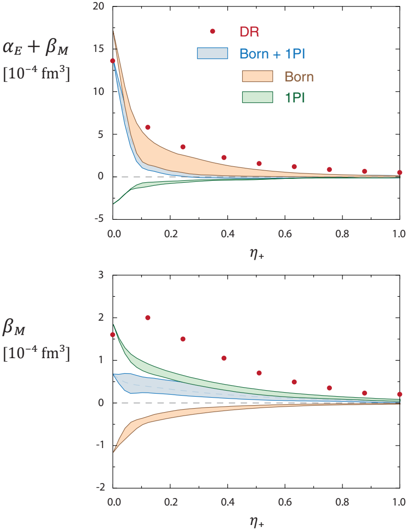


Preliminary results:

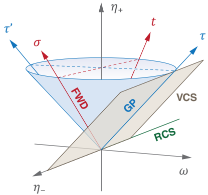
- band = result inside cone (70% of radius)
- compared to GPs from dispersion relation
  Pasquini et al., EPJ A11 (2001),
  Downie & Fonvieille, EPJ ST 198 (2011)
- α_E in ballpark, β_M too small (expect large pion effects)



Proton polarizabilities

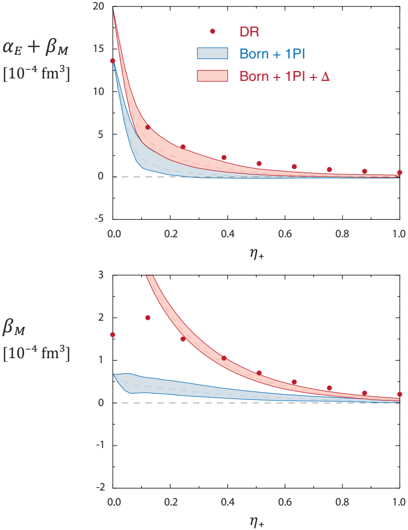


Preliminary results:

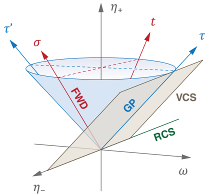


- band = result inside cone (70% of radius)
- compared to GPs from dispersion relation
  Pasquini et al., EPJ A11 (2001),
  Downie & Fonvieille, EPJ ST 198 (2011)
- α_E in ballpark, β_M too small (expect large pion effects)
- α_E dominated by Born (handbag), β_M small due to cancellation

Proton polarizabilities



Preliminary results:



- band = result inside cone (70% of radius)
- compared to GPs from dispersion relation
  Pasquini et al., EPJ A11 (2001),
  Downie & Fonvieille, EPJ ST 198 (2011)
- α_E in ballpark, β_M too small (expect large pion effects)
- α_E dominated by Born (handbag), β_M small due to cancellation
- What about Δ? Large contribution to β_M!
  Pascalutsa & Phillips, PRC 68 (2003)

In total: polarizabilities ≈

Born (handbag)
+ 1PI (t-channel meson poles)
+ nucleon resonances (mostly Δ)
+ pion cloud (at low η_+)?

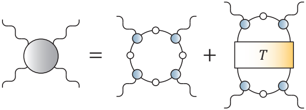
Summary & Outlook

- **Baryon masses, electromagnetic & transition form factors**
  reasonably well described, but need to include pion-cloud effects

- **Light scalar mesons** as **tetraquarks,**
  transition from four quarks to "meson molecule" **. . . see talk by Christian Fischer**
  GE, Fischer, Heupel, 1508.07178 [hep-ph]

- **Compton scattering:** looks promising
  ⇒ look into spin polarizabilities, structure functions, VCS, proton radius puzzle

- **Hadronic light-by-light:** almost same problem!
  gauge invariant calculation, but need transverse + gauge basis for meaningful predictions
  GE, Fischer, Heupel, 1505.06336 [hep-ph], to appear in PRD



- **Other scattering processes:** microscopically the same!
  $N\pi$ scattering, $\pi\pi$ scattering, pion electroproduction, . . .