

# *Nucleon polarizabilities from Compton scattering*

*Nadiia Krupina*

in collaboration with Vladimir Pascalutsa



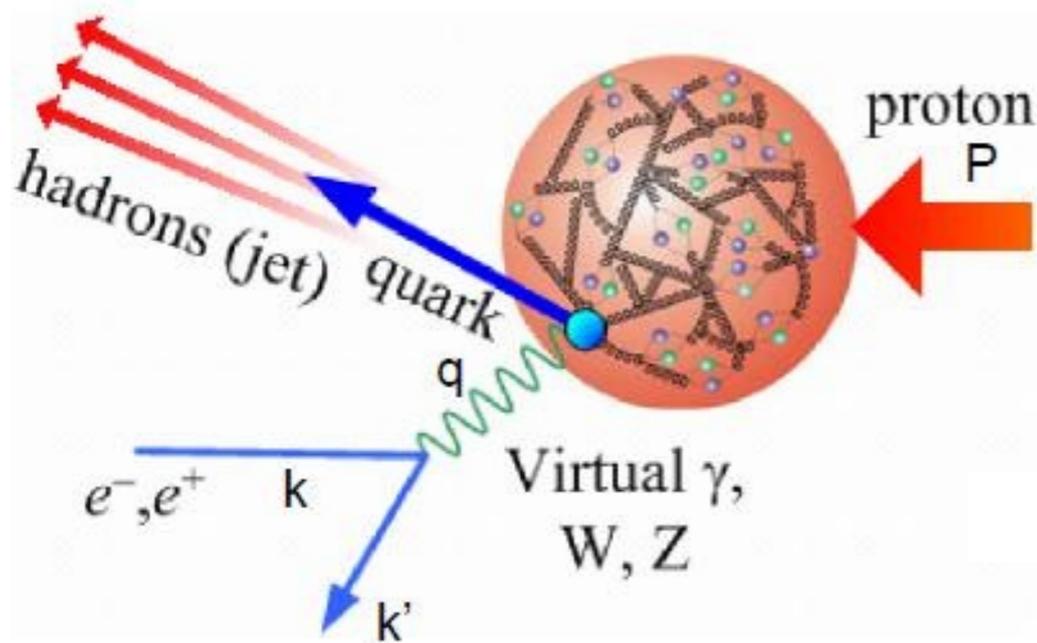
JOHANNES GUTENBERG  
UNIVERSITÄT MAINZ

International School of Nuclear Physics

Sept. 20, 2015, Erice

- 1) **Inelastic** structure, **polarizabilities**, and how they matter for the *proton size*
- 2) Status of dipole scalar polarizabilities
- 3) Disentangling the **magnetic polarizability** by **beam asymmetry** of Compton scattering
- 4) Multipole analysis - in progress..
- 5) Summary

## Electron-proton scattering



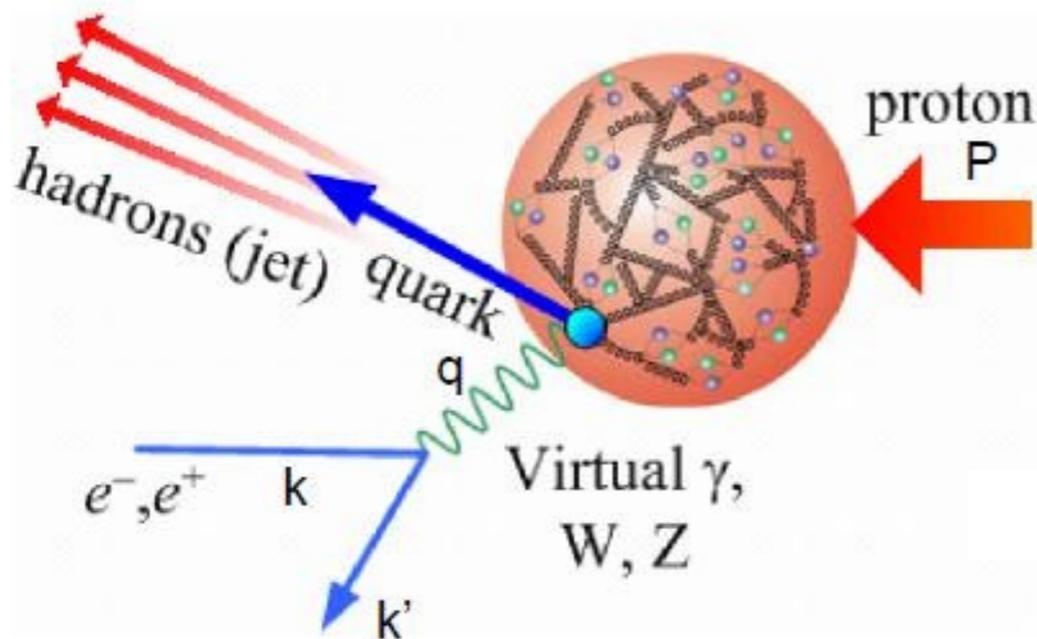
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$$x = Q^2 / (2M_N \nu)$$

Yields 4 **Structure functions**:

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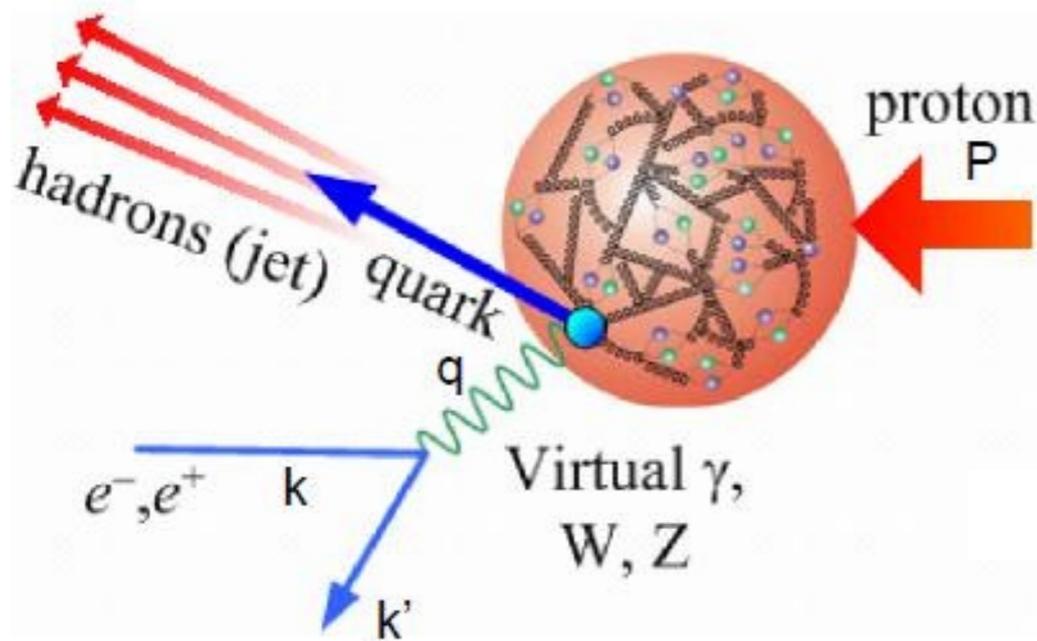
$$f_2^{\text{el}}(\nu, Q^2) = \frac{1}{1 + \tau} [G_E^2(Q^2) + \tau G_M^2(Q^2)] \delta(1 - x),$$

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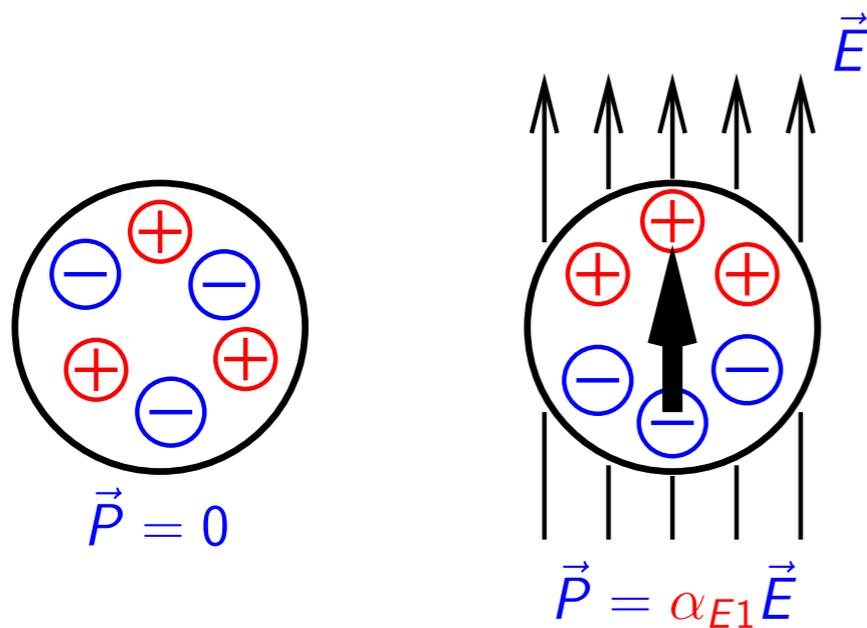
where  $\tau = Q^2 / 4M^2$  and  $G_E(Q^2), G_M(Q^2)$  are the Sachs FFs

(ii) Moments of the inelastic structure functions are related to **polarizabilities**, e.g.

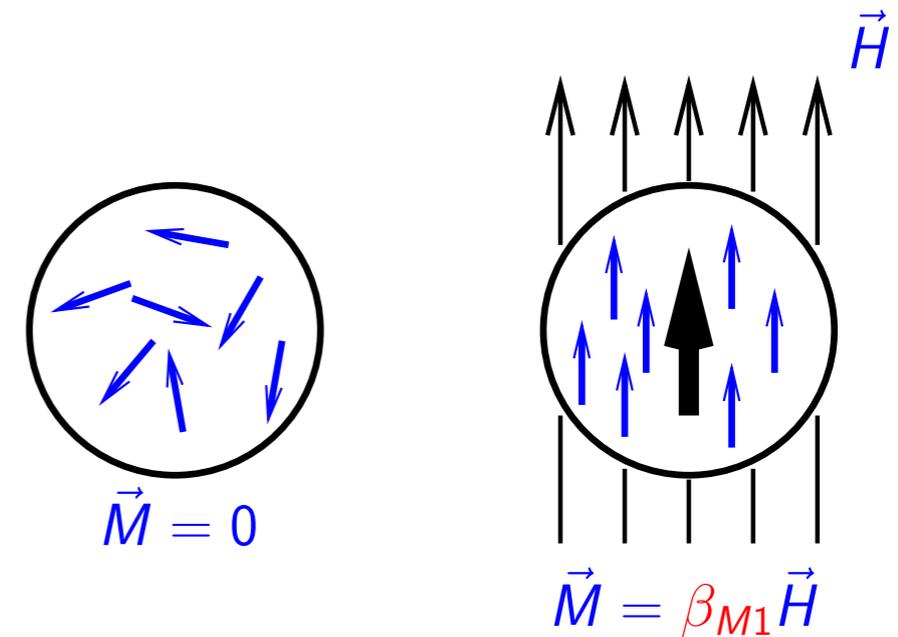
$$\alpha_{E1}(Q^2) + \beta_{M1}(Q^2) = \frac{8\alpha M_N}{Q^4} \int_0^{x_0} dx x f_1(x, Q^2).$$

(i) Describe the response of internal structure to applied external field

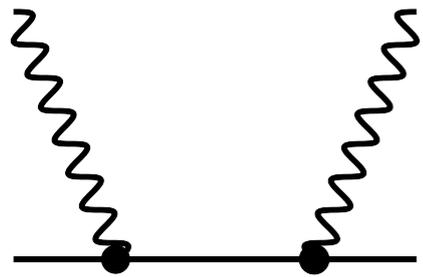
Electric dipole polarizability



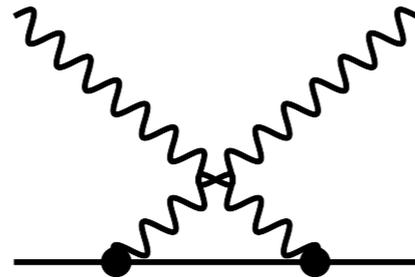
Magnetic dipole polarizability



(ii) Accessed experimentally in Compton scattering

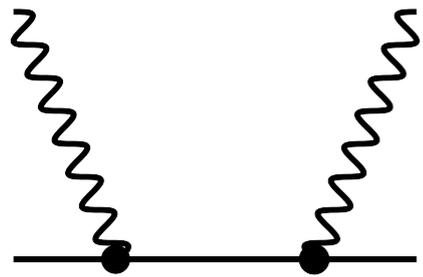


Born (elastic) contributions

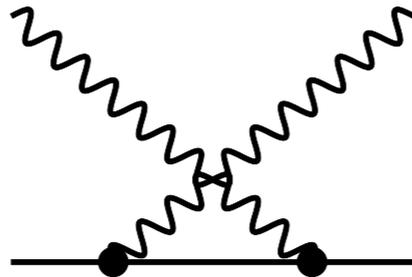


non-Born contribution, given by polarizabilities

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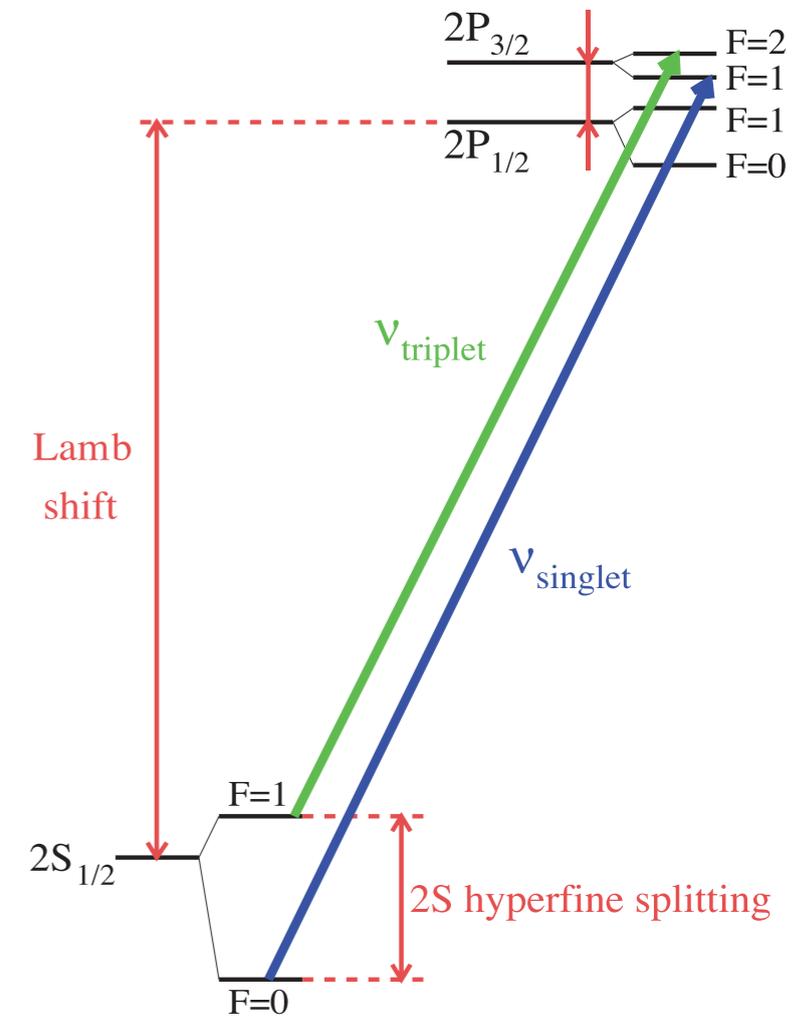
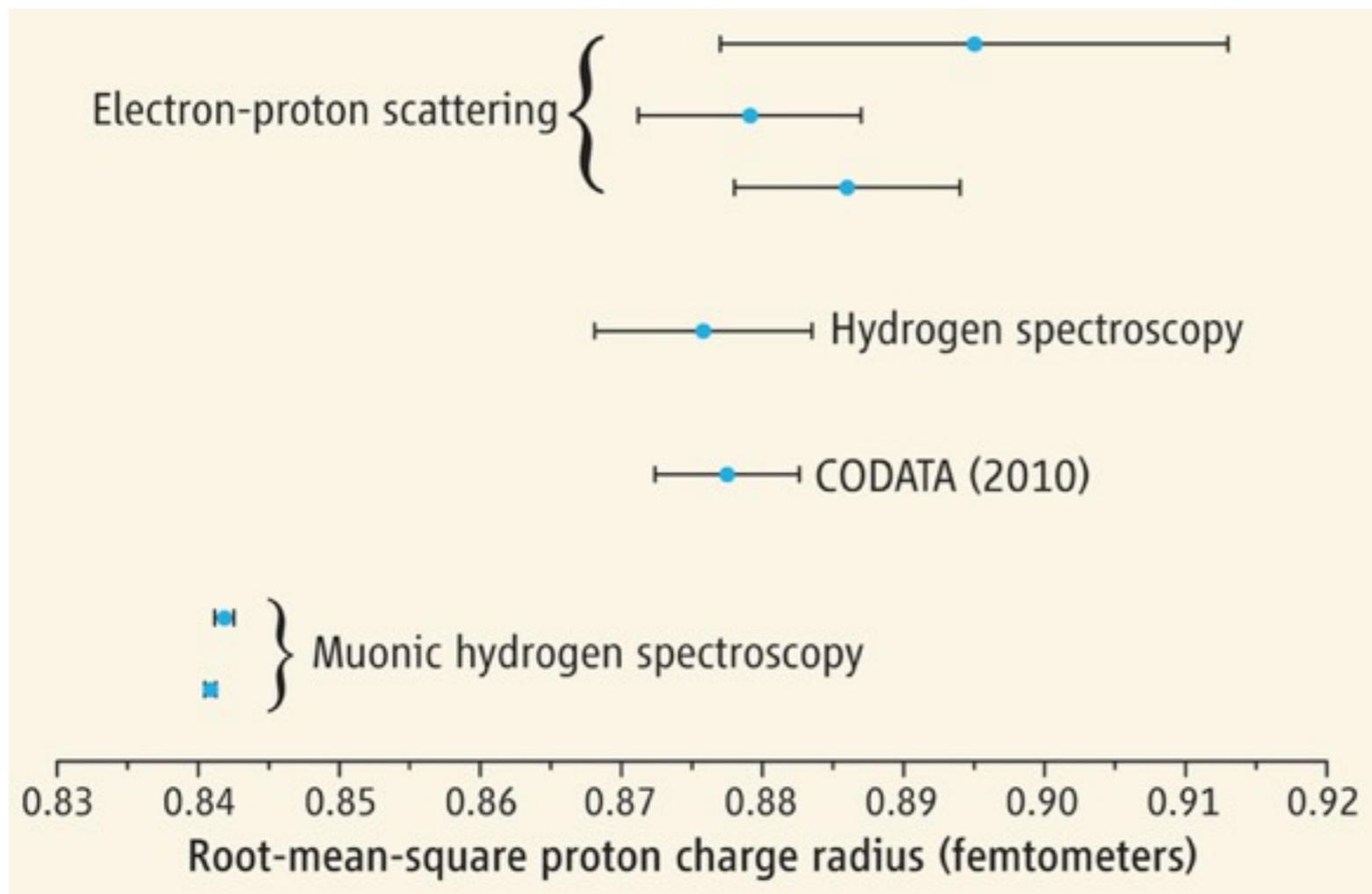
(iii) Theoretical approaches:

Effective field theory

Dispersion relations

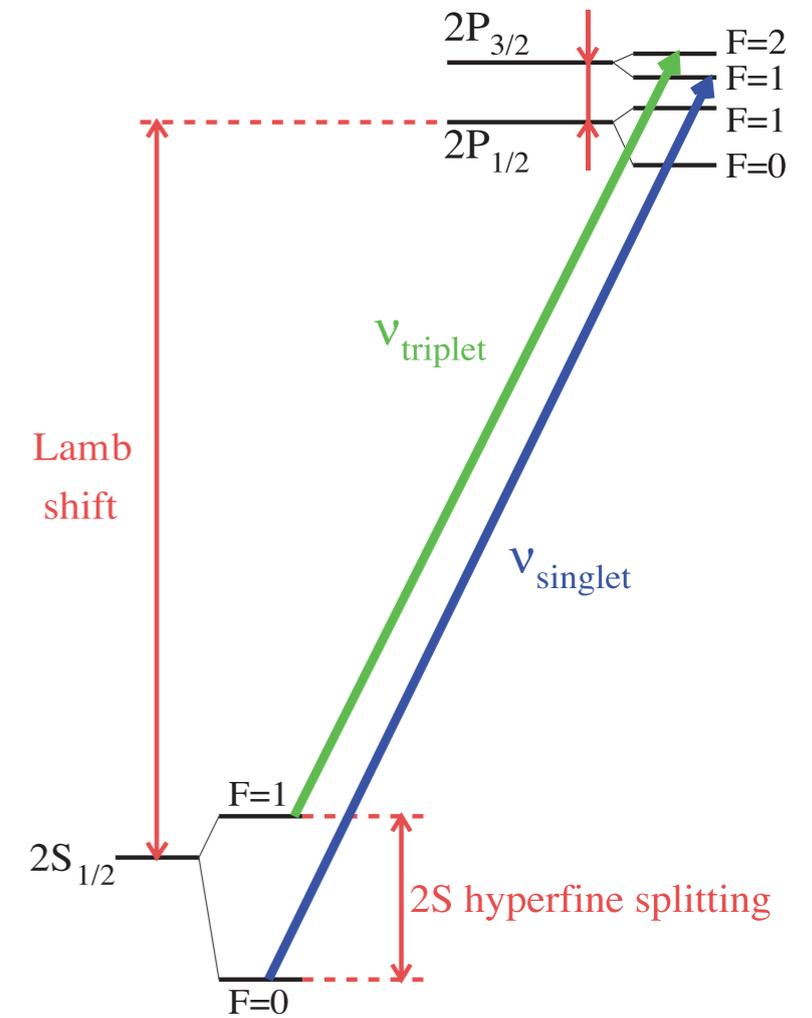
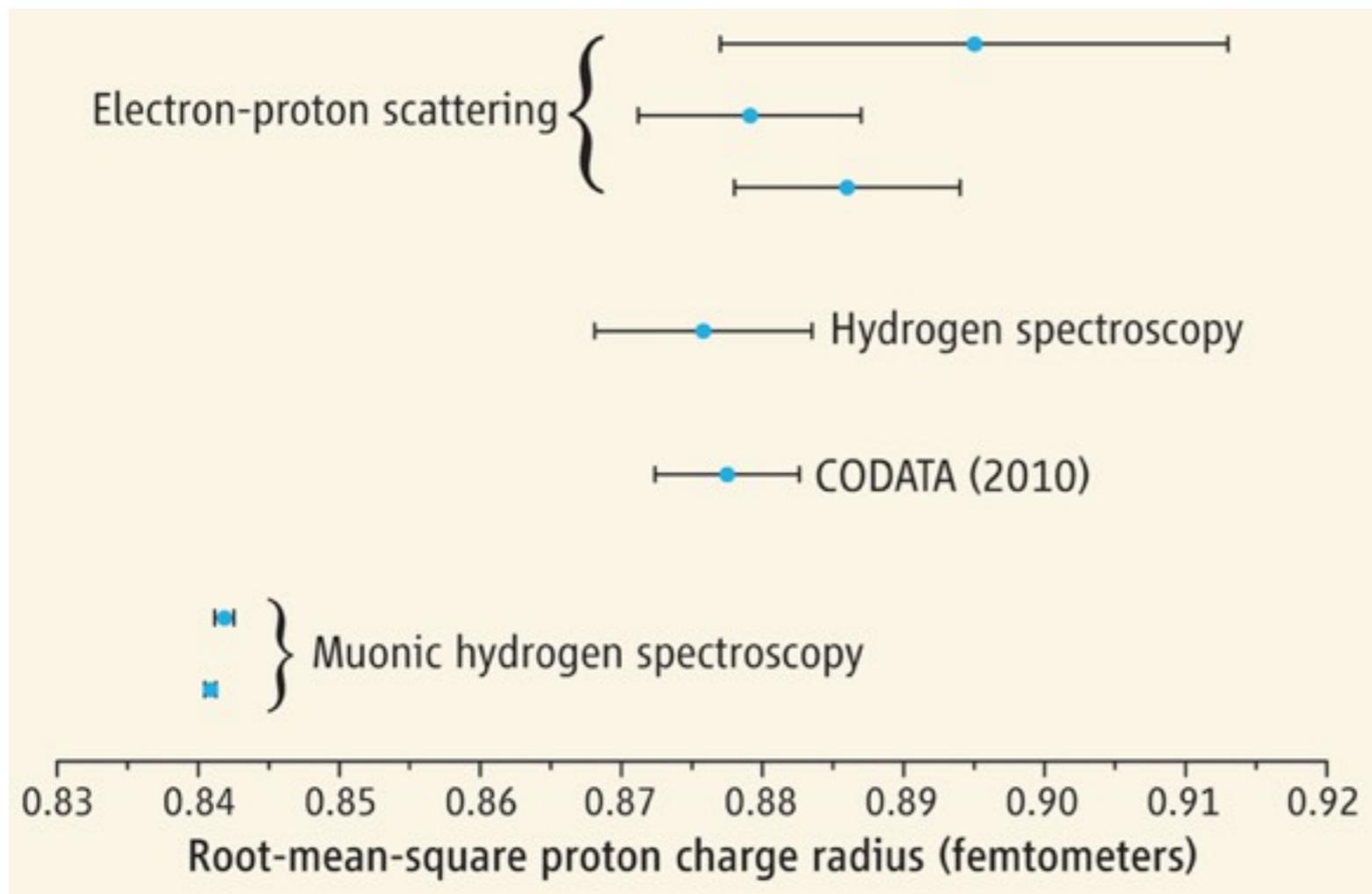
Lattice QCD

# Proton radius puzzle



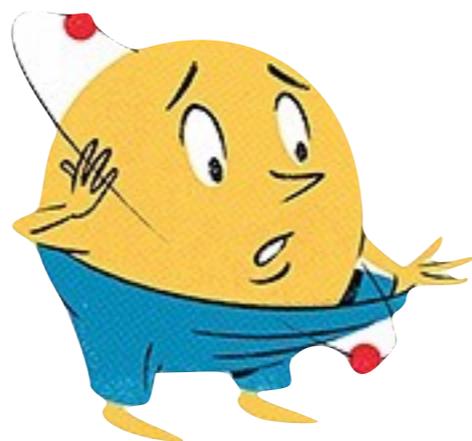
A. Antognini *et al.*, Science **339**, 417 (2013).

# Proton radius puzzle



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$$[R_E^{\mu\text{H}} = 0.84087(39) \text{ fm}]$$

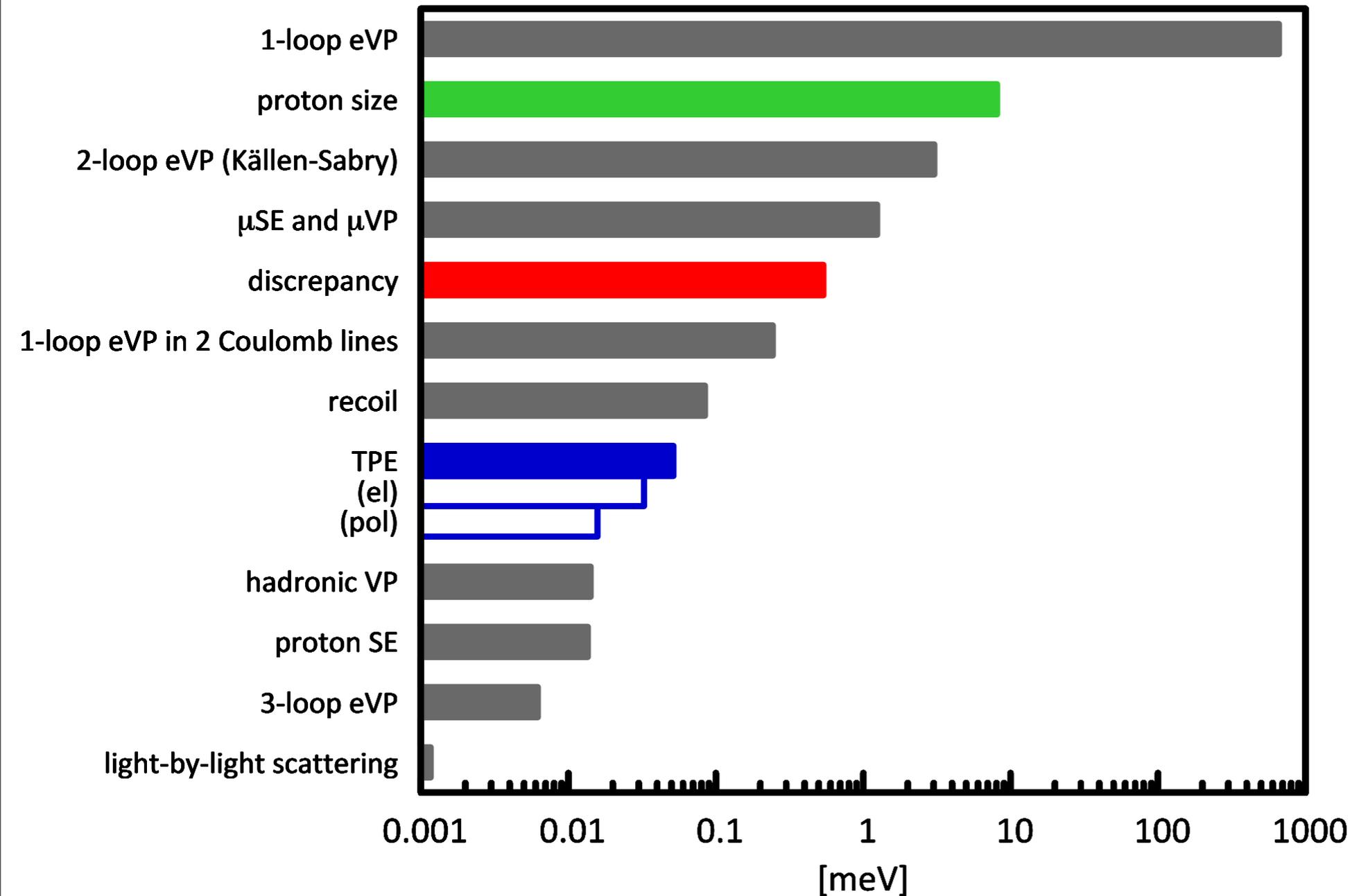


$$[R_E^{\text{CODATA 2010}} = 0.8775(51) \text{ fm}]$$

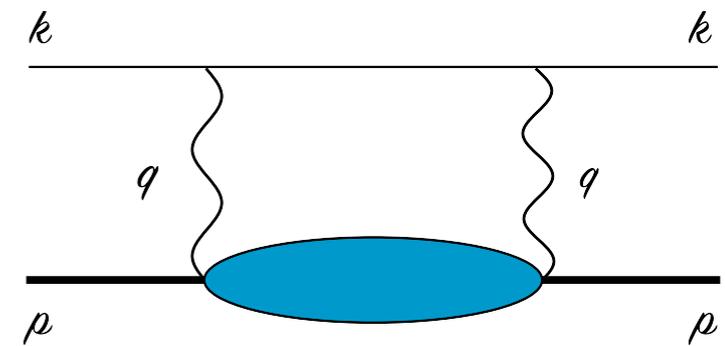
# Muonic Hydrogen Lamb shift

$$\Delta E_{\text{LS}}^{\text{th}} = 206.0668(25) - 5.2275(10) (R_E/\text{fm})^2$$

theory uncertainty:  
0.0025 meV

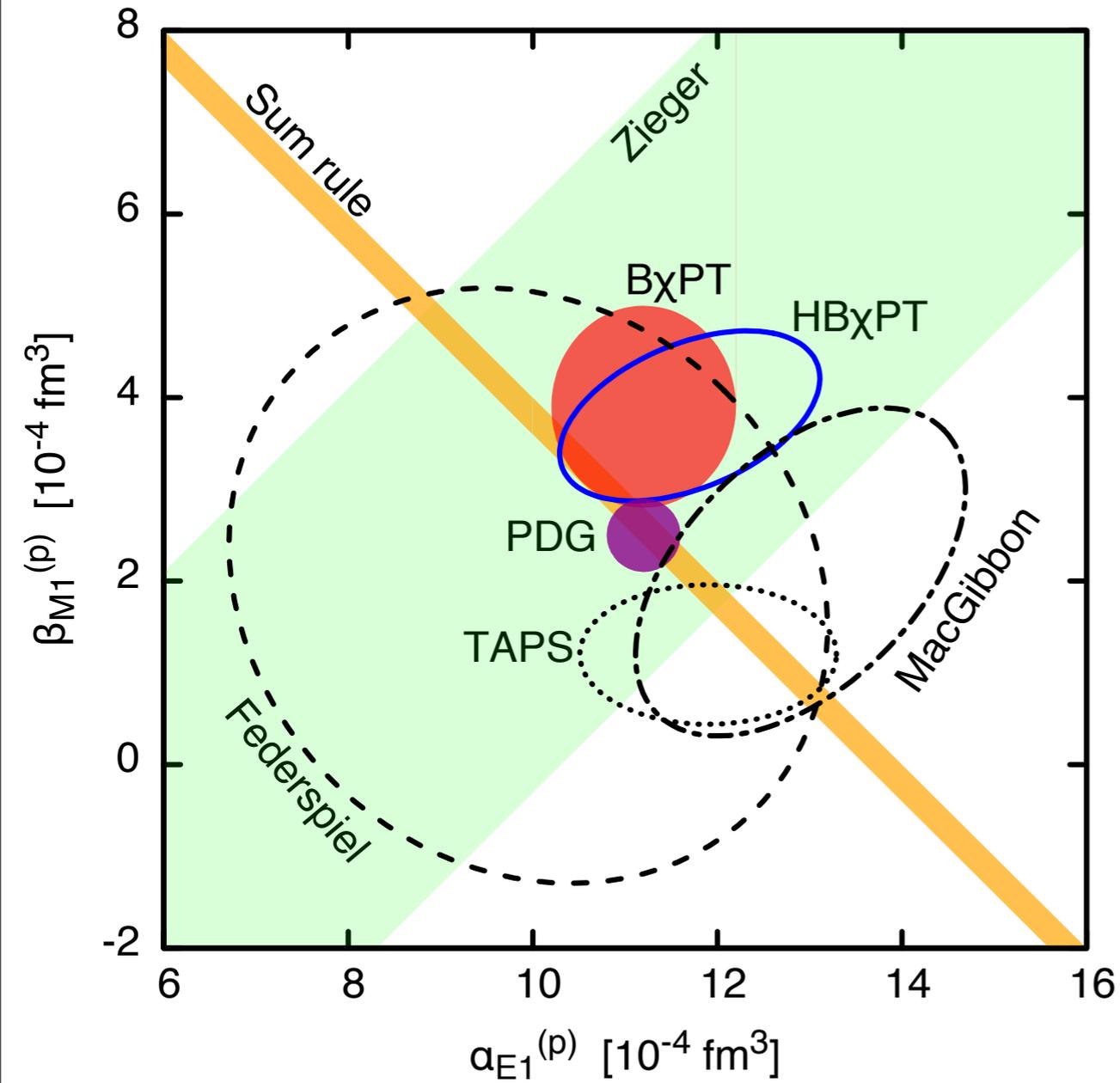


Two photon  
exchange (TPE)



$$\delta V^{(2\gamma)} = \delta V_{\text{elastic}}^{(2\gamma)} + \delta V_{\text{polariz.}}^{(2\gamma)}$$

# Proton dipole polarizabilities



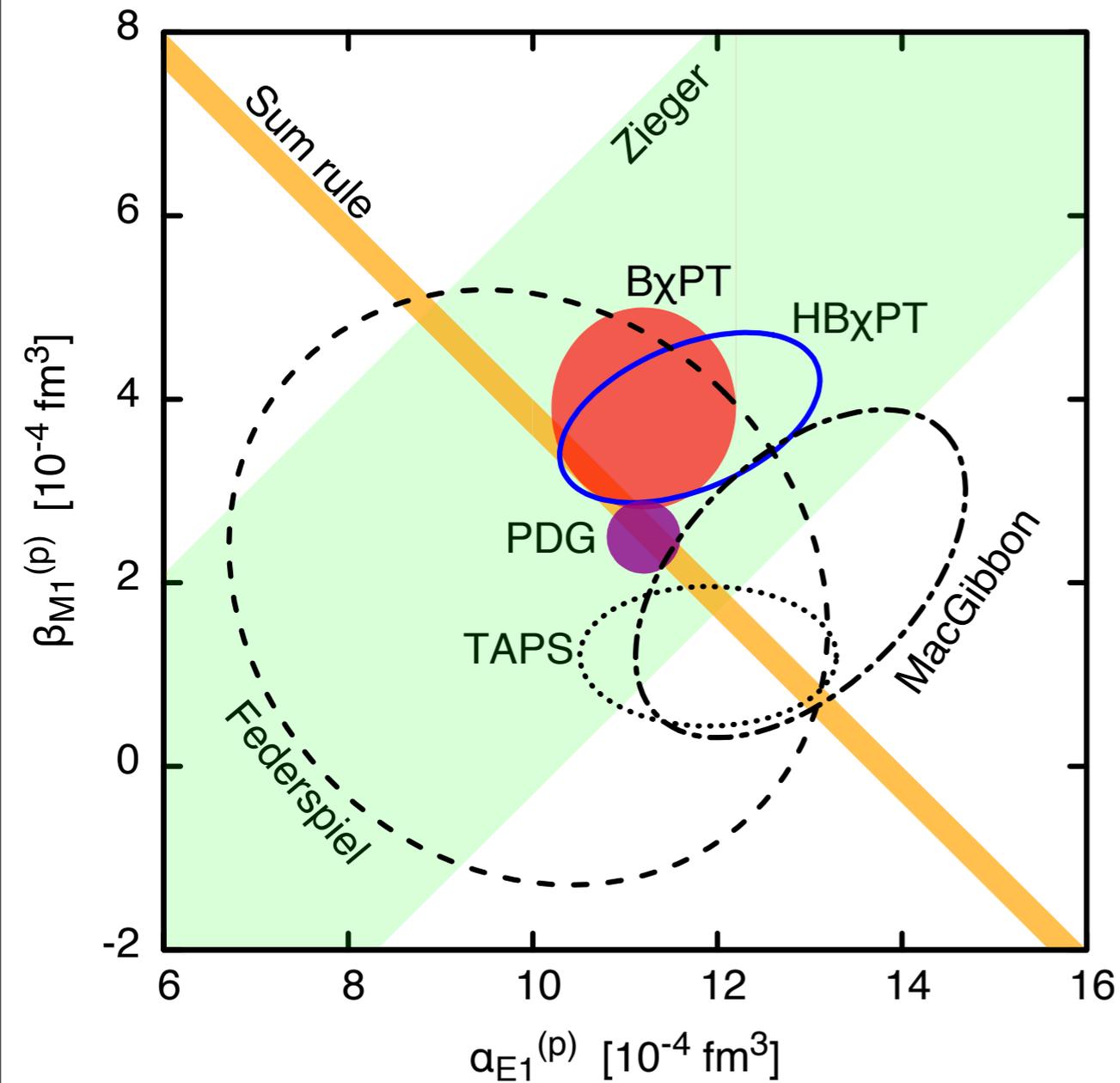
Theory:

**BChPT** - Lensky & Pascalutsa, EPJC (2010)

**HBChPT** - McGovern, Phillips & Griesshammer,

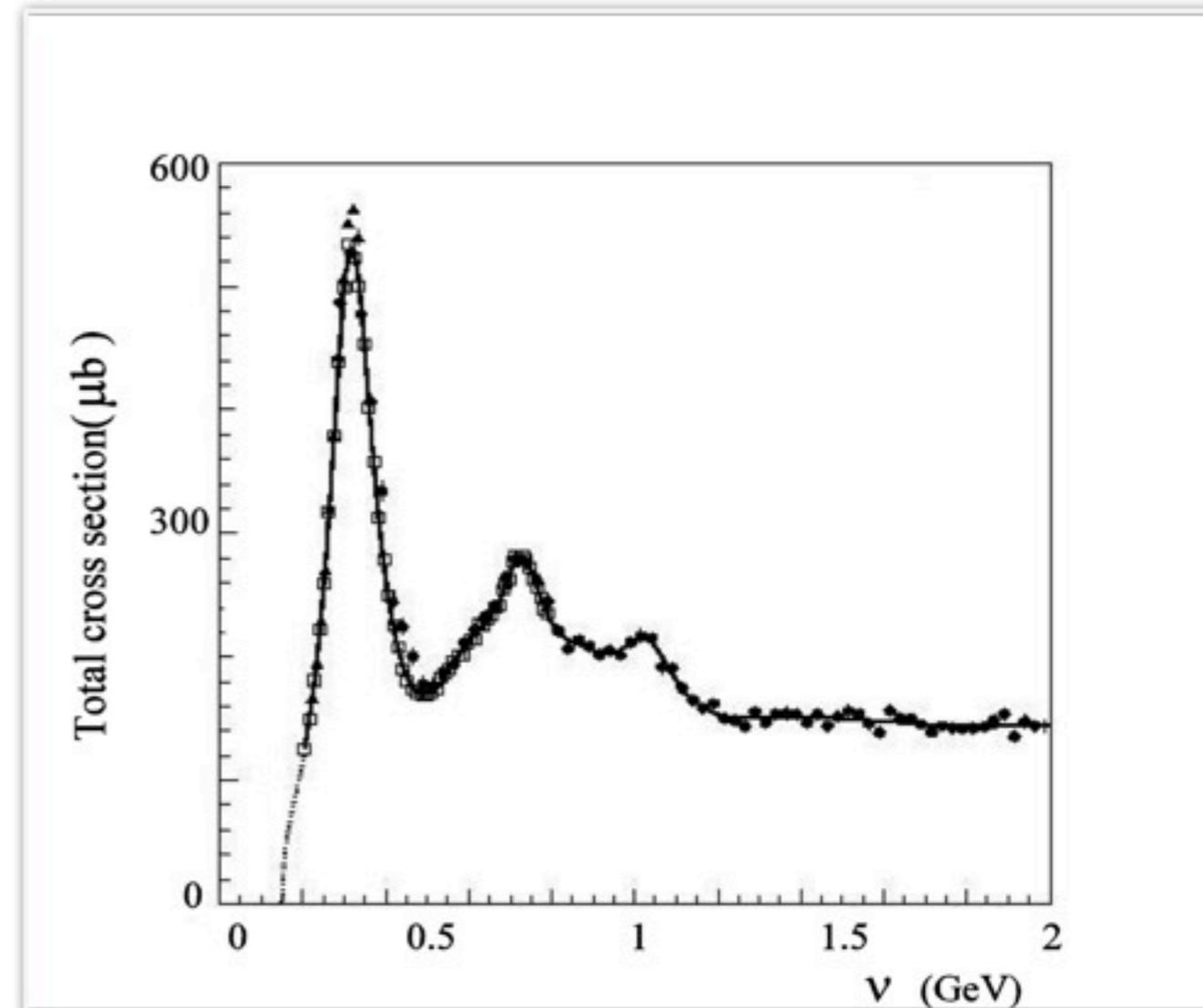
EPJA (2013)

# Proton dipole polarizabilities



Baldin sum rule

$$\alpha_{E1} + \beta_{M1} = \frac{1}{2\pi^2} \int_{\nu_{thr}}^{\infty} d\nu' \frac{\sigma_{tot}(\nu')}{\nu'^2} \simeq 14 \times 10^{-4} \text{ fm}^3$$



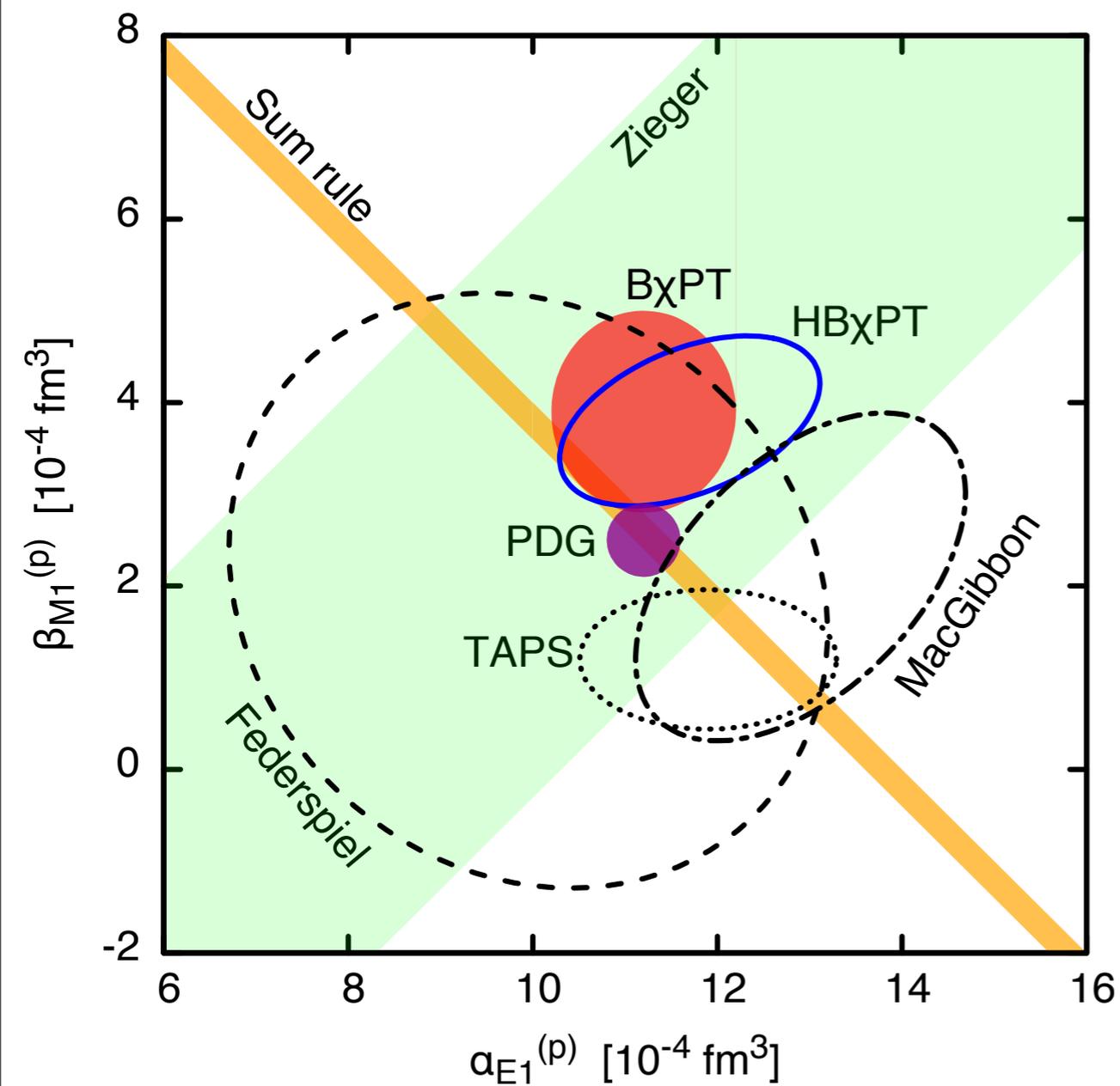
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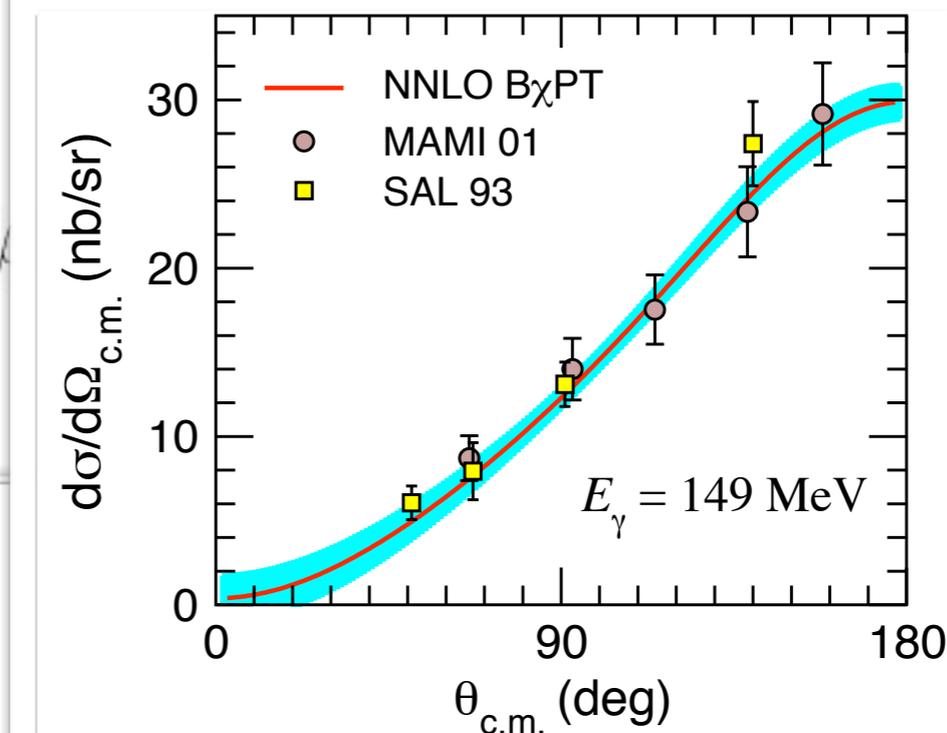
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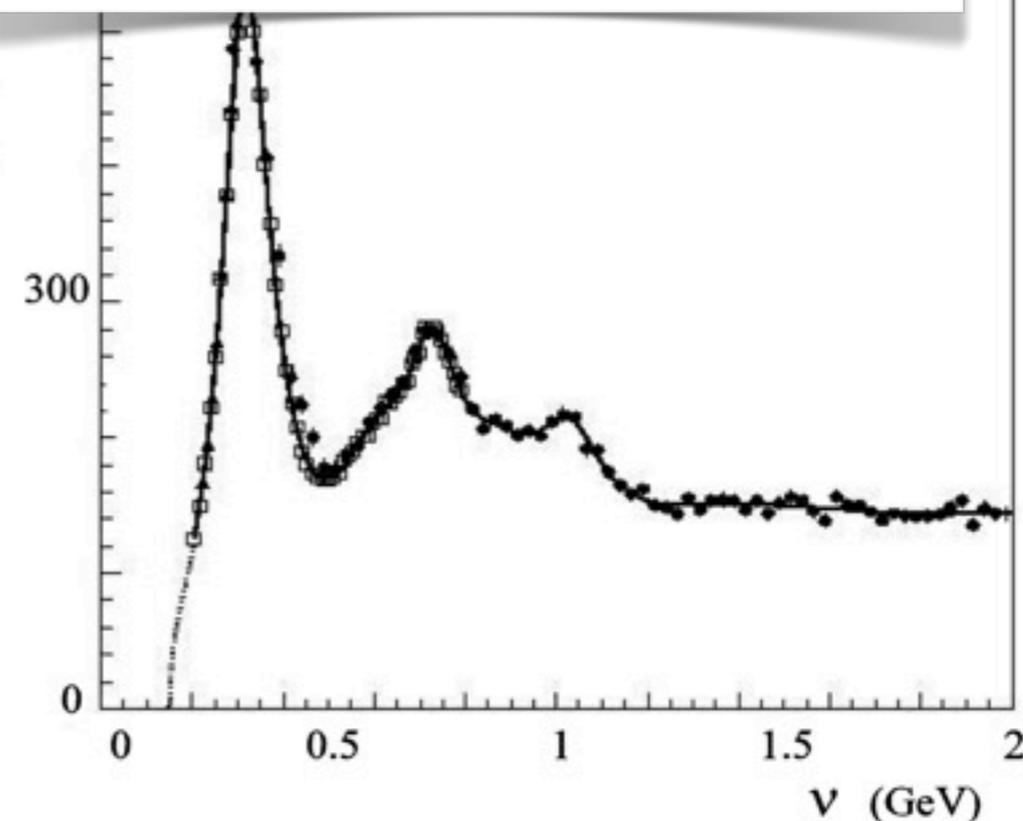
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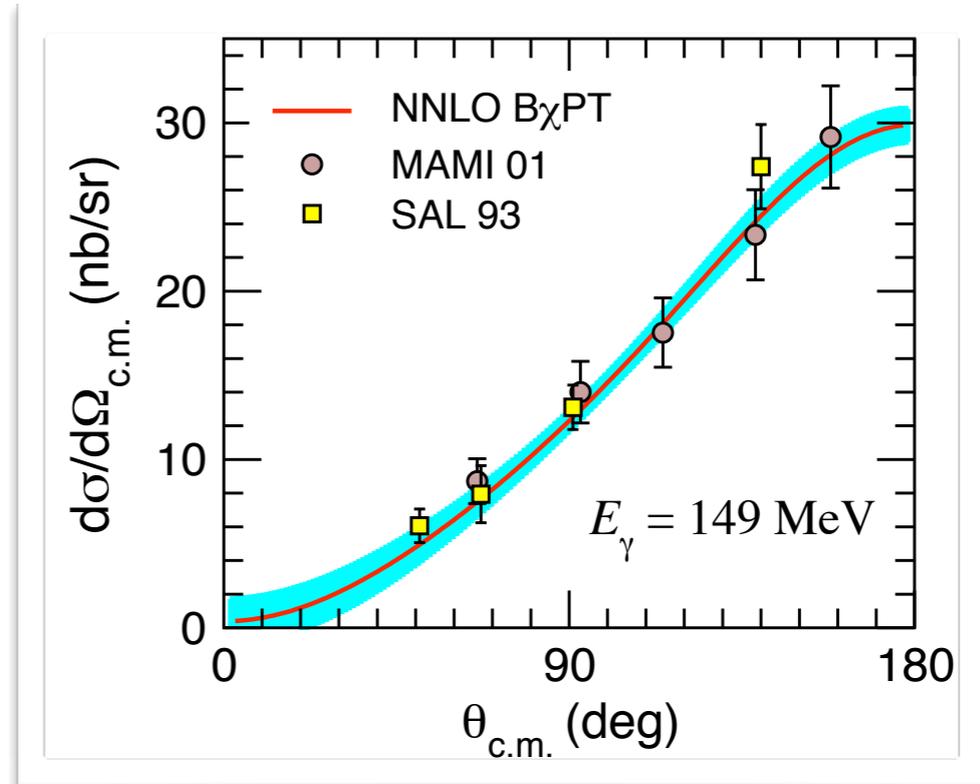
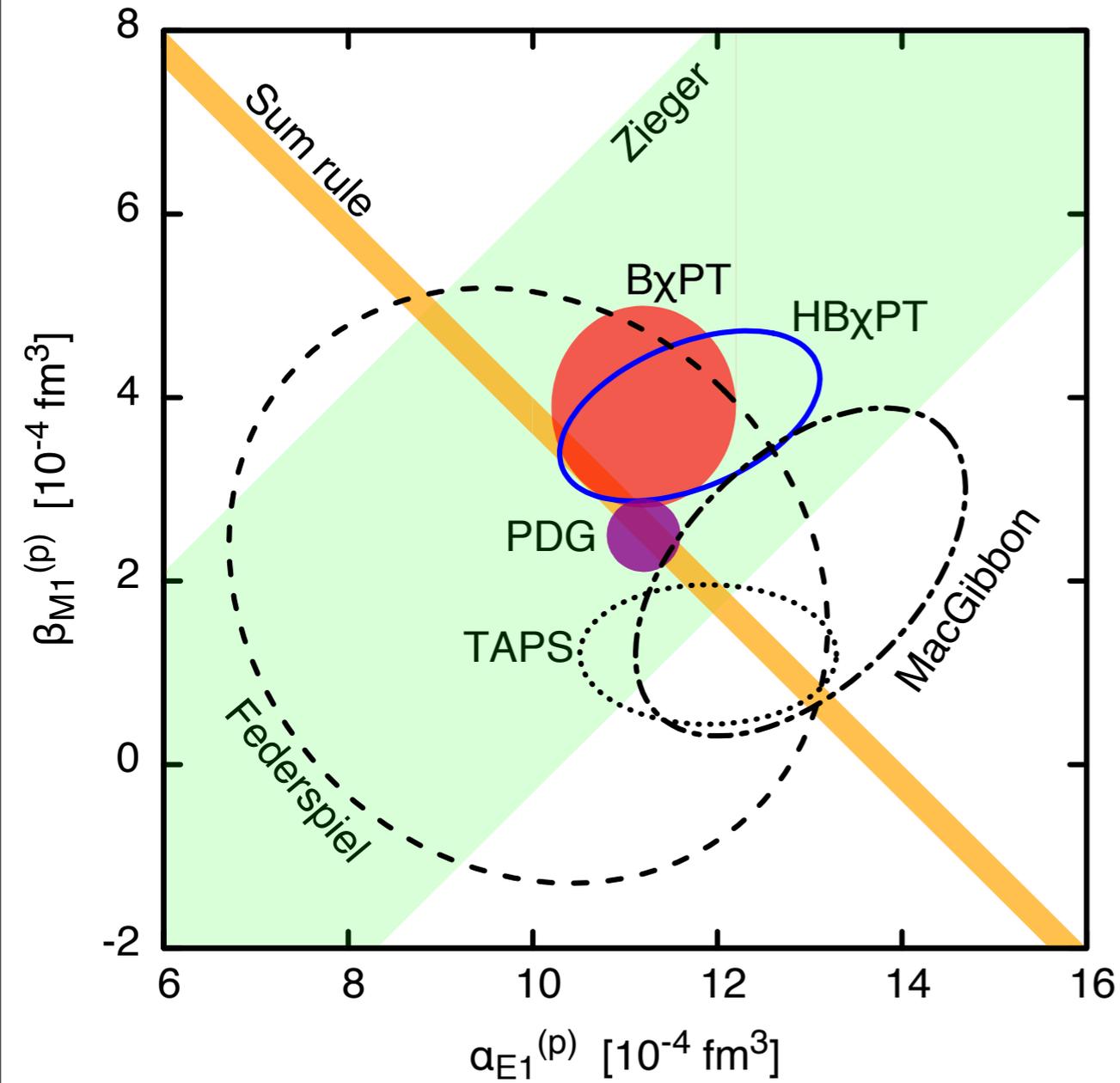
$\alpha_{E1} + \beta_{M1}$



Total cross section ( $\mu\text{b}$ )



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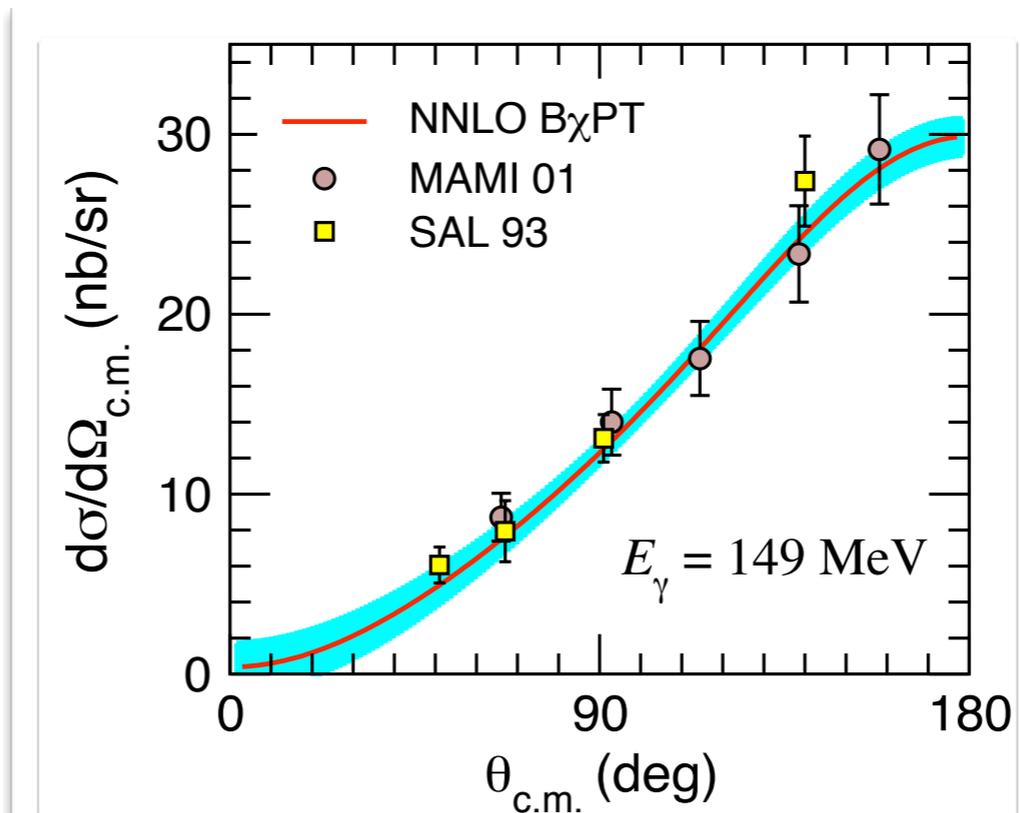
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# Experiments at MAMI and HIGS facilities

Past experiments: measured **unpolarized cross sections** of Compton scattering to extract alpha and beta from the angular dependence.  
(a la Rothenbluth separation of GE, GM)

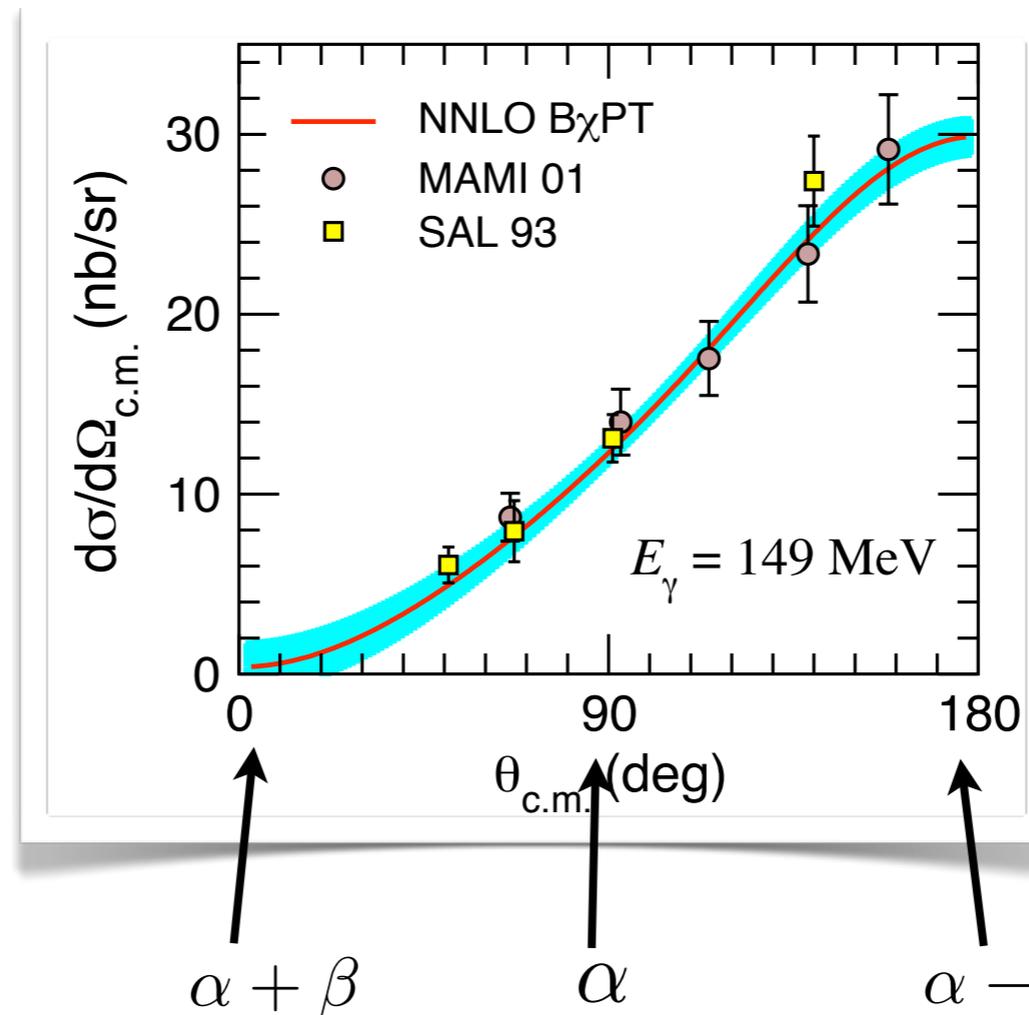
LEX: 
$$\frac{d\sigma^{(\text{NB})}}{d\Omega} = -2\pi Z^2 \frac{\alpha}{M} \left(\frac{\nu'}{\nu}\right)^2 \nu\nu' [2\alpha_{E1} (1 + \cos^2 \theta) + 4\beta_{M1} \cos \theta] + O(\nu^4)$$



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## Separation of Proton Polarizabilities with the Beam Asymmetry of Compton Scattering

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*PRISMA Cluster of Excellence Institut für Kernphysik, Johannes Gutenberg–Universität Mainz, 55128 Mainz, Germany*

(Received 3 April 2013; published 25 June 2013)

We propose to determine the magnetic dipole polarizability of the proton from the beam asymmetry of low-energy Compton scattering based on the fact that the leading non-Born contribution to the asymmetry is given by the magnetic polarizability alone; the electric polarizability cancels out. The beam asymmetry thus provides a simple and clean separation of the magnetic polarizability from the electric one. Introducing polarizabilities in a Lorentz-invariant fashion, we compute the higher-order (recoil) effects of polarizabilities on beam asymmetry and show that these effects are suppressed in forward kinematics. With the prospects of precision Compton experiments at the Mainz Microtron and High Intensity Gamma Source facilities in mind, we argue why the beam asymmetry could be the best way to measure the elusive magnetic polarizability of the proton.

DOI: [10.1103/PhysRevLett.110.262001](https://doi.org/10.1103/PhysRevLett.110.262001)

PACS numbers: 13.60.Fz, 14.20.Dh, 25.20.Dc

**Definition:**  $\Sigma_3 \equiv \frac{d\sigma_{||} - d\sigma_{\perp}}{d\sigma_{||} + d\sigma_{\perp}}$

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Energy < 100 MeV)

Low energy measurement of beam asymmetry can be used to extract magnetic polarizability independently of electric one

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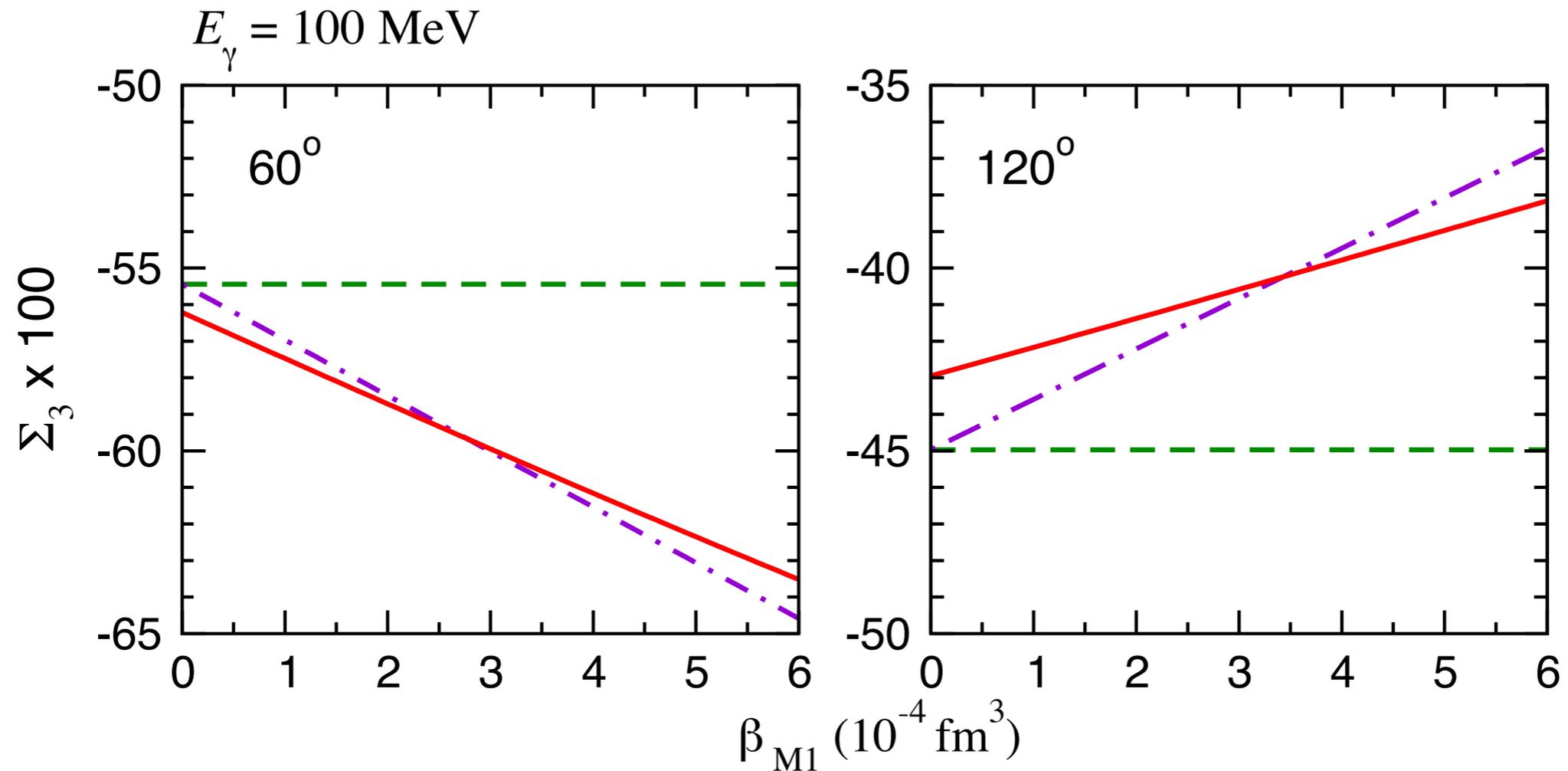
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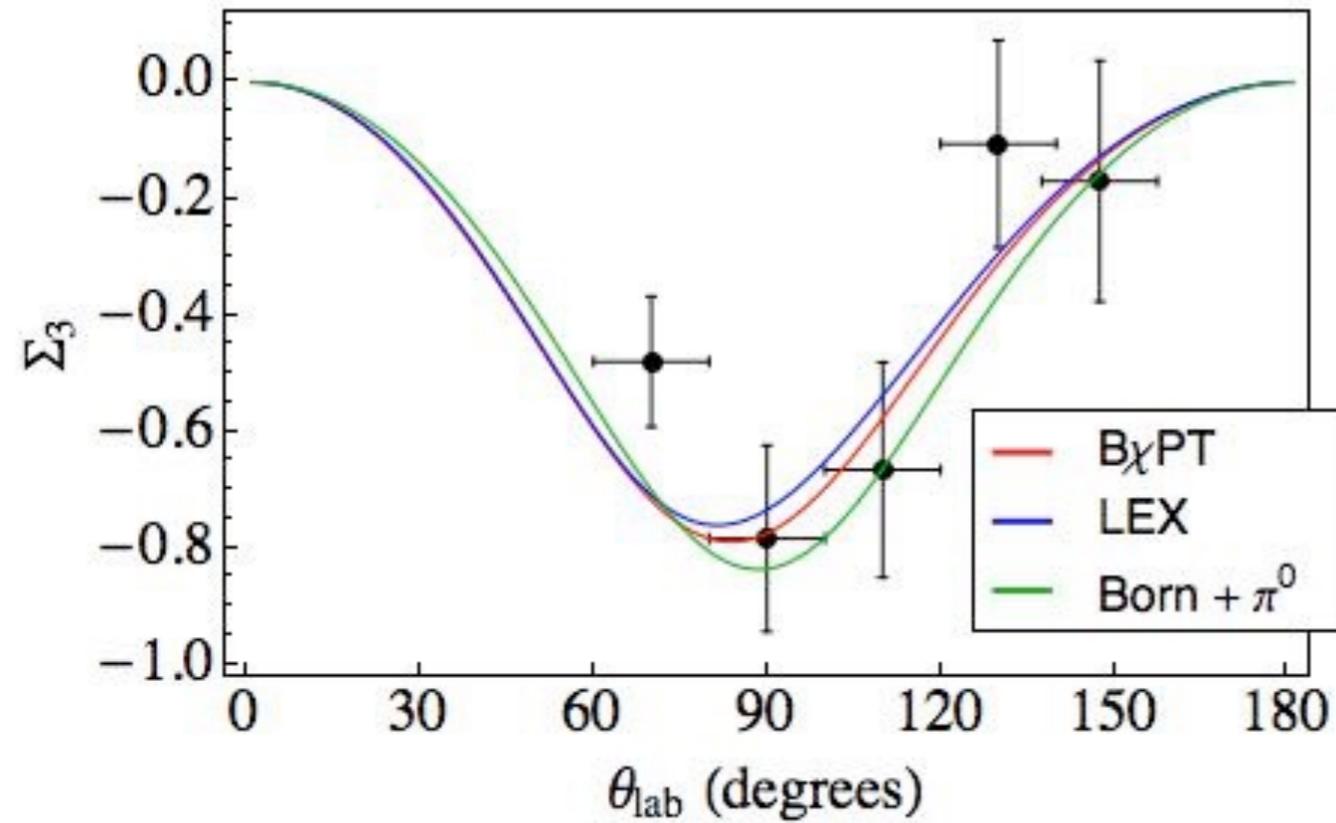
# Applicability of LEX



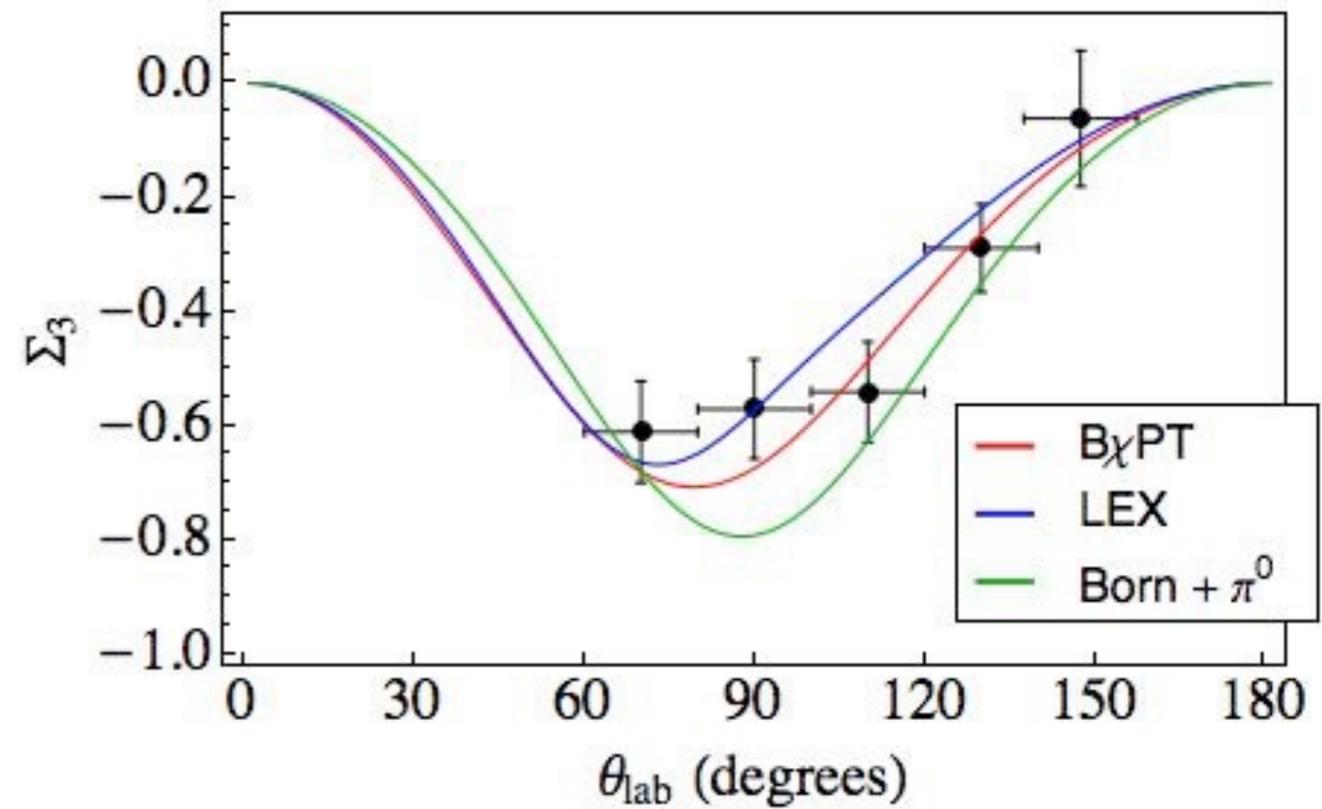
At 100 MeV *NNLO BChPT and LEX curves coincide* for forward angles - 'LEX regime'

# Preliminary results from MAMI

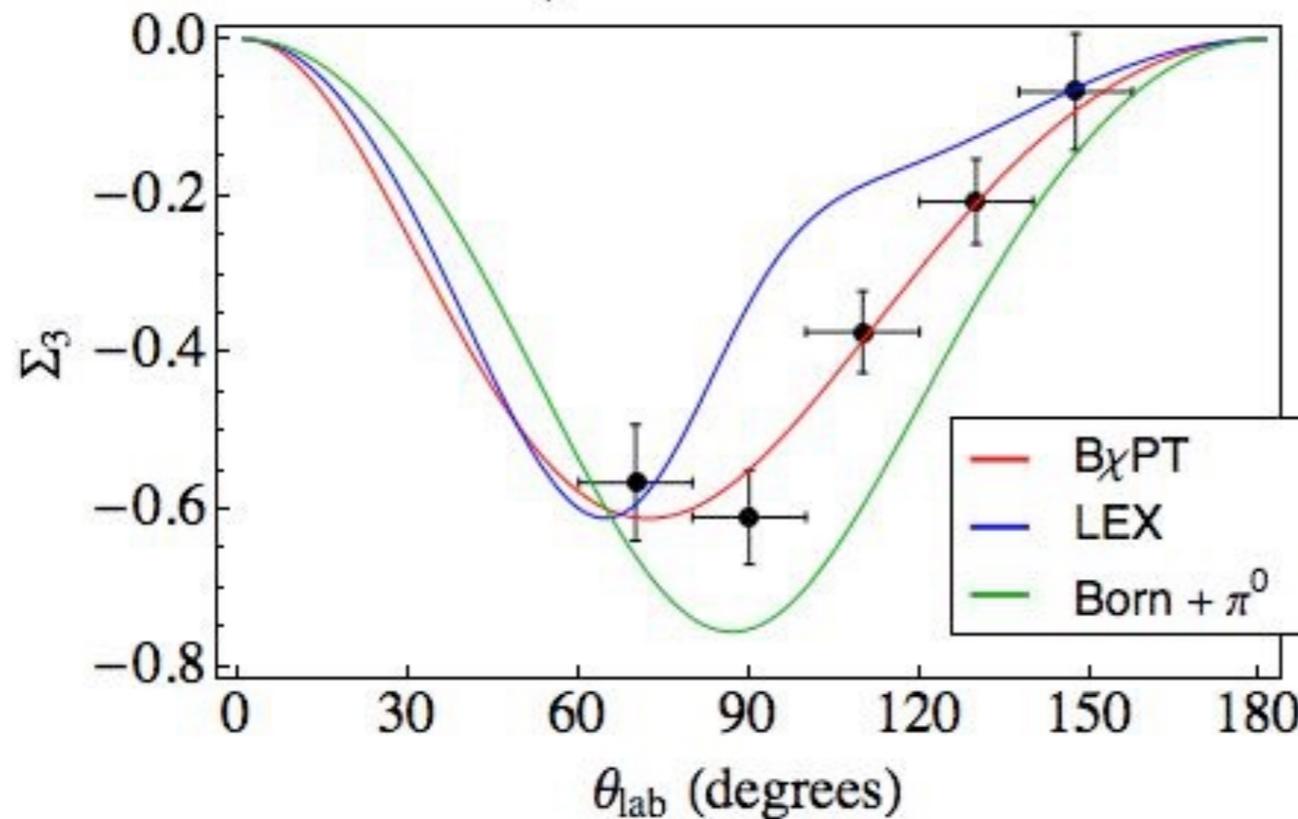
$E_\gamma = 80-100$  MeV



$E_\gamma = 100-120$  MeV



$E_\gamma = 120-140$  MeV



More data is needed to extract the values of polarizabilities

# Multipole expansion

Dynamical polarizabilities:

$$\alpha_{E1}(\omega_{cm}) = \omega_{cm}^{-2} (2f_{EE}^{1+} + f_{EE}^{1-}),$$

$$\beta_{M1}(\omega_{cm}) = \omega_{cm}^{-2} (2f_{MM}^{1+} + f_{MM}^{1-})$$



$\alpha_{E1}(0)$  and  $\beta_{M1}(0)$   
are the **scalar polarizabilities**

where  $f_{EE}^{L+}$  denotes the multipole with the angular momentum of the initial photon  $L$  and the initial and final photons are both in an electrical mode.

*The values of scalar polarizabilities can be obtained by extrapolation of dynamical polarizabilities to zero energy.*

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# Summary

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Thank you for listening