A natural solution of the proton charge radius puzzle.

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INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS 37th Course
Probing Hadron Structure with Lepton and Hadron Beams
Erice-Sicily: September 16-24, 2015

Outline

- Introduction

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- Introduction
  - the puzzle

Outline

- Introduction
  - the puzzle
  - some ideas for solutions

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- Introduction
  - the puzzle
  - some ideas for solutions
  - my idea

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- Introduction
- Idea for a solution
- Extension of the wave equation

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- Results

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  - Proton

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  - Proton
  - prediction Deuteron

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  - prediction ^4He

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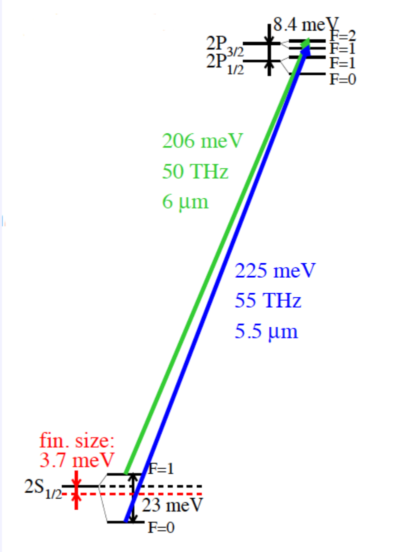
- Results
- Conclusions

The muonic/electronic puzzle of the charge radius



courtesy of Randolf Pohl

LS ≡ Lamb shift
HFS ≡ hyperfine structure

$$2S_{1/2}^{F=1} \rightarrow 2P_{3/2}^{F=2}$$

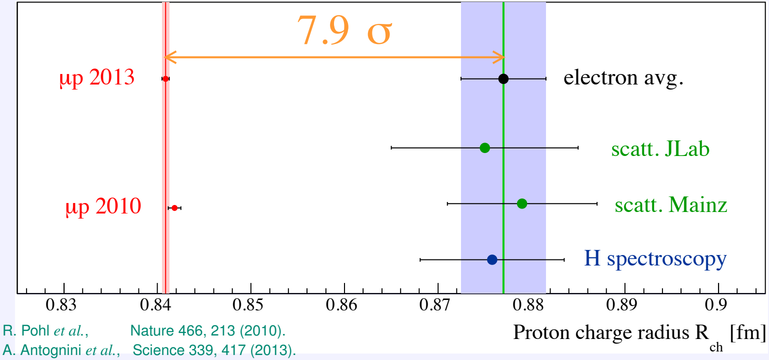
$$2S_{1/2}^{F=0} \rightarrow 2P_{3/2}^{F=1}$$

$$\Delta E_{LS}^{exper.} = 202.3706(23)meV$$

$$\Delta E_{HFS}^{exper.} = 22.8089(51)meV$$

The muonic/electronic puzzle of the charge radius

$$\Delta E_{LS}^{theory} = [206.0336(15) - 5.2275(10) r_p^2 + 0.0332(20)]\, meV; \quad r_p = \sqrt{\langle r_p^2 \rangle}$$



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electrons: $r_p^e = 0.8770 \pm 0.0045 fm$ muons: $r_p^\mu = 0.8409 \pm 0.0004 fm$

$$\Delta E_{LS}(r_p^\mu) - \Delta E_{LS}(r_p^e) = 202.371\, meV - 202.046\, meV = 0.325\, meV$$

The muonic/electronic puzzle of the charge radius

What could be wrong? or Is it “new” physics?

Akin to three std. dev. difference between experiment and theory of muon magnetic moment?

electron scattering:

- very small $0 \leqslant Q^2 \lesssim 0.003\,\mathrm{GeV}^2$ region not measured
  wiggles, bumps, spikes? But what would be their physics?
- Models don't extrapolate right to $Q^2 \to 0$?
  But, a plethora of models tried. All give same result.
- Coulomb corrections, resp. two photon exchange (TPE) is incomplete?
  But, effect on charge radius r_p is very small at $Q^2 \lesssim 0.3\,\mathrm{GeV}^2$ for all TPE calculations.

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muonic hydrogen:

- QED calculations are still - after 50 years - not good enough?
  Many checks after discovery of discrepancy, only small corrections, no solution
- relativistic Dirac wave functions have to be used
  small effects, no explanation
- Hadronic corrections incomplete?
  numerous calculations, no solution
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new physics:

- Some fancy new particle?
  e.g. couples to muon and electron differently!
- QED has a problem?

my idea

radiative and vacuum polarization corrections , i.e. Lamb shift need
"self-consistent renormalization" in external Coulomb potential

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The muonic/electronic puzzle of the charge radius

basic problem:

Lamb shift is a QED correction to bound states in a central external Coulomb potential described with

- scattering amplitudes ≡ non stationary states
  i.e. Feynman diagrams
- but, bound states ≡ stationary states
  i.e. solution of a wave equation

question:

How does one marry scattering states with bound states?

The problem: describe hydrogen atom by QED

task:

- sum up all possible Feynman diagrams (time ordering, symmetry, all orders in α) and determine the poles
- position of poles give the energy of the bound states including radiative corrections, i.e. Lamb-Shift

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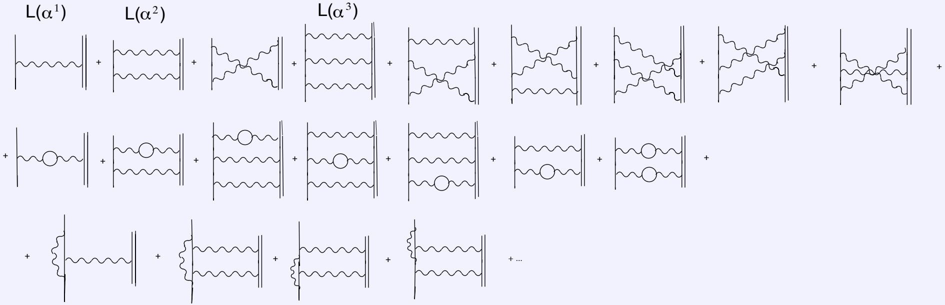

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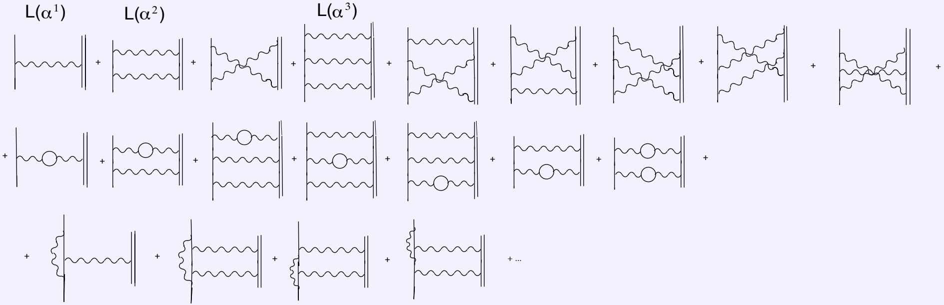

of course, not possible $\curvearrowright$ approximations needed:

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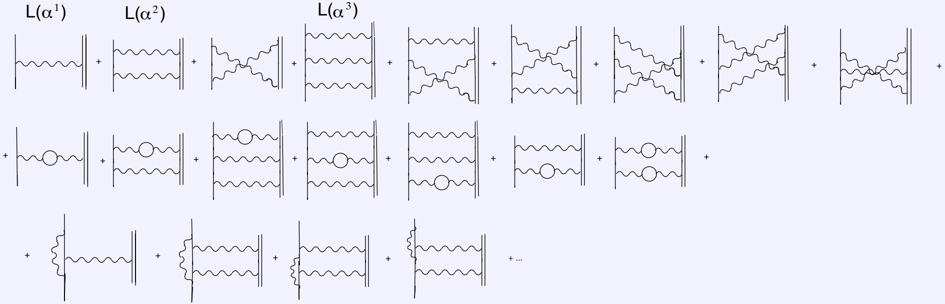

of course, not possible $\curvearrowright$ approximations needed:

- muon and proton in hydrogen are non-relativistic $\curvearrowright$ only ladder diagrams $L(\alpha^n)$ are important
- other diagrams are treated as perturbative relativistic corrections (spin-orbit interaction) and radiative corrections

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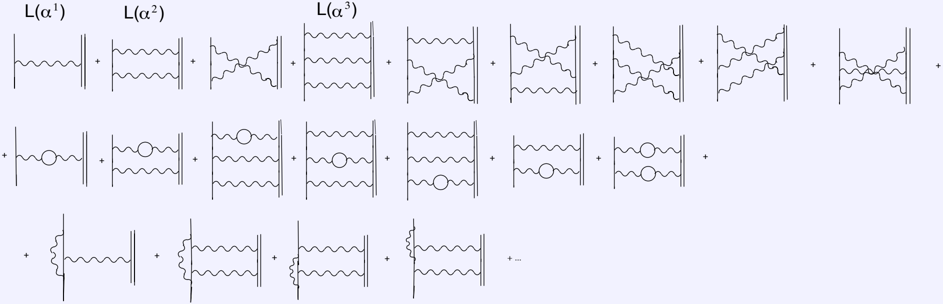

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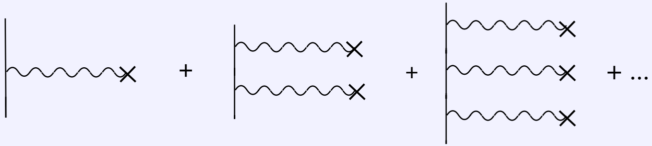
The Coulomb potential as external potential

ladder diagrams for photon exchanges:

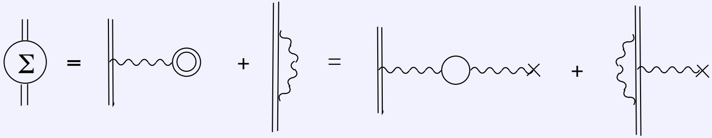
- factorize
- any number of photons: 1 to ∞
- sum all permutations



- gives the usual <u>external</u> Coulomb potential (proof Weinberg, Vol. I, chpt. 13.6)
- can be put as such into a wave equation (Schroedinger, Dirac)

Lamb shift

Lamb-Shift is given by scattering diagrams for radiative corrections and vacuum polarization ≡ ”self energy” in external Coulomb potential

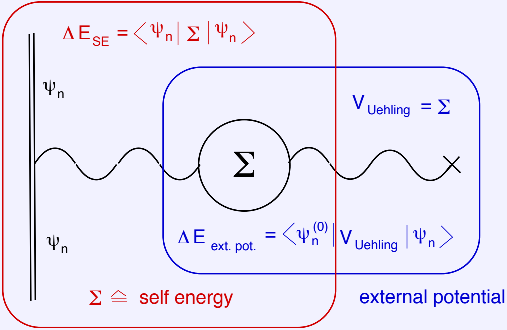


but:

- bound states ≡ stationary states $\curvearrowright$ equal in- and out-states
- no change of wf. in space $\curvearrowright$ forward scattering amplitudes
- change of energy via interaction with many body system, here the physical vacuum: ”self energy” of quasi particle

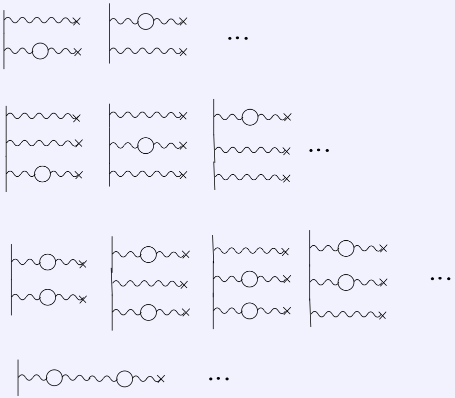
Lamb shift

two views:



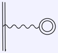
Lamb shift

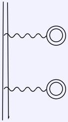
higher orders:



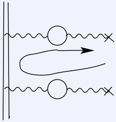
photons are put into bound state wave functions $\psi_n^{(0)}$
Weinberg's proof for external potential not valid any more
How can we put "self energy" into wave equation?

Lamb shift

salient point: diagrams next to leading order :



$\neq$



$$\Delta G(\vec{k}, E) \propto \langle\psi_n|\Sigma|\psi_n\rangle\langle\psi_n|\Sigma|\psi_n\rangle$$

$$\Delta G(\vec{k}, E) \propto \sum_{k\neq n}\int_{E_k\geqslant 0} \frac{\langle\psi_n^{(0)}|\Sigma|\psi_k^{(0)}\rangle\langle\psi_k^{(0)}|\Sigma|\psi_n^{(0)}\rangle}{E_n^{(0)} - E_k^{(0)}}$$

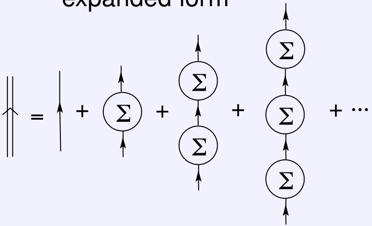
forward scattering
in- = intermediate = out-state
probabilities multiply

scattering through intermediate states,
asymptotic in- = out-states
approximated by bound states,
integration and summation
over intermediate states
excluding left diagram

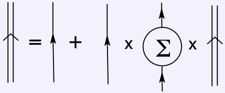
Lamb shift

The generalized Dyson series in the presence of the external Coulomb potential in three different forms.

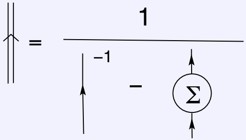
expanded form



reiterated form



geometric sum

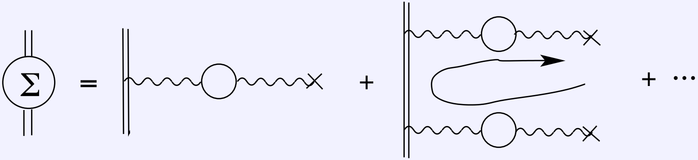


$$G(\vec{k}, E) = \frac{1}{E - E_k^{(0)} - \langle \psi_n | \Sigma(\vec{k}, E) | \psi_n \rangle}$$

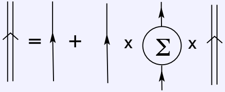
$$\Delta E = \langle \psi_n | \Sigma(\vec{k}, E) | \psi_n \rangle$$

Lamb shift

in principle Σ comprises all diagrams with self-energy parts



Self consistent solution



$$G(\vec{k},E) = G_0(\vec{k},E) + G_0(\vec{k},E)\,\Sigma(\vec{k},E)\,G(\vec{k},E)$$

fully equivalent to Hartree-Fock equation
[cit. R.D. Matuck "A Guide to Feynman Diagrams
in the Many-Body Problem" (1976), chpt. 11.1]

$$(T + V_{Coulomb})\,\widetilde{\psi}_n + \langle\widetilde{\psi}_n|\Sigma|\,\widetilde{\psi}_n\rangle\,\widetilde{\psi}_n = \widetilde{E}_n\,\widetilde{\psi}_n\,, \quad \langle\widetilde{\psi}_n|\widetilde{\psi}_n\rangle = 1$$

to be solved iteratively for self consistent solution $\widetilde{\psi}_n$

"self-consistent renormalization" in external field (Matuck, chpt. 11.1)

Lamb shift

task: solve integro-differential wave equation

$$(T + V_{Coulomb})\,\psi_n + \frac{\langle\psi_n|\Sigma|\,\psi_n\rangle}{\langle\psi_n|\psi_n\rangle}\,\psi_n = E_n\,\psi_n$$

multiplication with $\langle\psi_n^{(0)}|$ from the left

$$\frac{\langle\psi_n^{(0)}|T + V_{\mathrm{Coulomb}}|\,\psi_n\rangle}{\langle\psi_n^{(0)}|\psi_n\rangle} + \frac{\langle\psi_n|\Sigma|\psi_n\rangle}{\langle\psi_n|\psi_n\rangle} = E_n$$

or

$$\Delta E_{n\,LS} = E_n - E_n^{(0)} = \frac{\langle\psi_n|\Sigma|\,\psi_n\rangle}{\langle\psi_n|\,\psi_n\rangle}$$

now iterate so that $\psi_n \to \widetilde{\psi}_n$ and $E_n \to \widetilde{E_n}$

Lamb shift

solution by iteration:

- initial step:

$$(T + V_{\mathrm{Coulomb}})\,\psi'_n + V_{\mathrm{Uehling}}\,\psi'_n = E'_n\,\psi'_n.$$

  solve for E'_n and ψ'_n by numerical integration of differential equation

$$\text{i.e. with } \Sigma = V_{\mathrm{Uehling}} \quad \curvearrowright \quad E'_n = E_n^{(0)} + \frac{\langle \psi_n^{(0)} | \Sigma | \psi'_n \rangle}{\langle \psi_n^{(0)} | \psi'_n \rangle}$$

- first step:

$$\frac{\langle \psi_n^{(0)} | T + V_{\mathrm{Coulomb}} | \psi''_n \rangle}{\langle \psi_n^{(0)} | \psi''_n \rangle} + \frac{\langle \psi''_n | \Sigma | \psi''_n \rangle}{\langle \psi''_n | \psi''_n \rangle} = E''_n$$

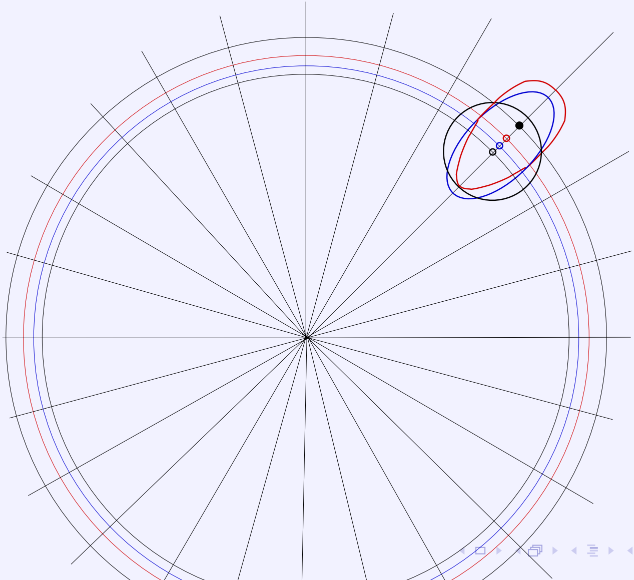
$$E''_n = E_n^{(0)} + \frac{\langle \psi'_n | \Sigma | \psi'_n \rangle}{\langle \psi'_n | \psi'_n \rangle}$$

  where $\psi''_n = \psi'_n + \lambda\,\delta\psi'_n,\ \delta\psi'_n \perp \psi'_n$, vary λ that equation is satisfied

- second step: $\cdots$

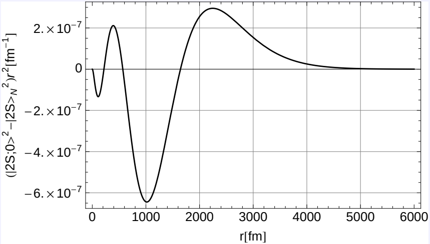
Illustrative Interpretation

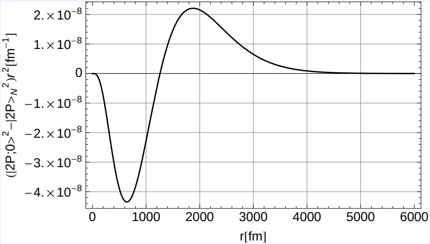
Classical analog: moon in external gravitational central field



Results for Proton

$$\Delta E_{LS}^{(0)} = E_{LS,2\,P}^{(0)} - E_{LS,2\,S}^{(0)} = 205.005\,meV$$





$$\Delta E_{LS}^{\prime\prime\prime\;self\;consist.\,approx.\,point\;charge} = \langle \psi_n^{\prime\prime} | \Sigma | \psi_n^{\prime\prime} \rangle = 205.307(1)\,meV$$

$$\delta(\Delta E_{LS}) = \Delta E_{LS}^{\prime\prime\prime\;self\;consist.\,approx.\,point\;charge} - \Delta E_{LS}^{(0)} = 0.302(1)\,meV$$

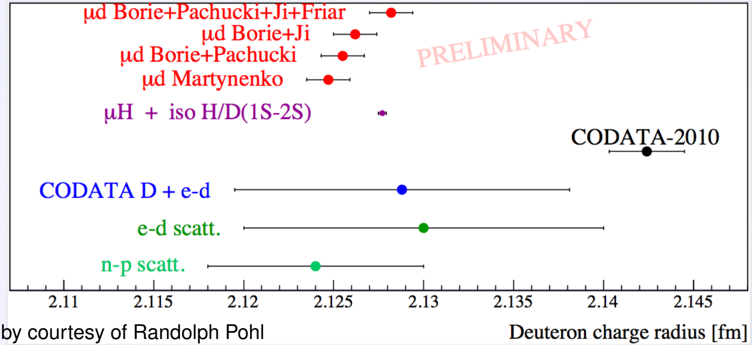
electrons: $r_p^e = 0.8770 \pm 0.0045\,fm$ muons: $r_p^\mu = 0.8409 \pm 0.0004 fm$

$$\Delta E_{LS}(r_p^\mu) - \Delta E_{LS}(r_p^e) = 202.3706(23)\,meV - 202.046(46)\,meV = 0.325(46)\,meV$$

Results for Deuteron

4 independent measurements;

- $r_p^e = 0.8770(45) fm$
- $r_p^\mu = 0.84087(39) fm$
- H/D isotope transitions: $r_d^{e\,2} - r_p^{e\,2} = 3.82007(65) fm^2$
  (CODATA 2010 and C.G. Parthey, et al., PRL 104, 233001 (2010))
- muonic Lamb shift in deuteron preliminary 2014: $r_d^\mu = 2.1282(12) fm$



by courtesy of Randolph Pohl

Results for Deuteron

- H/D isotope shift

$$r_d^{e\,2} - r_p^{e\,2} = 3.82007(65)\, fm^2$$
$$r_d^{\mu\,2} - r_p^{\mu\,2} = 3.8221(51)\, fm^2$$

since e doesn't couple to neutron $\curvearrowright$ μ doesn't couple to neutron

- e-μ radius difference

$$r_p^{e\,2} - r_p^{\mu\,2} = 0.0620(79)\, fm^2$$
$$r_d^{e\,2} - r_d^{\mu\,2} = 0.0600(94)\, fm^2$$

$$\Delta E_{LS} = p_1 + p_2\, r^2 + p_3 \quad \curvearrowright$$

energy difference between e and μ measurements on the proton is the same for the deuteron

Results for Deuteron

natural explanation:

- no muon coupling to neutron
  the Lamb shift depends on the charge $Z = 1$ only
  (corrections due to smearing of Uehling effect are small
  J. Caroll et al., Phys.Rev. A84, 012506 (2011))
- the radius difference is due to the missing correction
  for self-consistent wave functions as for the proton

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Results for Helium

measurement on muonic Helium:

Aldo Antognini et al., Mainz, June 2014

$$\Delta E_{LS}^{exp.}(2S_{1/2} \rightarrow 2P_{3/2}) = 1378.xx(8)\, meV$$

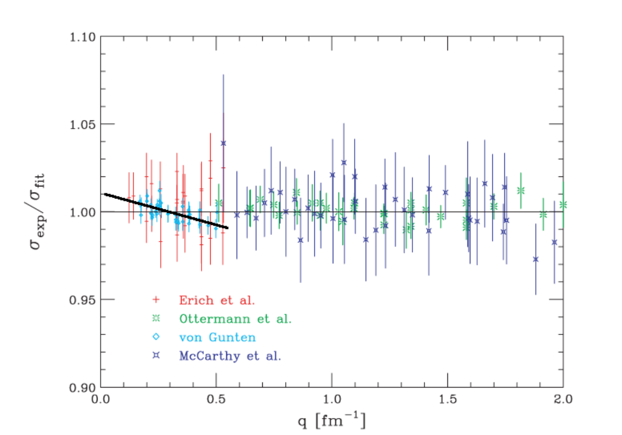
$$\Delta E_{LS}^{theor.} = [1668.598(100) - 106.340(xx) r^2 + 1.40(4) r^3 + 2.470(150)]\, meV$$

preliminary!! muonic result

$$r^{\mu}_{{}^4He} = 1.677(1) fm$$

Results for Helium $r^e_{^4He}$

preliminary!! muonic result	1.677(1) fm
elastic electron scattering:	
model independent average 4 experiments	1.689(7) fm
constrains for charge distribution (Ingo Sick)	1.681(4) fm
average both together	1.683(3) fm



Results for Helium

$$r^e_{^4He} - r^\mu_{^4He} = 0.006(3)\,fm \quad \curvearrowright \quad \delta(\Delta E_{LS}) = 2.1 \pm 1.2\,meV$$

$$\begin{array}{llll} Err.[\delta(\Delta E_{LS})] = & \pm\, 1.2\,meV & \pm & 0.5\,meV \\ & \uparrow & & \uparrow \\ & \delta r^e_{^4He} & & \textit{theoretical} \\ & \pm 0.003\,fm & & \textit{uncertainty}? \end{array}$$

selfconsistent calculation for Z=2 : $\delta(\Delta E''''_{LS}) = 2.9158\,meV$

no conclusive statement yet possible, but intriguing tendency
also still quite a few theoretical problems:

- finite size effect on ”Uehling effect”
- two photon exchange effect
- contribution of polarization in intermediate states of ^4He and of nucleons

Conclusions

- In the language of many body physics the muon is a quasi particle in a medium, here the physical vacuum, and an external potential, the Coulomb potential.
- If one calculates the self energy self consistently the proton radius puzzle disappears.
- This realization is in accord with the preliminary results on deuterium and ^{4}He reported June 2014, Schloss Waldhausen, Mainz.

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